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1. Introduction

Like immunotherapy, chemotherapy, and surgery, radiotherapy is one of the major tools in fighting against cancer. As acute IR is applied, cell can trigger its self-defensive mechanisms in response to genome stresses [1]. As one of the pivotal anticancer genes within the cell, P53 can control the transcription and translation of series genes, and trigger cell cycle arrest and apoptosis through interaction with downstream genes and their complicated signal pathways [2]. Under radiotherapy, the outcomes of cellular response depend on the presence of functional P53 proteins to induce tumor regression through apoptotic pathways [3]. Conversely, the P53 tumor suppressor is the most commonly known specific target of mutation in tumorigenesis [4]. Abnormalities in the P53 have been identified in over 60% of human cancers and the status of P53 within tumor cells has been proposed to be one of the determinant response to anticancer therapies [3,4]. Controlled radiotherapy studies show the existence of a strong biologic basis for considering P53 status as a radiation predictor [3,5]. Therefore, the status of P53 in tumor cell can be considered as a predictor for long-term biochemical control during and after radiotherapy [6-8].

Recently, several models have been proposed to explain the damped oscillations of P53 in cell populations [9-12]. However, the dynamic mechanism of the single-cell responses is not completely clear yet, and the complicated regulations among genes and their signal pathways need to be further addressed, particularly under the condition of acute IR.

Many studies have indicated that introducing novel mathematical and computational approaches can stimulate in-depth investigation into various complicated biological systems (see, e.g., [13-23]). These methods have provided useful tools for both basic research and
drug development [24-33], helping understanding many marvelous action mechanisms in various biomacromolecular systems (see, e.g., [21,34-39]).

Based on the existing models [9-12] and inspired by the aforementioned mathematical and computational approaches in studying biological systems, here a new model is proposed for studying the P53 stress response networks under radiotherapy at the cellular level, along with the kinetics of DNA double-strand breaks (DSBs) generation and repair, ATM and ARF activation, as well as the regulating oscillations of P53-MDM2 feedback loop (MDM2 is an important negative regulator of the p53 tumor suppressor). Furthermore, the kinetics of the oncogenes degradation, as well as the eliminations of the mutation of P53 (mP53) and the toxins were presented. Also, the plausible outcomes of cellular response were analyzed under different IR dose domains.

It is instructive to mention that using differential equations and graphic approaches to study various dynamical and kinetic processes of biological systems can provide useful insights, as indicated by many previous studies on a series of important biological topics, such as enzyme-catalyzed reactions [18,40], low-frequency internal motions of biomacromolecules [41-46], protein folding kinetics [47,48], analysis of codon usage [49,50], base distribution in the anti-sense strands [51], hepatitis B viral infections [52], HBV virus gene missense mutation [53], GPCR type prediction [54], protein subcellular location prediction [55], and visual analysis of SARS-CoV [56,57].

In the present study, we are to use differential equations and directed graphic approaches to investigate the dynamic and kinetic processes of the cellular responding radiotherapy.

2. Method

2.1. Model review

Under the genome stresses, many efforts have been made to enhance P53-mediated transcription through some models [58,59] [9-12]. However, the interactions in a real system would make these models [60] extremely complicated. Therefore, a new feasible model is needed in order to incorporate more biochemical information. To realize this, let us take the following criteria or assumptions for the new model: (1) only the vital components and interactions are taken into account; (2) all the localization issues are ignored; (3) the simple linear relations are used to describe the interactions among the components concerned; and (4) there are enough substances to keep the system “workable” [58].

The new integrated model thus established for the P53 stress response networks under radiotherapy is illustrated in Fig.1. Compared with the previous models [9-12], the current model contains more vital components, such as oncogenes, ARF and mP53, as well as their related regulating pathways. In the DSBs generation and repair module, the acute IR induces DSBs stochastically and forms DSB-protein complexes (DSBCs) at each of the damage sites after interacting with the DNA repair proteins [2,3]. As a sensor of genome stress, ATM is activated by the DSBCs signal transferred from DSBs. Meanwhile, the over-expression of oncogenes prompted by acute IR can trigger the activation of ARF, further
prompting the ATM activation [2] [7]. The cooperating effects of active ATM (ATM*) and active ARF (ARF*) switch on or off the P53-MDM2 feedback loop [2] [7,9], further regulating the downstream genes to control the cell cycle arrest and the cell apoptosis in response to genome stresses [8]. Here, we use the superscript * to represent the activate state as done in [61].

**Figure 1.** Illustration showing the integrated model of P53 stress response networks under radiotherapy. It is composed of three modules, including DNA damage generation and repair, ATM and ARF activation, as well as P53-MDM2 feedback loop. As acute IR is applied, ARF is activated by the over-expression of oncogenes, and ATM is activated with the cooperation of DSBCs and ARF*. ATM* and ARF* corporately trigger the responding mechanism of P53-MDM2 feedback loop.

### 2.2. DSBs generation and repair

Under the continuous effect of acute IR dose, DSBs occur and trigger two major repair mechanisms in eukaryotic cells: homologous recombination (HR) and nonhomologous end joining (NHEJ) [62,63]. About 60-80% of DSBs are rejoined quickly, whereas the remaining 20-40% of DSBs are rejoined more slowly [64,65]. As shown in **Fig.2**, the module of DSBs generation and repair process contains both the fast and slow kinetics, with each being composed of a reversible binding of repair proteins and DSB lesions into DSBCs, and an irreversible process from the DSBCs to the fixed DSBs [62,65]. DSBCs are synthesized by binding the resulting DSBs with repair proteins (RP), which is the main signal source to transfer the DNA damage to P53-MDM2 feedback loop by ATM activation [2].

Due to the misrepair part of DSBs (F_w) having the profound consequences on the subsequent cellular viability and the cellular response in fighting against genome stresses [1,3], we obviously distinguish between correct repair part of DSBs (F_r) and F_w [9,10,12]. Moreover, we further deal the total F_w in both repair processes as a part of toxins within the cell [2,4,11], which can be eliminated by the regulatory functions of P53 during and after radiotherapy, and treated as an indicator of outcomes in cellular response to genome stresses [2].

Some experimental data suggest that the quantity of the resulting DSBs within different IR dose domains obey a Poisson distribution [11]. In accordance with the experiments, we
assume that the stochastic number of the resulting DSBs per time scale is proportional to the number generated by a Poisson random function during the period of acute radiation [11].

The DSBs generation process is formulated as follows:

$$\frac{d[DT]}{dt} = k_i \times \text{Poissrnd}(a_{ir} \times \text{IR})$$

(1)

where $[DT]$ is the concentration of total resulting DSBs induced by IR in both fast and slow repair processes. $k_i$ is the parameter to set the number of DSBs per time scale, and $a_{ir}$ is the parameter to set the number of DSBs per IR dose.

Moreover, we assume that the limited repair proteins are available around DSBs sites, and 70% of the initial DSBs are fixed by the fast repair process. Each DSB can be in one of the four states: intact DSB, DSBC, $F_r$, and $F_w$. Subscripts ‘1’ and ‘2’ refer to the fast kinetics and slow one.

The following differential equations:

$$\frac{d[D_1]}{dt} = a_1[D_1] + k_{d11}C_1 - [RP](k_{d12}[D_1] + k_{cross}(C_1 + [D_2]))$$

(2)

$$\frac{d[D_2]}{dt} = a_2[D_2] + k_{d22}C_2 - [RP](k_{d21}[D_2] + k_{cross}(C_1 + [D_2]))$$

(3)

$$\frac{d[C_1]}{dt} = k_{d11}[D_1] - k_{d11}C_1 - k_{d12}C_1$$

(4)

$$\frac{d[C_2]}{dt} = k_{d22}[D_2] - k_{d22}C_2 - k_{d21}C_2$$

(5)

$$\frac{d[RP]}{dt} = S_p + k_{d12}[C_1] + k_{d22}[C_2] - [RP](k_{d11}[D_1] + k_{d21}[D_2] + k_{cross}([D_1] + [D_2]))$$

(6)
\[
\frac{d[F_w]}{dt} = k_{w1}[C_1] + k_{w2}[C_2]
\]

where \([D]\), \([C]\), and \([F_w]\) represent the concentrations of DSBs, DSBCs, and \(F_w\) in the fast and the slow repair kinetics respectively, \(k_{dc}\), \(k_{cd}\), \(k_{cf}\), and \(k_{fw}\) are the transition rates among the above three states; \(k_{dc}\), and \(k_{cross}\) represent the first-order and second-order rate constants in both the fast and the slow repair kinetics respectively [65]. \(S_p\) is the basal induction rate of repair mRNA, and subscripts ‘1’ and ‘2’ refer to the fast and the slow kinetics.

### 2.3. ATM and ARF activation

As a DNA damage detector, ATM exists as a dimer in unstressed cells. After IR is applied, intermolecular autophosphorylation occurs, causing the dimer to dissociate rapidly into the active monomers. The active ATM monomer (\(ATM^*\)) can prompt the P53 expression further [64]. Meanwhile, ARF, another tumor suppressor, is activated by hyperproliferative signals emanating from oncogenes, such as Ras, c-myc etc., further prompting the ATM activation [2,7,10]. Based on the existing model of ATM switch [11], we present an ATM and ARF activation module under IR. Shown in Fig.3 is the module scheme of ATM and ARF activation, which includes five components: ATM dimer, inactive ATM monomer, ATM*, ARF, and ARF*. Compared with the previous studies in [9-12], ARF, oncogenes, and the related signal pathways are involved in this module [2,7]. Here, let us assume that DSBCs is the main signal transduction from DSBs to P53-MDM2 feedback loop through ATM activation, and the rate of ATM activation is a function of the amount of DSBCs, ARF*, and the self-feedback of ATM*. Furthermore, the total concentration of ATM is a constant, including ATM dimer, ATM monomer and ATM, as treated in [Ma, 2005 #1194].

![Figure 3](image-url)

**Figure 3.** Illustration showing the module scheme of ATM and ARF activation under constant IR. ARF is activated by the over-expression of oncogenes induced by acute IR, and ATM is activated from ATM monomers under the cooperating effects of DSBCs, ARF*, and self-feedback of ATM*. 

Oncogenes over-expression

\[ \text{ARF} \xrightarrow{k_{onf}} \text{ARF}^* \]

\[ \text{C (DSBCs)} \]

\[ F(C, [\text{ARF}^*], [\text{ATM}^*]) \]

\[ \text{ATM} \xrightarrow{k_{cat}} \text{ATM}^* \]

\[ \text{ATM}_D \xrightarrow{k_{undim}} \text{ATM} \]

\[ \text{ARF} \xrightarrow{k_{undim}} \text{ARF}^* \]

\[ \text{ATM} \xrightarrow{k_{cat}} \text{ATM}^* \]

\[ \text{Oncogenes over-expression} \]

\[ \text{ARF} \xrightarrow{k_{onf}} \text{ARF}^* \]

\[ \text{C (DSBCs)} \]

\[ F(C, [\text{ARF}^*], [\text{ATM}^*]) \]

\[ \text{ATM} \xrightarrow{k_{cat}} \text{ATM}^* \]

\[ \text{ATM}_D \xrightarrow{k_{undim}} \text{ATM} \]

\[ \text{ARF} \xrightarrow{k_{undim}} \text{ARF}^* \]

\[ \text{ATM} \xrightarrow{k_{cat}} \text{ATM}^* \]
As a detector of DNA damage, ATM activation plays an important role in triggering the regulatory mechanisms of P53 stress response networks [2,65]. After the acute IR is applied, phosphorylation of inactive ATM monomers is promoted first by DSBCs and then rapidly by means of the positive feedback from ATM*, accounting for the intermolecular autophosphorylation [11]. Meanwhile, under the circumstance of continuous IR dose, ARF, a detector of over-expression of oncogenes is activated by hyperproliferative signals emanating from oncogenes, further prompting the ATM activation [2,7,10], as can be formulated as follows:

\[
\frac{d[\text{ATM}_d]}{dt} = \frac{1}{2} k_{\text{dim}}[\text{ATM}_m]^2 - k_{\text{undim}}[\text{ATM}_d] 
\]

\[
\frac{d[\text{ATM}_m]}{dt} = 2k_{\text{undim}}[\text{ATM}_d] - k_{\text{dim}}[\text{ATM}_m]^2 - k_{\text{ar}}f[\text{ATM}_m] + k_{\text{af}}[\text{ATM}^*] 
\]

\[
\frac{d[\text{ATM}^*]}{dt} = k_{\text{af}}f[\text{ATM}_m] - k_{\text{ar}}[\text{ATM}^*] 
\]

\[
\frac{d[\text{ARF}]}{dt} = S_{\text{arf}} - k_{\text{arf}}[\text{ARF}] - k_{\text{onf}}[\text{Onco}][\text{ARF}] 
\]

\[
\frac{d[\text{ARF}^*]}{dt} = k_{\text{onf}}[\text{Onco}][\text{ARF}] - k_{\text{pad}}[\text{ARF}^*] 
\]

\[
f(C, [\text{ATM}^*]) = a_1C + a_2[\text{ATM}^*] + a_3C[\text{ATM}^*] + a_4[\text{ARF}^*] 
\]

where \([\text{ATM}_d]\), \([\text{ATM}]\) and \([\text{ATM}^*]\) represent the concentrations of ATM dimer, ATM monomer, and active ATM monomer respectively; \([\text{Onco}]\), \([\text{ARF}]\) and \([\text{ARF}^*]\) represent the concentrations of oncogenes, ARF, and active ARF respectively; \(k_{\text{undim}}, k_{\text{dim}}, k_{\text{ar}},\) and \(k_{\text{af}}\) are the rates of ATM undimerization, ATM dimerization, ATM monomer inactivation, and ATM monomer activation, respectively. \(S_{\text{arf}}, k_{\text{arf}}, k_{\text{onf}}\) and \(k_{\text{pad}}\) are the rates of ARF basal induction, ARF activation triggered by Oncogenes, ARF degradation, and ARF* degradation, respectively. In addition, \(f\) is the function of ATM activation, the term \(a_1C\) implies the fact that DSBs somehow activate ATM molecules at a distance, \(a_2[\text{ATM}^*]\) indicates the mechanism of autophosphorylation of ATM, \(a_3C[\text{ATM}^*]\) represents the interaction between the DSBCs and ATM* [9-12,66], and \(a_4[\text{ARF}^*]\) represents the regulating function of ARF* to ATM activation [1,3,7].

### 2.4. Regulation of P53-MDM2 feedback loop

As shown in Fig.4, P53 and its principal antagonist, MDM2 transactivated by P53, form a P53-MDM2 feedback loop, which is the core part in the integrated networks [9-12]. ATM* elevates the transcriptional activity of P53 by prompting phosphorylation of P53 and degradation of MDM2 protein [67]. Also, ARF* can indirectly prompt the transcriptional
activity of P53 by inhibiting the expression of MDM2 and preventing P53 degradation [2,7,9]. With the cooperating regulations of ATM* and ARF*, this negative feedback loop can produce oscillations in response to the sufficiently strong IR dose [11].

Figure 4. The directed graph of P53-MDM2 feedback loop under radiotherapy. P53 is translated from P53mRNA and phosphorylated by ATM* and ARF*. MDM2 protein promotes a fast degradation of P53 protein and a slow degradation of P53*. In addition, ATM* and ARF* stimulate the degradation of MDM2, and then indirectly increase the regulatory activation of P53* further. Especially, oncogenes, toxins and mP53 are decreased directly by the regulatory functions of P53*.

Especially, the mutation of P53 (mP53) triggered by oncogenes is added in this module, and mP53 is further dealt as another detector of outcomes in cellular response to acute IR. To account for a decreased binding affinity between inactive P53 and P53*, we assume that MDM2-induced degradation of inactive P53 is faster than that of P53*, and only P53* can induce target genes to depress the over-expression of oncogenes and further eliminate the toxins within the cell [3,4,9-12]. The main differential equations used in this module are as follows:

\[
\frac{d[P53]}{dt} = S_{P53} - d_{pp}[P53] - k_{pp}[P53] \\
\frac{d[P53*]}{dt} = k_p[P53] + k_{pp}[P53*] - d_{pp}[P53*] \\
- k_{app}[ATM]* \frac{[P53]}{[P53*]+k_p} - k_{app}[MDM2*] \frac{[P53]}{[P53*]+k_d} \\
\frac{d[mP53]}{dt} = k_{app}[ATM*] \frac{[P53]}{[P53*]+k_p} - k_{pp}[mP53] - d_{pp}[mP53]
\]
\[ -k_{pp}[P53^*] - d_{pp}[P53^*] - k_{mp}[MDM2_p] \frac{[P53^*]}{[P53^*]+k_p} \]

\[
\frac{d[MDM2_e]}{dt} = S_{mdm2} + k_{pp} \frac{[P53^*]^p}{[P53^*]^p+k_p}
\]

\[ -k_{mp}[MDM2_e] - d_{mp}[MDM2_e] \]

\[
\frac{d[MDM2_e]}{dt} = k_{mp}[MDM2_e] - d_{mp}[MDM2_e]
\]

\[ -(k_{mat} \frac{[ATM^*]}{[ATM^*]+k_{mat}} + k_{mat} \frac{[ARF^*]}{[ARF^*]+k_{mat}})[MDM2_p] \]

\[
\frac{d[Onco]}{dt} = k_{onc}[Onco][IR] - k_{mp}[Onco][P53^*]
\]

\[
\frac{d[Toxins]}{dt} = k_{onc}[P53^*][Toxins]
\]

\[
\frac{d[mP53]}{dt} = k_{mp}[P53_p][Onco] - k_{pmd}[P53_p][mP53]
\]

where \([P53_e], [P53_p], [P53^*], [MDM2_e], \) and \([MDM2_p]\) represent the concentrations of P53 mRNA, P53 protein, active P53, MDM2 mRNA, and MDM2 protein, respectively; \([Onco]\), \([Toxins]\), and \([mP53]\) represent the concentrations of oncogenes, F\(w\) and mP53, respectively. \(S_{P53}\) and \(S_{MDM2}\) represent the basal induction rates of P53 mRNA and MDM2 mRNA, respectively; \(k\) and \(d\) represent the regulation and degradation rates among genes and proteins, respectively. The other parameters are presented in Tables 1-3.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Constant</th>
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</thead>
<tbody>
<tr>
<td>(k_1)</td>
<td>Rate of DSBs generation per time scale</td>
<td>0.01</td>
</tr>
<tr>
<td>(a_1)</td>
<td>Number of DSBs generation per IR dose</td>
<td>35</td>
</tr>
<tr>
<td>(a_2)</td>
<td>Percentage of DSBs processed by fast repair</td>
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</tr>
<tr>
<td>(a_2)</td>
<td>Percentage of DSBs processed by slow repair</td>
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<tr>
<td>(k_{o1})</td>
<td>Rate of DSBs transition to DSBCs in fast repair process</td>
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</tr>
<tr>
<td>(k_{o1})</td>
<td>Rate of DSBs transition to DSBCs in slow repair process</td>
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<tr>
<td>(k_{o1})</td>
<td>Rate of DSBCs transition to DSBs in fast repair process</td>
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<td>Rate of DSBCs transition to DSBs in slow repair process</td>
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<td>(k_{o1})</td>
<td>Rate of DSCs transition to F(i) in fast repair process</td>
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<td>(k_{o2})</td>
<td>Rate of DSCs transition to F(i) in slow repair process</td>
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<tr>
<td>(k_{o1})</td>
<td>Rate of DSB binary mismatch in second order repair process</td>
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Table 1. The parameters used in the DSBs generation and repair processes
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<thead>
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<th>Parameters</th>
<th>Description</th>
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<td>$k_{\text{dim}}$</td>
<td>ATM dimerization rate</td>
<td>8</td>
</tr>
<tr>
<td>$k_{\text{undim}}$</td>
<td>ATM undimerization rate</td>
<td>1</td>
</tr>
<tr>
<td>$k_{\text{at}}$</td>
<td>ATM phosphorylation rate</td>
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</tr>
<tr>
<td>$k_{\text{ar}}$</td>
<td>ATM dephosphorylation rate</td>
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<tr>
<td>$S_{\text{arf}}$</td>
<td>Basal induction rate of ARF mRNA</td>
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<tr>
<td>$k_{\text{onf}}$</td>
<td>ARF activation rate triggered by Oncogenes</td>
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<tr>
<td>$k_{\text{onp}}$</td>
<td>ARF degradation rate</td>
<td>0.015</td>
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<tr>
<td>$a_1$</td>
<td>Scale of the activation function of ATM phosphorylation</td>
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</tr>
<tr>
<td>$a_2$</td>
<td>Scale of the activation function of ATM phosphorylation</td>
<td>0.08</td>
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<tr>
<td>$a_3$</td>
<td>Scale of the activation function of ATM phosphorylation</td>
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Table 2. The parameters used in the process of ATM and ARF activation

<table>
<thead>
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<th>Description</th>
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<tr>
<td>$S_{\text{p53}}$</td>
<td>Basal induction rate of P53 mRNA</td>
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<td>$d_{\text{rp}}$</td>
<td>Degradation rate of P53 mRNA</td>
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<td>Translation rate of P53 mRNA</td>
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<tr>
<td>$k_{\text{rp}^*}$</td>
<td>Dephosphorylation rate of P53*</td>
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<tr>
<td>$k_{\text{ar}^*}$</td>
<td>ATM*-dependent phosphorylation rate of P53</td>
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<tr>
<td>$k_{\text{mrp}}$</td>
<td>MDM2-dependent degradation rate of P53</td>
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<tr>
<td>$k_{\text{ar}^*}$</td>
<td>MDM2-dependent degradation rate of P53*</td>
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<tr>
<td>$d_{\text{rp}}$</td>
<td>Basal degradation rate of P53</td>
<td>0.02</td>
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<tr>
<td>$d_{\text{pp}}$</td>
<td>Basal degradation rate of P53*</td>
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<td>$S_{\text{mdm2}}$</td>
<td>Basal induction rate of MDM2 mRNA</td>
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<td>P53-dependent MDM2 transcription rate</td>
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</tr>
<tr>
<td>$k_{\text{mrp}}$</td>
<td>Translation rate of MDM2 mRNA</td>
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<tr>
<td>$d_{\text{mm}}$</td>
<td>Degradation rate of MDM2</td>
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<tr>
<td>$d_{\text{mp}}$</td>
<td>Basal degradation rate of MDM2</td>
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<td>$k_{\text{ar}^*}$</td>
<td>ATM*-dependent degradation rate of MDM2</td>
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</tr>
<tr>
<td>$k_{\text{ar}^*}$</td>
<td>ARF*-dependent degradation rate of MDM2</td>
<td>0.02</td>
</tr>
<tr>
<td>$k_{\text{p}}$</td>
<td>Michaelis constant of ATM*-dependent P53 phosphorylation</td>
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<tr>
<td>$k_{\text{p}}$</td>
<td>Michaelis constant of P53-dependent MDM2 transcription</td>
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<td>$k_{\text{at}}$</td>
<td>Threshold concentration for MDM2-dependent P53 degradation</td>
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<td>Threshold concentration for ATM*-dependent MDM2 transcription</td>
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<tr>
<td>$k_{\text{ar}}$</td>
<td>Threshold concentration for ARF*-dependent MDM2 degradation</td>
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<tr>
<td>$k_{\text{ar}}$</td>
<td>Threshold concentration for MDM2-dependent P53* degradation</td>
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<td>$k_{\text{ar}}$</td>
<td>Threshold concentration for MDM2-dependent P53* degradation</td>
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<td>Activation rate of oncogenes induced by IR</td>
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<td>$k_{\text{onp}}$</td>
<td>Degradation rate of oncogenes induced by P53*</td>
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### Parameters Description Constant

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<th>Parameter</th>
<th>Description</th>
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<tr>
<td>$k_{tfo}$</td>
<td>Toxins accumulation rate triggered by IR</td>
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<tr>
<td>$k_{pt}$</td>
<td>Toxins elimination rate triggered by P53*</td>
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<tr>
<td>$k_{mipo}$</td>
<td>Induction rate of mP53 induced by oncogenes over-expression</td>
<td>0.03</td>
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<tr>
<td>$k_{mpd}$</td>
<td>Elimination rate of mP53 triggered by P53*</td>
<td>0.015</td>
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</table>

**Table 3.** The parameters used in the process of P53-MDM2 loop and toxins degradation

### 3. Results and discussion

To ensure the accuracy of the simulation results, we consider that the valid parameter sets should obey the following rules [2,11,67].

1. The model must contain oscillations because there has been experimental evidence that oscillations occur between P53 and MDM2 after cell stress.
2. The mechanism used to mathematically describe the degradation of P53 by MDM2 is accurate only for low concentrations of P53.
3. The concentration of P53* is much higher than that of inactive P53 after the system reaching an equilibrium.

Based on the above three rules and the existing parameter sets used in [11], we obtained the kinetics of P53 stress response networks and cellular response under acute IR dose through simulation platform in MATLAB7.0. The detailed parameters used for the current model are given in Tables 1-3.

#### 3.1. Kinetics of DSBCs synthesizing

During the simulation process, the continuous 2, 5, and 7Gy IR are applied into a cell respectively. As shown in Fig.5a, owing to the condition that many DSBs occur and the limited RP are available around damage sites, the concentration of RP begins to decrease as IR dose overtakes 5Gy, and trends to zero versus radiation time. Meanwhile, the kinetics of DSBCs synthesizing is shown in Fig.5b. We can see that the rates of DSBCs synthesis keep increasing under 2, and 5Gy IR, whereas, it begins to decrease and trend to constant after about 120min under 7Gy IR dose.

#### 3.2. Kinetics of ARF and ATM activation

The ARF activation is used to describe the mechanisms in cellular response to the over-expression of oncogenes induced by acute IR [2,7]. The kinetics of ARF activation is shown in Fig.6a. Owing to the over-expression of oncogenes without depressing functions of P53*, ARF is activated fast and ARF* keeps increasing followed by trending to dynamic equilibrium versus radiation time.

Meanwhile, the ATM activation module was established to describe the switch-like dynamics of the ATM activation in response to DSBCs increasing, and the regulation mechanisms during the process of the ATM transferring DNA damage signals to the P53-MDM2 feedback loop. Under the cooperative function of DSBCs, ARF*, and the positive self-feedback of ATM*, the ATM would reach the equilibrium state within minutes due to
Figure 5. The kinetics of DSBs repairing and transferring under continuous effect of 2, 5, 7Gy IR.
(a) The dynamics of RP available around the resulting DSBs under different IR dose domains.
(b) The kinetics of DSBCs synthesized by DSBs and RP versus continuous radiation time under different IR dose domains.
Figure 6. The kinetics of ARF and ATM activation under 2, 5, 7Gy IR. (a) The kinetics of ARF activation in response to over-expression of oncogenes induced by different IR dose. (b) The switch-like kinetics of ATM activation, ATM$^*$ reach saturation and trend to constant state in response to continuous radiation time of different IR dose domains.
the fast phosphorylation [2,11,67]. Kinetics of ATM activation is shown in Fig.6b. ATM is activated rapidly and switches to “on” state with respective rates, and then trends to the saturation state. The step-like traces suggest that the ATM module can produce an on-off switching signal, and transfer the damage signal to the P53-MDM2 feedback loop [3]. Furthermore, under the cooperation effects of ATM* and ARF*, DNA damage signals can be further transferred to the downstream genes and their signal pathways more efficiently [2,7].

3.3. Outcomes of cellular responding radiotherapy

The P53-MDM2 feedback loop is a vital part in controlling the downstream genes and regulation pathways to fight against the genome stresses [6,67,68]. In response to the input signal of ATM* and ARF*, the P53-MDM2 module generates one or more oscillations. The response traces of P53 and MDM2 protein under continuous application of 2, 5, and 7Gy IR from time 0 are shown in Fig. 7a. Upon the activation by ATM*, ARF* and decreased degradation by MDM2, the total amount of P53 proteins increases quickly. Due to the P53-dependent induction of MDM2 transcription, the increase of MDM2 proteins is sufficiently large to lower the P53 level, which in turn reduces the amount of the MDM2 proteins.

The oscillation pulses shown in Fig.7a have a period of 400 min, and the phase difference between P53 and MDM2 is about 100 min. Moreover, the first pulse is slightly higher than the second, quite consistent with the experimental observations [2,7,11] as well as the previous simulation results [9,10,12,69].

Also, by comparing these simulation results, we can see that the strength and swing of these oscillations begin to decrease as IR overtakes 7Gy, suggesting that the ability of cellular responding genome stresses begin to decrease as IR dose exceeds a certain threshold.

Furthermore, because in the current model the toxins, mP53 and oncogenes can be degraded directly by P53* in this module, we can plot the predictable outcomes of cellular response in fighting against genome stresses under different IR dose domains. As shown in Fig.7b, Fw remaining within the cell keeps decreasing with respective rate, and trends to zero versus continuous radiation time under 2 and 5Gy IR. Whereas, when IR exceeds 7Gy, Fw begins to increase slightly with some oscillations. Also, the kinetics of oncogenes degrading is plotted in Fig.7c. As we can see, owing to the negative regulations of P53*, the expression level of oncogenes keeps decreasing after the first climate under 2 and 5Gy IR dose, and then begins to increase slowly under 7Gy IR dose. Meanwhile, as shown in Fig.7d, quite similar to the results in Fig.7b and Fig.7c, mP53 keeps decrease after reaching the first maximum under 2 and 5Gy IR dose, and then begins to increase slowly under 7Gy IR dose. All these results obtained by the above simulations based on the new model indicate that that P53* indeed acts an important role in regulating downstream genes and their signal pathways, whereas its capabilities in cellular responding DNA damage under radiotherapy begin to decrease as the strength of IR exceeds a certain maximal threshold.
the oscillating kinetics of P53* and MDM2 in response to different IR dose domains

(a) Radiation Time (Min)  Concentration

- P53* under IR=2Gy
- P53* under IR=5Gy
- P53* under IR=7Gy
- MDM2 under IR=2Gy
- MDM2 under IR=5Gy
- MDM2 under IR=7Gy

the kinetics of Fw elimination with the regulating functions of P53* under different IR

(b) Radiation Time (Min)  Concentration

- Fw under IR=2Gy
- Fw under IR=5Gy
- Fw under IR=7Gy
Figure 7. The outcomes of cellular responding 2, 5, 7Gy IR under radiotherapy. (a) The oscillating kinetics of P53* and MDM2 in response to the cooperative effect of ATM* and ARF* under different IR dose domains. (b) The kinetics of toxins elimination triggered by the functions of P53*. (c) The depressing dynamics of oncogenes over-expression with the regulations of P53*. (d) The kinetics of mP53 elimination triggered by the effect of P53*. 
4. Conclusion

A new model was proposed to simulate the P53 stress response network under radiotherapy. It is demonstrated according to our model that ATM and ARF exhibits a strong sensitivity and switch-like behavior in response to the number of DSBs, fully consistent with the experimental observations. Interestingly, it is shown in this study that after the DNA damage signals transferring, P53-MDM2 feedback loop will produce oscillations, then triggering the cellular self-defense mechanisms to degrade the toxins remaining within the cell, such as F$_w$, oncogenes, and mP53. Particularly, under different IR dose domains, the new model can reasonably predict outcomes of cellular response in fighting against genome stresses, and hence providing a framework for analyzing the complicated regulations of P53 stress response networks, as well as the mechanisms of the cellular self-defense under radiotherapy.

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5. References


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