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Chapter

More Functions Associated with Neutrosophic gs*$\alpha$- Closed Sets in Neutrosophic Topological Spaces

P. Anbarasi Rodrigo and S. Maheswari

Abstract

The concept of neutrosophic continuous function was very first introduced by A.A. Salama et al. The main aim of this paper is to introduce a new concept of Neutrosophic continuous function namely Strongly Neutrosophic gs*$\alpha$* - continuous functions, Perfectly Neutrosophic gs*$\alpha$* - continuous functions and Totally Neutrosophic gs*$\alpha$* - continuous functions in Neutrosophic topological spaces. These concepts are derived from strongly generalized neutrosophic continuous function and perfectly generalized neutrosophic continuous function. Several interesting properties and characterizations are derived and compared with already existing neutrosophic functions.

Keywords: Neutrosophic gs*$\alpha$*- closed set, Neutrosophic gs*$\alpha$*- open set, Strongly Neutrosophic gs*$\alpha$*- continuous function, Perfectly Neutrosophic gs*$\alpha$*- continuous function, Totally Neutrosophic gs*$\alpha$*- continuous function

1. Introduction

The concept of Neutrosophic set theory was introduced by F. Smarandache [1] and it comes from two concept, one is intuitionistic fuzzy sets introduced by K. Atanassov’s [2] and the other is fuzzy sets introduced by L.A. Zadeh’s [3]. It includes three components, truth, indeterminacy and false membership function. R. Dhavaseelan and S. Jafari [4] has discussed about the concept of strongly generalized neutrosophic continuous function. Further he also introduced the topic of perfectly generalized neutrosophic continuous function. The real life application of neutrosophic topology is applied in Information Systems, Applied Mathematics etc.

In this paper, we introduce some new concepts related to Neutrosophic gs*$\alpha$*— continuous function namely Strongly Neutrosophic gs*$\alpha$*— continuous function, Perfectly Neutrosophic gs*$\alpha$*— continuous function, Totally Neutrosophic gs*$\alpha$*— continuous function.

2. Preliminaries

Definition 2.1: [5] Let $\mathbb{P}$ be a non-empty fixed set. A Neutrosophic set $H$ on the universe $\mathbb{P}$ is defined as $H = \{(p, i_H(p), t_H(p), f_H(p)) : p \in \mathbb{P}\}$ where $i_H(p), t_H(p), f_H(p)$ represent the degree of indeterminacy $i_H(p)$ and the degree of non-membership function $f_H(p)$ respectively for each element $p \in \mathbb{P}$ to the set $H$. Also, $i_H, t_H, f_H : \mathbb{P} \to [0, 1]$ and $-0$
\[ t_B(p) + i_B(p) + f_B(p) \leq 3. \] Set of all neutrosophic set over \( P \) is denoted by \( N_{eu}(P) \).

**Definition 2.2:** [8] Let \( P \) be a non-empty set.
\[ A = \{(p, (t_A(p), i_A(p), f_A(p))) : p \in P\} \text{ and } B = \{(p, (t_B(p), i_B(p), f_B(p))) : p \in P\} \]
are neutrosophic sets, then

i. \( A \subseteq B \) if \( t_A(p) \leq t_B(p), i_A(p) \leq i_B(p), f_A(p) \geq f_B(p) \) for all \( p \in P \).

ii. Intersection of two neutrosophic set \( A \) and \( B \) is defined as \( A \cap B = \{(p, (\min(t_A(p), t_B(p)), \min(i_A(p), i_B(p))) \max(f_A(p), f_B(p))) : p \in P \} \).

iii. Union of two neutrosophic set \( A \) and \( B \) is defined as \( A \cup B = \{(p, (\max(t_A(p), t_B(p)), \max(i_A(p), i_B(p))) \min(f_A(p), f_B(p))) : p \in P \} \).

iv. \( A^c = \{(p, (f_A(p), 1 - i_A(p), t_A(p))) : p \in P\} \).

v. \( 0_{N_{eu}} = \{(p, (0, 0, 1)) : p \in P\} \) and \( 1_{N_{eu}} = \{(p, (1, 1, 0)) : p \in P\} \).

**Definition 2.3:** [5] A neutrosophic topological \( (N_{eu}, T) \) on a non-empty set \( P \) is a family \( \tau_{N_{eu}} \) of neutrosophic sets in \( P \) satisfying the following axioms,

i. \( 0_{N_{eu}}, 1_{N_{eu}} \in \tau_{N_{eu}} \).

ii. \( A_1 \cap A_2 \in \tau_{N_{eu}} \) for any \( A_1, A_2 \in \tau_{N_{eu}} \).

iii. \( \bigcup A_i \in \tau_{N_{eu}} \) for every family \( \{A_i / i \in \Omega\} \subseteq \tau_{N_{eu}} \).

In this case, the ordered pair \( (P, \tau_{N_{eu}}) \) or simply \( P \) is called a neutrosophic topological space \( (N_{eu}, TS) \). The elements of \( \tau_{N_{eu}} \) is neutrosophic open set \( (N_{eu} - OS) \) and \( \tau_{N_{eu}} \) is neutrosophic closed set \( (N_{eu} - CS) \).

**Definition 2.4:** [6] A neutrosophic set \( A \) in a \( N_{eu} \) TS \( (P, \tau_{N_{eu}}) \) is called a neutrosophic generalized semi alpha star closed set \( (N_{eu}gs\alpha^* - CS) \) if \( N_{eu} - int(N_{eu}gs\alpha^* - CS) \) if \( N_{eu} - int(N_{eu}gs\alpha^* - cl(A)) \subseteq N_{eu} - int(G) \), whenever \( A \subseteq G \) and \( G \) is \( N_{eu}gs\alpha^* \) open set.

**Definition 2.5:** [7] A neutrosophic topological space \( (P, \tau_{N_{eu}}) \) is called a \( N_{eu}gs\alpha^* - T_\beta \) space if every \( N_{eu}gs\alpha^* \) - closed set \( (P, \tau_{N_{eu}}) \) is an \( N_{eu} - CS \) in \( (P, \tau_{N_{eu}}) \).

**Definition 2.6:** A neutrosophic function \( f : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}}) \) is said to be

1. neutrosophic continuous [8] if the inverse image of each \( N_{eu} - CS \) in \( (Q, \sigma_{N_{eu}}) \) is a \( N_{eu} - CS \) in \( (P, \tau_{N_{eu}}) \).

2. \( N_{eu}gs\alpha^* \) - continuous [7] if the inverse image of each neutrosophic closed set in \( (Q, \sigma_{N_{eu}}) \) is a \( N_{eu}gs\alpha^* \) - closed set in \( (P, \tau_{N_{eu}}) \).

3. \( N_{eu}gs\alpha^* \) - irresolute map [7] if the inverse image of each \( N_{eu}gs\alpha^* \) - closed set in \( (Q, \sigma_{N_{eu}}) \) is a \( N_{eu}gs\alpha^* \) - closed set in \( (P, \tau_{N_{eu}}) \).

4. strongly neutrosophic continuous [4] if the inverse image of each neutrosophic set in \( (Q, \sigma_{N_{eu}}) \) is both \( N_{eu} - OS \) and \( N_{eu} - CS \) in \( (P, \tau_{N_{eu}}) \).

5. perfectly neutrosophic continuous [4] if the inverse image of each \( N_{eu} - CS \) in \( (Q, \sigma_{N_{eu}}) \) is both \( N_{eu} - OS \) and \( N_{eu} - CS \) in \( (P, \tau_{N_{eu}}) \).
3. Strongly neutrosophic $g_\alpha^*$-continuous function

Definition 3.1: A neutrosophic function $f : (\mathbb{P}, \tau_{\mathbb{P}}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{Q}})$ is said to be strongly $N_{g_\alpha}g_\alpha^*$-continuous if the inverse image of every $N_{g_\alpha}g_\alpha^*$-closed set in $(\mathbb{Q}, \sigma_{\mathbb{Q}})$ is a $N_{g_\alpha}g_\alpha^*$-closed set in $(\mathbb{P}, \tau_{\mathbb{P}})$. (i.e.) if $f^{-1}(A)$ is a $N_{g_\alpha}g_\alpha^*$-closed set in $(\mathbb{P}, \tau_{\mathbb{P}})$ for every $N_{g_\alpha}g_\alpha^*$-closed set $A$ in $(\mathbb{Q}, \sigma_{\mathbb{Q}})$.

Theorem 3.2: Every strongly $N_{g_\alpha}g_\alpha^*$-continuous is not conversely, but not conversely.

Proof: Let $f : (\mathbb{P}, \tau_{\mathbb{P}}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{Q}})$ be any neutrosophic function. Let $A$ be any $N_{g_\alpha}g_\alpha^*$-closed set in $(\mathbb{Q}, \sigma_{\mathbb{Q}})$. Then $f^{-1}(A)$ is a $N_{g_\alpha}g_\alpha^*$-closed set in $(\mathbb{P}, \tau_{\mathbb{P}})$. Therefore, $f$ is neutrosophic continuous.

Example 3.3: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{\mathbb{P}} = \{0_{\mathbb{P}}, 1_{\mathbb{P}}\}$ and $\sigma_{\mathbb{Q}} = \{0_{\mathbb{Q}}, 1_{\mathbb{Q}}\}$ are $\mathbb{P}$-TS and $\mathbb{Q}$-TS respectively. Also $A = \{\{p, 0.6, 0.4, 0.4\}\}$ and $B = \{\{q, 0.4, 0.6, 0.2\}\}$ are $\mathbb{P}$-closeds and $\mathbb{Q}$-closeds. Define a map $f : (\mathbb{P}, \tau_{\mathbb{P}}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{Q}})$ by $f(p) = q + 0.2$. Let $B^c = \{\{q, 0.2, 0.4, 0.4\}\}$ be a $N_{g_\alpha}g_\alpha^*$-closed set in $(\mathbb{Q}, \sigma_{\mathbb{Q}})$. Now, $f^{-1}(B^c)$ is strongly $N_{g_\alpha}g_\alpha^*$-continuous in $(\mathbb{P}, \tau_{\mathbb{P}})$. Therefore, $f$ is neutrosophic continuous, but not conversely.

Theorem 3.4: Let $f : (\mathbb{P}, \tau_{\mathbb{P}}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{Q}})$ be strongly $N_{g_\alpha}g_\alpha^*$-continuous iff the inverse image of every $N_{g_\alpha}g_\alpha^*$-$\alpha$-closed in $(\mathbb{Q}, \sigma_{\mathbb{Q}})$ is $N_{g_\alpha}g_\alpha^*$-$\alpha$-closed in $(\mathbb{P}, \tau_{\mathbb{P}})$.

Proof: Assume that $f$ is strongly $N_{g_\alpha}g_\alpha^*$-continuous function. Let $A$ be any $N_{g_\alpha}g_\alpha^*$-$\alpha$-closed in $(\mathbb{Q}, \sigma_{\mathbb{Q}})$. Then $f^{-1}(A)$ is $N_{g_\alpha}g_\alpha^*$-$\alpha$-closed in $(\mathbb{P}, \tau_{\mathbb{P}})$. Therefore, $f$ is neutrosophic continuous.

Theorem 3.5: Every strongly $N_{g_\alpha}g_\alpha^*$-continuous is not conversely, but not conversely.

Proof: Let $f : (\mathbb{P}, \tau_{\mathbb{P}}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{Q}})$ be any neutrosophic function. Let $A$ be any $N_{g_\alpha}g_\alpha^*$-$\alpha$-closed in $(\mathbb{Q}, \sigma_{\mathbb{Q}})$. Then $f^{-1}(A)$ is $N_{g_\alpha}g_\alpha^*$-$\alpha$-closed in $(\mathbb{P}, \tau_{\mathbb{P}})$. Therefore, $f$ is neutrosophic continuous.

Example 3.6: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{\mathbb{P}} = \{0_{\mathbb{P}}, 1_{\mathbb{P}}\}$ and $\sigma_{\mathbb{Q}} = \{0_{\mathbb{Q}}, 1_{\mathbb{Q}}\}$ are $\mathbb{P}$-TS and $\mathbb{Q}$-TS respectively. Also $A = \{\{p, 0.4, 0.5, 0.7\}\}$ and $B = \{\{q, 0.6, 0.8, 0.4\}\}$ are $\mathbb{P}$-closeds and $\mathbb{Q}$-closeds.
Define a map \( f : (\mathbb{P}, \tau_{\mathbb{N}_a}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{N}_a}) \) by \( f(p) = q \). Let \( B = \{(q, (0.4, 0.2, 0.6))\} \) be a \( N_{\mathbb{N}_a} \)-CS in \((\mathbb{Q}, \sigma_{\mathbb{N}_a})\). Then \( f^{-1}(B) = \{(p, (0.4, 0.2, 0.6))\}, N_{\mathbb{N}_a} a - CS = N_{\mathbb{N}_a} a - OS = \{N_{\mathbb{N}_a}, N_{\mathbb{N}_a}, A\} \) and \( N_{\mathbb{N}_a} a - CS = \{0_{\mathbb{N}_a}, 1_{\mathbb{N}_a}, A^c\} \).

\[ N_{\mathbb{N}_a} a - cl (f^{-1}(B)) = A \cap 1_{\mathbb{N}_a} = A^c. \]

Now, \( N_{\mathbb{N}_a} a - int(N_{\mathbb{N}_a} a - cl (f^{-1}(B))) = A \subseteq N_{\mathbb{N}_a} - int(N_{\mathbb{N}_a} a - cl (f^{-1}(B))) \subseteq 1_{\mathbb{N}_a} \Rightarrow f^{-1}(B) \) is \( N_{\mathbb{N}_a} a^* - CS \) in \((\mathbb{P}, \tau_{\mathbb{N}_a})\). Therefore, \( f \) is \( N_{\mathbb{N}_a} a^* - \) continuous. But \( f \) is not strongly \( N_{\mathbb{N}_a} a^* - \) continuous.

**Example 3.8:** Let \( \mathbb{P} = \{p\} \) and \( \mathbb{Q} = \{q\} \). \( \tau_{\mathbb{N}_a} = \{0_{\mathbb{N}_a}, 1_{\mathbb{N}_a}, A, C\} \) and \( \sigma_{\mathbb{N}_a} = \{N_{\mathbb{N}_a}, N_{\mathbb{N}_a}, B\} \) are \( N_{\mathbb{N}_a} a \)-TS on \((\mathbb{P}, \tau_{\mathbb{N}_a})\) and \((\mathbb{Q}, \sigma_{\mathbb{N}_a})\) respectively. Also \( A = \{(p, (0.4, 0.6, 0.2))\}, C = \{(p, (0.4, 1), [0.6, 1], [0, 0.2])\} \) and \( B = \{(q, (0.4, 0.6, 0.2))\} \) are \( N_{\mathbb{N}_a}(\mathbb{P}) \) and \( N_{\mathbb{N}_a}(\mathbb{Q}) \). Define a map \( f : (\mathbb{P}, \tau_{\mathbb{N}_a}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{N}_a}) \) by \( f(p) = q \). Let \( T = \{(q, (0, 0.2), [0, 0.4], [0.4, 1])\} \) be a \( N_{\mathbb{N}_a} a^* - \) CS in \((\mathbb{Q}, \sigma_{\mathbb{N}_a})\). Then \( f^{-1}(T) = \{(p, (0, 0.2), [0, 0.4], [0.4, 1])\} \). Now \( N_{\mathbb{N}_a} - cl(f^{-1}(T)) = A^c \cap C^c \cap 1_{\mathbb{N}_a} = C^c = f^{-1}(T) \). Therefore, \( f \) is strongly \( N_{\mathbb{N}_a} a^* - \) continuous. But \( f \) is not strongly neutrosophic continuous.

**Remark 3.9:** Every strongly neutrosophic continuous is \( N_{\mathbb{N}_a} a^* - \) continuous, but not conversely. (by Theorem 3.5 & 3.7).

**Theorem 3.10:** Let \( f : (\mathbb{P}, \tau_{\mathbb{N}_a}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{N}_a}) \) be neutrosophic function and \((\mathbb{Q}, \sigma_{\mathbb{N}_a})\) be \( N_{\mathbb{N}_a} a^* - T_{12} \) space. Then the following are equivalent.

1. \( f \) is strongly \( N_{\mathbb{N}_a} a^* - \) continuous.
2. \( f \) is neutrosophic continuous.

**Proof:**

1. \( \Rightarrow \) (2), Proof follows from theorem 3.2.

2. \( \Rightarrow \) (1), Let \( A \) be any \( N_{\mathbb{N}_a} a^* - \) CS in \((\mathbb{Q}, \sigma_{\mathbb{N}_a})\). Since \((\mathbb{Q}, \sigma_{\mathbb{N}_a})\) is \( N_{\mathbb{N}_a} a^* - T_{12} \) space, then \( A \) is \( N_{\mathbb{N}_a} a - CS \) in \((\mathbb{Q}, \sigma_{\mathbb{N}_a})\). Since \( f \) is neutrosophic continuous, then \( f^{-1}(A) \) is \( N_{\mathbb{N}_a} a - CS \) in \((\mathbb{P}, \tau_{\mathbb{N}_a})\). Therefore, \( f \) is strongly \( N_{\mathbb{N}_a} a^* - \) continuous.

**Theorem 3.11:** Let \( f : (\mathbb{P}, \tau_{\mathbb{N}_a}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{N}_a}) \) be \( N_{\mathbb{N}_a} a^* - \) continuous. Both \((\mathbb{P}, \tau_{\mathbb{N}_a})\) and \((\mathbb{Q}, \sigma_{\mathbb{N}_a})\) are \( N_{\mathbb{N}_a} a^* - T_{12} \) space, then \( f \) is strongly \( N_{\mathbb{N}_a} a^* - \) continuous.
Proof:

Let A be any $\text{N}_{u}\text{gs}^a \ast - \text{CS}$ in $(Q, \sigma_{N_u})$. Since $(Q, \sigma_{N_u})$ is $\text{N}_{u}\text{gs}^a \ast - T_{12}$ space, then A is $\text{N}_{u} \ast - \text{CS}$ in $(Q, \sigma_{N_u})$. Since f is $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$, then $f^{-1}(A)$ is $\text{N}_{u}\text{gs}^a \ast - \text{CS}$ in $(P, \tau_{N_u})$. Hence, $f$ is strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$.

Theorem 3.12: Let $f : (P, \tau_{N_u}) \to (Q, \sigma_{N_u})$ be strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$, then $f$ is $\text{N}_{u}\text{gs}^a \ast - \text{irresolute}$.

Proof:

Let A be any $\text{N}_{u}\text{gs}^a \ast - \text{CS}$ in $(Q, \sigma_{N_u})$. Since f is strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$, then $f^{-1}(A)$ is $\text{N}_{u} \ast - \text{CS}$ in $(P, \tau_{N_u})$. Since $(P, \tau_{N_u})$ is $\text{N}_{u}\text{gs}^a \ast - T_{12}$ space, then $f^{-1}(A)$ is $\text{N}_{u}\text{gs}^a \ast - \text{CS}$ in $(P, \tau_{N_u})$. Hence, $f$ is strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$.

Theorem 3.13: Let $f : (P, \tau_{N_u}) \to (Q, \sigma_{N_u})$ be $\text{N}_{u}\text{gs}^a \ast - \text{irresolute}$ and $(P, \tau_{N_u})$ be $\text{N}_{u}\text{gs}^a \ast - T_{12}$ space, then $f$ is strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$.

Proof:

Let A be any $\text{N}_{u}\text{gs}^a \ast - \text{CS}$ in $(Q, \sigma_{N_u})$. Since f is $\text{N}_{u}\text{gs}^a \ast - \text{irresolute}$, then $f^{-1}(A)$ is $\text{N}_{u}\text{gs}^a \ast - \text{CS}$ in $(P, \tau_{N_u})$. Since $(P, \tau_{N_u})$ is $\text{N}_{u}\text{gs}^a \ast - T_{12}$ space, then $f^{-1}(A)$ is $\text{N}_{u}\text{gs}^a \ast - \text{CS}$ in $(P, \tau_{N_u})$. Therefore, $f$ is strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$.

Theorem 3.14: Let $f : (P, \tau_{N_u}) \to (Q, \sigma_{N_u})$ and $g : (Q, \sigma_{N_u}) \to (R, \gamma_{N_u})$ be strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$ and $g : (Q, \sigma_{N_u}) \to (R, \gamma_{N_u})$ be $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$, then $gof : (P, \tau_{N_u}) \to (R, \gamma_{N_u})$ is $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$.

Proof:

Let A be any $\text{N}_{u}\text{gs}^a \ast - \text{CS}$ in $(R, \gamma_{N_u})$. Since g is strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$, then $g^{-1}(A)$ is $\text{N}_{u}\text{gs}^a \ast - \text{CS}$ in $(Q, \sigma_{N_u})$. Since f is strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is $\text{N}_{u} \ast - \text{CS}$ in $(P, \tau_{N_u})$. Therefore, $gof$ is strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$.

Theorem 3.15: Let $f : (P, \tau_{N_u}) \to (Q, \sigma_{N_u})$ be strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$ and $g : (Q, \sigma_{N_u}) \to (R, \gamma_{N_u})$ be $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$, then $gof : (P, \tau_{N_u}) \to (R, \gamma_{N_u})$ is $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$.

Proof:

Let A be any $\text{N}_{u}\text{gs}^a \ast - \text{CS}$ in $(R, \gamma_{N_u})$. Since g is $\text{N}_{u}\text{gs}^a \ast - \text{irresolute}$, then $g^{-1}(A)$ is $\text{N}_{u}\text{gs}^a \ast - \text{CS}$ in $(Q, \sigma_{N_u})$. Since f is strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is $\text{N}_{u} \ast - \text{CS}$ in $(P, \tau_{N_u})$. Therefore, $gof$ is strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$.

Theorem 3.16: Let $f : (P, \tau_{N_u}) \to (Q, \sigma_{N_u})$ be strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$ and $g : (Q, \sigma_{N_u}) \to (R, \gamma_{N_u})$ be $\text{N}_{u}\text{gs}^a \ast - \text{irresolute}$, then $gof : (P, \tau_{N_u}) \to (R, \gamma_{N_u})$ is strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$.

Proof:

Let A be any $\text{N}_{u}\text{gs}^a \ast - \text{CS}$ in $(R, \gamma_{N_u})$. Since g is $\text{N}_{u}\text{gs}^a \ast - \text{irresolute}$, then $g^{-1}(A)$ is $\text{N}_{u}\text{gs}^a \ast - \text{CS}$ in $(Q, \sigma_{N_u})$. Since f is strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is $\text{N}_{u} \ast - \text{CS}$ in $(P, \tau_{N_u})$. Therefore, $gof$ is strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$.

Theorem 3.17: Let $f : (P, \tau_{N_u}) \to (Q, \sigma_{N_u})$ be $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$ and $g : (Q, \sigma_{N_u}) \to (R, \gamma_{N_u})$ be strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$, then $gof : (P, \tau_{N_u}) \to (R, \gamma_{N_u})$ is $\text{N}_{u}\text{gs}^a \ast - \text{irresolute}$.

Proof:

Let A be any $\text{N}_{u}\text{gs}^a \ast - \text{CS}$ in $(R, \gamma_{N_u})$. Since g is strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$, then $g^{-1}(A)$ is $\text{N}_{u} \ast - \text{CS}$ in $(Q, \sigma_{N_u})$. Since f is strongly $\text{N}_{u}\text{gs}^a \ast - \text{continuous}$, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is $\text{N}_{u}\text{gs}^a \ast - \text{CS}$ in $(P, \tau_{N_u})$. Hence, $gof$ is $\text{N}_{u}\text{gs}^a \ast - \text{irresolute}$.
**Theorem 3.18:** Let \( f : (\mathbb{P}, \tau_{\mathbb{P}_u}) \to (\mathbb{Q}, \sigma_{\mathbb{Q}_u}) \) be neutrosophic continuous and \( g : (\mathbb{Q}, \sigma_{\mathbb{Q}_u}) \to (\mathbb{R}, \tau_{\mathbb{R}_u}) \) be strongly \( N_{\alpha}g \sigma^* \) continuous, then \( gof : (\mathbb{P}, \tau_{\mathbb{P}_u}) \to (\mathbb{R}, \tau_{\mathbb{R}_u}) \) is strongly \( N_{\alpha}g \sigma^* \) continuous.

**Proof:**
Let \( A \) be any \( N_{\alpha}g \sigma^* \) CS in \((\mathbb{R}, \tau_{\mathbb{R}_u})\). Since \( g \) is strongly \( N_{\alpha}g \sigma^* \) continuous, then \( g^{-1}(A) \) is \( N_{\alpha u} \) CS in \((\mathbb{Q}, \sigma_{\mathbb{Q}_u})\). Since \( f \) is neutrosophic continuous, then \( f^{-1}(g^{-1}(A)) = (gof)^{-1}(A) \) is \( N_{\alpha u} \) CS in \((\mathbb{P}, \tau_{\mathbb{P}_u})\). Hence, \( gof \) is strongly \( N_{\alpha}g \sigma^* \) continuous.

**Inter-relationship 3.19:**

<table>
<thead>
<tr>
<th>Strongly neutrosophic</th>
<th>Neutrosophic continuous</th>
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<tbody>
<tr>
<td>( N_{\alpha}gs \sigma^* ) – continuous</td>
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<td>( N_{\alpha}gs \sigma^* ) – irresolute</td>
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4. Perfectly neutrosophic \( g \sigma^* \)-continuous function

**Definition 4.1:** A neutrosophic function \( f : (\mathbb{P}, \tau_{\mathbb{P}_u}) \to (\mathbb{Q}, \sigma_{\mathbb{Q}_u}) \) is said to be perfectly \( N_{\alpha}g \sigma^* \) – continuous if the inverse image of every \( N_{\alpha}g \sigma^* \) – CS in \((\mathbb{Q}, \sigma_{\mathbb{Q}_u})\) is both \( N_{\alpha u} \) OS and \( N_{\alpha u} \) CS (ie, \( N_{\alpha u} \) clopen set) in \((\mathbb{P}, \tau_{\mathbb{P}_u})\).

**Theorem 4.2:** Every perfectly \( N_{\alpha}g \sigma^* \) – continuous is strongly \( N_{\alpha}g \sigma^* \) – continuous, but not conversely.

**Proof:**
Let \( f : (\mathbb{P}, \tau_{\mathbb{P}_u}) \to (\mathbb{Q}, \sigma_{\mathbb{Q}_u}) \) be any neutrosophic function. Let \( A \) be any \( N_{\alpha}g \sigma^* \) – CS in \((\mathbb{Q}, \sigma_{\mathbb{Q}_u})\). Since \( f \) is perfectly \( N_{\alpha}g \sigma^* \) – continuous, then \( f^{-1}(A) \) is both \( N_{\alpha u} \) OS and \( N_{\alpha u} \) CS in \((\mathbb{P}, \tau_{\mathbb{P}_u})\). Therefore, \( f \) is strongly \( N_{\alpha}g \sigma^* \) – continuous.

**Example 4.3:** Let \( P = \{p\} \) and \( Q = \{q\} \), \( \tau_{\mathbb{P}_u} = \{0_{\mathbb{P}_u}, 1_{\mathbb{P}_u}, A, C\} \) and \( \sigma_{\mathbb{Q}_u} = \{0_{\mathbb{Q}_u}, 1_{\mathbb{Q}_u}, B\} \) are \( N_{\alpha}TS \) on \((\mathbb{P}, \tau_{\mathbb{P}_u})\) and \((\mathbb{Q}, \sigma_{\mathbb{Q}_u})\) respectively. Also, \( A = \{\{p, (0.7, 0.8, 0.3)\}\}, C = \{\{p, (0.7, 1), (0.8, 1), (0, 0.3)\}\} \) and \( B = \{\{q, (0.7, 0.8, 0.3)\}\} \) are \( N_{\alpha u}(\mathbb{P}) \) and \( N_{\alpha u}(\mathbb{Q}) \). Define a map \( f : (\mathbb{P}, \tau_{\mathbb{P}_u}) \to (\mathbb{Q}, \sigma_{\mathbb{Q}_u}) \) by \( f(p) = q \). Let \( T = \{\{q, (0, 0.3), (0, 0.2), (0.7, 1)\}\} \) be a \( N_{\alpha}g \sigma^* \) – CS in \((\mathbb{Q}, \sigma_{\mathbb{Q}_u})\). Then \( f^{-1}(T) = \{\{p, (0, 0.3), (0, 0.2), (0.7, 1)\}\} \). Now \( N_{\alpha u} = cl( f^{-1}(T) ) = A^c \cap C^c \cap 1_{\mathbb{P}_u} = C^c = f^{-1}(T) \). Therefore, \( f \) is strongly \( N_{\alpha}g \sigma^* \) – continuous. But \( f \) is not perfectly \( N_{\alpha}g \sigma^* \) – continuous, because \( f^{-1}(T) \) is not both \( N_{\alpha u} \) OS and \( N_{\alpha u} \) CS in \((\mathbb{P}, \tau_{\mathbb{P}_u})\). Since, \( N_{\alpha u} \) int \( ( f^{-1}(T) ) = 0_{\mathbb{P}_u} \neq f^{-1}(T) \Rightarrow f^{-1}(T) \) is not \( N_{\alpha u} \) CS in \((\mathbb{P}, \tau_{\mathbb{P}_u})\). Therefore, \( f^{-1}(T) \) is not both \( N_{\alpha u} \) OS and \( N_{\alpha u} \) CS in \((\mathbb{P}, \tau_{\mathbb{P}_u})\).

**Theorem 4.4:** Every perfectly \( N_{\alpha}g \sigma^* \) – continuous is perfectly neutrosophic continuous, but not conversely.
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**Proof:**
Let \( f : (\mathbb{P}, \tau_{\mathbb{P}}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{Q}}) \) be any neutrosophic function. Let \( A \) be any \( N_{\mathbb{P}} \subset \text{CS} \) in \((\mathbb{Q}, \sigma_{\mathbb{Q}})\). Then \( A \) is \( N_{\mathbb{P}} \text{gsa}^* - \text{CS} \) in \((\mathbb{Q}, \sigma_{\mathbb{Q}})\). Since \( f \) is perfectly \( N_{\mathbb{P}} \text{gsa}^* \) – continuous, then \( f^{-1}(A) \) is both \( N_{\mathbb{P}} \subset \text{OS} \) and \( N_{\mathbb{P}} \subset \text{CS} \) in \((\mathbb{P}, \tau_{\mathbb{P}})\). Therefore, \( f \) is perfectly neutrosophic continuous.

**Example 4.5:** Let \( \mathbb{P} = \{p\} \) and \( \mathbb{Q} = \{q\}. \) \( \tau_{\mathbb{P}} = \{0_{\mathbb{P}}, 1_{\mathbb{P}}, A, C, E\} \) and \( \sigma_{\mathbb{Q}} = \{0_{\mathbb{Q}}, 1_{\mathbb{Q}}, B\} \) are \( N_{\mathbb{P}} \text{TS} \) on \((\mathbb{P}, \tau_{\mathbb{P}})\) and \((\mathbb{Q}, \sigma_{\mathbb{Q}})\) respectively. Also \( A = \{(p, (0, 0.4, 0.2, 0.6))\}, C = \{(p, (0.6, 0.8, 0.4))\}, \) \( E = \{(q, (0, 0.4, 0.2, 0.6)), \} \). \( N_{\mathbb{P}}(p) \) and \( N_{\mathbb{Q}}(q). \) Define a map \( f : (\mathbb{P}, \tau_{\mathbb{P}}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{Q}}) \) by \( f(p) = q. \) Let \( B = \{(q, (0, 0.4, 0.2, 0.6))\} \) be a \( N_{\mathbb{Q}} \subset \text{CS} \) in \((\mathbb{Q}, \sigma_{\mathbb{Q}})\). Then \( f^{-1}(B) = \{(q, (0, 0.4, 0.2, 0.6))\}. \) Now \( N_{\mathbb{P}} - \text{cl}(f^{-1}(B)) = A \cap C^c \cap E^c \cap 1_{\mathbb{Q}} = C^c = f^{-1}(B^c). \) Also, \( N_{\mathbb{P}} - \text{int}(f^{-1}(B)) = A \cup E \cup 0_{\mathbb{Q}} = A = f^{-1}(B^c) \Rightarrow f^{-1}(B^c) \) is both \( N_{\mathbb{P}} \subset \text{OS} \) and \( N_{\mathbb{P}} \subset \text{CS} \) in \((\mathbb{P}, \tau_{\mathbb{P}})\). Therefore, \( f \) is perfectly neutrosophic continuous. But \( f \) is not perfectly \( N_{\mathbb{Q}} \text{gsa}^* \) – continuous. Let \( T = \{(q, (0, 0.4, 0.2, 0.6))\} \) be \( N_{\mathbb{Q}} \text{gsa}^* \) – CS in \((\mathbb{Q}, \sigma_{\mathbb{Q}})\). Then \( f^{-1}(T) = \{(q, (0, 0.4, 0.2, 0.6))\}. \) Since \( N_{\mathbb{P}} - \text{int}(f^{-1}(T)) = E \cup 0_{\mathbb{Q}} = E = f^{-1}(T) \Rightarrow f^{-1}(T) \) is \( N_{\mathbb{P}} \subset \text{OS} \) in \((\mathbb{P}, \tau_{\mathbb{P}})\). Also, \( N_{\mathbb{Q}} - \text{cl}(f^{-1}(T)) = A^c \cap C^c \cap E^c \cap 1_{\mathbb{Q}} = C^c \neq f^{-1}(T) \Rightarrow f^{-1}(T) \) is not \( N_{\mathbb{P}} \subset \text{CS} \) in \((\mathbb{P}, \tau_{\mathbb{P}})\). Therefore, \( f^{-1}(T) \) is not both \( N_{\mathbb{P}} \subset \text{OS} \) and \( N_{\mathbb{P}} \subset \text{CS} \) in \((\mathbb{P}, \tau_{\mathbb{P}})\).

**Theorem 4.6:** Let \( f : (\mathbb{P}, \tau_{\mathbb{P}}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{Q}}) \) be perfectly \( N_{\mathbb{Q}} \text{gsa}^* \) – continuous iff the inverse image of every \( N_{\mathbb{Q}} \text{gsa}^* \) – OS in \((\mathbb{Q}, \sigma_{\mathbb{Q}})\) is both \( N_{\mathbb{P}} \subset \text{OS} \) and \( N_{\mathbb{P}} \subset \text{CS} \) in \((\mathbb{P}, \tau_{\mathbb{P}})\).

**Proof:**
Assume that \( f \) is perfectly \( N_{\mathbb{Q}} \text{gsa}^* \) – continuous function. Let \( A \) be any \( N_{\mathbb{Q}} \text{gsa}^* \) – OS in \((\mathbb{Q}, \sigma_{\mathbb{Q}})\). Then \( A^c \) is \( N_{\mathbb{Q}} \text{gsa}^* \) – CS in \((\mathbb{Q}, \sigma_{\mathbb{Q}})\). Since \( f \) is perfectly \( N_{\mathbb{Q}} \text{gsa}^* \) – continuous, then \( f^{-1}(A^c) \) is both \( N_{\mathbb{P}} \subset \text{OS} \) and \( N_{\mathbb{P}} \subset \text{CS} \) in \((\mathbb{P}, \tau_{\mathbb{P}})\). Conversely, Let \( A \) be any \( N_{\mathbb{Q}} \text{gsa}^* \) – CS in \((\mathbb{Q}, \sigma_{\mathbb{Q}})\). Then \( A^c \) is \( N_{\mathbb{Q}} \text{gsa}^* \) – OS in \((\mathbb{Q}, \sigma_{\mathbb{Q}})\). By hypothesis, \( f^{-1}(A^c) \) is both \( N_{\mathbb{P}} \subset \text{OS} \) and \( N_{\mathbb{P}} \subset \text{CS} \) in \((\mathbb{P}, \tau_{\mathbb{P}})\) \Rightarrow \( f^{-1}(A^c) \) is both \( N_{\mathbb{P}} \subset \text{OS} \) and \( N_{\mathbb{P}} \subset \text{CS} \) in \((\mathbb{P}, \tau_{\mathbb{P}})\). Therefore, \( f \) is perfectly \( N_{\mathbb{Q}} \text{gsa}^* \) – continuous.

**Theorem 4.7:** Let \( (\mathbb{P}, \tau_{\mathbb{P}}) \) be a neutrosophic discrete topological space and \((\mathbb{Q}, \sigma_{\mathbb{Q}})\) be any neutrosophic topological space. Let \( f : (\mathbb{P}, \tau_{\mathbb{P}}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{Q}}) \) be a neutrosophic function, then the following statements are true.

1. \( f \) is strongly \( N_{\mathbb{Q}} \text{gsa}^* \) – continuous.

2. \( f \) is perfectly \( N_{\mathbb{Q}} \text{gsa}^* \) – continuous.

**Proof:**
1. \( \Rightarrow (2) \), Let \( A \) be any \( N_{\mathbb{Q}} \text{gsa}^* \) – CS in \((\mathbb{Q}, \sigma_{\mathbb{Q}})\). Since \( f \) is strongly \( N_{\mathbb{Q}} \text{gsa}^* \) – continuous, then \( f^{-1}(A) \) is \( N_{\mathbb{P}} \subset \text{CS} \) in \((\mathbb{P}, \tau_{\mathbb{P}})\). Since \((\mathbb{P}, \tau_{\mathbb{P}})\) is neutrosophic discrete topological space, then \( f^{-1}(A) \) is \( N_{\mathbb{P}} \subset \text{OS} \) in \((\mathbb{P}, \tau_{\mathbb{P}})\) \Rightarrow \( f^{-1}(A) \) is both \( N_{\mathbb{P}} \subset \text{OS} \) and \( N_{\mathbb{P}} \subset \text{CS} \) in \((\mathbb{P}, \tau_{\mathbb{P}})\). Therefore, \( f \) is perfectly \( N_{\mathbb{Q}} \text{gsa}^* \) – continuous.

2. \( \Rightarrow (1) \), Let \( A \) be any \( N_{\mathbb{Q}} \text{gsa}^* \) – CS in \((\mathbb{Q}, \sigma_{\mathbb{Q}})\). Since \( f \) is perfectly \( N_{\mathbb{Q}} \text{gsa}^* \) – continuous, then \( f^{-1}(A) \) is both \( N_{\mathbb{P}} \subset \text{OS} \) and \( N_{\mathbb{P}} \subset \text{CS} \) in \((\mathbb{P}, \tau_{\mathbb{P}})\) \Rightarrow \( f^{-1}(A) \) is both \( N_{\mathbb{P}} \subset \text{OS} \) and \( N_{\mathbb{P}} \subset \text{CS} \) in \((\mathbb{P}, \tau_{\mathbb{P}})\). Therefore, \( f \) is perfectly \( N_{\mathbb{Q}} \text{gsa}^* \) – continuous.
Theorem 4.8: Let $f : (\mathbb{P}, T_{\mathbb{N}_0}) \to (\mathbb{Q}, \sigma_{\mathbb{N}_0})$ be perfectly neutrosophic continuous and $(\mathbb{Q}, \sigma_{\mathbb{N}_0})$ be $N_{\cap g\alpha^*} - T_{12}$ space, then $f$ is perfectly neutrosophic continuous.

Proof:
Let $A$ be any $N_{\cap g\alpha^*} - CS$ in $(\mathbb{Q}, \sigma_{\mathbb{N}_0})$. Since $(\mathbb{Q}, \sigma_{\mathbb{N}_0})$ is $N_{\cap g\alpha^*} - T_{12}$ space, then
$\overline{A}$ is $N_{\cap g\alpha^*} - CS$ in $(\mathbb{Q}, \sigma_{\mathbb{N}_0})$. Since $f$ is perfectly neutrosophic continuous, then $f^{-1}(A)$ is both $N_{\cap g\alpha^*} - OS$ and $N_{\cap g\alpha^*} - CS$ in $(\mathbb{P}, T_{\mathbb{N}_0})$. Therefore, $f$ is perfectly neutrosophic continuous.

Theorem 4.9: Let $f : (\mathbb{P}, T_{\mathbb{N}_0}) \to (\mathbb{Q}, \sigma_{\mathbb{N}_0})$ and $g : (\mathbb{Q}, \sigma_{\mathbb{N}_0}) \to (\mathbb{R}, T_{\mathbb{N}_0})$ be perfectly neutrosophic continuous, then $g\circ f : (\mathbb{P}, T_{\mathbb{N}_0}) \to (\mathbb{R}, T_{\mathbb{N}_0})$ is perfectly neutrosophic continuous.

Proof:
Let $A$ be any $N_{\cap g\alpha^*} - CS$ in $(\mathbb{R}, T_{\mathbb{N}_0})$. Since $g$ is perfectly neutrosophic continuous, then $g^{-1}(A)$ is both $N_{\cap g\alpha^*} - OS$ and $N_{\cap g\alpha^*} - CS$ in $(\mathbb{Q}, \sigma_{\mathbb{N}_0})$. Since $f$ is neutrosophic continuous, then $f^{-1}(g^{-1}(A)) = \{ (p, g\circ f, (p, T_{\mathbb{N}_0})) \} \in (\mathbb{P}, T_{\mathbb{N}_0})$. Therefore, $g\circ f$ is perfectly neutrosophic continuous.

5. Totally neutrosophic $g\alpha^*$ - continuous function

Definition 5.1: A neutrosophic function $f : (\mathbb{P}, T_{\mathbb{N}_0}) \to (\mathbb{Q}, \sigma_{\mathbb{N}_0})$ is said to be totally neutrosophic continuous if the inverse image of every $N_{\cap g\alpha^*} - CS$ in $(\mathbb{Q}, \sigma_{\mathbb{N}_0})$ is both $N_{\cap g\alpha^*} - OS$ and $N_{\cap g\alpha^*} - CS$ (ie, $N_{\cap g\alpha^*} - cl$ set) in $(\mathbb{P}, T_{\mathbb{N}_0})$.

Definition 5.2: A neutrosophic topological space $(\mathbb{P}, T_{\mathbb{N}_0})$ is called a $N_{\cap g\alpha^*} - cl$ set $(N_{\cap g\alpha^*} - cl$ set) if it is both $N_{\cap g\alpha^*} - OS$ and $N_{\cap g\alpha^*} - CS$ in $(\mathbb{P}, T_{\mathbb{N}_0})$.

Example 5.3: Let $\mathbb{P} = \{ p \}$ and $\mathbb{Q} = \{ q \}$. $T_{\mathbb{N}_0} = \{ \emptyset, \mathbb{N}_0, \mathbb{N}_0, B \}$ are neutrosophic TS on $(\mathbb{P}, T_{\mathbb{N}_0})$ and $(\mathbb{Q}, \sigma_{\mathbb{N}_0})$ respectively. Also $A = \{ (p, \{ 0.4, 0.5, 0.7 \}) \}$ and $B = \{ (q, \{ 0.2, 0.7, 0.8 \}) \}$ are neutrosophic $\mathbb{N}_0(\mathbb{P})$ and $\mathbb{N}_0(\mathbb{Q})$.

Define a map $f : (\mathbb{P}, T_{\mathbb{N}_0}) \to (\mathbb{Q}, \sigma_{\mathbb{N}_0})$ by $f(p) = q$. Let $B' = \{ (q, \{ 0.8, 0.3, 0.2 \}) \}$ be a $N_{\cap g\alpha^*} - CS$ in $(\mathbb{Q}, \sigma_{\mathbb{N}_0})$. Then $f^{-1}(B') = \{ (p, \{ 0.8, 0.3, 0.2 \}) \}$. $\mathbb{N}_0\alpha^* - OS = N_{\cap g\alpha^*} - OS = \{ \emptyset, \mathbb{N}_0, \mathbb{N}_0, A \}$ and $N_{\cap g\alpha^*} - CS = \{ 0_{\mathbb{N}_0}, 1_{\mathbb{N}_0}, A \}$. $N_{\cap g\alpha^*} - cl (f^{-1}(B')) = 1_{\mathbb{N}_0}$. Now, $N_{\cap g\alpha^*} - cl (f^{-1}(B')) = 1_{\mathbb{N}_0} \subseteq N_{\cap g\alpha^*} - int(1_{\mathbb{N}_0}) = 1_{\mathbb{N}_0}$, whenever $f^{-1}(B') \subseteq 1_{\mathbb{N}_0} \Rightarrow f^{-1}(B')$ is $N_{\cap g\alpha^*} - CS$ in $(\mathbb{P}, T_{\mathbb{N}_0})$.
Also, $N_{eu}a - \text{int} \left( f^{-1}(B') \right) = 0_{N_{eu}}$. Now, $N_{eu}a - \text{cl} \left( N_{eu}a - \text{int} \left( f^{-1}(B') \right) \right) = 0_{N_{eu}} \supseteq N_{eu} - \text{cl}(0_{N_{eu}}) = 0_{N_{eu}}$, whenever $f^{-1}(B') \supseteq 0_{N_{eu}} \Rightarrow f^{-1}(B')$ is $N_{eu}gsa^* - \text{OS}$ in $(P, \tau_{N_{eu}})$. Therefore, $f$ is totally $N_{eu}gsa^* - \text{continuous}$.

**Theorem 5.4**: Every perfectly $N_{eu}gsa^* - \text{continuous}$ is totally $N_{eu}gsa^* - \text{continuous}$, but not conversely.

**Proof**: Let $f : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}})$ be any neutrosophic function. Let $A$ be any $N_{eu} - \text{CS}$ in $(Q, \sigma_{N_{eu}})$. Then $A$ is $N_{eu}gsa^* - \text{CS}$ in $(Q, \sigma_{N_{eu}})$. Since $f$ is perfectly $N_{eu}gsa^* - \text{continuous}$, then $f^{-1}(A)$ is both $N_{eu} - \text{OS}$ and $N_{eu} - \text{CS}$ in $(P, \tau_{N_{eu}}) \Rightarrow f^{-1}(A)$ is both $N_{eu}gsa^* - \text{OS}$ and $N_{eu}gsa^* - \text{CS}$ in $(P, \tau_{N_{eu}})$. Therefore, $f$ is totally $N_{eu}gsa^* - \text{continuous}$.

**Example 5.5**: Let $P = \{ \phi \}$ and $Q = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, A\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, B\}$ are $N_{eu}TS$ on $(P, \tau_{N_{eu}})$ and $(Q, \sigma_{N_{eu}})$ respectively. Also $A = \{\phi, (0.2, 0.4, 0.6)\}$ and $B = \{q, (0.6, 0.8, 0.4)\}$ are $N_{eu}(P)$ and $N_{eu}(Q)$. Define a map $f : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}})$ by $f(\phi) = q$. Let $B' = \{(q, (0.4, 0.2, 0.6))\}$ be a $N_{eu} - \text{CS}$ in $(Q, \sigma_{N_{eu}})$. Then $f^{-1}(B') = \{\phi, (0.4, 0.2, 0.6)\}$. $N_{eu}gsa^* - \text{OS} = N_{eu}a - \text{OS} = \{0_{N_{eu}}, 1_{N_{eu}}, A\}$ and $N_{eu}a - \text{CS} = \{0_{N_{eu}}, 1_{N_{eu}}, A'\}$. $N_{eu}a - \text{cl} \left( f^{-1}(B') \right) = A' \cap 1_{N_{eu}} = A'$. Now, $N_{eu}a - \text{int} (N_{eu}a - \text{cl} \left( f^{-1}(B') \right)) = \emptyset \supseteq N_{eu}a - \text{int} (\text{cl}(f^{-1}(B'))) = N_{eu}gsa^* - \text{CS}$ in $(P, \tau_{N_{eu}})$. Also, $N_{eu}gsa^* - \text{CS}$ in $(P, \tau_{N_{eu}})$. Therefore, $f$ is totally $N_{eu}gsa^* - \text{continuous}$. But $f$ is not perfectly $N_{eu}gsa^* - \text{continuous}$. Let $T = \{(q, (0.3, 0.1, 0.8))\}$ be $N_{eu}gsa^* - \text{CS}$ in $(Q, \sigma_{N_{eu}})$. Then $f^{-1}(T) = \emptyset$. Now, $N_{eu}gsa^* - \text{CS}$ in $(P, \tau_{N_{eu}})$. Therefore, $f$ is totally $N_{eu}gsa^* - \text{continuous}$.

**Theorem 5.6**: Every totally $N_{eu}gsa^* - \text{continuous}$ is $N_{eu}gsa^* - \text{continuous}$.

**Proof**: Let $f : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}})$ be any neutrosophic function. Let $A$ be any $N_{eu} - \text{CS}$ in $(Q, \sigma_{N_{eu}})$. Since $f$ is totally $N_{eu}gsa^* - \text{continuous}$, then $f^{-1}(A)$ is both $N_{eu}gsa^* - \text{OS}$ and $N_{eu}gsa^* - \text{CS}$ in $(P, \tau_{N_{eu}})$. Therefore, $f$ is totally $N_{eu}gsa^* - \text{continuous}$. But $f$ is not perfectly $N_{eu}gsa^* - \text{continuous}$. Let $T = \{(q, (0.3, 0.1, 0.8))\}$ be $N_{eu}gsa^* - \text{CS}$ in $(Q, \sigma_{N_{eu}})$. Then $f^{-1}(T) = \emptyset$. Now, $N_{eu}gsa^* - \text{CS}$ in $(P, \tau_{N_{eu}})$. Therefore, $f$ is totally $N_{eu}gsa^* - \text{continuous}$.
Inter-relationship 5.8:

Perfectly $N_{eu}gsa^*$ - continuous

Totally $N_{eu}gsa^*$ - continuous

$N_{eu}gsa^*$ - continuous

**Theorem 5.9:** Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be totally $N_{eu}gsa^*$ - continuous and $(\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gsa^*$ - $T_{12}$ space, then $f$ is $N_{eu}gsa^*$ - irresolute.

**Proof:**
Let $A$ be any $N_{eu}gsa^*$ - CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since $(\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gsa^*$ - $T_{12}$ space, then $A$ is $N_{eu}gsa^*$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since $f$ is totally $N_{eu}gsa^*$ - continuous, then $f^{-1}(A)$ is both $N_{eu}gsa^*$ - OS and $N_{eu}gsa^*$ - CS in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, $f$ is $N_{eu}gsa^*$ - irresolute.

**Theorem 5.10:** Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ and $g : (\mathbb{Q}, \sigma_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ be totally $N_{eu}gsa^*$ - continuous and $(\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gsa^*$ - $T_{12}$ space, then $gof : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ is totally $N_{eu}gsa^*$ - continuous.

**Proof:**
Let $A$ be any $N_{eu}$ - CS in $(\mathbb{R}, \gamma_{N_{eu}})$. Since $g$ is totally $N_{eu}gsa^*$ - continuous, then $g^{-1}(A)$ is both $N_{eu}gsa^*$ - OS and $N_{eu}gsa^*$ - CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since $(\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gsa^*$ - $T_{12}$ space, then $g^{-1}(A)$ is $N_{eu}$ - CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Therefore, $gof$ is totally $N_{eu}gsa^*$ - continuous.

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