

We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

4,900

Open access books available

124,000

International authors and editors

140M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com



Energy Transfer and Dissipation in non-Newtonian Flows in non-Circular Tubes

Mario F. Letelier

Additional information is available at the end of the chapter

<http://dx.doi.org/10.5772/59907>

1. Introduction

In general, physiological processes are basically isothermal and isobaric. Several mechanisms contribute to keep constant pressure in systems such as blood vessels, among them diameter distribution and peristaltic motion. In this, it is to be understood that pressure constancy refers to time-averaged or extreme values.

Many pathologies make the human body develop fever, which raises temporarily body temperature. It is common practice to use, in some cases, medicines that restore, albeit for a given period of time, normal temperature. Fever is not the only cause of body heat-transfer. This process also occurs with body adjustment to changes in ambient temperature, physical exertion, metabolism acceleration due to food ingestion, drug effects, and others. During these body changes, blood and other fluid vessels undergo heat-transfer processes that pose considerable difficulties to physical modeling. Some pertinent variables are fluid composition, vessel elasticity, peristaltic motion in some cases, unsteadiness, and complex vessel geometry.

Heat-transfer in tube-flow has been investigated for several decades. One main practical driving force for such studies is the need to understand and design efficient heat-transfer equipment, widely used in several industrial areas. Models of simple tube-flow heat-transfer problems are now textbook's standard contents.

These include laminar Newtonian steady flow in round tubes with simple boundary conditions. More recently, as it is reviewed in the next section, Newtonian and non-Newtonian flows in tubes of geometry other than circular have been modeled and analyzed. One important finding as to this chapter's objective is the effect of the interplay between tube geometry and non-linear viscoelasticity.

Such coupling leads to the development of secondary flows, or helicoidal flows, that increase the transversal transport capacity of the flow, a phenomenon that can be applied, or related, to heat-transfer enhancement, or to other transversal transport processes, such as cross-sectional transport of particles immersed in the fluid.

Especially in blood flow, cells and other components introduce viscoelasticity [1, 2] that becomes relevant in smaller blood vessels. The understanding of its effect on the flow characteristics may become very relevant in cases such as heart arteries, and when artificial implants affect the blood flow. Also blood plastic effects appear in smaller vessels due to the aggregation of red blood cells at low shear rates, which develop a yield stress to be overcome for the flow to ensue.

Drawing on the above results, in this chapter it is presented a summary of relatively new analytical findings that may be useful for the better understanding of heat-transfer and complex flow phenomena in vessels that share some characteristics with biological vessels, particularly when these work under abnormal conditions. Also it is analyzed the effect of geometry in energy dissipation. This chapter is closely related to [3].

2. Mathematical models

Non-linear viscoelasticity and fluid plasticity are related in the following to heat-transfer and energy dissipation processes. Next are presented the mathematical models to be considered together with some remarks as to the corresponding state-of-the-art, and relevant developments with related references.

2.1. Viscoelastic flow

The physical model considered is a straight tube of arbitrary cross-sectional shape, in which a non-Newtonian fluid moves along the axial coordinate z impelled by a pressure gradient, which can be a function of $t =$ time. Secondary flows are induced when the necessary conditions operate. The flow is assumed laminar, incompressible and with constant properties. Considering Cartesian coordinates, the velocity field can be expressed as (u, v, w) in which the velocity components align with the (x, y, z) axes respectively. The temperature and velocity fields are, in general, dependent on x, y, z and t .

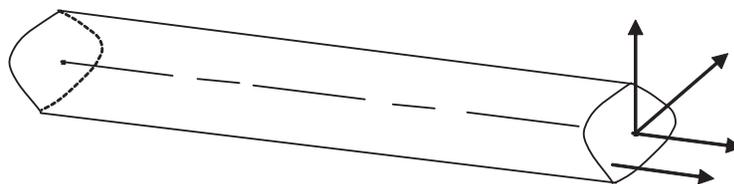


Figure 1. Definition diagram

Two well-known problems are here relevant, i.e.,

1. Flow with constant wall heat-flux, in which the wall temperature is assumed constant at any cross-section, with prescribed axial variation.
2. The “Graetz problem”, in which the flow is assumed isothermal up to a point $z=0$, from which on a different constant temperature is applied at the wall, so that the fluid is progressively heated or cooled.

Experimental findings concerning heat-transfer characteristics of aqueous polymer solutions flowing in straight tubes point at considerable enhancement as compared to its Newtonian counterpart driven by the same conditions and in the same geometry. Specifically it is reported that heat-transfer results for viscoelastic aqueous polymer solutions are considerably higher in flows fully developed both hydrodynamically and thermally, as much as by an order of magnitude depending primarily on the constitutive elasticity of the fluid and to some extent on the boundary conditions, than those found for water in laminar flow in rectangular ducts [4, 5]. Heat-transfer phenomena in laminar flow of non-linear fluids has not been the subject of many investigations with the exception of round pipes, and the case of inelastic shear-thinning fluids in tubes of rectangular cross-section, in spite of the widespread use of some specific contours in industry such as flattened elliptical tubes.

This statement is true for all cross-sectional shapes for both steady and unsteady phenomena including quasi-periodic flows. Heat-transfer with viscoelastic fluids has been declared to be a new challenge in heat-transfer research in the early nineties [6], but progress has been limited since that time. The physics of the phenomenon has not been entirely clarified.

Highly enhanced heat-transfer to aqueous solutions of polyacrylamide and polyethylene of the order of 40–45% as compared to the case of pure water in flattened copper tubes was observed by Oliver [7] and later by Oliver and co-workers as early as 1969. Recent numerical investigations in rectangular cross-sections of Gao and Hartnett [8, 9], Naccache and Souza Mendes [10], Payvar [11] and Syrjala [12] establish the connection between the enhanced heat-transfer observed and the secondary flows induced by viscoelastic effects. The former researchers as well as Naccache and Souza Mendes predict for instance viscoelastic Nusselt numbers as high as three times their Newtonian counterparts. Gao and Hartnett [8, 9].report numerical results in rectangular contours which provide evidence that the stronger the secondary flow (as represented by the dimensionless second normal stress coefficient Ψ_2) the higher the value of the heat-transfer (as represented by the Nusselt number Nu) regardless the combination of thermal boundary conditions on the four walls. Constant heat flux is imposed everywhere on the heated walls in their numerical experiments with the remaining walls being adiabatic. The combination of boundary conditions plays some role in the enhancement reported with the largest enhancement occurring when two opposing walls are heated. Despite these efforts heat-transfer characteristics of viscoelastic fluids in steady laminar flow in rectangular tubes remains very much an open question (quoted from Siginer and Letelier [13]).

Coelho et al [14] presented an analytical solution for the Graetz problem for the MPTT fluid for a circular tube, steady flow, including several computations for negative heat flux (flow cooling). Valko [15] published a solution of the Graetz problem using a power-law fluid model;

he determined the influence of the Brinkman number for several flow conditions in circular tubes. Kin and Özisik [17] published work on transient laminar forced convection of a power-law fluid in ducts with sudden change in wall temperature. In these and related references it is reflected the actual state-of-the-art in this subject.

Concerning numerical analysis, there is only one available commercial package for viscoelastic fluid flow computations, POLYFLOW. However POLYFLOW cannot handle even relatively high Weissenberg number flows and although convergent, gives erroneous results of the order of 400% as compared to analytical test cases, Filali et al. (2012). In addition it cannot handle heat-transfer in steady viscoelastic fluid flow in tubes. Thus to study problems of this type a numerical algorithm has to be built from scratch and tested for stability (Hadamard-type). Numerical analysis is not considered in this chapter.

Two well-known models of non-linear viscoelastic fluids are next described:

Modified Phan-Thien-Tanner (MPTT)

$$2\eta_m \mathbf{D} = \left(1 + \frac{\alpha\lambda}{\eta_{m0}} \text{tr } \boldsymbol{\tau}\right) \boldsymbol{\tau} + \lambda (\mathbf{V} \cdot \nabla \boldsymbol{\tau} - \boldsymbol{\varphi} \boldsymbol{\tau} - \boldsymbol{\tau} \boldsymbol{\varphi}^T) \quad (1)$$

Giesekus

$$\begin{aligned} \boldsymbol{\tau} + \lambda \check{\boldsymbol{\tau}} + \frac{\alpha\lambda}{\eta_0} \boldsymbol{\tau} \cdot \boldsymbol{\tau} &= \eta_0 \mathbf{D} \\ \check{\boldsymbol{\tau}} &= -((\nabla \cdot \mathbf{V})^T \cdot \boldsymbol{\tau} + \boldsymbol{\tau} \cdot \nabla \mathbf{V}) \end{aligned} \quad (2)$$

In the following, the MPTT model is applied to the flow field analysis. For the purposes of this presentation, there is some evidence [17] that both models lead to qualitatively similar results.

The applicable equation of motion for 3-D, steady and incompressible flow are, in cylindrical coordinates:

Continuity

$$\nabla \cdot \mathbf{V} = 0 \quad (3)$$

Momentum

$$\mathbf{V} \cdot \nabla \mathbf{V} = \nabla \cdot \boldsymbol{\sigma} \quad (4)$$

or, in expanded form

$$u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} = \frac{1}{r} \left[\frac{\partial}{\partial r} (r \sigma_{rr}) + \frac{\partial}{\partial \theta} (\sigma_{\theta r}) + \frac{\partial}{\partial z} (r \sigma_{zr}) \right] - \frac{\sigma_{\theta\theta}}{r} \quad (5)$$

$$u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \theta} + w \frac{\partial v}{\partial z} - \frac{uv}{r} = \frac{1}{r} \left[\frac{\partial}{\partial r} (r\sigma_{r\theta}) + \frac{\partial}{\partial \theta} (\sigma_{\theta\theta}) + \frac{\partial}{\partial z} (r\sigma_{z\theta}) \right] - \frac{\sigma_{r\theta}}{r} \quad (6)$$

$$u \frac{\partial w}{\partial r} + \frac{v}{r} \frac{\partial w}{\partial \theta} + w \frac{\partial w}{\partial z} = \frac{1}{r} \left[\frac{\partial}{\partial r} (r\sigma_{rz}) + \frac{\partial}{\partial \theta} (\sigma_{\theta z}) + \frac{\partial}{\partial z} (r\sigma_{zz}) \right] \quad (7)$$

In the above, u , v , w are the radial, tangential and axial velocity components, σ is the stress matrix and P is the piezometric pressure. Scale factors applied are a (base radius) for r , w_0 for the velocity components, and $\eta_N w_0/a$ for the stress components, in which η_N is the Newtonian viscosity.

The MPTT model of viscoelastic fluid is next expressed, in dimensional variables, through the following equations, i.e. [18].

$$\sigma = -PI + 2\eta_N D + \tau \quad (8)$$

$$2\eta_m D = f(\varepsilon_0, tr\tau)\tau + \lambda \tau^\nabla \quad (9)$$

$$\tau^\nabla = \frac{\partial \tau}{\partial t} + V \cdot \nabla \tau - (\nabla V^T - \xi D)\tau - \tau(\nabla V^T - \xi D)^T \quad (10)$$

where D is the rate of deformation tensor and τ is the non-Newtonian component of the shear stress.

Defining

$$\varphi = \nabla \cdot \nabla V^T - \xi D \quad (11)$$

then, a more compact form of (10), is

$$\tau^\nabla = V \cdot \nabla \tau - \varphi \tau - \tau \varphi^T \quad (12)$$

In this η_m is a viscosity, ε_0 , ξ are material parameters, and λ is the relaxation time.

The function f , as defined in the MPTT model can be simplified for small values of ε_0 , yielding

$$f(\varepsilon_0, tr\tau) = \exp\left(\frac{\varepsilon_0 \lambda}{\eta_{m0}} tr\tau\right) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{\varepsilon_0 \lambda}{\eta_{m0}} tr\tau\right)^n = 1 + \frac{\varepsilon_0 \lambda}{\eta_{m0}} tr\tau + O(\varepsilon_0^2) \quad (13)$$

where from

$$2\eta_m D = \left(1 + \frac{\varepsilon_0 \lambda}{\eta_{m0}} tr\tau\right)\tau + \lambda(V \cdot \nabla \tau - \varphi \tau - \tau \varphi^T) \quad (14)$$

For this constitutive equation the viscosity is defined as

$$\eta_m = \eta_{m0} \frac{1 + \xi(2 - \xi)\lambda^2 \kappa^2}{(1 + \lambda^2 \kappa^2)^{(1-m)/2}} \quad (15)$$

in which

$$\kappa = \sqrt{2trD^2} \quad (16)$$

and where η_{m0} and m are additional material parameters. If $m=1$, then (15) reduces to

$$\eta_m = \eta_{m0} [1 + \lambda^2 \xi(2 - \xi) \kappa^2] \quad (17)$$

Coming back to dimensionless variables and parameters (14) becomes

$$2(1 + 2\xi(2 - \xi)Wi^2 trD^2)D = (1 + \varepsilon_0 Wi tr\tau)\tau + Wi \tau^\nabla \quad (18)$$

in which Wi is the Weissenberg number, or dimensionless relaxation time, defined as

$$Wi = \frac{w_0 \lambda}{a} \quad (19)$$

By combining equations (18), (8) and (5-7), it is obtained the set of working equations in full, i.e.

$$u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} = F_r - \frac{\partial P}{\partial r} + \nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2} \frac{\partial v}{\partial \theta} \quad (20)$$

$$u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \theta} + w \frac{\partial v}{\partial z} - \frac{uv}{r} = F_\theta - \frac{1}{r} \frac{\partial P}{\partial \theta} + \nabla^2 v - \frac{v}{r^2} - \frac{2}{r^2} \frac{\partial u}{\partial \theta} \quad (21)$$

$$u \frac{\partial w}{\partial r} + \frac{v}{r} \frac{\partial w}{\partial \theta} + w \frac{\partial w}{\partial z} = F_z - \frac{\partial P}{\partial z} + \nabla^2 w \quad (22)$$

in which the functions F_r , F_θ and F_z are the viscoelastic forcing functions that determine the transversal flow. For developed flow, all derivatives with respect to z , excepted for the pressure, must be put equal to zero.

The explicit expressions for the extra forcing terms are:

$$F_r = (\nabla \cdot \tau^\nabla)_r = \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rr}^\nabla) + \frac{1}{r} \frac{\partial \tau_{\theta r}^\nabla}{\partial \theta} - \frac{\tau_{\theta\theta}^\nabla}{r} \quad (23)$$

$$F_{\theta} = (\nabla \cdot \boldsymbol{\tau})_{\theta} = \frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{2}{r} \tau_{r\theta} \nabla_{r\theta} \quad (24)$$

$$F_z = (\nabla \cdot \boldsymbol{\tau})_z = \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rz}) + \frac{1}{r} \frac{\partial \tau_{\theta z}}{\partial \theta} \quad (25)$$

Energy

For steady flow of an incompressible fluid with constant properties, the energy equation, in terms of the temperature T can be written as

$$\rho_0 C_p \frac{DT}{Dt} = \Phi + \nabla \cdot (k \nabla T) \quad (26)$$

Where ρ_0 is the density, C_p is the specific heat at constant pressure, ϕ is the dissipation function, and k is the thermal conductivity coefficient. Further, it is assumed negligible dissipation and constant k . Under these assumptions (26) becomes

$$\rho C_p \left(u \frac{\partial T}{\partial r} + \frac{v}{r} \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad (27)$$

2.2. Viscoplastic flow

Viscoplastic fluids are fluids that exhibit yield stress, which must be overcome before the material develops deformation. As already mentioned plasticity appears in small blood vessels due to red cell aggregation.

Also, many industrial fluids exhibit yield stress, and are found in areas such as mining (slurries), food (pastes), construction (concrete and mud), cosmetics, etc. Flow and heat-transfer description in tubes and other configurations is compounded by their geometry, which brings in non-linear constitutive equations, except for very simple shapes. Some recent research in this field include flow around a cylinder by Tokpavi et al. [19] particle sedimentation, Yu and Wachs [20], flow in an eccentric annular tube, Wachs [21], and Walton and Bittleston [22], a general analysis for flow in non-circular ducts, Letelier and Siginer [23] and a preliminary analysis of the velocity field in non-circular pipes, Letelier, et al. [24]. Other pertinent references can be found in the previous ones.

The constitutive characteristics of viscoplastic flow determine complex structures of velocity and shear fields in tube flow when the tube cross-sectional contour differs from circular. This is true even for the case of the Bingham model of viscoplastic fluid, which is one of the simplest mathematical expressions for this kind of fluids.

Plasticity implies existence of fluid yield stress, which may induce both plug zones and stagnant zones within the tube cross-section, when in there the tube geometry determines zones of shear stress below the value of the yield stress. Such solid regions of the flow make it necessary to apply greater pressure gradients in order to keep a given rate of flow which, in turn, leads to greater energy dissipation and heating inside the fluid.

In the following Bingham's model of fluid is used. Simple plastic flows in straight non-circular tubes do not develop secondary flows. The flow is, therefore, parallel when the motion is laminar, and only the axial component of the velocity exists. Under these conditions, the momentum equation, in terms of shear stress components, is

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} + \frac{1}{r} \frac{\partial \tau_{\theta z}}{\partial \theta} = - \frac{\partial P}{\partial z} \quad (28)$$

The constitutive relations are

$$\tau_{rz} = - \left(1 + \frac{N}{I} \right) \frac{\partial w}{\partial r} \quad (29)$$

$$\tau_{\theta z} = - \left(1 + \frac{N}{I} \right) \frac{1}{r} \frac{\partial w}{\partial \theta} \quad (30)$$

in which

$$I = \left[\left(\frac{\partial w}{\partial r} \right)^2 + \left(\frac{1}{r} \frac{\partial w}{\partial \theta} \right)^2 \right]^{\frac{1}{2}} \quad (31)$$

is an invariant related to the rate of deformation matrix. Also

$$N = \frac{a}{\eta w_0} \tau_y \quad (32)$$

where τ_y is the dimensional yield stress. The parameter N is a dimensionless yield stress that greatly influence the flow characteristics.

The momentum and constitutive equations can be written in more compact form by using natural coordinates, i.e.

$$\frac{d \tau_{nz}}{dn} + \frac{\tau_{nz}}{\rho} = - \frac{\partial P}{\partial z} \quad (33)$$

$$\tau_{nz} = N - \frac{dw}{dn} \quad (34)$$

In the above, n is a coordinate normal to isovels and ρ = radius of curvature of isovels

The dissipation function ϕ can be used as an index of energy dissipation. For parallel flow ϕ is given by

$$\phi = \tau_n \frac{dw}{dn} = \tau_n (\tau_n - N) \quad (35)$$

3. Analysis of secondary flows and their effect on heat-transfer

The equations of motion and energy are next solved by using a double perturbation method, as follows. First, and for all boundary conditions, the tube cross-section boundary is defined by

$$G = 1 - r^2 + \epsilon r^n \sin n\theta = 0 \quad (36)$$

In which G is here labeled as a “shape factor”, that can describe a wide array of shapes according to the given values of the parameters n and ϵ [1]. The shape perturbation parameter is ϵ , which can take values in between 0 and a limiting values for the curve (36) staying closed. The parameter n must be given integer values in order to get regular shapes.

Next all velocity components are expanded in series in terms of the Weissenberg number, i.e.

$$\begin{aligned} u &= Wi u_1 + Wi^2 u_2 + \dots \\ v &= Wi v_1 + Wi^2 v_2 + \dots \\ w &= w_0 + Wi w_1 + Wi^2 w_2 + \dots \end{aligned} \quad (37)$$

Complementarily, all velocity component, at any order, are defined as

$$V = G (f_0 + \epsilon f_1 + \epsilon^2 f_2 + \dots) \quad (38)$$

where V is a generic velocity component, at any order in Wi , and f_0, f_1, \dots are functions specific for every V , to be determined by solving the momentum equations. Similarly, all other dependent variables are expressed in series, i.e.

$$\begin{aligned} \tau &= \tau_0 + Wi \tau_1 + Wi^2 \tau_2 + \dots \\ P &= P_0 + Wi P_1 + Wi^2 P_2 + \dots \end{aligned} \quad (39)$$

More details can be found in references [13, 25]. The velocity field is first found by substituting (37-39) in (20-25) and related equations. For the axial velocity, the following expressions can be determined, i.e.

$$w_0 = p(1 - r^2 + \epsilon r^n \sin n\theta) = pG \quad (40)$$

$$p = -\frac{1}{4} \frac{\partial P}{\partial z} \quad (41)$$

$$w_1=0$$

$$w_2(r, \theta)=w_0 p^3 \left[4(1-\xi)_0 - \xi(2-\xi) \right] (1+r^2) + \left[\frac{\xi(2-\xi)(3n-1) - 4(1-\xi)(n^2+2n-1)_0}{(n+1)} \right] r^n \sin(n\theta) \quad (42)$$

Further, albeit more involved, exact solutions can be found for w_3 and higher order terms in W_i . In these results, since the parameter ε has a maximum value of 0,3849 for $n=3$, which decreases as n increases, the series in ε shown in brackets in (38) were developed up to (ε) yielding accurate results.

Computations show that up to $O(Wi^2)$ the transversal velocity field is zero. Viscoelastic forcing terms in (20-22) are non-zero from $O(Wi^3)$ upwards, which implies $u_1=u_2=v_1=v_2=0$.

At third order in Wi , the viscoelastic forcing terms become non-zero. These are

$$F_{r3} = (\nabla \cdot \tau \nabla)_r$$

$$F_{\theta3} = (\nabla \cdot \tau \nabla)_\theta$$

$$F_{z3} = (\nabla \cdot \tau \nabla)_z \quad (43)$$

If the velocity components at third order in Wi are expressed in terms of a stream-function Ψ_3 , i.e.

$$u_3 = \frac{1}{r} \frac{\partial \Psi_3}{\partial \theta} \quad v_3 = - \frac{\partial \Psi_3}{\partial r} \quad (44)$$

then it is found

$$r \nabla^4 \Psi_3 = \frac{\partial(r F_{\theta3})}{\partial r} - \frac{\partial F_{r3}}{\partial \theta} \quad (45)$$

and

$$r \nabla^4 \Psi_3 = 8 \varepsilon (\xi - 2) \xi^2 (n-1) n (n+4) p^4 r^{n+1} \cos(n\theta) \quad (46)$$

The solution of (46) is

$$\Psi_3(r, \theta) = \frac{1}{4} \xi^2 (2-\xi) p^4 [1 - r^2 + r^n \sin(n\theta)]^2 \frac{n(n-1)(n+4)}{(n+1)(n+2)} r^n \cos(n\theta) \quad (47)$$

Where from

$$u_3(r, \theta) = - \frac{\xi^2 (2-\xi) p^4 n^2 (n+4)(n-1) w_0^2 r^{n-1} \sin(n\theta)}{4(n+1)(n+2)}$$

$$v_3(r, \theta) = \frac{\xi^2 (2-\xi) n (n+4)(n-1) p^4 w_0 [n - (n+4)r^2] r^{n-1} \cos(n\theta)}{4(n+1)(n+2)} \quad (48)$$

Plots of (47) are shown in figures 2, 3, 4 and 5

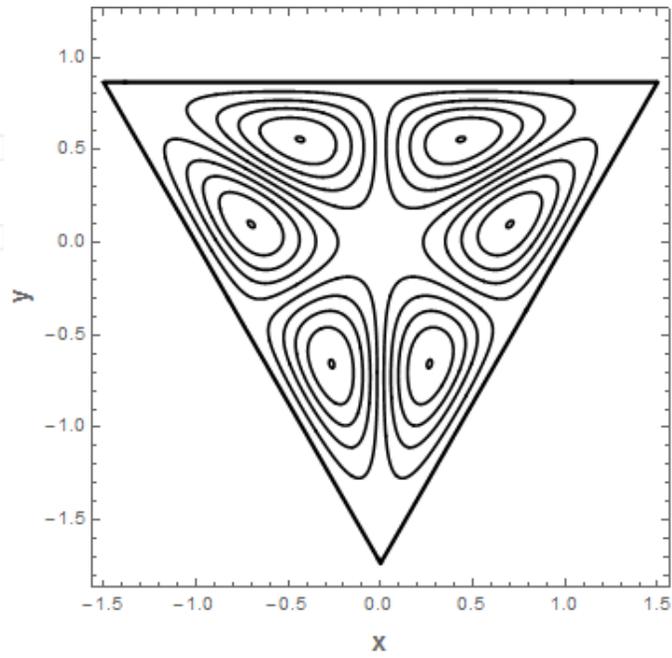


Figure 2. Characteristics plots of transversal streamlines for $n=3$ and $\varepsilon=0.3849$

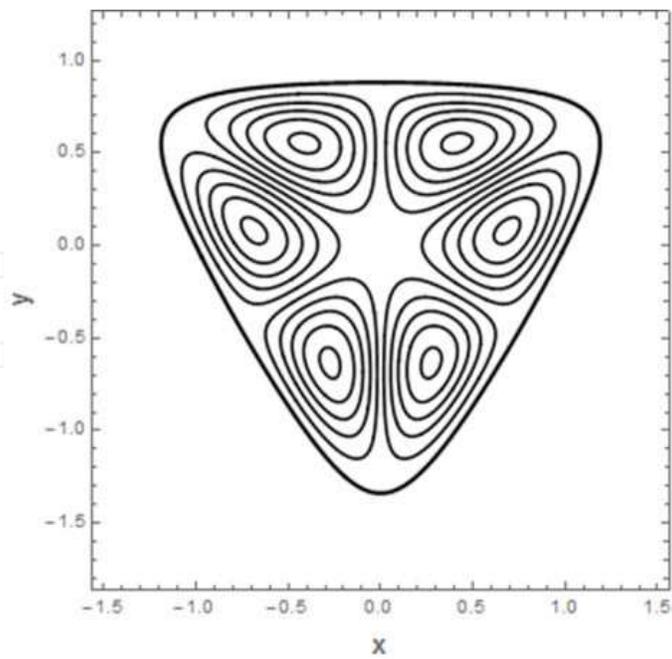


Figure 3. Characteristics plots of transversal streamlines for $n=3$ and $\varepsilon=0.33$

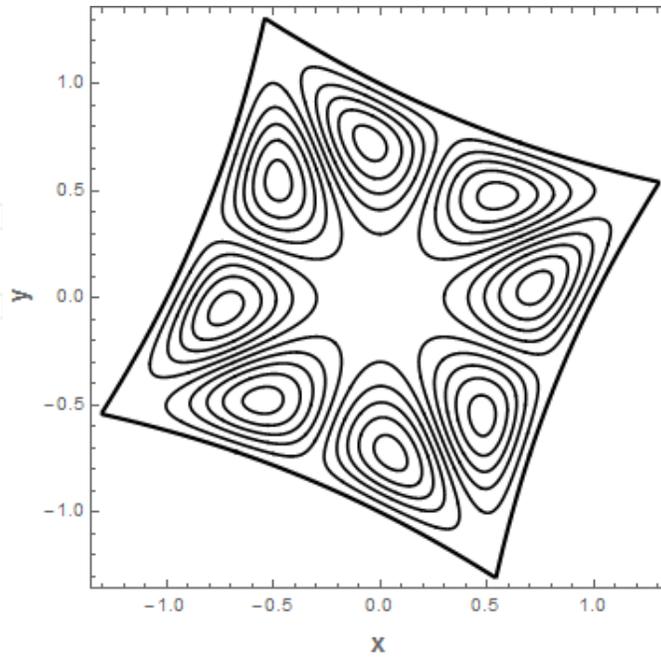


Figure 4. Characteristics plots of transversal streamlines for $n = 4$ and $\varepsilon = 0.25$

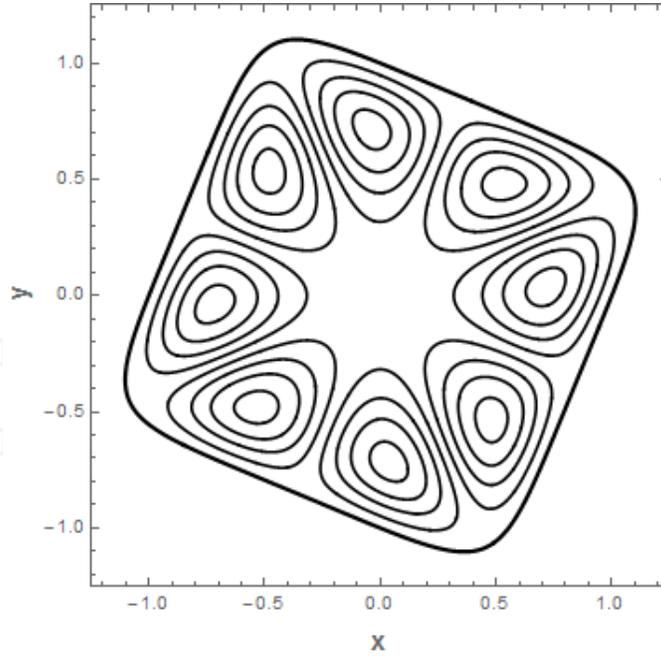


Figure 5. Characteristics plots of transversal streamlines for $n = 4$ and $\varepsilon = 0.2$

It is to be noted that the vortical shape does not change with the slip parameter ξ , the pressure coefficient p or the Weissenberg number. As these parameters change, the strength of the

vortices change. Also, other parameters being equal, the strength of the vortices increases significantly with ε , and thus transversal transport capacity. In all these cases the Reynolds number is $Re = 180$. Similar results are found for $n = 4, 5...$ in which the number of vortices is $2n$.

In Figure 6 is shown the internal distribution of the normal axial shear stress for the case of $\varepsilon = 0.3849$

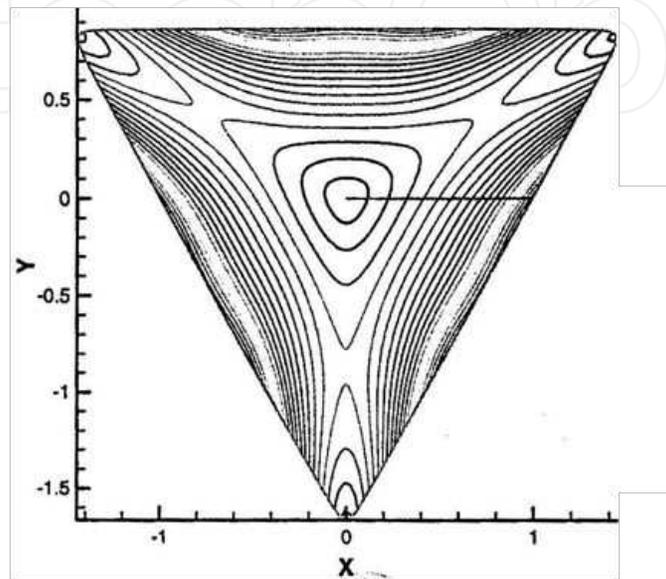


Figure 6. Characteristics plots of normal axial shear stress for $n=3$, $\varepsilon=0.3849$, $Wi=0.3$ and $\xi=0.2$

Curves are similar for all values of parameters, but the values of the normal axial shear stress vary with Wi , being of the order of 70% greater than the Newtonian counterpart for $Re=180$ and $Wi=0,3$.

The temperature field can be computed once the velocity field is known, through the energy equation (27). To this end, the temperature T is expressed as

$$T = T_0 + Wi T_1 + Wi^2 T_2 + \dots \quad (49)$$

In the following the temperature field and heat-transfer are computed for the case in which there is a constant heat flux through de tube wall, so that the temperature difference between de wall and the average temperature, i.e., $T_w - T_a$ remains constant and also $\partial T_a / \partial z$ is constant. This is problem 1 described in the section Mathematical Models.

The above leads to

$$\nabla^2 T_0 = a_0 Pr w_0 \quad (50)$$

in which

$$a_0 = \frac{1}{T_w - T_a} \frac{\partial T_a}{\partial z} \quad (51)$$

and Pr is the Prandtl number, i.e.

$$Pr = \frac{C_p \mu}{k} \quad (52)$$

Applying the condition that the temperature, at all orders in Wi be zero at the tube contour, i.e., that the fluid temperature is equal to T_w at the wall, then it is found

$$T_0 = \frac{P_r w_0 p a_0}{16} \left[r^2 - 3 + \varepsilon \frac{(n-3)}{(n-1)} r^n \sin n\theta \right] \quad (53)$$

Higher order terms of T can be found in similar way, but the expressions become very involved and are omitted here.

Heat exchange is computed through the Nusselt number Nu , i. e.

$$Nu = \frac{D_n h}{k} \quad (54)$$

where

$$h = \frac{k \int \frac{dT}{dn} d\bar{p}}{\bar{p}(T_w - T_a)} \quad (55)$$

$$D_n = \frac{4S}{\bar{p}} = \text{hydraulic diameter} \quad (56)$$

$$S = \iint r dr d\theta \quad (57)$$

In the above \bar{p} is de contour perimeter and S is de cross-sectional area. A plot of Nu in terms of Wi is shown in Figure 7 for different values of the Reynolds number Re , defined as

$$Re = \frac{\rho_0 w_a D h}{\eta_{m0}} \quad (58)$$

in which w_a is the axial average velocity.

Numerical values related to Figure 7 are given in the following table, where they can be compared with the Newtonian case

Typical plots of temperature distribution appears in Figures 8 and 9.

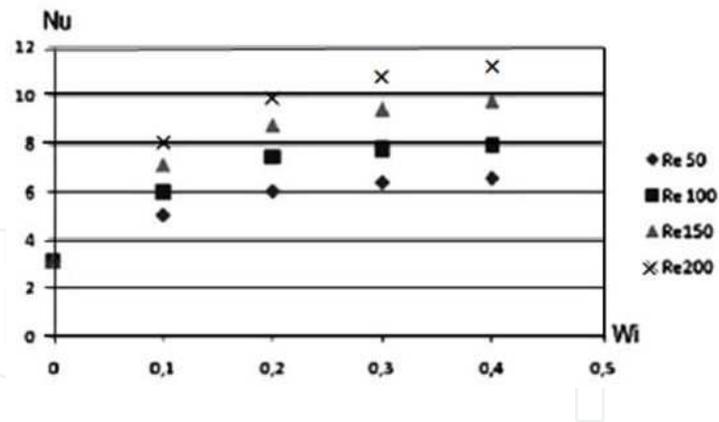


Figure 7. Please add caption

Nu	Re	Re	Re	Re
Wi	50	100	150	200
0	3,1623	3,1623	3,1623	3,1623
0.1	5,0498	6,0012	7,1403	8,046
0.2	6,0340	7,4544	8,7734	9,895
0.3	6,3993	7,7702	9,4245	10,7913
0.4	6,5746	7,9199	9,7712	11,2114

Table 1. Nusselt number values

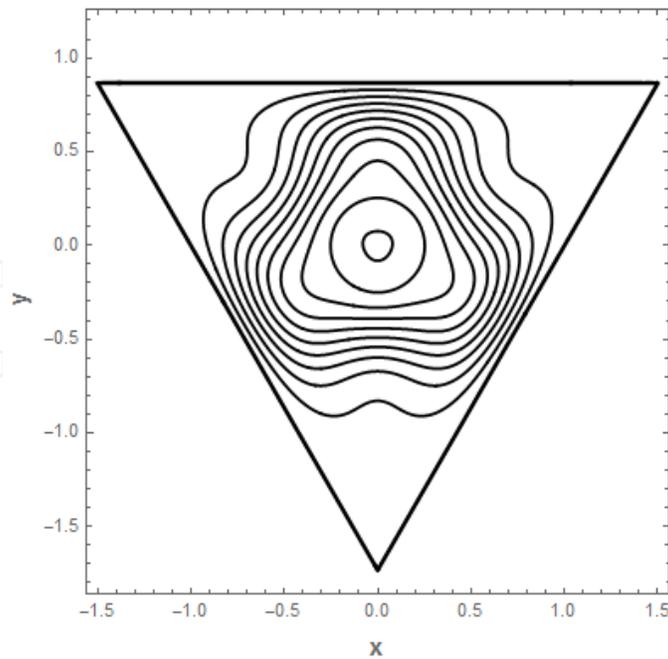


Figure 8. Characteristics plots of isothermal curves for $n = 3$ and $\varepsilon = 0.3849$

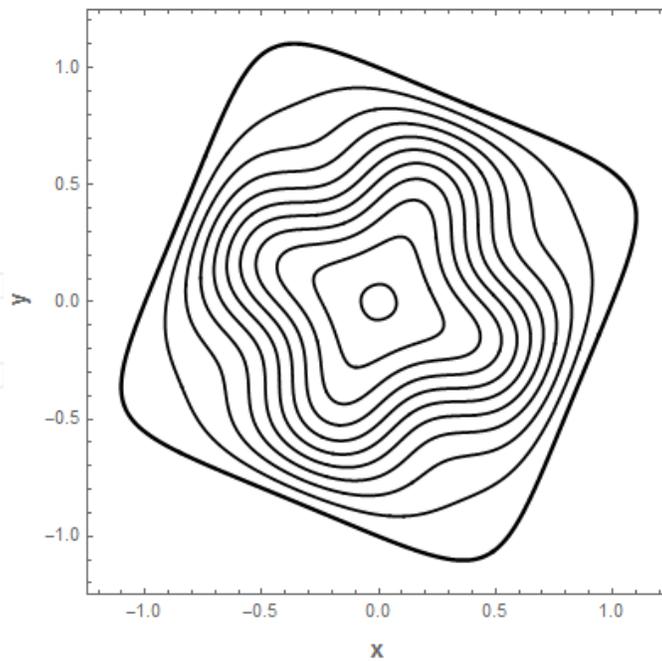


Figure 9. Characteristics plots of isothermal curves for $n=3$ and $\varepsilon=0.2$

These results, as initially presented and discussed in [13, 25, 26] show that the Nusselt number, i.e., the heat-transfer between fluid and wall, increases as the viscoelastic parameter Wi increases, with an asymptotic trend, for a given value of the Reynolds number. As R_e increases, also N_u increases for a given value of Wi .

The relevance of these findings for biological flows may be related to vessel deformation due to wall elasticity and peristaltic motion.

The preceding analysis shows that a very small deviation of the cross-section contour from the circular geometry, as represented by the value of the parameter ε , may induce secondary flows, and so increase the transversal transport capacity of biological flows. It seems to be an open research area the study of physiological and morphological changes that, in this context, may induce pathologies that accelerate heat transfer inside the human body. Such changes may create conditions that improve or worsen transport processes that should lead to restore normal physiological states.

Previous results associated to the flow of viscoelastic fluid in channels of axially-varying cross-section [27] show also that axial change of geometry, as found in biological vessels, also augment transport capacity.

4. Plastic flow

The velocity field and related variables for the flow of a Bingham fluid in non-circular tubes is next found using a similar technique. The axial velocity is defined as

$$w = w_0 + \varepsilon w_1 + \varepsilon^2 w_2 + \dots \quad (59)$$

The constitutive equations (29, 30) can be expanded in series around the parameter ε by using (58), where from [24]

$$\tau_{rz} = N - \frac{\partial w_0}{\partial r} - \varepsilon \frac{\partial w_1}{\partial r} + O(\varepsilon^2) \quad (60)$$

$$\tau_{\theta z} = -\frac{\varepsilon}{r} \left(\frac{\partial w_0}{\partial r} - N \right) \frac{\frac{\partial w_1}{\partial \theta}}{\frac{\partial w_0}{\partial r}} + O(\varepsilon^2) \quad (61)$$

From the above the governing equation for w_1 are found, i.e.

$$r \left(r - \frac{N}{2} \right) \left(\frac{\partial^2 w_1}{\partial r^2} + \frac{1}{r} \frac{\partial w_1}{\partial r} \right) + \frac{\partial^2 w_1}{\partial \theta^2} = 0 \quad (62)$$

which can be solved by separation of variables.

The dissipation function, in terms of the normal axial shear stress, is given in equation (35) and can be computed, as an indicator of energy dissipation due to friction.

Typical results for the velocity field and dissipation function are shown in the following figures.

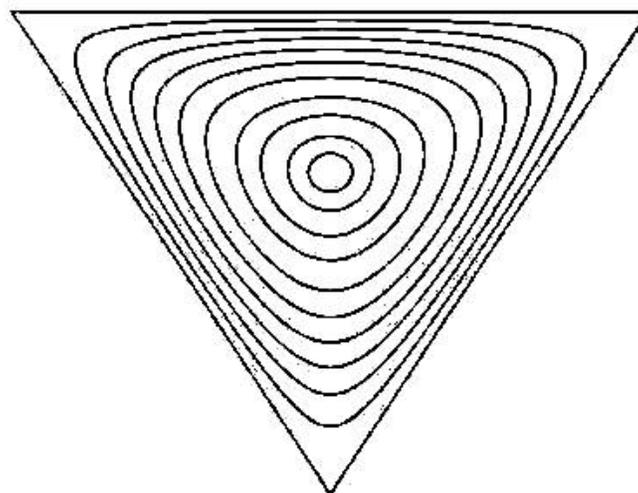


Figure 10. Plots of isovels for $n=3$, $\varepsilon=0.3849$ and $N=0.2$

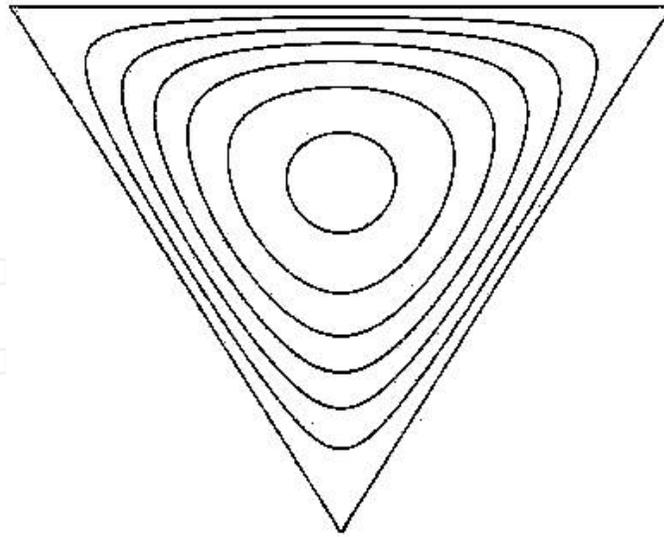


Figure 11. Plots of isovels for $n=3$, $\varepsilon=0.3849$ and $N=0.5$

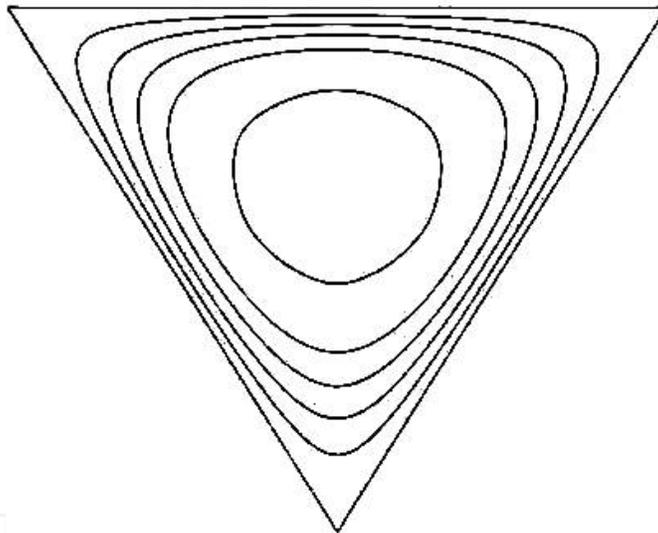


Figure 12. Plots of isovels for $n=3$, $\varepsilon=0.3849$ and $N=1.0$

One significant feature of plastic flows are plug (or solid) and stagnant zones, which, as a general rule, tend to decrease the rate of flow for a given pressure gradient, or require that more energy be supplied in order to keep constant the rate of flow.

Figure 13 shows that energy dissipation is greatly affected by the tube geometry. The maximum energy dissipation in the case of an equilateral triangle appears in the middle of the sides, where the axial shear stress is maximum. Also the shear stress is zero at the corners, which is a factor that determines stagnant areas close to such corners.

The plug zones are defined through the following joint conditions

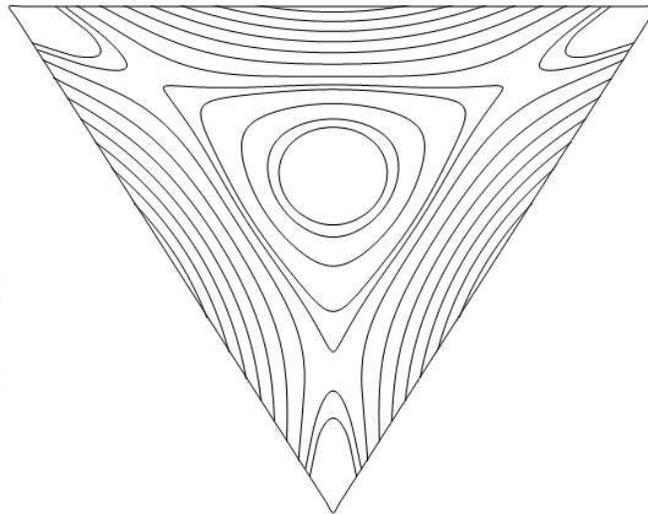


Figure 13. Plots of the dissipation function (35) for $n=3$, $\varepsilon=0.3849$ and $N=0.5$

$$\frac{dw}{dn} = 0 \quad ; \quad w = const \quad (63)$$

which can be met up to certain value of N according to the cross-section shape.

Abnormal vessel geometries may occur in biological flow arising from many sources. Especially in the case of stenosis, or geometry change due to solid deposition in artery walls, plastic effects may lead to blood clotting in corners, when the arteries are small.

5. Concluding remarks

Some analytical results concerning the effects of fluid elasticity and plasticity, coupled with tube cross-sectional geometry variation have been presented in this chapter.

The analysis is unified through some concepts that make it possible to explore, in a rather general fashion, the mechanisms of transversal transport that arise with the coupling of viscoelasticity and non-circular shape.

The circular shape is the most energy-efficient shape when only longitudinal mass transfer is considered. However, this not the case when transversal motion becomes important, as in heat-exchange processes, or in cases when also particle distribution is relevant.

General results indicate that viscoelasticity tends to increase transversal transport, which can be demonstrated by analytical means. These in particular, show that starting at third order in W_i , secondary flows appear when the fluid exhibits non-linear viscoelasticity, as prescribed by the fluid models of Giesekus and Phan-Tien-Tanner. Such secondary, or transversal motions, when coupled with the temperature, through the energy equation, determine a temperature field that improve heat-transfer between the fluid and the tube wall.

Plastic effects, on another perspective, do not induce transversal motion if the cross-section is not circular. Rather plasticity increases energy consumption when maintaining a given rate of flow is a priority. Shapes that include sharp corners lead to stagnant zones of fluids in the vicinity of those corners, thus decreasing the flow. The analytical method herein applied can be used for determining plug-zones of limiting values of the yield parameter N for which such zones exist properly.

The application of these findings to biological flows have been commented in previous sections of the chapter.

Acknowledgements

The author acknowledges the financial support of the National Fund for Scientific and Technological Development-FONDECYT, and of the Department of Scientific and Technological Research-DICYT at the University of Santiago, Chile.

Author details

Mario F. Letelier

Address all correspondence to: mario.letelier@usach.cl

Department of Mechanical Engineering, Center for Research in Creativity and Higher Education, University of Santiago, Chile

References

- [1] M. Marinkovic, K. Turner, J. Butler, J. Fredberg, S. Suresh, Viscoelasticity of the Human Red Blood Cell. *American Journal of Physiology - Cell Physiology* 293 (2007), 597-605.
- [2] A. Ündar, W. Vaughn, J. Calhoon, The effects of cardiopulmonary bypass and deep hypothermic circulatory arrest on blood viscoelasticity and cerebral blood flow in a neonatal piglet model. *Perfusion* 15 (2000) 121–128.
- [3] M. Letelier, J. Stockle, A Shape-Factor method for Modeling Parallel and Axially-Varying Flow in Tubes and Channels of Complex Cross- Section Shapes. *Biomedical Science. Engineering and Technology: InTech* 19 (2011) 469-487.

- [4] J.P. Hartnett, M. Kostic, Heat transfer to a Viscoelastic Fluid in Laminar Flow Through a Rectangular Channel. *International Journal Heat Mass Transfer* 28 (1985) 1147–1155.
- [5] J.P. Hartnett, M. Kostic, Heat Transfer to Newtonian and Non-Newtonian Fluids in Rectangular Ducts, *Adv. Heat Transfer* 19 (1989) 247–355.
- [6] J.P. Hartnett, Viscoelastic Fluids: a New Challenge in Heat Transfer (1990 Max Jacob Memorial Award Lecture). *Journal Heat Transfer* 114 (1992) 296–303.
- [7] D.R. Oliver, Non-Newtonian Heat Transfer: an Interesting Effect Observed in non-Circular Tubes, *Trans. Inst. Chem. Eng.* (1969) 47 (T18).
- [8] S.X. Gao, J.P. Hartnett, Steady Flow of non-Newtonian Fluids Through Rectangular Ducts, *International Commun. Heat Mass Transfer* 20 (1993) 197–210.
- [9] S.X. Gao, J.P. Hartnett, Heat Transfer Behavior of Reiner-Rivlin Fluids in Rectangular Ducts, *International Journal Heat Mass Transfer* 3 (1996) 1317–1324.
- [10] M.F. Naccache, P.R. Souza Mendes, Heat Transfer to non-Newtonian Fluids in Laminar Flow Through Rectangular Ducts, *International Journal Heat Fluid Flow* 17(6) (1996) 613–620.
- [11] P. Payvar, Heat Transfer Enhancement in Laminar Flow of Viscoelastic Fluids Through Rectangular Ducts, *International Journal Heat Mass Transfer* 40(3) (1997) 745–756.
- [12] S. Syrjala, Laminar Flow of Viscoelastic Fluids in Rectangular Ducts with Heat Transfer: a Finite Element Analysis, *International. Commun. Heat Mass Transfer* 25(2) (1998) 191–204.
- [13] D.A. Siginer, M.F. Letelier, Laminar Flow of Non-Linear Viscoelastic Fluids in Straight Tubes of Arbitrary Contour, *International Journal Heat Mass Transfer* 54(9-10) (2011) 2188–2202.
- [14] P.M. Coelho, F.T. Pinho, P.J. Oliveira, Thermal entry flow for a Viscoelastic fluid: the Graetz Problem for the PTT Model, *International Journal of Heat and Mass Transfer* 46 (2003) 3865–3880.
- [15] P.P. Valko, Solution of the Graetz–Brinkman problem with the Laplace transform Galerkin method, *International Journal of Heat and Mass Transfer* 48 (2005) 1874–1882.
- [16] W.S. Kiln, M.N. Özisik, Transient laminar forced convection to power-law fluids inside ducts resulting from a sudden change in wall temperature, *Wärme- und Stoffübertragung* 2 (1998) 159–165.
- [17] M. Letelier, D.A. Siginer, J.S. Stockle, Secondary Flows in Straight Non-Circular Tubes-Giesekus Fluid. *Asme International Mechanical Engineering Congress and Exposition*, November 2013, San Diego, California, USA. 2013.

- [18] S.C. Xue, N. Phan-Thien, R.I. Tanner. Numerical Study of Secondary Flows of Viscoelastic Fluid in Straight Pipes by an Implicit Finite Volume Method. *Journal of Non-Newtonian Fluid Mech.* 59 (1995) 191-213.
- [19] D.L. Tokpavi, A. Magnin, P. Jay, Very Slow Flow of Bingham Viscoplastic Fluid Around a Circular Cylinder. *Journal of Non-Newtonian Fluid Mechanics* 154 (2008) 65-76.
- [20] Z. Yu, A. Wachs, A Fictitious Domain Method for Dynamic Simulation of Particle Sedimentation in Bingham Fluids. *Journal of Non-Newtonian Fluid Mechanics* 145 (2007) 78-91.
- [21] A. Wachs, Numerical Simulation of Steady Bingham Flow Through an Eccentric Annular Cross- Section by Distributed Lagrange Multiplier Fictitious Domain and Augmented Lagrangian Methods. *Journal of Non-Newtonian Fluid Mechanics* 142 (2007) 183-198.
- [22] I.C. Walton, S.H. Bittleston, The Axial Flow a Bingham Plastic in a Narrow Eccentric Annulus. *Journal Fluid Mechanics* 222 (1991) 39-60.
- [23] M. Letelier, D. Siginer, On the Flow of a Class of Viscoelastic-Viscoplastic Fluids in Tubes of Non-Circular Contour. *International Journal of Engineering Science* 45 (2007) 873-881.
- [24] M. Letelier, F. Godoy, C. Rosas, On the Existence Conditions for Plug Zones in Plastic Flow in Tubes of Non-Circular Cross-Section. 14th Pan-American Congress of Applied Mechanics PACAM XIV 2014, Santiago, Chile.
- [25] D.A. Siginer, M.F. Letelier, Heat Transfer Asymptote in Laminar Flow of Non-Linear Viscoelastic Fluids in Straight Non-Circular Tubes, *Int. J. Engineering Science* 48(11) (2010) 1544 -1562.
- [26] M. Letelier, F.N. Zapata, D.A. Siginer, J.S. Stockle, Analysis of Secondary Flows and Heat Transfer in Viscoelastic Flow with Viscous Dissipation in Non-Circular Tubes. ASME International Mechanical Engineering Congress and Exposition, November 2012, Houston, Texas, USA 2012.
- [27] M.F. Letelier, R. Gaete, D.A. Siginer, Steady Viscoelastic Flow Channels of Arbitrarily Varying Cross-Section Shape. Asme International Mechanical Engineering Congress and Exposition, DC., ISBN N°0-7918-4663-6, Washington 2003.