Chapter 8

# A Parametric Model for Potential Evapotranspiration Estimation Based on a Simplified Formulation of the Penman-Monteith Equation

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Additional information is available at the end of the chapter

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## 1. Introduction

The estimation of evapotranspiration is a typical task in several disciplines of geosciences and engineering, including hydrology, meteorology, climatology, ecology and agricultural engineering. According to the specific area of interest, we distinguish between three different aspects of evapotranspiration, i.e. actual, potential and reference crop. In particular, the actual evapotranspiration refers to the quantity of water that is actually removed from a surface due to the combined processes of evaporation and transpiration. At the global scale, it is estimated that about 60% of precipitation reaching the Earth's terrain returns to the atmosphere by means of water losses due to evapotranspiration [1]. Obviously this percentage is not constant but exhibits significant variability both in space and time. In particular, in semiarid basins the annual percentage of evapotranspiration losses may be 70-80% of precipitation, while in cold hydroclimates this percentage is lower that 30%.

The actual evapotranspiration is one of the most difficult processes to measure in the field, except for experimental, small-scale areas, for which lysimeter observations can be assumed representative. For the spatial scale of practical applications (e.g., river basin), there is a growing interest on remote sensing technologies, which attempt to estimate actual evapotranspiration at regional scales, by combining ground measurements with satellite-derived data [2]. Otherwise, the only reliable method is based on the calculation of the water balance of the basin. Apparently, this approach is valid only when actual evapotranspiration is the single unknown component of the water balance. In addition, it can be implemented only for large time scales (annual and over-annual), for which storage regulation effects can be neglected. Alternatively, the spatiotemporal representation of the water balance can be evaluated through hydrological models. The latter simulate the main hydrological mechanisms of the river basin, using areal rainfall and potential evapotranspiration as input data. In order to obtain the actual evapotranspiration, which is output of the model, it is therefore essential to estimate the potential evapotranspiration across the basin.

Potential, as well as and crop reference evapotranspiration, are two key concepts, for which different definitions have been proposed, often leading to confusing interpretations [3]. The term "potential evapotranspiration", introduced by Thornthwaite [4], generally refers to the maximum amount of water that could evaporate and transpire from a large and uniformly vegetated landscape, without restrictions other than the atmospheric demand [5, 6]. On the other hand, the reference crop evapotranspiration (or reference evapotranspiration) is strictly defined as the evapotranspiration rate from a reference surface, not short of water, of a hypothetical grass crop with a height of 0.12 m, a fixed surface resistance of 70 s/m and an albedo of 0.23. This description closely resembles a surface of green, well-watered grass of uniform height, actively growing and completely shading the ground [7].

Potential and reference evapotranspiration can be theoretically retrieved on the basis of mass transfer and energy balance approaches. In this respect, a wide number of methods and models have been developed, of different levels of complexity. Lu *et al.* [6] report that there exist over 50 of such methods, which can be distinguished according to their mathematical framework and data requirements. The most integrated approaches are the analytical (also called combination) ones, which combine energy drivers, i.e. solar radiation and temperature, with atmospheric drivers, i.e. vapour pressure deficit and surface wind speed, towards a physically-based representation of the phenomenon [8]. In particular, Penman [9] developed the classical method named after him, which combines the energy balance and aerodynamic processes to predict the evaporation through open water, bare soil and grass. Later, Monteith [10] expanded the Penman equation to also account for the role of vegetation in controlling transpiration, particularly through the opening and closing of stomata.

The subsequently referred to as the Penman-Monteith method is by far the most recognized among all evapotranspiration models, as reported in numerous investigations worldwide. For instance, this approach provided the optimal estimates on the daily and monthly scales and was the most consistent across all locations [11]. It was also indicated that the Penman-Monteith method exhibited excellent performance in both arid and humid climates [12] and was the most accurate in estimating the water needs of turfgrass [13]. Its suitability was confirmed even when relative humidity and wind velocity data are missing [14].

The Penman-Monteith method requires complex calculations and it is data demanding. In this respect, a number of radically simplified methods have been proposed that require fewer meteorological variables. The latter can be divided into two categories: radiation-based approaches, which account for both the variability of solar radiation and temperature [15, 16, 17, 18], and elementary empirical methods that use as single meteorological input temperature [4, 19, 20]. A comprehensive review of them is provided in [21] and [22]. Yet, the predictive capacity of most of these models remains questionable, since they are based on empirical assumptions that do not ensure consistency with the physics of the natural phenomenon. Moreover, their applicability is usually restricted to specific geographical locations and climatic regimes, because their parameters are derived from experiments employed at the local scale.

Except for temperature, long records of input time series for the Penman-Monteith method are rarely available, especially in older meteorological stations. Thus, a typical problem is the estimation of evapotranspiration, in case of missing meteorological data. The Food and Agriculture Organization (FAO) emphatically suggests employing the Penman-Monteith method even under limited data availability. In this context, it provides a number of procedures for dealing with missing information, while it discourages the use of empirical approaches [7]. Yet, most of the proposed procedures still require meteorological data that are barely accessible, or require arbitrary assumptions with regard to the climatic regime of the study area.

Here we propose an alternative parametric approach, which is founded on a simplified parsimonious expression of the Penman-Monteith formula. The method can be assigned to radiation-based approaches, yet its key difference is the use of free variables (i.e. parameters) instead of constants, which are fitted to local climatic conditions. The method is parsimonious both in terms of meteorological data requirements (temperature) and with regard to the number of parameters (one to three).

For the identification of parameters, it is essential to have reference time series of satisfactory accuracy, preferably estimated through the Penman-Monteith method. Under this premise, the model parameters can be obtained through calibration, thus by minimizing the departures between the modelled evaporation data and the reference data. This task was applied in 37 meteorological stations that are maintained by the National Meteorological Service of Greece. As will be shown, the proposed method is clearly superior to commonly-used empirical approaches. Moreover, by mapping the parameter values over the entire Greek territory, using typical interpolation tools, we provide a flexible framework for the direct and reliable estimation of evapotranspiration anywhere in Greece.

The article is organized as follows: In section 2, we review the Penman-Monteith method and its simplifications, which estimate evapotranspiration on the basis of temperature and radiation data. In section 3 we present the new parametric model, which compromises the requirements for parsimony and consistency. In section 4, we calibrate the model at the point scale, using historical meteorological data, and evaluate it against other empirical approaches. In addition, we investigate the geographical distribution of its parameters over Greece. Finally, in section 5 we summarize the outcomes of our research and discuss next research steps.

## 2. The Penman-Monteith method and its simplifications

### 2.1. The Penman method and its physical background

Evaporation can be viewed both as energy (heat) exchange and an aerodynamic process. According to the energy balance approach, the net radiation at the Earth's surface ($R_n = S_n - L_n$, where $S_n$ and $L_n$ are the shortwave—solar—and longwave—earth—radiation, respectively) is mainly transformed to latent heat flux, $\Lambda$, and sensible heat flux to the air, $H$. The evaporation rate, expressed in terms of mass per unit area and time (e.g. kg/m²/d), is given by the ratio $E' := \Lambda / \lambda$, where $\lambda$ is the latent heat of vaporization, with typical value 2460 kJ/kg. By ignoring fluxes of lower importance, such as soil heat flux, the heat balance equation is solved for evaporation, yielding:

$$E' = (R_n - H) / \lambda = R_n / \lambda(1 + b) \tag{1}$$

where $b := H / \Lambda$ is the co-called Bowen ratio. The estimation of $b$ requires the measurement of temperature at two levels (surface and atmosphere), as well as the measurement of humidity at the atmosphere. On the other hand, the estimation of the net radiation $R_n$ is based on a radiation balance approach to determine the components $S_n$ and $L_n$. Typical input data required (in addition to latitude and time of the year), are solar radiation (direct and diffuse, or, in absence of them, sunshine duration data or cloud cover observations), temperature and relative humidity. The net radiation also depends of surface properties (i.e. albedo) and topographical characteristics, in terms of slope, aspect and shadowing. Recent studies proved that the impacts of topography are important at all spatial scales, although they are usually neglected in calculations [23].

From the aerodynamic viewpoint, evaporation is a mass diffusion process. In this context, the rate of evaporation is related to the difference in the water vapor content of the air at two levels above the evaporating surface and a function of the wind speed $F(u)$ in the diffusion equation. Theoretically, $F(u)$ can be computed on the basis of elevation, wind velocity, aerodynamic resistance and temperature. Yet, for simplicity it is usually given by empirical formulas, derived through linear regression, for a standard measurement level of 2 meters.

Penman [9] combined the energy balance with the mass transfer approaches, thus allowing the use of temperature, humidity and wind speed measurements at a single level. His classical formula for computing evaporation from an open water surface is written as:

$$E' = \frac{\Delta}{\Delta + \gamma} \frac{R_n}{\lambda} + \frac{\gamma}{\Delta + \gamma} F(u) D \tag{2}$$

where $\Delta$ is the slope of vapor pressure/temperature curve at equilibrium temperature (hPa/K), $\gamma$ is a psychrometrcic coefficient, with typical value 0.67 hPa/K, and $D$ is the vapor pressure deficit of the air (hPa), defined as the difference between the saturation vapor pressure $e_a$ and the actual vapor pressure $e_s$, which are functions of temperature and relative humidity. We remind that (2) estimates the evaporation rate (mass per unit area per day), which is expressed in terms of equivalent water depth by dividing by the water density $\rho$ (1000 kg/m³). Next we will use symbols $E'$ for evaporation rates, and $E := E' / \rho$ for equivalent depths per unit time.

Summarizing, the Penman equation requires records of solar radiation (alternatively, sunshine duration or cloud cover), temperature, relative humidity and wind speed, as well as a number of parameters accounting for surface characteristics (latitude, Julian day index, albedo, elevation, etc.).

### 2.2. The Penman-Monteith method

Penman's method was extended to cropped surfaces, by accounting for various resistance factors, aerodynamic and surface. As mentioned in the introduction, Monteith [10] introduced the concept of the so-called "bulk" surface resistance that describes the resistance of vapor flow through the transpiring crop and evaporating soil surface [7]. This depends on multiple factors, such as the solar radiation, the vapor pressure deficit, the temperature of leafs, the water status of the canopy, the height of vegetation, etc. Combining the two resistance components, the following generalized expression of the Penman equation is obtained, referred to as Penman–Monteith formula:

$$E' = \frac{\Delta}{\Delta + \gamma'} \frac{R_n}{\lambda} + \frac{\gamma}{\Delta + \gamma'} F(u) D \tag{3}$$

where:

$$\gamma' = \gamma(1 + r_s / r_a) \tag{4}$$

In the above relationship, $r_s$ and $r_a$ are the surface and aerodynamic resistance factors, respectively. For water, the surface resistance is zero, thus eq. (3) is identical to the Penman formula. For any other type of surface, the evapotranspiration can be analytically computed, provided that the two resistance parameters are known. In practice, it is extremely difficult to obtain parameter $r_s$ on the basis of typical field data, even more so because this is time-varying. Thus, its evaluation is only possible for some specific cases.

In this context, FAO proposed the application of the Penman–Monteith method for the hypothetical reference crop, thus introducing the concept of reference evapotranspiration. With standardized height for wind speed, temperature and humidity measurements at 2.0 m and the crop height of 0.12 m, the aerodynamic and surface resistances become $r_a = 208 / u_2$ (where $u_2$ is the wind velocity, in m/s) and $r_s = 70$ s/m. The experts of FAO suggested using the Penman–Monteith method as the standard for reference evapotranspiration and advised on procedures for calculation of the various meteorological inputs and parameters [7].

### 2.3. Radiation-based approaches

The complexity of the Penman–Monteith method stimulated many researchers to seek for alternative, simplified expressions, based on limited and easily accessible data. Given that the main sources of variability in evapotranspiration are temperature and solar radiation, the two variables are introduced in a number of such models, typically referred as radiation-based. We note that the classification to "radiation-based" instead of the more complete "radiation-temperature-based" is made for highlighting the difference with even simpler approaches that use temperature as single input variable.

A well-known simplification is the Priestley–Taylor formula [17], which is expressed in terms of equivalent depth (mm/d):

$$E = \alpha_e \frac{\Delta}{\Delta + \gamma} \frac{R_n}{\lambda \rho} \tag{5}$$

where $a_e$ is a numerical coefficient, with values from 1.26 to 1.28. Thus, in the above expression, the energy term of the Penman–Monteith equation is increased by about 30%, in order to skip over the aerodynamic term. This assumption allows for omitting the use of wind velocity and surface resistance in evapotranspiration calculations.

Despite its simplifications, the Priestley–Taylor method is still physically-based, as opposed to most of radiation-based approaches that are rather empirical. In addition, many of them use either total solar radiation or even extraterrestrial radiation, instead of net radiation data. For instance, Jensen & Haise [15], based on evapotranspiration measurements from irrigated areas across the United States, proposed the following expression:

$$E = \frac{R_a T_a}{40 \lambda \rho} \tag{6}$$

This equation only uses average daily temperature, $T_a$, and extraterrestrial radiation, $R_a$. Later, McGuiness & Bordne [16] suggested a slight modification of the Jensen–Haise expression, known as McGuinness model:

$$E = \frac{R_a \left(T_a + 5\right)}{68 \lambda \rho} \tag{7}$$

Another widely used approach is the Hargreaves model [18] that estimates the reference evapotranspiration at the monthly scale by:

$$E = 0.0023 \, (R_a / \lambda) \left(T_a + 17.8\right)\left(T_{max} - T_{min}\right)^{0.5} \tag{8}$$

where $T_a$ is the average monthly temperature and $T_{max} - T_{min}$ is the difference between the maximum and minimum monthly temperature (°C). This method provides satisfactory results, with typical error of 10-15% or 1.0 mm/d, and it is suggested when only temperature data is available [24].

Many studies proved the superiority of radiation-based approaches against temperature-based ones. For instance, Lu *et al.* [6] compared three models from each category across SE

United States and showed that radiation-based methods produced clearly better results, with reference to pan evaporation measurements. Oudin *et al.* [25] investigated the suitability of such approaches, as input to rainfall-runoff models. In this context, they evaluated a number of evapotranspiration methods, on the basis of precipitation and streamflow data from a large number of catchments in U.S., France and Australia. Within their research, they provided a generalized expression of the Jensen–Haise and McGuinness models, i.e.

$$E = \frac{R_a\left(T_a + K_2\right)}{K_1\,\lambda\rho} \tag{9}$$

where $K_1$ (°C) is a scale parameter and $K_2$ (°C) is a parameter related to the threshold for air temperature, i.e. the minimum value of air temperature for which evapotranspiration is not zero. The researchers tested several values of $K_1$ and $K_2$ for four well-known hydrological models, and kept the combination that gave the best streamflow simulations over the entire catchment sample. After extended analysis, they concluded that the parameters are model-specific. As general recommendation, they proposed the values $K_1$ = 100°C and $K_2$ = 5°C.

### 2.4. Temperature-based approaches

The most elementary approaches only use temperature data as input, while the radiation term is indirectly accounted for, by means of empirical or semi-empirical expressions. The most recognized method of this category is the Thornthwaite formula [4]:

$$E = 16\left(\frac{10T_a}{I}\right)^a\left(\frac{d}{12}\right)\left(\frac{N}{30}\right) \tag{10}$$

where $T_a$ is the average monthly temperature, *d* is the average number of daylight hours per day for each month, *N* is the number of days in the month, *I* is the so-called annual heat index, which is estimated on the basis on monthly temperature values, and *a* is another parameter, which is function of *I*. Initially, the method was applied in USA, but later it was also tested in a number of regions worldwide [26]. Today, this method is not recommended, since it results in unacceptably large errors. In dry climates, evapotranspiration is underestimated, while in wet climates it is significantly overestimated [27].

Another widely known temperature-based method is the Blaney & Criddle formula [19]:

$$E = 0.254p\left(32 + 1.8T_a\right) \tag{11}$$

where *p* is the mean daily percentage of annual daytime hours. The method estimates the reference crop evapotranspiration on a monthly basis, and has been widely applied in Greece for the assessment of irrigation needs. Yet, the Blaney–Criddle method is only suitable for rough estimations. Especially under "extreme" climatic conditions, the method is inaccurate: in windy, dry, sunny areas, the evapotranspiration is underestimated by up to 60%, while in calm, humid, clouded areas it is overestimated up to 40% [26]. For this reason, several improvements of the method have been proposed. A well-known one is the modified Blaney-Criddle formula by Doorenbos & Pruitt [28], which however require additional meteorological inputs, thus loosing the advantage of data parsimony.

Linacre [20] has also developed simplified formulas to estimate open water evaporation and reference evapotranspiration on the basis of temperature. For the calculation of evaporation, the Linacre formula is:

$$E = \frac{700\ (\mathrm{T} + 0.006z)\,/\,(100 - \varphi) + 15\ \left(T - T_d\right)}{\left(80 - T\right)} \tag{12}$$

where $z$ is the elevation, $\varphi$ is the latitude and $T_\mathrm{d}$ is the dew point, which is also approximately derived though air temperature. For the reference evapotranspiration, the constant 700 in numerator is replaced by 500. In Greece, the Linacre method generally seriously overestimates the evapotranspiration, and it is not recommended [33]. Its major drawback is the assumption of a one-to-one relationship between evaporation and radiation, which is not realistic.

We notify that all the above approaches are only valid for monthly or daily scales, which are sufficient for most of engineering applications. For shorter time scales (e.g. hourly), the estimation of reference evapotranspiration requires additional astronomical calculations as well as finely-resolved meteorological data. Empirical formulas for such time scales are also available in the literature, e.g. in [29].

## 3. Development of parsimonious parametric formulas for evapotranspiration modelling

### 3.1. The need for a parametric model

A typical shortcoming of most of simplified approaches (i.e. empirical models that use solar radiation and temperature data) for evapotranspiration modeling is their poor predictive capacity against the entire range of climatic regimes and locations. For, these models are built and validated in specific conditions. Thus, while they work well in the areas and over the periods for which they have been developed, they provide inaccurate results when they are applied to other climatic areas, without reevaluating the constants involved in the empirical formulas [30]. Hence, the key idea behind the development of a parametric model is the replacement of some of the constants that are used in such formulas by free parameters, which are regionally-varying. The optimal values of parameters can be obtained through calibration, i.e. by fitting model outputs to “reference” evapotranspiration time series. Nowadays, calibration is a rather straightforward task, thanks to the development of powerful global optimization techniques, such as evolutionary algorithms. The reference data can be provided either directly (e.g.

through remote sensing) or estimated by well-established methods (preferably the Penman-Monteith model). Optimization is carried out against a goodness-of-fit criterion, which aggregates the departures between the simulated and reference data (e.g. coefficient of efficiency).

In this context, a parametric expression can be used as a generalized regression formula, for infilling and extrapolating evapotranspiration records. This is of high importance, since in many conventional meteorological stations, the temperature records usually overlap the records of the rest of variables that are required by the Penman-Monteith method. The use of a parametric model, with parameters calibrated on the basis of reliable evapotranspiration data, is obviously preferable to empirical models, which cannot account for the whole available meteorological information, i.e. the past observations of humidity, solar radiation and wind velocity.

### 3.2. Ensuring parsimony and consistency

The need for parsimonious models is essential in all aspects of water engineering and management [31]. Generally, the concept of parsimony refers to both the model structure, which should be as simple as possible, and the input data, which should be as fewer as possible and easily available. Yet, as indicated in the Section 2, many of the empirical models for evapotranspiration estimations, although they are data parsimonious, fail to provide realistic results (see also Section 4 below), since they do not account for important aspects of the natural phenomenon. Therefore, before building the model it is crucial to ensure that simplicity will not be in contrast with consistency. This requires: (a) the choice of the most important explanatory variables in the equations, and (b) the determination of the number of parameters, which should be identified through calibration.

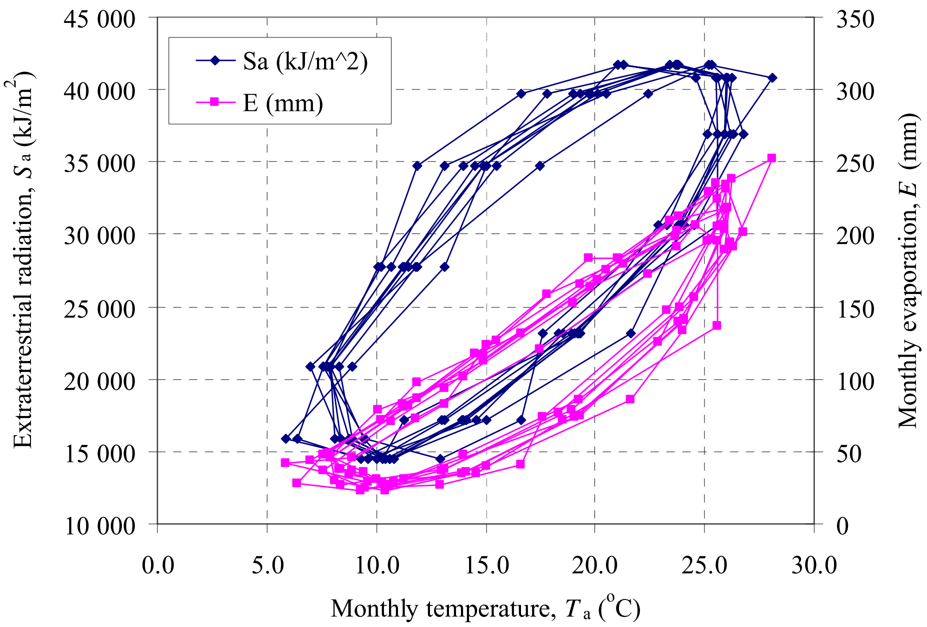


**Figure 1.** Scatter plots of mean monthly temperature data vs. extraterrestrial solar radiation and Penman evaporation (data from Agrinio station)

It is well-known that the variability of evapotranspiration is mainly explained by the variability of solar radiation and temperature. Yet, instead of measured solar radiation, which is rarely available, and following the practice of most of the empirical methods, we decided to use extraterrestrial radiation, together with temperature, as explanatory variables in the new formula. In order to justify this option, we examined several combinations of variables, using data from a number of meteorological stations in Greece. In the example of Figure 1 are illustrated the scatter plots of mean monthly temperature $T_a$, against extraterrestrial solar radiation $S_a$, and against monthly evaporation $E$, which is estimated through the Penman method. The data are from the Agrinio station, Western Greece, and cover a ten-year period (1979-1988). The graphical representation of $E$ vs. $T_a$ and $S_a$ vs. $T_a$ exhibit characteristic loop shapes, which clearly indicate that there is no one-to-one relationship between evapotranspiration and temperature, due to the influence of thermal inertia. Thus, to ensure consistent estimations, the knowledge of the temperature is not sufficient, but requires extraterrestrial radiation as additional explanatory variable.

Next, in order to determine the essential number of parameters, we took advantage of the associated experience with conceptual hydrological models. For, in lumped rainfall-runoff models, it is argued that no more than five to six parameters are adequate to describe the variability of streamflow [32]. Since the variability of monthly evapotranspiration is clearly less than of runoff, in the proposed model we restricted the maximum number of parameters to three. This number is reasonable, also because three are the missing meteorological variables. Within the case study, we also examine even more parsimonious formulations, using two or even one parameter.

### 3.3. Model formulation and justification

By dividing both the numerator and the denominator by $\Delta$, the Penman-Monteith equation (now expressed in mm/d) is written in the form:

$$E = \frac{1}{\lambda\rho} \frac{R_n + \gamma\lambda F(u) D}{1 + \gamma' / \Delta} \tag{13}$$

In the above expression, the numerator is the sum of a term related to solar radiation and a term related to the rest of meteorological variables, while the denominator is function of temperature. Koutsoyiannis & Xanthopoulos [33] proposed a parametric simplification of the Penman-Monteith formula, where the numerator is approximated by a linear function of extraterrestrial solar radiation, $R_a$ (kJ/m$^2$), while the denominator is approximated by a linear descending function of temperature $T_a$ (°C) i.e.:

$$E = \frac{aR_a + b}{1 - cT_a} \tag{14}$$

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The formula contains three parameters, i.e. $a$ (kg/kJ), $b$ (kg/m$^2$) and $c$ (°C$^{-1}$), to which a physical interpretation can be assigned. Since extraterrestrial solar radiation is the upper bound of net shortwave radiation, the dimensionless term $a^* = a / \lambda\rho$ represents the average percentage of the energy provided by the sun (in terms of $R_a$) and, after reaching the Earth's terrain, is transformed to latent heat, thus driving the evapotranspiration process. Parameter $b$ lumps the missing information associated with aerodynamic processes, driven by the wind and the vapour deficit in the atmosphere. Finally, the expression $1 - c\ T_a$ approximates the more complex $1 + \gamma' / \Delta$. We remind that $\gamma'$ is function of the surface and aerodynamic resistance (eq. 4) and $\Delta$ is the slope vapour pressure curve, which is function of $T_a$. As shown in Fig. 2, $T_a$ and $-(1 / \Delta)$ are highly correlated, thus justifying the adoption of the linear approximation.

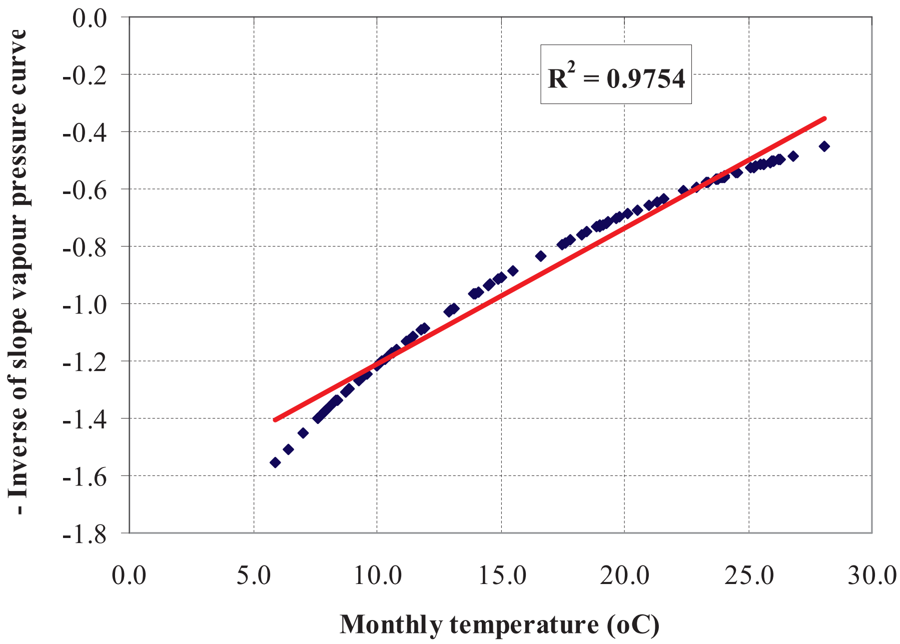


**Figure 2.** Scatter plot of $T_a$ and $-1 / \Delta$ (data from Agrinio station)

## 4. Application of the parametric formula over Greece

### 4.1. Meteorological data and computational tools

We used monthly meteorological data from 37 stations well-distributed over Greece, run by the National Meteorological Service of Greece. The locations of the stations are shown in Figure 3 and their characteristics are summarized in Table 1. Most of stations are close to the sea. Their latitudes range from 35.0° to 41.5°, while their elevations range from 2 to 663 m. The original data include mean temperature, relative humidity, sunshine percent (i.e. ratio of actual sunshine duration to maximum potential daylight hours per month) and wind velocity records. These records cover the period 1968-1989 with very few missing values, which have not been considered in calculations. At all study locations, the monthly potential evapotranspiration was calculated with the Penman-Monteith formula, assumed as reference model for the evaluation of the examined methodologies. The mean annual potential evapotranspiration values, shown in Table 2, range from 912 mm (Florina station, North Greece, altitude 662 m) to 1628 mm (Ierapetra station, Crete Island, South Greece).

The handling of the time series and the related calculations were carried out using the Hydrognomon software, a powerful tool for the processing and management of hydrometeorological data [34]. The software is open-access and freely available at http://www.hydrognomon.org/.

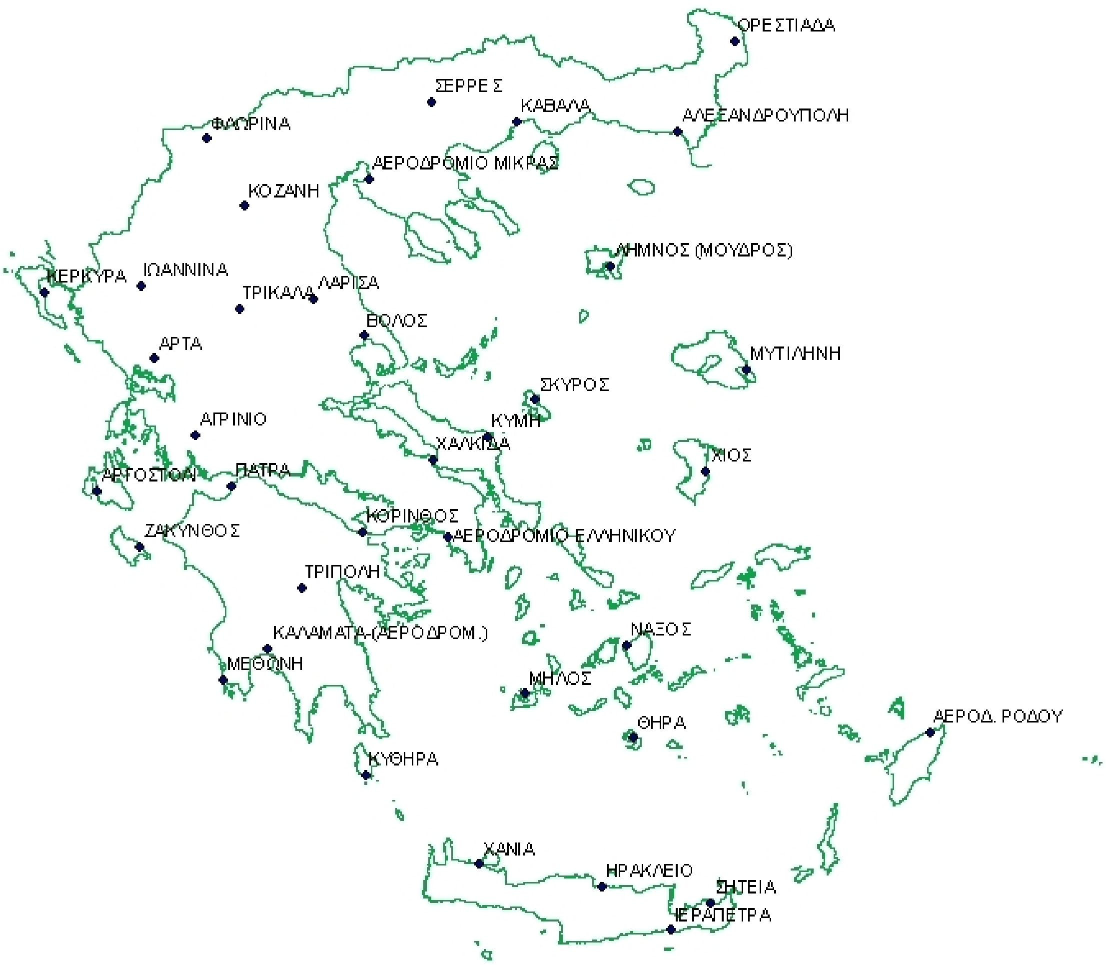


**Figure 3.** Locations of meteorological stations

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| No | Station | φ (°) | z (m) | No | Station | φ (°) | z (m) |
|---|---|---|---|---|---|---|---|
| 1 | Agrinio | 38.37 | 46 | 20 | Larisa | 39.39 | 74 |
| 2 | Alexandroupoli | 40.51 | 4 | 21 | Limnos | 39.54 | 17 |
| 3 | Argostoli | 38.11 | 2 | 22 | Methoni | 36.50 | 34 |
| 4 | Arta | 39.10 | 38 | 23 | Milos | 36.41 | 183 |
| 5 | Chalkida | 38.28 | 6 | 24 | Mytiline | 39.30 | 5 |
| 6 | Chania | 35.30 | 63 | 25 | Naxos | 37.60 | 9 |
| 7 | Chios | 38.20 | 4 | 26 | Orestiada | 41.50 | 48 |
| 8 | Florina | 40.47 | 662 | 27 | Patra | 38.15 | 3 |
| 9 | Hellinico | 37.54 | 15 | 28 | Rhodes | 36.24 | 37 |
| 10 | Heracleio | 35.20 | 39 | 29 | Serres | 41.40 | 35 |
| 11 | Ierapetra | 35.00 | 14 | 30 | Siteia | 35.12 | 28 |
| 12 | Ioannina | 39.42 | 484 | 31 | Skyros | 38.54 | 5 |
| 13 | Kalamata | 37.40 | 8 | 32 | Thera | 36.25 | 208 |
| 14 | Kavala | 40.54 | 63 | 33 | Thessaloniki | 40.31 | 5 |
| 15 | Kerkyra | 39.37 | 2 | 34 | Trikala | 39.33 | 116 |
| 16 | Korinthos | 38.20 | 15 | 35 | Tripoli | 37.32 | 663 |
| 17 | Kozani | 40.18 | 626 | 36 | Volos | 39.23 | 7 |
| 18 | Kymi | 38.38 | 221 | 37 | Zakynthos | 37.47 | 8 |
| 19 | Kythira | 36.10 | 167 | | | | |

**Table 1.** List of meteorological stations (latitude, $\varphi$, and elevation, $z$)

### 4.2. Model calibration and validation

The entire sample was split into two control periods, 1968-1983 (calibration) and 1984-1989 (validation). At each station, the three parameters of eq. (14) were calibrated against the reference potential evapotranspiration time series. This task was automatically employed via a least square optimization technique, embedded in the evapotranspiration module of Hydrognomon.

The optimized values of $a$, $b$ and $c$ were next introduced into the parametric model, which ran for the six-year validation period, in order to evaluate its predictive capacity against the Penman-Monteith method (detailed results are provided in [36]).

| No | MAPET (mm)> | CE (cal.) | CE (val.) | No | MAPET (mm) | CE (cal.) | CE (val.) |
|---|---|---|---|---|---|---|---|
| 1 | 1108.4 | 0.989 | 0.975 | 20 | 1039.0 | 0.987 | 0.980 |
| 2 | 1164.0 | 0.971 | 0.970 | 21 | 1345.5 | 0.964 | 0.971 |
| 3 | 1277.9 | 0.982 | 0.980 | 22 | 1286.4 | 0.962 | 0.970 |
| 4 | 1195.0 | 0.981 | 0.871 | 23 | 1461.9 | 0.972 | 0.980 |
| 5 | 1274.5 | 0.950 | 0.953 | 24 | 1458.6 | 0.988 | 0.970 |
| 6 | 1296.0 | 0.973 | 0.963 | 25 | 1459.0 | 0.975 | 0.980 |
| 7 | 1412.9 | 0.919 | 0.953 | 26 | 1028.4 | 0.981 | 0.970 |
| 8 | 911.7 | 0.967 | 0.960 | 27 | 1205.4 | 0.987 | 0.960 |
| 9 | 1476.6 | 0.983 | 0.980 | 28 | 1551.9 | 0.972 | 0.970 |
| 10 | 1521.8 | 0.980 | 0.980 | 29 | 997.8 | 0.982 | 0.970 |
| 11 | 1628.4 | 0.962 | 0.940 | 30 | 1477.9 | 0.985 | 0.990 |
| 12 | 939.5 | 0.987 | 0.980 | 31 | 1297.9 | 0.926 | 0.910 |
| 13 | 1221.0 | 0.983 | 0.980 | 32 | 1437.1 | 0.971 | 0.940 |
| 14 | 962.4 | 0.983 | 0.980 | 33 | 1157.8 | 0.983 | 0.980 |
| 15 | 1089.5 | 0.989 | 0.990 | 34 | 1118.7 | 0.973 | 0.970 |
| 16 | 1483.1 | 0.957 | 0.820 | 35 | 1133.5 | 0.942 | 0.950 |
| 17 | 979.7 | 0.982 | 0.980 | 36 | 1240.6 | 0.981 | 0.910 |
| 18 | 1307.3 | 0.975 | 0.910 | 37 | 1370.4 | 0.974 | 0.980 |
| 19 | 1508.2 | 0.957 | 0.970 | | | | |

**Table 2.** Mean annual potential evapotranspiration (MAPET) and coefficients of efficiency (CE) for calibration and validation periods

For both calibration and validation, we used as performance measure the coefficient of efficiency, given by:

$$CE = 1 - \frac{\sum_{t=1}^{N}(E_{PM}(t) - E(t))^2}{\sum_{t=1}^{N}(E_{PM}(t) - E_{mean})^2} \tag{15}$$

where $E_{PM}(t)$ and $E(t)$ are the potential evapotranspiration values of month $t$, computed by the Penman-Monteith method and the parametric relationship, respectively, $E_{mean}$ is the monthly average over the common data period, estimated by the Penman-Monteith formula, and $N$ is the sample size.

The coefficients of efficiency for all stations, during the two control periods are given in Table 2. For all stations the model fitting is very satisfactory, since the CE values are very high. Specifically, the average CE values of the 37 stations are 97.2% during calibration and 95.9% during validation, while their minimum values are 91.9 and 82.0%, respectively.

### 4.3. Comparison with other empirical methods

At all stations, we compared the performance of the parametric method (in terms of coefficients of efficiency) with two radiation-based approaches, i.e. the McGuiness model (eq. 7), and the generalized eq. (9), proposed by Oudin *et al.* [25]. In the latter, we applied the recommended values $K_1$ = 100°C and $K_2$ = 5°C. The distribution of the CE values for the two control periods are summarized in Table 3. The suitability of the parametric model is obvious, while the other two methods exhibit much less satisfactory performance. Moreover, the improvement of the empirical expression by Oudin *et al.* against the classical McGuiness model is rather marginal.

| | **Parametric** | | **McGuiness** | | **Oudin *et al.*** | |
|---|---|---|---|---|---|---|
| **CE** | Cal. | Val. | Cal. | Val. | Cal. | Val. |
| **>0.95** | 37 | 30 | 0 | 2 | 5 | 2 |
| **0.90-0.95** | 0 | 6 | 8 | 9 | 5 | 9 |
| **0.70-0.90** | 0 | 1 | 12 | 19 | 12 | 15 |
| **0.50-0.70** | 0 | 0 | 15 | 6 | 12 | 7 |
| **<0.50** | 0 | 0 | 2 | 1 | 3 | 4 |

**Table 3.** Distribution of CE values of radiation-based approaches

We also compared the performance of the new parametric approach against the most widely used methods in Greece, i.e. Thornthwaite, Blaney-Criddle, and Hargreaves. We remind that the first two are temperature-based, while the latter also uses extraterrestrial radiation and monthly average minimum and maximum temperature as inputs. The calculations of monthly potential evapotranspiration, using the Hydrognomon software, were made for ten representative stations, with different climatic characteristics. For each station Table 4 gives the coefficient of efficiency and the bias, i.e. the relative difference of the monthly average against the average of the Penman-Monteith method (both refer to the entire control period 1968-1988). The proposed formula is substantially more accurate, while all other commonly used approaches do not provide satisfactory results across all locations. The suitability of each method depends on local conditions. Thus, at each location, one method may be satisfactorily accurate, while another method may result to unacceptably large errors. In general, the Thornthwaite formula underestimates potential evapotranspiration by 30%, while the Blaney-Criddle method overestimates it by 30%. In stations that are far from the sea (Florina, Larisa), the deviation exceeds 50%. The Hargreaves method, although it uses additional inputs, totally fails to predict the potential evapotranspiration at some stations, resulting even in negative values of CE. On the other hand, the parametric method is impressively accurate, as indicated by both the CE values and the practically zero bias.

| Station | Thornthwaite | | Blaney-Criddle | | Hargreaves | | Parametric | |
|---|---|---|---|---|---|---|---|---|
| | CE | Bias | CE | Bias | CE | Bias | CE | Bias |
| Alexandroupoli | 0.747 | -0.287 | 0.645 | 0.321 | 0.935 | -0.055 | 0.973 | -0.006 |
| Chios | 0.440 | -0.384 | 0.725 | 0.153 | 0.568 | -0.274 | 0.898 | 0.006 |
| Florina | 0.883 | -0.200 | 0.339 | 0.557 | 0.842 | 0.200 | 0.966 | -0.002 |
| Heracleio | 0.068 | -0.402 | 0.807 | 0.113 | 0.170 | -0.341 | 0.981 | -0.001 |
| Kythira | 0.094 | -0.421 | 0.815 | 0.087 | -0.114 | -0.446 | 0.961 | 0.011 |
| Larisa | 0.920 | -0.167 | 0.413 | 0.504 | 0.843 | 0.227 | 0.986 | -0.004 |
| Milos | 0.180 | -0.406 | 0.806 | 0.116 | 0.105 | -0.398 | 0.974 | -0.009 |
| Patra | 0.686 | -0.258 | 0.381 | 0.379 | 0.942 | 0.046 | 0.987 | 0.009 |
| Thessaloniki | 0.816 | -0.250 | 0.614 | 0.363 | 0.937 | 0.003 | 0.983 | 0.002 |
| Tripoli | 0.711 | -0.303 | 0.651 | 0.325 | 0.957 | 0.088 | 0.948 | -0.008 |
| Average | 0.555 | -0.308 | 0.620 | 0.292 | 0.619 | -0.095 | 0.966 | 0.000 |

**Table 4.** Performance characteristics (efficiency, bias) of various evapotranspiration models at representative meteorological stations

### 4.4. Investigation of alternative parameterizations

In order to provide a further parsimonious expression, we investigated two alternative parameterizations by simplifying eq. (14). In the first approach, we omitted parameter $b$, thus using a two-parameter relationship of the form:

$$E = \frac{aR_a}{1 - cT_a} \tag{16}$$

The two parameters of eq. (16) were calibrated using as reference data the Penman-Monteith estimations. Next, we provided a single-parameter expression, in which we applied the spatially average value of $c$ (= 0.00234), i.e.:

$$E = \frac{aR_a}{1 - 0.00234T_a} \tag{17}$$

The two models (16) and (17) were evaluated on the basis of CE, for the entire control period (results not shown). Compared to the full expression (14), the reduction of CE is negligible when parameter $b$ is omitted, while the use of the simplest form (17) occasionally leads to a considerably reduced predictive capacity (in particular, in two out of 37 stations). This is because the spatial variability of *c* is very low (<10%), thus allowing for substituting the parameter by a constant, in terms of the average value of all stations. The statistical characteristics of the model parameters (average, standard deviation) for the three alternative expressions are given in Table 5.

| Parameter | Equation (14) | | Equation (16) | | Equation (17) | |
|---|---|---|---|---|---|---|
| | Average | St. dev. | Average | St. dev. | Average | St. dev. |
| *a* ($10^{-5}$ kg/kJ) | 5.486 | 1.487 | 6.362 | 0.913 | 6.257 | 0.735 |
| *b* (kg/m$^2$) | 0.1973 | 0.4330 | – | – | – | – |
| *c* (°C$^{-1}$) | 0.0241 | 0.0024 | 0.0234 | 0.0020 | – | – |

**Table 5.** Statistical characteristics of model parameters, for the three alternative expressions (for the 37 study locations)

### 4.5. Spatial variability of model parameters

For the simplified model (17), we investigated the spatial variability of its single parameter *a* (kg/kJ) over the Greek territory [35, 36]. Initially, we examined whether this parameter is correlated with two characteristic properties of the meteorological stations, i.e. latitude and elevation (Fig. 4). From the two scatter plots it is detected that parameter *a* is not correlated with elevation, while there is a notable reduction of the parameter value with the increase of latitude.

Next, we visualized the variation of the parameter over the Greek territory, using spatial integration procedures that are embedded in the ESRI ArcGIS toolbox. After preliminary investigations, we selected the Inverse Distance Weighting (IDW) method to interpolate between the known point values at the 37 meteorological stations, using a cell dimension of 0.5 km. The IDW method estimates the parameter values at the grid scale as the weighted sum of the point values, where the weights are decreasing functions of the distance between the centroid of each cell from the corresponding station. Since the parameter variability is not correlated with elevation, it was not necessary to employ more complex integration methods, such as co-kriging, which also account for the influence of altitude.

The mapping of parameter *a* over Greece through the IDW approach is illustrated in Fig. 5. We detect that the parameter values increase from SE to NW Greece. The physical interpretation of this systematic pattern is the increase of both the sunshine duration and the wind velocity as we move from the continental to insular Greece. A similar pattern appears for parameter *a* when accounting for the two-parameter expression (eq. 16). On the other hand, as shown in Fig. 6, the spatial distribution of *c* is irregular, which makes impossible to assign a physical interpretation. Obviously, this parameter is site-specific.

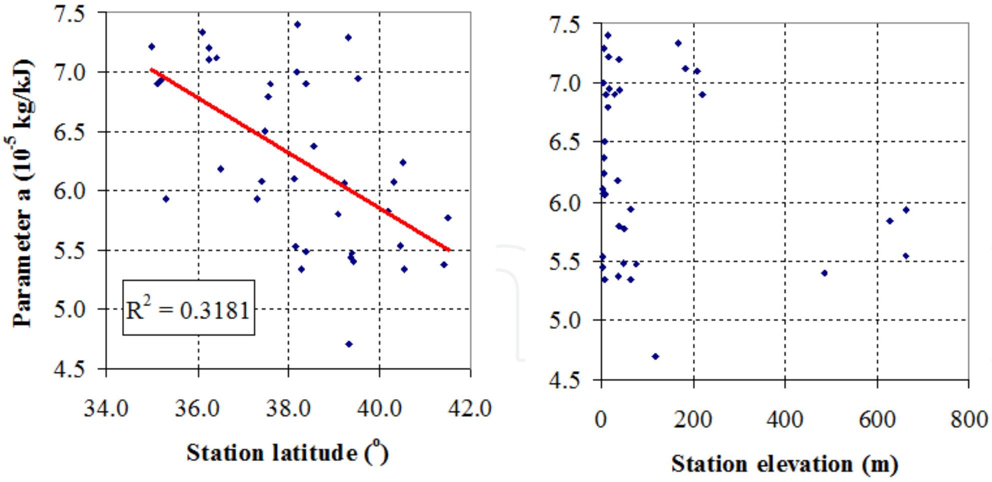


**Figure 4.** Scatter plots of parameter *a* (eq. 17) vs. latitude and elevation

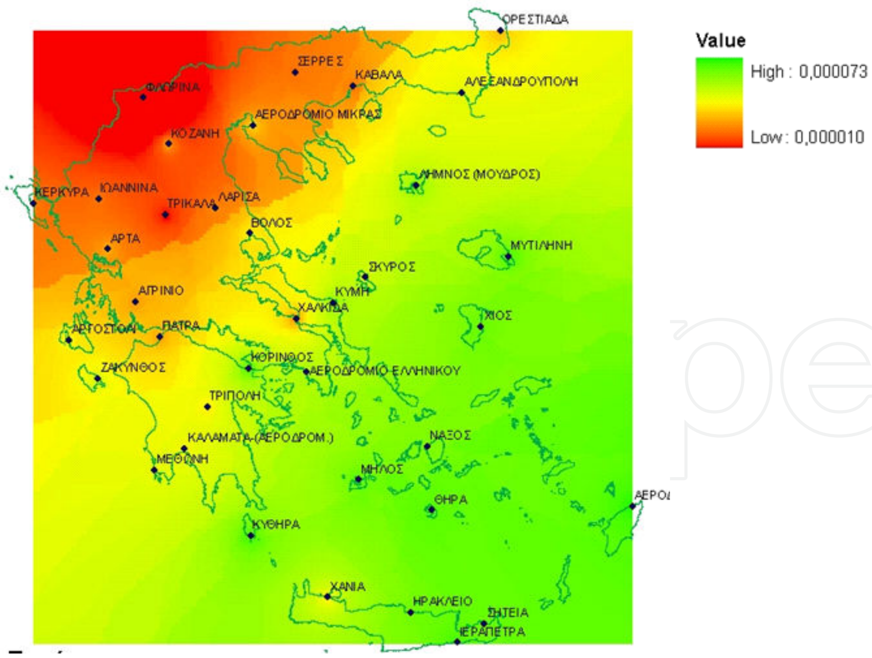


**Figure 5.** Geographical distribution of parameter *a* (eq. 17) over Greece

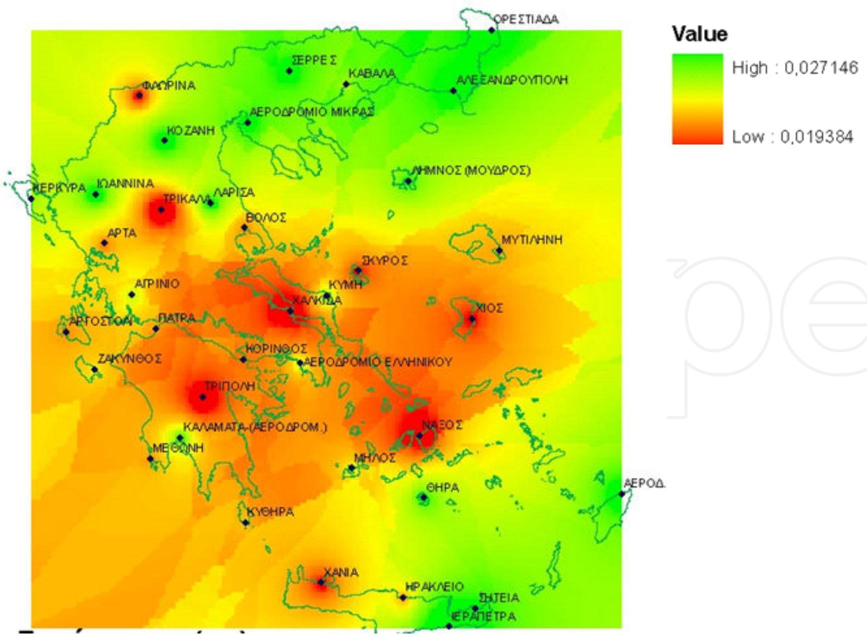


**Figure 6.** Geographical distribution of parameter *c* (eq. 16) over Greece

## 5. Synopsis and discussion

A parametric, radiation-based model for the estimation of potential evapotranspiration has been developed, which is parsimonious both in terms of data requirements and number of parameters. Its mathematical expression originates from consecutive simplifications of the Penman-Monteith formula, thus being physically-consistent. In its full form, the model requires three parameters to be calibrated. Yet, simpler formulations, with either one or two parameters, were also examined, without significant degradation of the model performance.

The model parameters were optimized on the basis of monthly evapotranspiration data, estimated with the Penman-Monteith method at 37 meteorological stations in Greece, using historical meteorological data for a 20-year period (1968-1988). The model exhibits excellent fitting to all locations. Its appropriateness is further revealed through extensive comparisons with other empirical approaches, i.e. two radiation-based methods and three temperature-based ones. In particular, common temperature-based methods that have been widely used in Greece provide rather disappointing results, while the new parametric model retains a systematically high predictive capacity across all study locations. This is the great advantage of parametric approaches against empirical ones, since calibration allows the coefficients that are involved in the mathematical formulas to be fitted to local climatic conditions.

The key model parameter was spatially interpolated throughout Greek territory, using a typical method in a GIS environment. The geographical distribution of the parameter exhibits a systematic increase from SE to NW, which is explained by the increase of sunshine duration and wind velocity as we move from the continental to insular Greece. Similar patterns were not found for the rest of model parameters. The practical value of this analysis is important, since it allows for establishing a specific formula for the estimation of potential evapotranspiration at any point. Thus, a minimalistic model of high accuracy is now available everywhere in Greece, which requires monthly temperature data as the only meteorological input. The other inputs are the monthly extraterrestrial radiation, which is function of latitude and time of the year, and a map of the parameter distribution.

To improve the model, it is essential to implement additional investigations, using a denser network of meteorological stations (especially stations in mountainous areas). In particular, it is necessary to examine the sensitivity of the various hydrometeorological variables that are involved in the evapotranspiration process, and provide alternative formulations for various combinations of missing meteorological data.

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