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Chapter

Transition Modeling for Low to High Speed Boundary Layer Flows with CFD Applications

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Abstract

Transition modeling as applied to CFD methods has followed certain line of evolution starting from simple linear stability methods to almost or fully predictive methods such as LES and DNS. One pragmatic approach among these methods, such as the local correlation-based transition modeling approach, is gaining more popularity due to its straightforward incorporation into RANS solvers. Such models are based on blending the laminar and turbulent regions of the flow field by introducing intermittency equations into the turbulence equations. Menter et al. pioneered this approach by their two-equation \( \gamma \)-Re\( \theta \) intermittency equation model that was incorporated into the \( k-\omega \) SST turbulence model that results in a total of four equations. Later, a range of various three-equation models was developed for super-/hypersonic flow applications. However, striking the idea that the Re\( \theta \)-equation was rather redundant, Menter produced a novel one-equation intermittency transport \( \gamma \)-equation model. In this report, yet another recently introduced transition model called as the Bas-Cakmakcioglu (B-C) algebraic model is elaborated. In this model, an algebraic \( \gamma \)-function, rather than the intermittency transport \( \gamma \)-equation, is incorporated into the one-equation Spalart-Allmaras turbulence model. Using the present B-C model, a number of two-dimensional test cases and three-dimensional test cases were simulated with quite successful results.

Keywords: transitional flow, correlation-based transition model, intermittency transport equation, boundary layer flow, turbulence modeling

1. Introduction

Industrial design aerodynamics heavily depends on development of new CFD methods that can be only as good as their experimental database. All these industrial design CFD codes, as they may be called, are constantly in search of better physical modeling starting with appropriate transition and turbulence modeling. To this end, although numerical representation of turbulence has reached the acceptable levels of accuracy for computational aerodynamics, transition modeling has yet to reach the level of turbulence modeling capability for routine calculations. Therefore, transition modeling as part of turbulence has always been standing as the crux of the matter with regard to turbulence modeling. Today, state of the art
Reynolds Averaged Navier-Stokes (RANS) solvers are widely available for numerically predicting fully turbulent part of flow fields by frequent use of, for instance, one- or two-equation turbulence closure models. However, none of these models are adequate to handle flows with significant transition effects due to the lack of practical transition modeling. Menter et al. [1] state that some of the main requirements for pragmatic transition modeling are the following: calibrated prediction of the onset and length of transition, allow inclusion of different mechanisms, allow local formulation, and allow a robust integration with background turbulence models.

Nevertheless, transition modeling as applied to CFD methods has followed certain line of evolution covering a range of methods starting from simple linear stability methods such as the e^N method [2, 3] to almost or fully predictive methods such as LES and DNS that are very costly for engineering applications [1]. The e^N method is the lowest level transition model based on linear stability theory. This method has found quite wide application in numerical boundary layer methods [4], but translating this into RANS methods has proven quite demanding as it requires a high-resolution boundary layer code that must work hand in hand with the RANS method. Also, this method is also dependent on the empirical factor-n that is not universal and depends on the type of flow.

Following the e^N method, a better level of complexity that is compatible with the CFD methods is the low Reynolds number turbulence models [5]. Yet, they do not reflect real flow physics and lack the true predictive capability. These methods take advantage of the fortuitous ability of the wall damping terms mimicking some of the effects of transition. Next in the line of increasing complexity comes the class of the so-called correlation-based transition models [1]. These models are based on the fundamental approach of blending the laminar and the turbulent regions of the flow field by introducing intermittency equations to the turbulence equations. In this line, based on the boundary layer methods, there are three similar examples of intermittency equation approach that was introduced by Dhawan and Narasimha [6], Steelant and Dick [7], and Cho and Chung [8]. First, Dhawan and Narasimha [6] used a generalized form of intermittency distribution function in order to combine the laminar and the turbulent flow regions. Second, Steelant and Dick [7] proposed an intermittency equation that behaves like an experimental correlation. Third, Cho and Chung [8] introduced the k-ε-γ model which was formulated by an additional transport equation-γ to the well-known k-ε turbulence model. Finally, Suzen and Huang [9] significantly improved intermittency equation approach for flow transition prediction by combining the last two methods with a model that simulates transition in both streamwise and cross-stream directions. However, these models all rely on nonlocal flow data, and it was difficult to embed these models into practical CFD codes. These models require calculating the momentum thickness Reynolds number-Re_θ, which is an integral parameter, and comparing it with a critical momentum thickness Reynolds number. For this reason, these early models are “nonlocal” methods that require exhausting search algorithms for flows with complex geometries.

After the success of the “nonlocal” transition models that use intermittency transport equations including experimental correlations, a range of new methods [10, 11] has been developed, called as the local correlation-based transition models (LCTM) by Menter et al. [1] that are compatible with the modern CFD codes. This compatibility has been achieved by the experimental observation that a locally calculated parameter called as the vorticity Reynolds number (Re_v) is proportional to the momentum thickness Reynolds number (Re_θ) in a Blasius boundary layer. This observation is also shown to be quite effective for a wide class of flow types with moderate pressure gradients. This is due to the fact that the relative error between
the two parameters is less than 10% for such flows [1]. Therefore, the vorticity Reynolds number-Re would be used in order to avoid all the troublesome work that existed in the nonlocal models.

Following the success of the $\gamma$-Re$_{\theta}$ two-equation transition model of Menter et al. [1], some other two-, or three-equation models are proposed, such as the near/freestream intermittency model by Lodefier et al. [12], variations of the k-$\omega$ models of Walters and Leylek [13] and Walters and Cokljat [14], and the k-$\omega$-$\gamma$ model of Fu and Wang [15] with super/hypersonic flow applications. In addition, some researchers proposed extensions to local correlation-based transition models (LCTM) in order to take more physical phenomena into account.

To this end, cross-flow instability effects by Seyfert and Krumbein [16], surface roughness effects by Dassler et al. [17], and compressibility effects by Kaynak [18] were included. Meanwhile, Bas et al. [19] proposed a very pragmatic approach by introducing an algebraic or a zero-equation model called later as the Bas-Cakmakcioglu (B-C) model [20]. Herein, it was shown that an equivalent level of prediction compared with the two- and three-equation models could be achieved with less equations provided that physics was correctly modeled. In parallel, Kubacki et al. [21] proposed yet another algebraic transition model with a good level of success vindicating this line of approach. Similarly, Menter et al. [22] proposed a new one-equation $\gamma$-model which is the simplification of their earlier two-equation $\gamma$-Re$_{\theta}$ model [11] without the Re$_{\theta}$ equation that produced equal level of results as in the original model. Following this logical trend for reducing the total number of equations, the Wray-Agarwal (WA) wall-distance-free one-equation turbulence model [23] was complemented with the Menter et al. [22] one-equation intermittency transport-$\gamma$ model to obtain the so-called two-equation Nagapetyan-Agarwal WA-$\gamma$ transition model [24]. In the following, a brief review of the transition modeling is made that covers the practical applications of a range of models that are currently used in the industrial design aerodynamics. Based on the present authors’ recent experiences, the Bas-Cakmakcioglu model [20] will be covered in some detail to display the viability of the algebraic intermittency equation approach vis-a-vis the one- and two-equation local correlation-based transition models (LCTM).

2. Review of transition models

2.1 $e^N$ Method

The well-known $e^N$ method is based on the linear stability theory [25], and it is developed by assuming that the flow is two-dimensional and steady, the boundary layer is thin and the level of disturbances in the flow region is initially very low. In this method, the Orr-Sommerfeld eigenvalue equations are solved by using the previously obtained velocity profiles over a surface in order to calculate the local instability amplification rates of the most unstable waves for each profile. By taking the integral of those rates after a certain point where the flow first becomes unstable along each streamline, an amplification factor is calculated. Transition is said to occur when the value of the amplification factor exceeds a threshold N value. Typical values of N vary between 7 and 9.

2.2 Low Reynolds number turbulence models

In the low Reynolds number turbulence models, the wall damping functions are modified in order to capture the transition effects [5]. To be able to predict the
transition onset, these models depend on the diffusion of the turbulence from freestream into the boundary layer and its interaction with the source terms of the turbulence models. For this reason, these models are more suitable for bypass transition flows. Nonetheless, due to the similarities between a developing laminar boundary layer and a viscous sublayer, their success is thought to be coincidental, and thus these modes are mostly unreliable. These models also lack sensitivity to adverse pressure gradients and convergence problems arise for separation-induced transition cases.

2.3 Intermittency equation transition models

It has been known from experiment that turbulence has an intermittent character with large fluctuations in flow variables like velocity, pressure, etc. Based on this observation, transition to turbulence has been tried to be modeled using the so-called intermittency function. One-, two- or three-equation partial differential equations have been derived to include the intermittency equation as one of the equations of the complete equation set including relevant experimental calibrations that mimic the actual physical behavior. To this end, “nonlocal” [7–9] and “local” [1, 10, 11] correlation transition models have been proposed. In the following, a systematic line of progress is presented that reveals the evolution of such models.

2.3.1 Models depending on nonlocal flow variables

2.3.1.1 Dhawan and Narasimha model

Dhawan and Narasimha [6] proposed a scalar intermittency function-γ that would provide some sort of a measure of progression toward a fully turbulent boundary layer. Based on the experimentally measured streamwise intermittency distributions on flat plate boundary layers, for instance, Dhawan and Narasimha [6] introduced the following function for streamwise intermittency profile:

\[
\gamma = \begin{cases} 
0 & x < x_t \\
1.0 - \exp \left( \frac{(x - x_t)^2 \cdot n \sigma}{U} \right) & x \leq x_t 
\end{cases}
\]

In the above function, \(x_t\) is the known transition onset location, \(n\) is the turbulence spot formation rate per unit time per unit distance in the spanwise direction, \(\sigma\) is a turbulence spot propagation parameter, and \(U\) is the freestream velocity.

2.3.1.2 Cho and Chung model

Cho and Chung [8] developed the k-\(\varepsilon\)-\(\gamma\) turbulence model that is not designed for prediction of transitional flows but for free shear flows. In this model, the intermittency effect is incorporated into the conventional k-\(\varepsilon\) turbulence model with the addition of an intermittency transport equation for the intermittency factor \(\gamma\). In this model, the turbulent viscosity is defined in terms of \(k\), \(\varepsilon\), and \(\gamma\). The intermittency transport equation is given as:

\[
u_j \frac{\partial \gamma}{\partial x_j} = D_\gamma + S_\gamma
\]

where \(D_\gamma\) is the diffusion term and \(S_\gamma\) is the source term. This model is tested for a plane jet, a round jet, a plane far-wake, and a mixing layer case. As mentioned
before, although the model was not designed for transition prediction, the $\gamma$ intermittency profile for the turbulent-free shear layer flows was quite realistic.

2.3.1.3 Steelant and Dick model

Steelant and Dick [7] developed an intermittency transport model that can be used with the so-called conditioned Navier-Stokes equations. In this model, the intermittency function of Dhawan and Narasimha [6] is first differentiated along the stream-line direction, $s$, and the following intermittency transport equation is obtained:

$$\frac{\partial \gamma}{\partial t} + \frac{\partial \rho u \gamma}{\partial x} + \frac{\partial \rho v \gamma}{\partial y} = (1 - \gamma) \rho \sqrt{u^2 + v^2} \beta(s)$$ (3)

In the above equation, $\beta(s)$ is a turbulent spot formation and propagation term, which is seen in the exponential function part of the Dhawan and Narasimha model. Steelant and Dick tested their model for zero, adverse and favorable pressure gradient flows by using two sets of the so-called conditioned averaged Navier-Stokes equations. Although their model reproduces the intermittency distribution of Dhawan and Narasimha for the streamwise direction, a uniform intermittency distribution in the cross-stream direction is assumed. Yet, this is inconsistent with the experimental observations of, for instance, Klebanoff [26] where a variation of the intermittency in the normal direction by means of an error function formula.

2.3.1.4 Suzen and Huang model

Suzen and Huang [9] proposed an intermittency transport equation model by mixing the production terms of the Cho and Chung [8] and Steelant and Dick [7] models by means of a new blending function. An extra diffusion-related production term due to Cho and Chung is also added to the resultant equation. This model successfully reproduces experimentally observed streamwise intermittency profiles and demonstrates a realistic profile for the cross-stream direction in the transition region. This model is coupled with the Menter’s $k-\omega$ SST turbulence model [27] in which the intermittency factor calculated by the Suzen and Huang model is used to scale the eddy viscosity field computed by the turbulence model. This model is successfully tested against several flat plate and low-pressure turbine experiments. However, as mentioned before, this model is not a fully local formulation, and thus it cannot be implemented in straightforward fashion in the modern CFD codes.

2.3.2 Models depending on local flow variables

2.3.2.1 Langtry and Menter $\gamma$-Re$_\theta$ model

Langtry and Menter’s formulation of the two-equation $\gamma$-Re$_\theta$ model [11] is one of the most widely used transition models as far as general CFD applications in aeronautics are concerned. This model is formulated in such a way that allows calibrated prediction of transition onset and length that are valid for both the 2-D and 3-D flows. It uses the so-called local variables and thus applicable to any type of grids generated around complex geometries with robust convergence characteristics. As mentioned in the introduction part, this model is based on an important experimental observation that a locally calculated parameter called as the vorticity Reynolds number (Re$_\theta$) and the momentum thickness Reynolds number (Re$_\infty$) where

$$Re_\theta = \frac{Re_{\text{max}}}{2.193} \quad \text{and} \quad Re_\infty = \frac{\rho d_w^2}{\mu} \Omega$$ (4)
are proportional in a Blasius boundary layer. For most of the flow types, the relative error between the scaled vorticity Reynolds number and momentum thickness Reynolds number is reported [1] to be around 10%.

The model solves for two additional equations besides the underlying two-equation k-ω SST turbulence model, an intermittency equation ($\gamma$) that is used to trigger the turbulence production term of the k-ω SST turbulence model and a momentum thickness Reynolds number transport equation ($Re_\theta$) that includes experimental correlations that relates important flow parameters such as turbulence intensity, freestream velocity, pressure gradients etc. and supplies it to the intermittency equation. The details of the model are available in the literature [1, 11].

2.3.2.2 Walters and Cokljat k-k_L-ω model

Walters and Cokljat’s three-equation k-k_L-ω model [14] is proposed by the introduction of a transport equation for the laminar kinetic energy ($k_L$) into the conventional k-ω turbulence model and is used for natural and bypass transitional flows. This model is based on the understanding that the freestream turbulence is the cause of the high amplitude streamwise fluctuations in the pretransitional boundary layer, and these fluctuations are quite distinctive from the classic turbulence fluctuations. Also, growth of the laminar kinetic energy correlates with low frequency wall-normal fluctuations of the freestream turbulence. In this model, the total kinetic energy is assumed to be the sum of the large-scale energy which contributes to laminar kinetic energy and the small-scale energy which contributes to turbulence production. Thus, the transport equation for laminar kinetic energy ($k_L$) is solved in conjunction with the turbulent kinetic energy ($k_T$). Since the k-k_L-ω model uses a fully local formulation, it is suitable for the modern CFD codes and appears to be the first local model to specifically address pretransitional growth mechanism that is responsible for bypass transition [14].

2.3.2.3 Menter one-equation $\gamma$ model

Menter’s one-equation $\gamma$ transition model [22] is a simplified version of the two-equation $\gamma$-Re_θ transition model [10, 11]. In the new model, the Re_θ equation is avoided, and the experimental correlations for transition onset is embedded into the $\gamma$ equation in a simplified fashion. In effect, the simplified one-equation $\gamma$ model still possesses the same level of predictive capabilities as the original model. Menter et al. [22] summarize the advantages and the key changes to the model as follows: the new model is still fully local with new correlations valid for nearly all types of transition mechanisms, solves for one less equation, which is computationally cheaper; it is Galilean invariant; it has less coefficients that makes the model easier to fine-tune for specific application areas; and the new model would be coupled to any turbulence model that has viscous sublayer formulation. Menter et al. tested their model against most of the test cases which they previously used for the two-equation model. The results show that the new one-equation model is quite successful, and it would be a viable replacement for the original model.

2.3.2.4 Nagapetyan and Agarwal two-equation WA-$\gamma$ transition model

Following the trend for reducing the number of transition equations, a novel method was developed by integrating the recent Wray-Agarwal (WA) wall-distance-free one-equation turbulence model [23] based on the k-ω closure, with the one-equation intermittency transport $\gamma$-equation of Menter et al. [22] to construct the so-called two-equation Nagapetyan-Agarwal transition model WA-$\gamma$ [24]. An
important difference between the one-equation turbulence model derived earlier from k-ω models and the baseline turbulence model is the addition of a new cross diffusion term and a blending function between two destruction terms [23]. It was reported that the presence of destruction terms enables the Wray-Agarwal (WA) model to switch between a one-equation k-ω or one equation k-ε model. The new two-equation model was quite successfully validated for computing a number of two-dimensional benchmark experiments such as the transitional flows past flat plates in zero and slowly varying pressure gradients, flows past airfoils such as the S809, Aerospatiale-A, and NLR-7301 two-element airfoils.

2.3.2.5 Bas and Cakmakcioglu algebraic transition model

Bas and Cakmakcioglu (B-C) model [20] is an algebraic or zero-equation model that solves for an intermittency function rather than an intermittency transport (differential) equation. The main approach behind the B-C model follows the pragmatic idea of further reducing the total number of equations. Rather than deriving extra equations for intermittency convection and diffusion, already present convection and diffusion terms of the underlying turbulence model could be used. From a philosophical point of view, the transition, as such, is just a phase of a general turbulent flow. Addition of, in a sense, artificially manufactured transition equations appear to be rather redundant. Yet, for most of industrial flow types, the experimentally evidenced close relation between the scaled vorticity Reynolds number and the momentum thickness Reynolds number stood out as the primary reason for the success of so many intermittency transport equation models following the Langtry and Menter’s original two-equation γ-Reθ model [11].

In the application, the production term of the underlying turbulence model is damped until a considerable amount of turbulent viscosity is generated, and the damping effect of the transition model would be disabled after this point. The Spalart-Allmaras (S-A) turbulence model [28] is used as the baseline turbulence model, and rather than using an intermittency equation, just an intermittency function is proposed to control its production term. To this end, the B-C model is also a local correlation transition model that can be easily implemented for both 2-D and 3-D flows with reduced number of equations. For instance, for a 3-D problem, the B-C model solves for six equations (1 continuity + 3 momentum + 1 energy + 1 turbulence), whereas the two-equation γ-Reθ model solves for nine equations (1 continuity + 3 momentum + 1 energy + 2 turbulence + 2 transition). In addition, in the B-C model formulation, the freestream turbulence intensity parameter is only present in the critical momentum thickness Reynolds number function that makes the calibration of the model quite easy for different problems. The details of the B-C model formulation are presented in the following.

The S-A one-equation turbulence model is used as the underlying turbulence model for the B-C model. The S-A model solves for a transport equation for a new working variable νT, which is related to the eddy viscosity. The B-C model’s transition effects are included into the turbulence model is provided by multiplying the intermittency distribution function (γBC) with the production term of the S-A equation given as:

\[
\frac{\partial \nu_T}{\partial t} + \frac{\partial}{\partial x_j}(\nu_T u_j) = \gamma_{BC} C_{b1} S \nu_T - C_{w1} f_w \left( \frac{\nu_T}{d} \right)^2 + \frac{1}{\sigma} \left\{ \frac{\partial}{\partial x_j} \left[ \nu_L + \nu_T \frac{\partial \nu_T}{\partial x_j} \right] \right\} + C_{b2} \frac{\partial \nu_T}{\partial x_j} \frac{\partial \nu_T}{\partial x_j}
\]

The γBC function works in such a way that the turbulence production is damped (γBC = 0) until some transition onset criteria is fulfilled. After a point at which the onset criteria is ensured, the damping effect of the intermittency function γBC is checked,
and the remaining part of the flow is taken to be fully turbulent ($\gamma_{BC} = 1$). For this purpose, an exponential function of the form $1 - e^{-x}$ is proposed for the $\gamma_{BC}$ as follows:

$$\gamma_{BC} = 1 - \exp \left(-\sqrt{\text{Term}_1} - \sqrt{\text{Term}_2}\right)$$  \hspace{1cm} (6)

where Term$_1$ and Term$_2$ are defined as:

$$\text{Term}_1 = \frac{\max(\text{Re}_θ - \text{Re}_θ^c, 0.0)}{\chi_1 \text{Re}_θ^c}, \quad \text{Term}_2 = \frac{\max(\nu_{BC} - \chi_2, 0.0)}{\chi_2}$$  \hspace{1cm} (7)

and,

$$\text{Re}_θ = \frac{\text{Re}_θ}{2.193} \quad \text{and} \quad \text{Re}_v = \frac{\rho d^2}{\mu} \Omega, \quad \nu_{BC} = \frac{\nu_t}{U d_w}$$  \hspace{1cm} (8)

In the above, $\rho$ is the density, $\mu$ is the molecular viscosity, $d_w$ is the distance from the nearest wall, $\nu_{BC}$ is a proposed turbulent viscosity-like nondimensional term where $\nu_t$ is the turbulent viscosity, $U$ is the local velocity magnitude, $d_w$ is the distance from the nearest wall, and $\chi_1$ and $\chi_2$ are calibration constants. $\text{Re}_θ^c$ is defined as the critical momentum thickness Reynolds number, which is a correlation that is based on a range of transition experiments. In effect, Term$_1$ checks for the transition onset point by comparing the locally calculated $\text{Re}_θ$ with the experimentally obtained critical momentum thickness Reynolds number $\text{Re}_θ^c$. As soon as the vorticity Reynolds number $\text{Re}_v$ exceeds a critical value, Term$_1$ becomes greater than zero and the intermittency function $\gamma_{BC}$ begins to increase. However, the vorticity Reynolds number $\text{Re}_v$ relation above is a function of the square of the wall distance $d_w$; therefore, it takes a very low value inside the boundary layer where the wall distance is quite low. Because of this, Term$_1$ alone is not enough for intermittency generation inside the boundary layer. To remedy this, Term$_2$ is introduced. Inspecting the Term$_2$ equation with the $\nu_{BC}$ relation shows that the regions close to wall is inversely related and the damping effect of the transition model would be disabled inside the boundary layer. In effect, Term$_2$ checks for the viscosity levels inside the boundary layer, and the turbulence production is activated wherever $\nu_{BC}$ exceeds a critical value $\chi_2$. In order to determine the calibration constants’ $\chi_1$ and $\chi_2$ values, the well-known zero pressure gradient flat plate test case of Schubauer and Klebanoff [29] is used. This test case represents a natural transition process due to the wind tunnel used in the experiment generates a freestream $Tu$ around 0.2%. The model calibration is done by numerical experimentation; setting $\chi_1$ and $\chi_2$ such that the transition occurs at the same location as in the experiment. As a result, the $\chi_1$ and $\chi_2$ values are set to be 0.002 and 5.0, respectively.

Any experimental $\text{Re}_θ^c$ correlation could be used in the model. However, it should be noted that, since the S-A turbulence model does not solve for the local turbulent kinetic energy, local turbulence intensity values cannot be calculated. Due to this reason, the turbulence intensity $Tu$ is assumed, for now, to be constant in the entire flow domain as Suluksna et al. [30] and Medida [31] have also suggested. For this lack of ability for calculating the local $Tu$ values, the B-C model has some deficiency in this respect that it cannot handle some physical effects compared with the models that can dynamically calculate the local $Tu$ levels. Whereas this deficiency makes the B-C model rather limited, there are quite a few aerodynamic flows for which the model is still viable. The transition onset correlation that was also used in the original two-equation $\gamma$-$\text{Re}_θ$ model [1] is given by:

$$\text{Re}_θ = 803.73 \left(Tu_w + 0.6067\right)^{-1.027}$$  \hspace{1cm} (9)
As mentioned before, any transition onset correlation would be incorporated into the B-C model. For instance, a class of potential transition onset correlations along with the one preferred in the present B-C model is shown in Figure 1.

Currently, the B-C model is available in the SU2 (Stanford University Unstructured) v6.0, an open-source CFD solver by the ADL of Stanford University [32]. The SU2 can solve two- and three-dimensional incompressible/compressible Euler/RANS equations using linear system solver methods.

3. Two- and three-dimensional test cases for low to high speeds

Some outstanding test cases that make a good platform for measuring novel transition model performances are simulated by the foregoing transition models. These cases cover a wide range of flows from low speed two-dimensional flat plate and airfoil test cases to three-dimensional wind turbine blade and aircraft wing test cases from low to high speeds.

3.1 Low speed flat plate test cases

Well-known benchmark experiments such as the Schubauer and Klebanoff natural transition flat plate experiment [29] and the ERCOFTAC T3 series flat plate experiments by Savill [33] are used. The T3 series flat plate experiments consist of three zero pressure flat plate cases (T3A, T3B, and T3A-) and five variable pressure flat plate cases (T3C1, T3C2, T3C3, T3C4, and T3C5), in which the pressure gradients are generated using an adjustable upper tunnel wall. In all ERCOFTAC T3 test cases, the free stream turbulence intensities vary between 0.1 and 6%. Table 1 summarizes the upstream conditions of the Schubauer and Klebanoff and the ERCOFTAC T3 flat plate experiments.

Figure 2 shows the numerical and experimental skin friction coefficients of the zero pressure gradient test cases of S&K, T3A, T3B and T3A-, respectively. The figures include numerical predictions of several researchers, including for instance Suzen and Huang [9], Langtry and Menter [11], Walters and Cokljat [14], Menter et al. [22], Nagapetyan and Agarwal [24], and Medida [31]. In the S&K calibration case, the B-C model displays a good agreement with the experiment for the transition onset point similar to other methods. For the T3A and T3B cases, the B-C model shows rather late transition onset, whereas the other models predict some early or late onset points. Specifically, Nagapetyan and Agarwal [24] show a very good agreement with the experiment as to the transition onset and rapid skin-friction
rise characteristic. Finally, for the T3A- case, the B-C [20], Menter et al. [22], Walters and Cokljat [14], and Nagapetyan and Agarwal [24] display early transition onset points with rather rapid rise in skin-friction, whereas two-equation Langtry and Menter [11] and Medida [31] models show quite good onset point and a gradual rise in the skin friction.

**Figure 3** depicts numerical and experimental skin friction coefficients for the T3C series variable pressure flat plate test cases. The T3C series flat plate test cases represent actual turbine characteristics by changing the pressure gradient by changing the upper wall profile of the wind tunnel over the flat plate. For the T3C1 case,
which represents the highest turbulence intensity test case among the T3C series test cases, the B-C model results are quite in agreement with the experimental data as the transition onset location is predicted with decent accuracy. For the T3C2 case, it is observed that although the B-C model predicted a good transition onset point, the turbulent stress abruptly rises after the onset. All other models predicted the transition onset location rather late in general.

For the T3C3 case, it is observed that the $\gamma$-Re$_{\theta}$ model [11], k-k$_{\omega}$ model [14], and WA-$\gamma$ model [24] outperform the other models as the B-C model prediction shows an early transition onset, whereas the one-equation $\gamma$ model [22] predicts a rather late transition onset. For the T3C4 case, which represents the lowest Reynolds number case, all the models except for the B-C and WA-$\gamma$ models show flow separation as their skin friction coefficients are below zero. Here, the B-C model obtained a quite good transition onset point that agreed with the experimental data although

Figure 3. Comparison of skin friction coefficients for the variable pressure gradient flat plate test cases.
the laminar region was rather inaccurate. Finally, for the T3C5 case, solution of the zero-equation B-C model [20], Menter et al. one-equation $\gamma$ model [22], and WA-$\gamma$ model [24] well agree with the experiment in the laminar region, the onset of transition is also fairly good with some delay, and again quite good agreement in the subsequent variable pressure gradient region is obtained.

3.2 Airfoil and turbomachinery test cases

3.2.1 S809 airfoil

The S809 airfoil is a 21% thick profile, which specifically designed for horizontal-axis wind turbine applications. The S809 airfoil was tested in a low-turbulence wind tunnel ($Tu = 0.2\%$) by Somers [34] at Re number of 2 million (based on chord length) and a Mach number of 0.15. Comparison of the numerical results by Langtry and Menter [11] $\gamma$-$Re_\theta$, Walters and Cokljat [14] $k$-$k$, $\omega$, and Medida [31] SA-$\gamma$-$Re_\theta$ and B-C models [20] with the experimental data is given in Figures 4–6. In general, all transition models agree well with the experimental data until the stall angle. Although the lift and drag coefficients (Figure 4) are rather inaccurate after the stall angle, it is observed that the experimental measurements of the transition locations are quite successfully predicted by all models (Figure 5). Also, comparing the experimental and numerical pressure coefficient distributions on the S809 airfoil at 1° angle of attack, it is observed that the separation bubble is predicted quite well by all the models (Figure 6).

3.2.2 T106 turbine cascade

T106 turbine cascade experiment was designed to investigate the interaction of a convected wake and a separation bubble on the suction surface of a highly loaded low-pressure turbine blade. In these experiments by Stieger et al. [35], five-blade cascade of T106 profile was placed downstream of a moving bar wake generator in order to simulate an unsteady wake passing environment of a turbomachine. In the experiment, the flow conditions correspond to a Reynolds number of nearly 91,000 based on the chord length of the T106 profile and the inlet velocity. The
experimental turbulence intensity is specified to be 0.1%. Geometric details of the experimental cascade setup are given in Table 2. Comparison of the experimental and numerical pressure coefficient distributions for T106 cascade for the steady case is depicted in Figure 7. Looking at Figure 7, it is observed that the separation bubble on the blade predicted by the B-C model and the two-equation $\gamma$-$Re_\theta$ model is slightly smaller in size than the experimentally measured bubble.

3.3 3-D wing test cases from low to high speeds

3.3.1 Low speed rotating wind turbine blade

Two twisted and tapered 10-meter diameter turbine blades that use the S809 airfoil profile are tested in the NASA Ames Research Center wind tunnels [36, 37]. In the experiments, the NREL wind turbine rotation speed was set to 72 RPM for all cases, whereas the wind speeds varied from 7 to 25 m/s. Figure 8 compares the pressure coefficient distributions over various spanwise locations on the turbine blades at the freestream velocity of 7 m/s. It is observed
that both fully turbulent and the transitional solutions differ very slightly and both agree well with the experimental data. The skin friction contours and the surface streamlines obtained by Medida [31], Potsdam et al. [38], and Aranake et al. [39] for the same freestream velocity are compared to the B-C model and the S-A model solutions in Figure 9.

3.3.2 High subsonic flow over 3-D swept wing

DLR-F5 wing tested by Sobieczky [40] is a 0.65 m span wing with 20° sweep angle and an average chord length of 150 mm. The wing is mounted to the tunnel wall with a smooth blending region, and the angle of attack is set to be 2°. The square cross-section wind tunnel has dimensions of 1 × 1 × 4 meters. The experimental inlet Mach number and the turbulence intensity are specified as $M = 0.82$ and $Tu <0.35\%$, respectively. The corresponding $Re$ number based on the average chord is 1.5 million. In the experiment, the transition locations are determined by the sublimation technique, whereas measurements of pressure coefficients at different spanwise stations are available. In 1987, a workshop with several researchers were took place in Gottingen [41], where the results were compared against the experimental data.

Figure 10 shows the pressure coefficient distributions at different span locations. It is observed that the fully turbulent and the transitional solutions are very
Figure 8. Comparison of pressure coefficient distributions for the NREL phase IV blade for U = 7 m/s freestream velocity.

Figure 9. Comparison of numerical skin friction contours obtained by several researchers.
similar to each other. **Figure 11** compares the skin friction contours of different numerical models with the experiment [40]. As seen, the B-C model predicts a somewhat similar transition and separation region with the experiment obtained by the sublimation and pressure measurement techniques.

Finally, in order to emphasize the difference between the fully turbulent and the transitional solutions, comparison of the skin friction coefficients at 80% span on

**Figure 11.** Skin-friction coefficient comparisons for the DLR-F5 wing.
the DLR-F5 wing is depicted in Figure 12. It can be clearly observed that the B-C model predicts marked extent of laminar regions for both the suction and pressure sides of the wing, which is in agreement with the contours shown in Figure 11.

4. Conclusions

Local correlation-based transition models in the sense of empirical correlations incorporated into Reynolds-averaged Navier-Stokes methods have been discussed. A logical path for the development of such models is highlighted such that a variety of combinations of turbulence and transition equations lead to different modeling alternatives. For instance, the pioneering work by Menter et al. [1] two-equation \( \gamma - \text{Re}_\theta \) transition model sums up to a total of four-equation model by the incorporation of the two-equation \( k-\omega \) SST turbulence model of Menter et al. [27]. In the same line of development but in a leaner approach, Walters and Cokljat [14] developed a three-equation \( k-k_\text{L}-\omega \) model. Similarly, Medida [31] developed a three-equation S-A-\( \gamma - \text{Re}_\theta \) transition model that is a sum of the Menter et al. [1] two-equation \( \gamma - \text{Re}_\theta \) transition model and the one-equation S-A turbulence model [28].

In fact, in a recent work, Menter [22] reached to the conclusion that the \( \text{Re}_\theta \) equation was rather redundant. Without any loss of accuracy, Menter produced a leaner three-equation \( k-\omega \) SST-\( \gamma \) transition model by incorporating a novel one-equation intermittency transport \( \gamma \)-model [22] with the two-equation \( k-\omega \) SST turbulence model of Menter et al. [27]. In the same line of thought, Nagapetyan-Agarwal constructed the so-called two-equation transition model of WA-\( \gamma \) [24] by incorporating the Wray-Agarwal (WA) wall-distance-free one-equation turbulence model [23] based on the \( k-\omega \) closure with the one-equation intermittency transport \( \gamma \)-equation of Menter et al. [22]. These two models paved the way for developing yet another leaner transition model by Bas et al. [19] with the introduction of the algebraic Bas-Cakmakcioglu (B-C) model by incorporating an algebraic \( \gamma \)-function with the one-equation S-A turbulence model [28].

The Bas-Cakmakcioglu (B-C) [19] model qualifies as a zero-equation model that solves for an intermittency function rather than an intermittency transport (differential) equation. The main approach behind the B-C model follows again the pragmatic idea of further reducing the total number of equations. Thus, rather than deriving extra equations for intermittency convection and diffusion, already
present convection and diffusion terms of the underlying turbulence model could have been used. From a philosophical point of view, the transition, as such, is just a phase of a general turbulent flow. In a sense, addition of artificially manufactured transition equations may appear to be rather redundant. Yet, for most of industrial flow types, there is experimental evidence that a close relation between the scaled vorticity Reynolds number and the momentum thickness Reynolds number exists. This fact stands out as the primary reason for the success of the class of so many intermittency transport equation models following the Menter's pioneering two-equation $\gamma$-$\text{Re}_\theta$ model [1]. Using the present B-C model, a number of two-dimensional test cases including flat plates, airfoils, turbomachinery blades, and three-dimensional low speed wind turbine and high-speed transport plane wing were simulated with quite successful results. These results may be regarded to vindicate this leaner approach of using even lesser equations for industrial design aerodynamics problems.

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