Chapter

Formative Assessment in Mathematics Education in the Twenty-First Century

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Abstract

Assessment does not always have to involve paper and pencil but can instead be a project, an observation, or a task that shows a student has acquired the concept and can make sound connections and linkages with other related concepts. Learning is meaningful when the student comprehends the relationship of what is being learned to other knowledge. Furthermore, concept map measures aspects of learning, which conventional tests cannot measure such as students' misconceptions. As such, the chapter shall focus on formative assessment in mathematics classroom mediated by a method of teaching (concept mapping) that promotes critical thinking, which assists teachers to teach and assess students' understanding and make connections between concepts explicitly.

Keywords: assessment, formative assessment, summative assessment, concept mapping, instructional tool

1. Introduction

In mathematics education, focus is on the interactions among the three components of an instructional unit, the teacher, material, and students. In other words, the capacity to deliver quality instruction depends not only on the individual teacher's intellectual and personal resources but also on his or her interaction with specific groups of students and materials. According to [1], all curricula exist to provide the basis for effective instruction, that is, instruction that maximizes learning. Effective instruction is a result of proper and extensive planning. Planning starts with organizing material from the mathematics content. After deciding what material will be used, the next step is sequencing that material in the way students will experience it [1]. If instruction requires all three components (the teacher, students, and materials), then the capacity to produce worthwhile learning must also be a function of the interactions among these three components. Students bring experience, prior knowledge, and habit of mind, and these influence how they apprehend, interpret, and respond to materials and teachers. Teacher's awareness of students' "capabilities, needs, and past experiences" and the ability to use this information to "create a learning situation in which students can meet their needs or solve a problem in an autonomous and independent way" is therefore important.

Assessment is the process of gathering information so as to monitor students' prior knowledge and progress and make sound instructional decisions.

As such, the primary purpose of assessment is to improve student learning of mathematics. Teachers examine the standards, assess where their students are in their knowledge base through some sort of pretest, and then plan their instruction based on the data collected (diagnostic assessment). During the process of teaching and learning in the classroom, teachers do assessment for learning (formative assessment) and assessment of learning (summative assessment). They then compare assessment for learning and assessment of learning in order to determine whether or not the implemented learning activity in class should be used again (or modified) [20]. A test score as feedback that measures whether a student has attained the expected standard cannot serve as formative assessment. Teachers need students' background information in order to modify teaching and learning activities to improve their learning. Therefore, feedback that involves a focus on the detailed content of what is being learnt has a central function of formative assessment [4]. Formative assessment or assessment for learning involves a continuous way of checks and balances in the teaching and learning process. It can be done at the beginning of instruction to tap prior knowledge in order to connect concepts when motivating for upcoming new concepts. The method allows teachers to check their students' progress and shortcomings as well as the effectiveness of their own practice, thus allowing for self-assessment.

The functional role of formative assessment (assessment for teaching) is often compromised in light of growing demand for external accountability related to performance and learning outcomes. Accountability pressures put many (mathematics) teachers between striking a balance between teaching mathematics facts and calculation procedures and also developing a conceptual understanding of mathematics. Due to accountability pressures, teachers have a tendency to focus on the preparation for examinations, where they opt to provide students with the necessary skills by working out problems similar to those that have occurred in past examination papers. This approach has dismally failed because student performance in mathematics remains depressed. Mindful of that, it is deplorable that the state of affairs concerning the functional role of formative assessment (assessment for teaching) is often overlooked. During the process of teaching and learning, teachers should assess the impact of their teaching on their students with the intention to create optimal learning spaces that meet the learning needs of each student. Therefore, teachers are discouraged from thinking of assessment as pencil and paper and embrace alternative forms of assessment in the teaching and learning of mathematics [2]. They should try to check on the performance of each student by giving class daily written exercises and mark the exercise books before the next day lesson. Also they should and always carry out weekly informal tests. Carrying out formative assessments in the form of informal tests, written classwork, or homework provides continual snapshots of students' progress throughout the week, month, or school year. By using these formative assessments, teachers can target students' specific problem areas derived from qualitative feedback (rather than scores), adapt instruction, and intervene earlier rather than later. The qualitative feedback about students and their abilities are likely to improve teachers' mathematics knowledge in teaching (which is demonstrated in the class by how well a teacher uses mathematical and pedagogical knowledge to help students learn mathematics) [5]. As teachers are guided by the qualitative feedback from the formative assessment, the critical component that must be present in any intervention is an opportunity for the students to discover the joy of creating knowledge from their own experience of the subject matter. Hence the activities that the teacher creates should be student-centered. Besides, when teachers' classroom assessments become an integral part of the instructional process and a central ingredient in efforts to help students learn, the benefits of assessment for both teachers and students will be boundless [5].

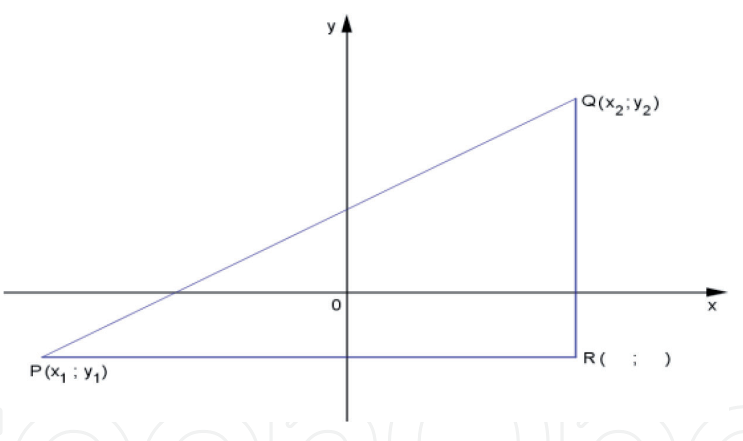
2. Formative assessment in mathematics classroom thrives on teaching approaches that promote critical thinking

In the process of teaching and learning, there are myriad factors that impact on student learning. However, how well a teacher uses mathematical and pedagogical knowledge to help students learn mathematics is one of the factors that influence student success. As such, teachers should strive to expose students to teaching practices that stimulate critical thinking process whose salient features are conceptualizing, applying, analyzing, synthesizing, and evaluating information [3]. It is therefore important for teachers to provide enquiry or problem-solving approaches in mathematics classes. Infusing critical thinking skills into didactic activities requires teachers to consciously integrate new knowledge with already existing knowledge schema of mathematics content [3]. Students battle to recall and apply basic concepts of mathematics in the resolution of mathematical problems, and this leads to the lack of understanding of mathematics [18]. Concept mapping can be used by teachers to stimulate critical thinking in students because it represents and organizes knowledge, helps retention and recall of concepts learnt, and provides feedback on the understanding of the concepts learnt [4]. Therefore, thinking of assessment as a task that shows a student has acquired the concept and can link with other related concepts becomes paramount [3]. Accordingly, learning is meaningful when the student comprehends the relationship of what is being learned to other knowledge. As such, there is a need for teachers to incorporate the concept mapping in the formative assessment process as this will help them diagnose students' misconceptions [4]. If students can link new information to their existing conceptual framework, they can construct new, meaningful interconnections, so that their existing conceptions are transformed, enriched, or revised, and conceptual change occurs. This is achieved by carrying out formative assessment where students are asked to summarize at the end of instruction to allow them to make connections [1]. Therefore, existing conceptions are transformed during construction of understanding [5]. Interaction, collaboration, cooperation, dialog, and discourse are key concepts facilitated by formative assessment for the effectiveness of instructional activities. As such, collaborative group learning fosters meaningful learning and new knowledge construction [4].

3. Assessment for learning versus assessment of learning in mathematics

Assessment is generally broken down into three categories: assessment before instruction (pre-assessment), assessment during instruction (formative assessment), and assessment after instruction (summative assessment). It could be argued that pre-assessment is both assessments of and for (as) learning—that is, it assesses "prior knowledge" (as a pre-assessment) and that data is then used to revise planned instruction (making it formative assessment). Assessment of learning is used to determine what students have learned, while assessment for learning is used to determine what students are learning. It should be clear that assessment for (as) learning is a process of gathering information about students learning and provide qualitative feedback to support individual student learning and improve teaching practice in the classroom. However, there is a significant overlap between assessment of and for learning. Therefore, learning for assessment (summative assessment) and learning from assessment (formative assessment) are two complementary purposes of assessment. For example, the same test given in one circumstance would be considered an assessment of learning, while in another circumstance be considered an assessment for (as) learning. In short then, the difference between assessment of learning and assessment for learning is a matter of function and purpose [17]. Hence assessment that occurs during the lesson to continuously assess learning throughout instruction is formative assessment. For example, in teaching mathematics, I often use concept learning where students are given an explanation, examples, and non-examples after which they engage in working problems on their own or in groups. Periodically during the lesson, I stop students and have them share their answers. This allows me to know if everyone is on the task, if everyone has understood, and whether I have to revisit the instruction in a different way if students are making several errors. At the end of instruction, assess whether or not the instruction was effective and whether the students have gained the knowledge as per lesson objective, and if they have not, then the instruction is redesigned to better cater for the students. Therefore, "if the students do not learn the way we teach them, we must teach them the way they learn [1]." I shall not give an example of *assessment of learning* task because it is predominant in high schools. However, hereunder is an example of *assessment of learning task* anchored on discovery-based learning. The objective of the task is to help students derive and apply the distance formula for calculating the length of a line segment joining any two given points.

1. Given the diagram below:



   a. Determine the coordinates of R.

   b. What is the horizontal distance PR?

   c. What is the vertical distance RQ?

   d. What type of triangle is PQR?

   e. Find the length of PQ.

2. Use the answer you found in 1(e) to calculate the length of the line segment joining points $A(3;4)$ and $B(-2;7)$.

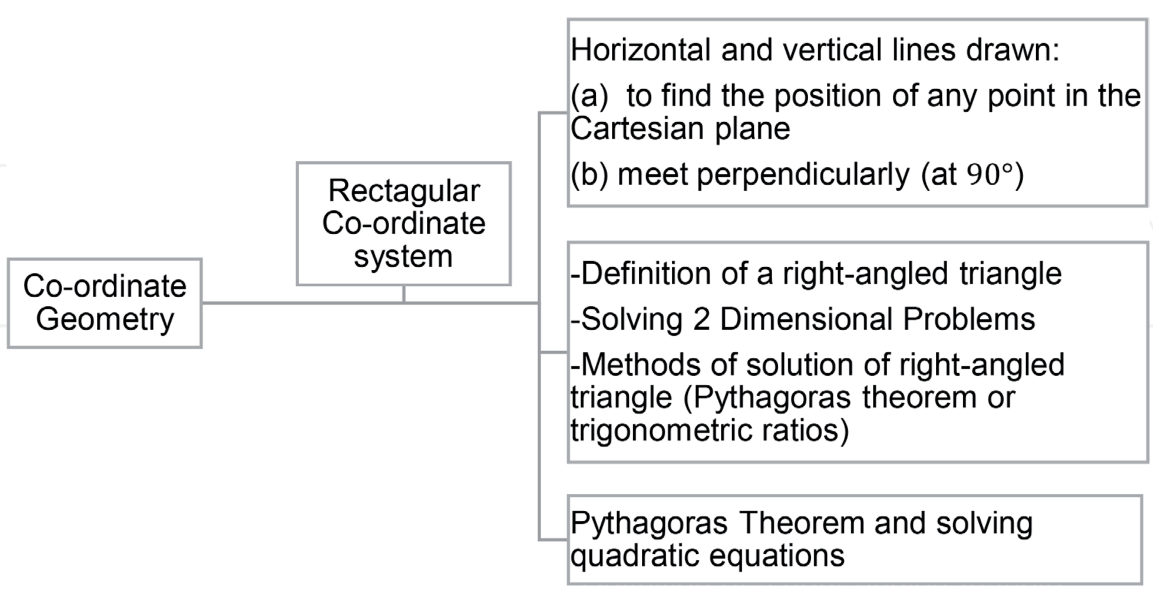
3. Generate a concept map in relation to concepts used to find the distance formula.

Solution strategy:

Question	Answer	Concepts employed to arrive at the answer
a)	$(x_2;y_1)$	*Rectangular coordinates
b)	$x_2 - x_1$	* Distance moved from point P to point R (horizontal displacement)
c)	$y_2 - y_1$	* Distance moved from point R to point Q (vertical displacement)
d)	Right-angled triangle	* Horizontal and vertical lines meet perpendicularly (at 90°); hence $PR \perp RQ$
e)	$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$	* Pythagoras theorem (result is a quadratic equation)
	$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	* Solution of quadratic equation (consider the positive solution because we are dealing with distance)
2.	$AB = \sqrt{(-2-3)^2 + (7-4)^2}$ $= \sqrt{(-5)^2 + 3^2}$ $= \sqrt{25+9}$ $= \sqrt{34}$	*Application of the distance formula *Leaving answer in surd form

In Question 3, students will be assessed on how they organized their knowledge when finding the distance formula. Also it allows the teacher to establish how students understand concepts related to Cartesian coordinate system such as rectangular coordinate system, and methods of solving a right-angled triangle and algebraic processes such as solving quadratic equations. Thus the number of links of concepts tells the teacher the knowledge gaps inherent in students' connections between and among mathematical ideas, thereby using concept map as a teaching and learning strategy [5].

The concept map below provides a summary of the different concept employed in the process of finding the distance formula:



4. What are the assessment tools in mathematics?

In the process of teaching and learning, teachers facilitate, observe, and assess student learning. This can be achieved by making learning practical through meaningful activities, embracing collaborative learning, using quizzes to engage students in reflections, or asking students to summarize lesson taught. The assessment tools they may choose to apply to assess student learning may differ depending on the stage of learning. However, assessment only *of* learning (summative assessment) and not *for* learning (formative assessment) is not enough to promote students' integrated understanding. They may use concept maps (connections between and among mathematical ideas), concept tests, examinations, oral and poster presentations (use different representations of mathematical ideas to support and deepen mathematical understanding), peer and self-assessment (introduce the peer or self-marking of home/classwork in the classroom, and allow for discussion if there is a disagreement of an answer), portfolios, rubrics, or written reports. All these forms of assessment tools in mathematics allow for ways of assessment that motivate students to learn and thereby avoid damage to student self-esteem [7]. Besides, these different forms of assessment tools give helpful feedback to students in that they are guided on how to avoid making similar mistakes in the main examination. Furthermore, students are guided on how to improve their performance, and this impacts positively on student learning [7].

5. What makes a good mathematics classroom assessment?

A good classroom assessment plan gathers evidence of student learning that informs teachers' instructional decisions. It provides teachers with information about what students know and can do. Students should, at all times, have access to the assessment, so they can use it to inform and guide their learning. However, how can assessment be used to improve mathematics teaching?

Using classroom assessment to improve both teaching and student learning is not a new concept. But assessments designed for evaluating student performance through scrutiny of examination results (test scores) will not help improve instructional practices of teachers that enhance students' learning. Assessment for learning should be treated as an integral part of an instructional process and as an essential element in teachers' effort to help students learn. It is encouraged that before teaching students, teachers should employ baseline assessment to see what students already know. During the learning experience, teachers should employ formative assessment to address the misconceptions that may arise and after the learning experience; summative assessment should be employed for evaluating the effect of the instructional process on student knowledge. There is then a need for balanced packages of assessment tools, with all the elements of fair testing in it. A teacher can use concept maps, concept tests, examinations, oral and poster presentations, peer and self-assessment, portfolios, rubrics or written reports investigations, projects, class activity, and weekly or fortnightly concept tests as forms of assessment instead of using only tests as forms of assessments. However, to use classroom assessment to make improvements, a teacher should employ all forms of assessments, i.e., baseline, formative, diagnostic, and summative assessments. The choice of methods of scoring students in these different forms of assessments is guided by the purpose of assessment. Methods of scoring students should be such that they enable the student to demonstrate what they know rather than what they do not know. A teacher may elect to use a rubric because it enables him/her to score students on all their thinking processes and not only focus on one correct answer [6]. In that way teachers would be able to identify the misconceptions that students have and employ appropriate teaching strategies when addressing the misconceptions. Very often testing is meant to find out what the students do not know. This is a rather negative approach, and it does not give the students a chance to show what they do know [6]. One result may be that the student loses confidence. Assessment should support learning; it should not be a judgment. Therefore the techniques a teacher may use in a mathematics classroom to make students understand better help them become more independent and stimulate their critical thinking [8]. Therefore, what stands out to be a good mathematics classroom assessment for teachers is to change their instructional approaches (techniques) in three ways:

- Use *assessments* to establish and describe the students' misconceptions.
- Turn these misconceptions into teaching and learning opportunities.
- *Give students second chances* to demonstrate success.

5.1 Use assessments to establish and describe the students' misconceptions

Teachers' knowledge of students' misconceptions should go a long way in equipping them to prepare mathematics activities in their classroom. This will allow them to plan instruction targeting the student misunderstandings. I can conclude that formative assessment, like homework, can be used to locate mistakes and to figure out why they were made and how to provide support to students by way of explanation and tutoring [19]. This approach can help teachers learn some pedagogical lessons from exploring the content of students' procedural knowledge and understanding [9]. That is, when students make mistakes, they must be considered as opportunities for reconstruction of their knowledge.

5.2 Turn these misconceptions into teaching and learning opportunities

Assessment must be followed by corrective instruction designed to help students remedy whatever learning errors are identified with the assessment [10]. Using corrective instruction is *not* the same as reteaching, which often consists simply of restating the original explanations louder and more slowly [12]. Instead, the teacher must use strategies that accommodate differences in learning styles and intelligences [13]; for example, to teach circle geometry, I gave the students all the different circle theorems and then showed them several circle questions to identify the theorems within and find missing angles using these theorems. There was no success in this type of instruction as students did not remember the theorems; hence, they could not identify or apply them in questions. I decided to alter instruction by creating different circle handouts where students were directed to draw lines to create the theorems, measure the angles with groups, and infer circle theorems based on what they observed. This new instruction of circle geometry gave far better results as students were remembering most of the theorem since they discovered them on their own. However, students who had few or no learning errors to correct also participated in the enrichment or extension activities and that helped them to broaden and expand their learning.

5.3 Give students second chances to demonstrate success

Teachers should strive to help their students become lifelong students and to develop learning-to-learn skills [10]. What better learning-to-learn skill is there than learning from one's mistakes? Mistakes should not mark the end of learning; rather, they can be the beginning. As such, assessments must be part of an ongoing effort to help students learn. If teachers follow assessments with corrective instruction, then students should be provided a second chance to demonstrate their new level of competence and understanding [11]. This second chance determines the effectiveness of the corrective intervention while giving students another opportunity to experience success in learning, thus providing additional motivation [11].

6. Conclusion

Is the assessment for the student or the teacher? If you are not clear about why you are assessing (and what you are going to do with the data the assessment provides) you are wasting a lot of time, energy, and resources—your own and that of the students [14]. Always attention should be given to the broader meanings present in the data such that, if need be, student debriefing should be done to shed more light on the thinking behind their identified misconceptions. Therefore, I hold the view that teachers have to have a plan of what they are going to do with what they learn from the assessment (the data) before they give the assessment—ideally, before one even designs the assessment to begin with. An important implication of this view is that there is a need for teachers to understand the importance of prior knowledge to learning in order to facilitate learning. Students build on what they already know and have come to understand through formal and informal experiences. As such, students' knowledge structure (or connected understanding) should be reinforced in all learning incidents. Therefore, it is important to identify the processes and associated domain knowledge that students activate and bring to the solution context [16]. To continuously connect concepts in the learning of mathematics, teachers then need to incorporate concept mapping in the formative assessment process. The use of concept map helps students identify their concept knowledge gaps in a nonjudgmental setting and then develop practical means for attaining that knowledge [15]. Also the use of concept map helps students to improve their skills in negotiating meaning and challenging each other's explanations. On the other hand, concept map (as a formative assessment tool) provides teachers with a snapshot of students' concept knowledge gaps during the teaching and learning process. The spin-off from incorporating concept mapping in the formative assessment process is informative and reflective feedbacks tailored to students' personal abilities. This information helps teachers to plan instructional experiences aligned to students' traits.

Unless mathematics teachers provide a learning environment that promotes understanding through interaction, students might only commit unassimilated information to their short-term memory through rote learning, and no meaningful learning will occur. Therefore, the use of extensive formative assessment, vis-à-vis concept mapping, to drive instruction and implement a variety of strategies for the purpose of differentiating the instruction is of paramount importance.

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References

[1] Henson KT. Curriculum Planning: Integrating Multiculturalism, Constructivism, and Education Reform. Long Grove, IL: Waveland Press; 2015

[2] Gravett S. Adult Learning: Designing and Implementing Learning Events: A Dialogic Approach. Pretoria: Van Schaik; 2005

[3] Huhta A. Innovations in diagnostic assessment and feedback: An analysis of the usefulness of the DIALANG language assessment system [unpublished PhD thesis]. University of Jyväskylä; 2010

[4] Mutodi P, Chigonga B. Concept map as an assessment tool in secondary school mathematics: An analysis of teachers' perspectives. Eurasia Journal of Mathematics, Science and Technology Education. 2016;**12**(10):2685-2696

[5] Black P, Wiliam D. Inside the black box: Raising standards through classroom assessment. Phi Delta Kappan. 2010;**92**(1):81-90

[6] Verhage H, de Lange J. Mathematics education and assessment. Pythagoras. 1997;**42**:14-20

[7] Mundia L. The assessment of math learning difficulties in a primary grade-4 child with high support needs: Mixed methods approach. International Electronic Journal of Elementary Education. 2017;**4**(2):347-366

[8] Hornsby TC. Examining the influence of pre-kindergarten attendance, first-grade literacy instructional practices, and instructional time on literacy growth and children's special education status (doctoral dissertation, oklahoma state university); 2017.

[9] Teixeira-Dias JJ, Pedrosa de Jesus H, Neri de Souza F, Watts M. Teaching for quality learning in chemistry. International Journal of Science Education. 2005;**27**(9):1123-1137

[10] Guskey TR. How Classroom Assessments Improve Learning. Alexandra, VA, USA: ASCD; 2003

[11] Guskey TR. Using assessments to improve teaching and learning. Ahead of the Curve: The Power of Assessment to Transform Teaching and Learning. Bloomington, IN: Solution Tree Press; 2007

[12] Bellert A. Effective re-teaching. Australian Journal of Learning Difficulties. 2015;**20**(2):163-183

[13] Edwards CJ, Carr S, Siegel W. Influences of experiences and training on effective teaching practices to meet the needs of diverse learners in schools. Education. 2006;**126**(3):580-592

[14] Baumert J, Kunter M, Blum W, Brunner M, Voss T, Jordan A, et al. Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. American Educational Research Journal. 2010;**47**(1):133-180

[15] Poling LL, Goodson-Espy T, Dean C, Lynch-Davis K, Quickenton A. Mapping the way to content knowledge. Teaching Children Mathematics. 2015;**21**(9):538-547

[16] Blackà P. The nature and value of formative assessment for learning. Improving Schools. 2003;**6**(3):7-22

[17] Wiggins G. A true test: Toward more authentic and equitable assessment. Phi Delta Kappan. 2011;**92**(7):81-93

[18] Novak JD, Canas AJ. Theoretical origins of concept maps, how to construct them, and uses in education. Reflecting Education. 2007;**3**(1):29-42

[19] Heck RH, Hallinger P. Assessing the contribution of distributed leadership to school improvement and growth in math achievement. American Educational Research Journal. 2009;**46**(3):659-689

[20] Schuwirth LW, Van der Vleuten CP. Programmatic assessment: From assessment of learning to assessment for learning. Medical Teacher. 2011;**33**(6):478-485