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The Effects of Hall and Joule Currents and Variable Properties on an Unsteady MHD Laminar Convective Flow Over a Porous Rotating Disk with Viscous Dissipation

Abdus Sattar and Mohammad Ferdows

Additional information is available at the end of the chapter

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1. Introduction

Heat transfer from convection in a rotating body is of theoretical as well as practical importance in the thermal analysis of rotating components of various types of mechanical devices. The rotating disk is one of a number of such geometrical configurations of rotating bodies which is of primary interest. Many practical systems can be modeled in terms of disk rotating in an infinite environment or in a housing. The importance of heat transfer from a rotating body can thus be ascertained in cases of many types of machineries, for example computer disk drives(Herrero *et al* 1994), rotating disk reactors for bio-fuel production and gas or marine turbines(Owen and Rogers 1989).

Heat transfer from a rotating disk by convection has been investigated theoretically by Wagner(1948), Millsaps and Pohlhausen(1952), Kreith and Taylor(1956), Kreith, Taylor and Chong(1959) and Sparrow and Gregg(1959). The theory thus established predicts that in the laminar flow regime heat and mass transfer coefficients are uniform over the entire surface of a rotating disk. Following pioneer treatment of von Karman(1921) for a flow over a rotating disk, an exact solution of complete Navier-Stokes' and energy equations was obtained by Millsaps and Pohlhausen for laminar convective flow. The rate of heat and mass transfer from a rotating disk at various speeds in an infinite environment in both laminar and turbulent flows were measured by Kreith, Taylor and Chong(1959). On the other hand Popiel and Boguslawski(1975) measured the heat transfer coefficient at a certain location over a disk rotating at different angular speeds.

The applied magnetic field effects on a steady flow due to the rotation of a disk of infinite or finite extent were studied by El-Mistikaway and Attia(1990) and El-Mistikaway *et al.* (1991). Aboul-Hassan and Attia(1997) also studied steady hydrodynamic flow due to an infinite disk rotating with uniform angular velocity in the presence of an axial magnetic field with Hall current. Attia(1998) separately studied the effects of suction as well as injection in the presence of a magnetic field on the unsteady flow past a rotating porous disk. It was observed by him that strong injection tends to destabilize the laminar boundary layer but when magnetic field works even with strong injection, it stabilizes the boundary layer.

The heat transfer phenomenon along with magneto-hydrodynamic effect on an unsteady incompressible flow due to an infinite rotating disk were studied by Maleque and Sattar(2003) using implicit finite difference scheme of Crank-Nicolson method. Later Maleque and Sattar(2005) investigated numerically the steady three-dimensional MHD free convective laminar incompressible boundary layer flow due to an infinite rotating disk in an axial uniform magnetic field taking into account the Hall current.

In classical treatment of thermal boundary layers, fluid properties such as density, viscosity, and thermal conductivity are assumed to be constant. But experiments indicate that the assumption of constant fluid property only makes sense if temperature does not change rapidly as far as application is concerned. To predict the flow behavior accurately, it may be necessary to take into account these properties as variables. It is of course known that these physical properties may change significantly with the change of temperature of the flow. Zakerullah and Ackroyd(1979) taking into account the variable properties analyzed the laminar natural convection boundary layer flow on a horizontal circular disk. Herwig(1985) analyzed the influence of variable properties on a laminar fully developed pipe flow with constant heat flux across the wall. He showed how the exponents in the property ratio method depend on the fluid properties. Herwig and Wikerer(1986) made a similar analysis in case of a wedge flow. In case of a fully developed flow in a concentric annuli, the effects of the variable properties have been investigated by Herwig and Klamp(1988). Maleque and Sattar(2002) , however, studied the effects of variable viscosity and Hall current on an unsteady MHD laminar convective flow due to a rotating disk. Similar unsteady hydromagnetic flow due to an infinite rotating disk was studied by Attia(2006) taking into account the temperature dependent viscosity in a porous medium with Hall and ion-slip currents. The effects of variable properties(density, viscosity and thermal conductivity) on the steady laminar convective flow due to a rotating disk were shown by Maleque and Sattar(2005a) while Maleque and Sattar(2005b) further investigated the same problem in presence of Hall current. Osalusi and Sibanda(2006) revisited the problem of Maleque and Sattar(2005b), considering magnetic effect. Osalusi *et al.*(2008) , however, considered an unsteady MHD flow over a porous rotating disk with variable properties in the presence of Hall and Ion-slip currents. Rahman(2010) recently made a similar study on the slip-flow with variable properties due to a porous rotating disk.

Most of the above studies were in cases of steady flows except few. The reason is that the theoretical treatment of unsteady problems is a difficult task. However, one can rely on the sophisticated numerical tools such as finite difference or finite element methods to solve the

unsteady problems but the solutions such obtained are non-similar. Problem of course arises when one tries to obtain similarity solutions of an unsteady flow. A similarity technique for unsteady boundary layer problems was thus fathered by Sattar and Hossain(1992), which has been incorporated here to investigate the effects of Hall and Joule currents on an unsteady MHD laminar convective flow due to a porous rotating disk with viscous dissipation.

2. Nomenclature

a, b, c arbitrary exponents

B_0 magnetic flux density

F dimensionless radial velocity

G dimensionless vertical velocity

H dimensionless tangential velocity

J_h Joule heating parameter

k thermal conductivity

k_∞ uniform condition of thermal conductivity

m Hall current

M magnetic parameter

p pressure of the fluid

p_∞ uniform condition of pressure

P_r Prandtl number

N_u Nusselt number

\mathbf{q} velocity vector

q_w rate of heat transfer

r radial axis

R_{ex} magnetic Reynolds number

R_e rotational Reynolds number

T temperature of the fluid

T_w uniform surface temperature

T_∞ free stream temperature

(u, v, w) velocity components along (r, ϕ, z) coordinates

U_0 mean velocity of the fluid

w_w uniform suction/injection velocity

z vertical axis

2.1. Greek letters

δ a time dependent length scale

- φ tangential axis
 γ relative temperature difference parameter
 λ a constant
 θ dimensionless temperature
 σ electrical conductivity
 η similarity variable
 τ_r, τ_t radial and tangential stresses
 ν kinematic coefficient of viscosity
 ν_∞ uniform condition of kinematic coefficient of viscosity
 μ viscosity
 μ_∞ uniform condition of viscosity
 ρ density of the fluid
 ρ_∞ uniform condition of density
 Ω angular velocity

3. Physical model

Let us consider a disk which is placed at $z = 0$ in a cylindrical polar coordinate system (r, φ, z) where z is the vertical axis and r and φ are the radial and tangential axes respectively. The disk is assumed to rotate with an angular velocity Ω and the fluid occupies the region $z > 0$. Let the components of the flow velocity $\mathbf{q} = (u, v, w)$ be in the directions of increasing (r, φ, z) respectively. Let p be the pressure, ρ be the density and T be the temperature of the fluid while the surface of the rotating disk is maintained at a uniform temperature T_w . For away from the wall, free stream is kept at a constant temperature T_∞ and at a constant pressure p_∞ . The fluid is assumed to be Newtonian, viscous and electrically conducting. An external magnetic field is applied in the z -direction having a constant magnetic flux density B_0 which is assumed unchanged by taking small magnetic Reynolds number ($R_{ex} \ll 1$). The electron-atom collision frequency is assumed to be relatively high, so that the Hall effect is assumed to exist. Geometry of the physical model is shown below.

The fluid properties viscosity (μ), thermal conductivity (k) and the density (ρ) are taken as functions of temperature alone and obey the following laws [Jayaraj (1995)]

$$\mu = \mu_\infty \left[\frac{T}{T_\infty} \right]^a, k = k_\infty \left[\frac{T}{T_\infty} \right]^b, \rho = \rho_\infty \left[\frac{T}{T_\infty} \right]^c \quad (1)$$

where a , b and c are arbitrary exponents and $\mu_\infty, k_\infty, \rho_\infty$ are the uniform conditions of viscosity, thermal conductivity and the density. For the present analysis the fluid considered in the flue gas. For flue gases the values of the exponent are $a = 0.7$, $b = 0.83$ and $c = -1.0$ (ideal gas).

Based on the above features , the Navier Stokes equations and Energy equation , which are the governing equations of the problem, due to unsteady axially symmetric, compressible MHD laminar flow of a homogenous fluid take the following form:

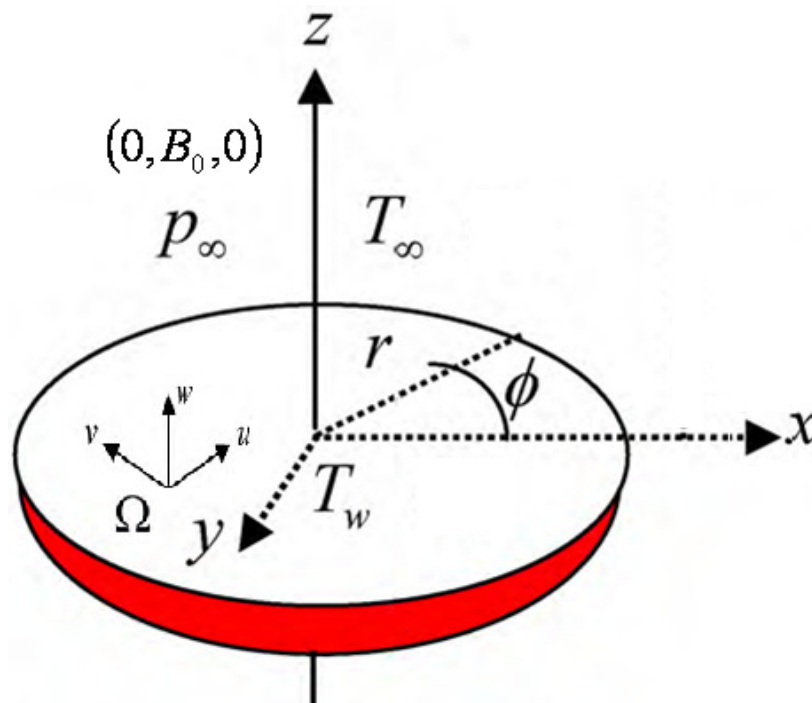
$$\frac{\partial}{\partial t}(\rho r) + \frac{\partial}{\partial r}(\rho r u) + \frac{\partial}{\partial z}(\rho r w) = 0 \quad (2)$$

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} - \frac{v^2}{r} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial r} + \frac{\partial}{\partial r} \left(\mu \frac{\partial u}{\partial r} \right) + \frac{\partial}{\partial r} \left(\mu \frac{u}{r} \right) + \frac{\partial}{\partial z} \left(\mu \frac{\partial u}{\partial z} \right) - \frac{\sigma B_0^2}{1+m^2} (u - mv) \quad (3)$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} - \frac{uv}{r} + w \frac{\partial v}{\partial z} \right) = \frac{\partial}{\partial r} \left(\mu \frac{\partial v}{\partial r} \right) + \frac{\partial}{\partial r} \left(\mu \frac{v}{r} \right) + \frac{\partial}{\partial z} \left(\mu \frac{\partial v}{\partial z} \right) - \frac{\sigma B_0^2}{1+m^2} (v + mu) \quad (4)$$

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial r} \left(\mu \frac{\partial w}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial r} (\mu w) + \frac{\partial}{\partial z} \left(\mu \frac{\partial w}{\partial z} \right) \quad (5)$$

$$\begin{aligned} \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \frac{\partial}{\partial r} \left(k \frac{\partial T}{\partial r} \right) + \frac{k}{r} \frac{\partial T}{\partial r} + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \frac{\sigma B_0^2}{1+m^2} (u^2 + v^2) + \\ + \mu \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right] \end{aligned} \quad (6)$$



Scheme 1. Geometry of the physical model

In the above equations (2)-(6), m represents the Hall current and in equation (6) the last two terms respectively represent magnetic and viscous dissipation terms.

The appropriate boundary conditions of the flow induced by the infinite disk ($z = 0$) which is started impulsively into steady rotation with constant angular velocity Ω and a uniform suction/injection velocity w_w through the disk are given by

$$\left. \begin{aligned} u = 0, v = \Omega r, w = w_w, T = T_w \text{ at } z = 0 \\ u \rightarrow 0, v \rightarrow 0, T \rightarrow T_\infty, p \rightarrow p_\infty \text{ as } z \rightarrow \infty. \end{aligned} \right\} \quad (7)$$

4. Similarity transformations

In order to tackle the unsteady character of the motion unlike other approaches for example that of Chamkha & Ahmed(2011), a new similarity parameter taken as a function of time is introduced as $\delta = \delta(t)$. Here δ is a time dependent length scale and is a new parameter that has been fathered by Sattar & Hossain(1992).

Hence to obtain similarity solutions of the above governing equations the following similarity transformations which are little deviated from the usual von-Karman transformations are introduced in terms of the similarity parameter δ :

$$\eta = \frac{z}{\delta}, u = r\Omega F(\eta), v = r\Omega H(\eta), w = r\Omega G(\eta), T = T_\infty + \Delta T\theta(\eta), p = p_\infty + 2\mu_\infty\Omega P(\eta), \quad (8)$$

where Ω is a constant angular velocity and $\Delta T = T_w - T_\infty$ and T_w is the temperature of the disk wall.

Following the laws in (1) the unsteady governing partial differential equations (2)-(6) are then transformed respectively to the following set of dimensionless nonlinear ordinary differential equations through the introduction of the transformations in (8):

$$c\gamma(1+\gamma\theta)^{-1} \left[R_e H\theta' - \frac{\delta}{v_\infty} \frac{d\delta}{dt} \eta\theta' \right] + 2R_e F + R_e H' = 0 \quad (9)$$

$$(1+\gamma\theta)^{c-a} \left[R_e (F^2 - G^2 + HF') - \frac{\delta}{v_\infty} \frac{d\delta}{dt} \eta F' \right] = \gamma a (1+\gamma\theta)^{-1} F'\theta' + F'' - \frac{M}{1+m^2} (F - mG)(1+\gamma\theta)^{-a} \quad (10)$$

$$(1+\gamma\theta)^{c-a} \left[R_e (2FG + HG') - \frac{\delta}{v_\infty} \frac{d\delta}{dt} \eta G' \right] = \gamma a (1+\gamma\theta)^{-1} G'\theta' + G'' -$$

$$\frac{M}{1+m^2}(G+mF)(1+\gamma\theta)^{-a} \quad (11)$$

$$\frac{\delta}{v_\infty} \frac{d\delta}{dt} (1+\gamma\theta)^{c-a} [H-H'] + P' = \gamma a (1+\gamma\theta)^{-1} H' \theta' + H'' \quad (12)$$

$$P_r (1+\gamma\theta)^{c-a} \left[R_e H \theta' - \frac{\delta}{v_\infty} \frac{d\delta}{dt} \eta \theta' \right] = \theta'' + \gamma b (1+\gamma\theta)^{-1} \theta^2 + \frac{M}{1+m^2} J_h P_r (G^2 + F^2) + J_h P_r (1+\gamma\theta)^{-a} (F'^2 + G'^2) \quad (13)$$

where $M = \frac{\sigma B_0^2 \delta^2}{\mu_\infty}$ is the magnetic parameter, $P_r = \frac{\rho_\infty v_\infty c_p}{k_\infty}$ is the Prandtl number, $R_e = \frac{\Omega \delta^2}{v_\infty}$ is the rotational Reynolds number, $J_h = \frac{r^2 \Omega^2}{c_p \Delta T}$ is the Joule heating parameter and $\gamma = \frac{\Delta T}{T_\infty}$ is the relative temperature difference parameter which is positive for heated surface and negative for cooled surface and zero for uniform properties.

The equations (10) to (14) are similar in time except for the term $\left(\frac{\delta}{v_\infty} \frac{d\delta}{dt} \right)$ where t appears explicitly. Thus the similarity conditions requires that $\left(\frac{\delta}{v_\infty} \frac{d\delta}{dt} \right)$ must be a constant. Hence following the work of Sattar & Hossain(1992) one can try a class of solutions of equations (10)-(14) by assuming

$$\frac{\delta}{v_\infty} \frac{d\delta}{dt} = \lambda \quad (\text{a constant}) . \quad (14)$$

Thus introducing (14) with the conditions that $\delta = 0$ when $t = 0$, one obtain

$$\delta = \sqrt{2\lambda vt} . \quad (15)$$

It thus appears from (15) that the length scale δ is consistent with the usual length scale considered for various non-steady flows(Schlichting,1958) . Since δ is a scaling factor as well as a similarity parameter, any value of λ in equation (15) would not change the nature of the solutions except that the scale would be different.

Now making a realistic choice of λ to be equal to 2 in equation (15), equations (9) to (13) finally become

$$c\gamma(1+\gamma\theta)^{-1} [R_e H \theta' - 2\eta \theta'] + 2R_e F + R_e H' = 0 \quad (16)$$

$$(1 + \gamma\theta)^{c-a} \left[R_e(F^2 - G^2 + HF') - 2\eta F' \right] = \gamma a(1 + \gamma\theta)^{-1} F' \theta' + F'' - \frac{M}{1+m^2} (F - mG)(1 + \gamma\theta)^{-a} \quad (17)$$

$$(1 + \gamma\theta)^{c-a} \left[R_e(2FG + HG') - 2\eta G' \right] = \gamma a(1 + \gamma\theta)^{-1} G' \theta' + G'' - \frac{M}{1+m^2} (G + mF)(1 + \gamma\theta)^{-a} \quad (18)$$

$$2(1 + \gamma\theta)^{c-a} \left[H - H' \right] + P' = \gamma a(1 + \gamma\theta)^{-1} H' \theta' + H'' \quad (19)$$

$$P_r((1 + \gamma\theta)^{c-b} \left[R_e H \theta' - 2\eta \theta' \right]) = \theta'' + \gamma b(1 + \gamma\theta)^{-1} \theta'^2 + \frac{M}{1+m^2} J_h P_r (G^2 + F^2) + J_h P_r (1 + \gamma\theta)^{-a} (F'^2 + G'^2) \quad (20)$$

With reference to the transformations (8), the boundary conditions (7) transform to

$$\begin{aligned} F(0) = 0, G(0) = 0, H(0) = W_s, \theta(0) = 1 \\ F(\infty) = 0, G(\infty) = 0, p(\infty) = 0, \theta(\infty) = 0 \end{aligned} \quad (21)$$

where, $W_s = \frac{w_w}{\Omega}$ represents a uniform suction ($W_s < 0$) or injection ($W_s > 0$) at the surface.

The quantities which are of physical interest relevant to our problem are the local skin-friction coefficients (radial and tangential) and the local Nusselt number.

Now since the radial (surface) and tangential stresses are respectively given by

$$\tau_r = \left[\mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right) \right]_{z=0} \quad \text{and} \quad \tau_t = \left[\mu \left(\frac{\partial v}{\partial z} + \frac{1}{r} \frac{\partial w}{\partial \phi} \right) \right]_{z=0} ,$$

the dimensionless radial and tangential skin-friction coefficients are respectively obtained as

$$\frac{U_0^2 \delta}{v_\infty \gamma \Omega} (1 + \gamma\theta)^{-a} = F'(0) \quad (22)$$

$$\frac{U_0^2 \delta}{v_\infty \gamma \Omega} (1 + \gamma\theta)^{a-c} = G'(0) \quad (23)$$

where U_0 is taken to be a mean velocity of the flow.

Again the rate of heat transfer from the disk surface to the fluid is given by

$$q_w = - \left(k \frac{\partial T}{\partial z} \right)_{z=0} .$$

Hence the Nusselt number defined by

$$N_u = \frac{\delta q_w}{k\Delta T}$$

is obtained as

$$N_u = -\theta'(0).$$

5. Numerical method

The nonlinear coupled ordinary differential equations (16) to (20) with the boundary conditions (21) have been solved numerically applying Natchtsheim-Swigert(1965) iteration technique(for detailed discussion of the method see Maleque and Sattar(2002)) along with sixth-order Runge-Kutta integration scheme. A step size of $\Delta\eta = .01$ was selected to be satisfactory for a convergence criteria of 10^{-7} . The value of η_∞ was found to each iteration loop by the statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞ was determined when the value of the unknown boundary conditions at $\eta = 0$ does not change to successful loop with an error less than 10^{-7} .

6. Steady case

When the flow is steady, δ is no longer a function of time rather can be considered to be a characteristic length scale such as L . Thus in equations (9) to (13) we can take

$$\frac{d\delta}{dt} = \frac{dL}{dt} = 0.$$

Thus putting $\frac{d\delta}{dt} = 0$ in equations (9) to (13) we obtain the following equations:

$$H' + 2F + c\gamma(1 + \gamma\theta)^{-1}H\theta = 0 \quad (24)$$

$$F'' + a\gamma(1 + \gamma\theta)^{-1}F'\theta' - [R_L(F^2 - G^2 + HF')] (1 + \gamma\theta)^{c-a} - \frac{M}{1+m^2}(F - mG)(1 + \gamma\theta)^{-a} = 0 \quad (25)$$

$$G'' + a\gamma(1 + \gamma\theta)^{-1}G'\theta' - [R_L(2FG + HG')] (1 + \gamma\theta)^{c-a} - \frac{M}{1+m^2}(G + mF)(1 + \gamma\theta)^{-a} = 0 \quad (26)$$

$$H' + \gamma a(1 + \gamma\theta)^{-1}H\theta' - P' = 0 \quad (27)$$

$$\theta'' + b\gamma(1 + \gamma\theta)^{-1}\theta^2 - P_r R_L(1 + \gamma\theta)^{c-a} + \frac{M}{1+m^2}J_h P_r(G^2 + F^2) + J_h P_r(1 + \gamma\theta)^{-a}(F'^2 + G'^2) = 0 \quad (28)$$

In the above equations

$$R_L = \frac{\Omega L^2}{\nu_\infty}.$$

The above equations exactly correspond to those of Maleque and Sattar(2005a) , therefore the solutions to the above equations have not been explored here for brevity. However, numerical values of the radial , tangential and rate of heat transfer coefficients for three different values of the relative temperature difference parameter γ is presented in Table-1 and compared with those of Maleque and Sattar(2005a).

γ	$F'(0)$	Maleque and Sattar(2005a)	$-G'(0)$	Maleque and Sattar(2005a)	$-\theta'(0)$	Maleque and Sattar(2005a)
-0.5	0.457	0.468	2.084	2.086	0.868	0.867
0.0	0.371	0.372	1.234	1.233	0.721	0.720
0.5	0.167	0.168	0.624	0.622	0.559	0.559

Table 1. Values of $F'(0)$, $-G'(0)$ and $-\theta'(0)$ for various values of γ when

$$R_L = 1 , J_h = 0 , \lambda = 0 , m = M = 0.1 .$$

Although the comparison should show exact values, due to the differences in the present code and that of Maleque and Sattar(2005a) there are differences in the calculated values of $F'(0)$, $-G'(0)$, and $-\theta'(0)$. Percentage wise differences have therefore been calculated and found to be maximum 2.35% and minimum 0.0% w.r.t three decimal places of the calculated values which shows a good agreement between our calculated results and that of Maleque and Sattar(2005a).

7. Unsteady solutions

As a result of the numerical calculations the radial, tangential and axial velocity profiles and temperature profiles are displayed in Figures 1-24 for various values of the governing parameters. In the analysis the fluid considered is flue gas for which $P_r = 0.64$ and the values of the exponents a, b and c are taken to be as $a = 0.7$, $b = 0.83$ and $c = -1.0$.

Variation of the radial, tangential and axial velocity profiles and temperature profiles under the influence of γ are shown in Figures 1-4. From Fig.1 it can be seen that due to the existence of the centrifugal force the radial velocity attains maximum values $0.23_{\gamma=-0.5}$, $0.13_{\gamma=0.0}$ and $0.10_{\gamma=0.5}$ close to the surface of the disk(approximately at $\eta = 0.75$). Thus at $\eta = 0.75$ the boundary layer thickness of the surface of the disk is reduced due to the increase in γ . From Fig-2, it is seen that the tangential velocity profile decreases in the interval $\eta \in [0, 0.75]$, but for

$\eta > 0.75$ this situation breaks down and the consequence is that the tangential velocity increases with the increase of γ . From Fig-3, it is seen that close to the disk surface γ has a tendency to reduce the motion and induce more flow far from the boundary indicating that there is a

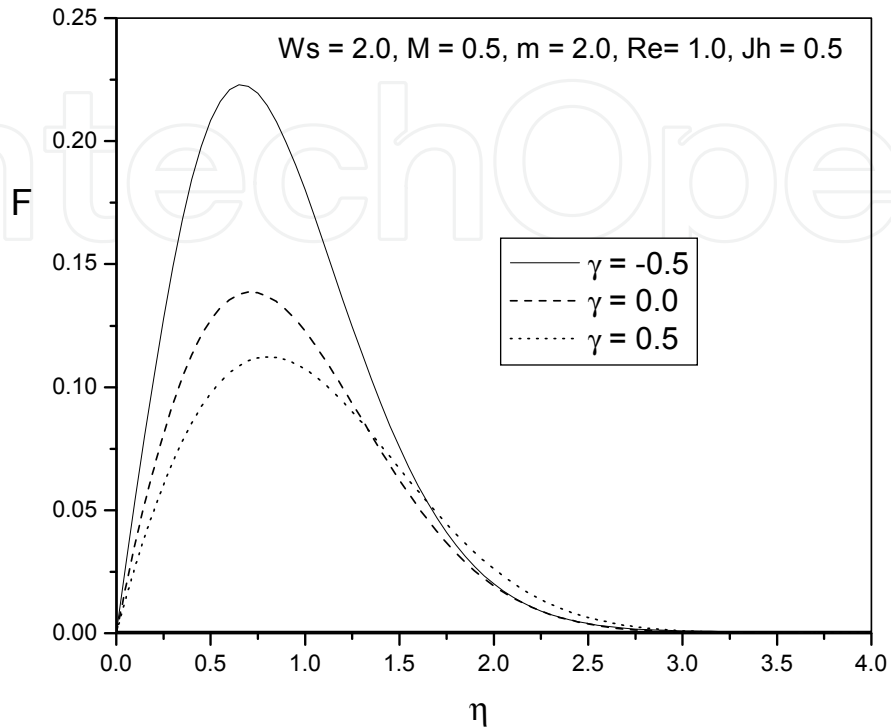


Figure 1. The dimensional radial velocity profiles against η for different values of γ

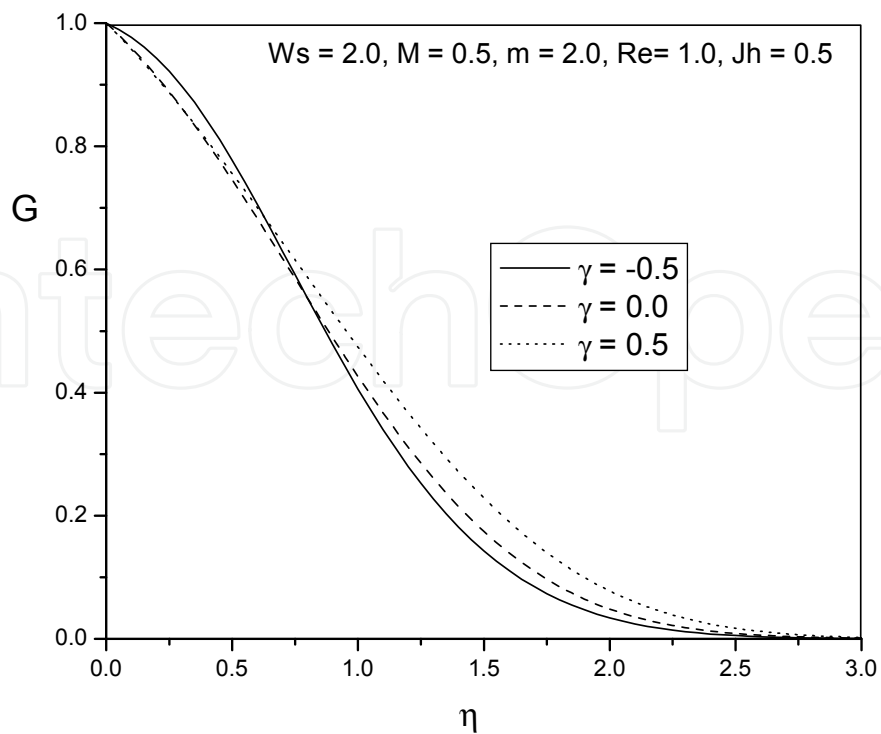


Figure 2. The dimensional tangential velocity profiles against η for different values of γ

separation flow and is detected at $\eta = 0.75$ (approximately). On the other hand, from Fig-4 it is observed that there is a small rate of decrease of the temperature close to the surface and then the temperature distributions starts increasing with increasing γ . This means that the thermal boundary layer induces more flow far from the surface of the disk.

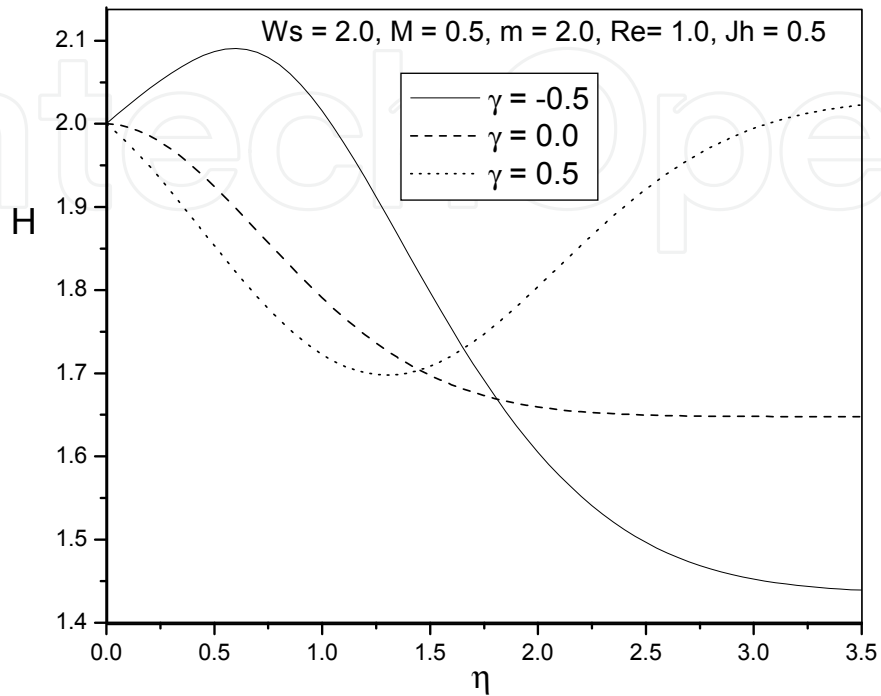


Figure 3. The dimensional axial velocity profiles against η for different values of γ

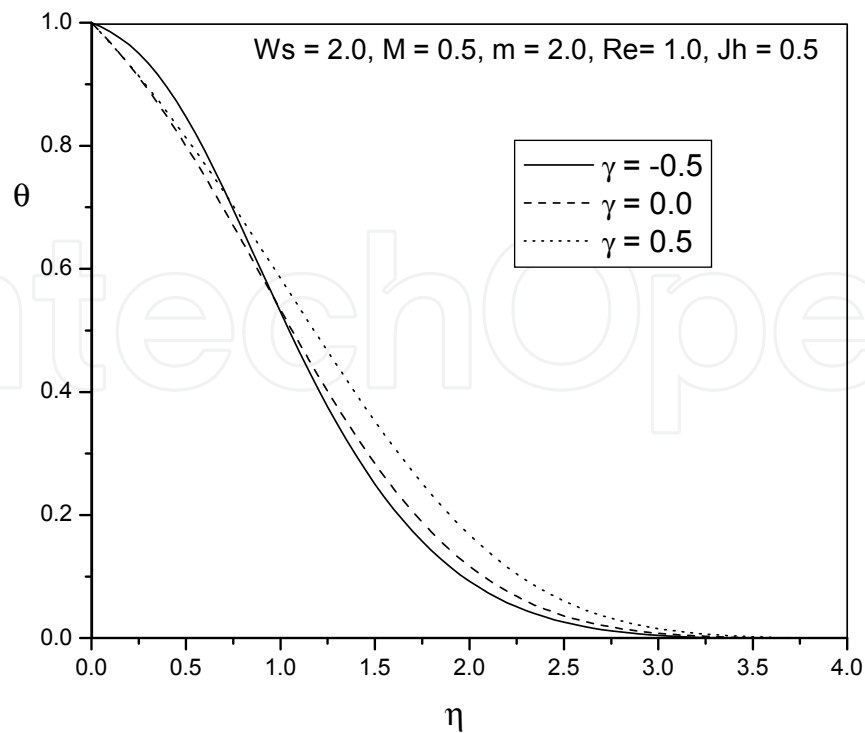


Figure 4. The dimensional temperature profiles against η for different values of γ

Figures 5-8, present the effects of uniform suction as well as injection(W_s) on the flow, which characterizes the flow behavior. It is evident from these figures that the boundary layer is increasingly blown away from the disk to form an interlayer between the injection and the outer flow regions. Also all flow profiles increase monotonically with increasing W_s . From Fig.5 it is apparent that radial velocity in this case attains maximum values approximately $0.19|_{W_s=4,\eta=1}$, $0.11|_{W_s=2,\eta=0.75}$, $0.05|_{W_s=0,\eta=0.5}$, $0.03|_{W_s=-2,\eta=0.25}$. This implies that the momentum boundary layer thickness decreases due to an increase in the values of W_s in different regions like $\eta = 1, 0.75, 0.5, 0.25$. Thus reduced flows are observed for increase in injection ($W_s < 0$) and induced flows are observed for increase in suction ($W_s > 0$) in the total flow behavior.

The effects of the magnetic parameter M on the radial, tangential and axial velocities and temperature profiles are depicted in Figures 9-12. We see that the radial velocity increases with the increase in M . It can also be seen that at each value of M there exists local maxima in radial velocity distributions. The maximum values of velocities are approximately $0.13|_{M=1,\eta=1}$, $0.11|_{M=0.5,\eta=1}$, $0.08|_{M=0,\eta=1}$. Thus we can say that at $\eta = 1$ the boundary layer thickness increases. From Fig.-10, we see that tangential velocity decreases with increasing M . On the other hand, axial velocity decreases with increasing M and seen that at each value of M there exists local minima in the profiles. The effect of M is found to be almost not significant for temperature distributions.

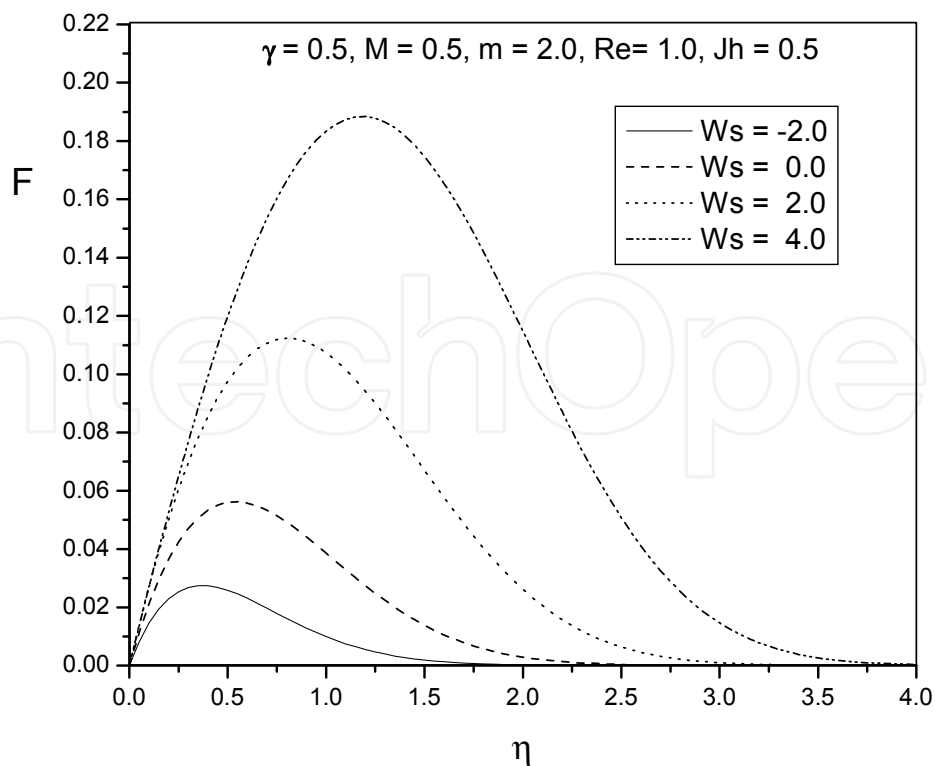


Figure 5. Fig-5: The dimensional radial velocity profiles against η for different values of W_s .

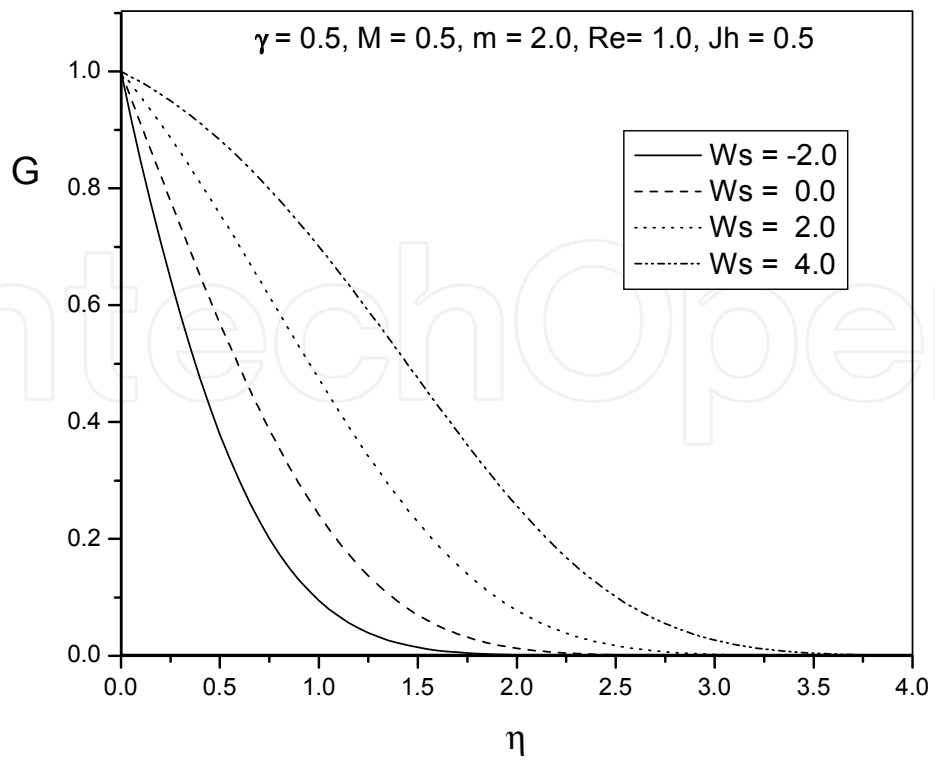


Figure 6. The dimensional tangential velocity profiles against η for different values of W_s .

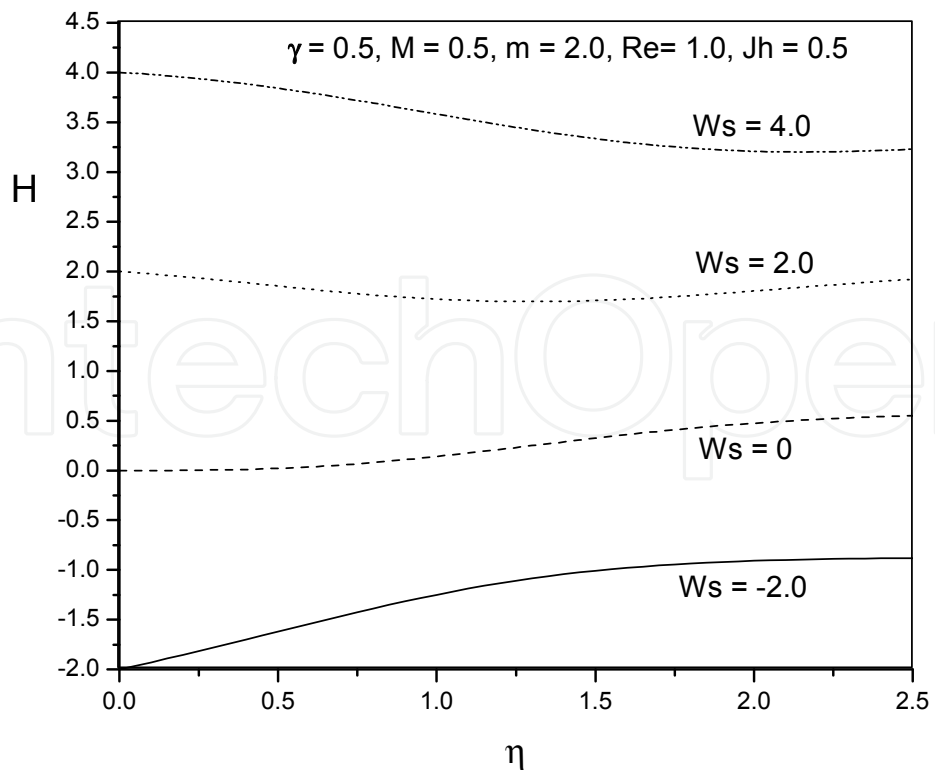


Figure 7. The dimensional axial velocity profiles against η for different values of W_s .

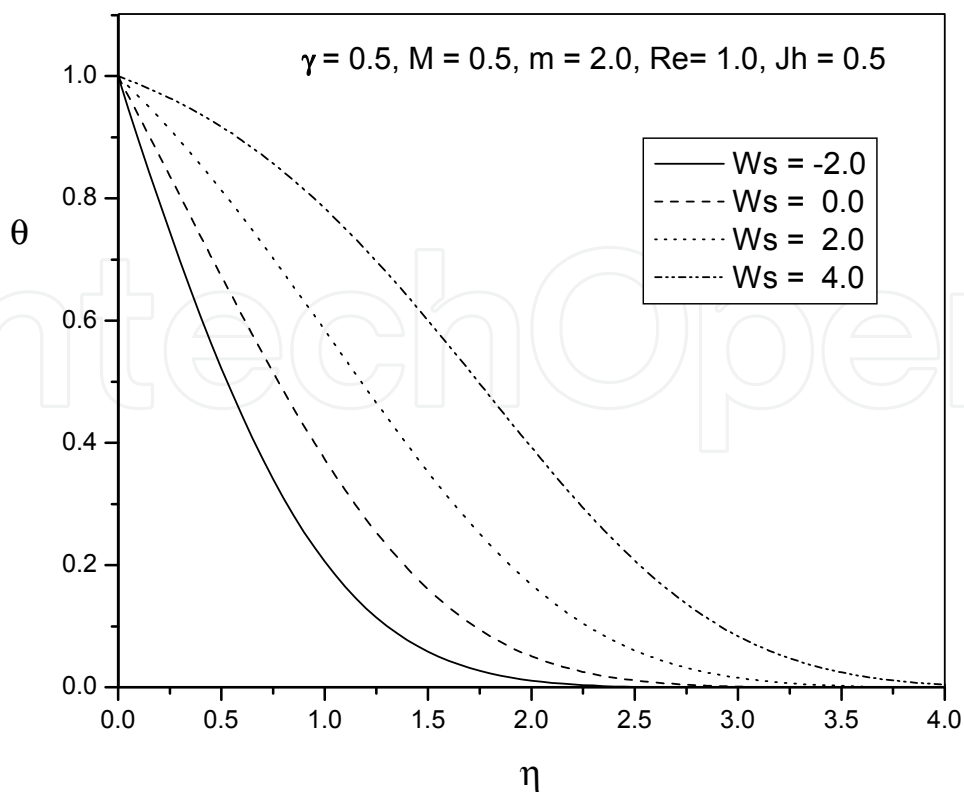


Figure 8. The dimensional temperature profiles against η for different values of W_s .

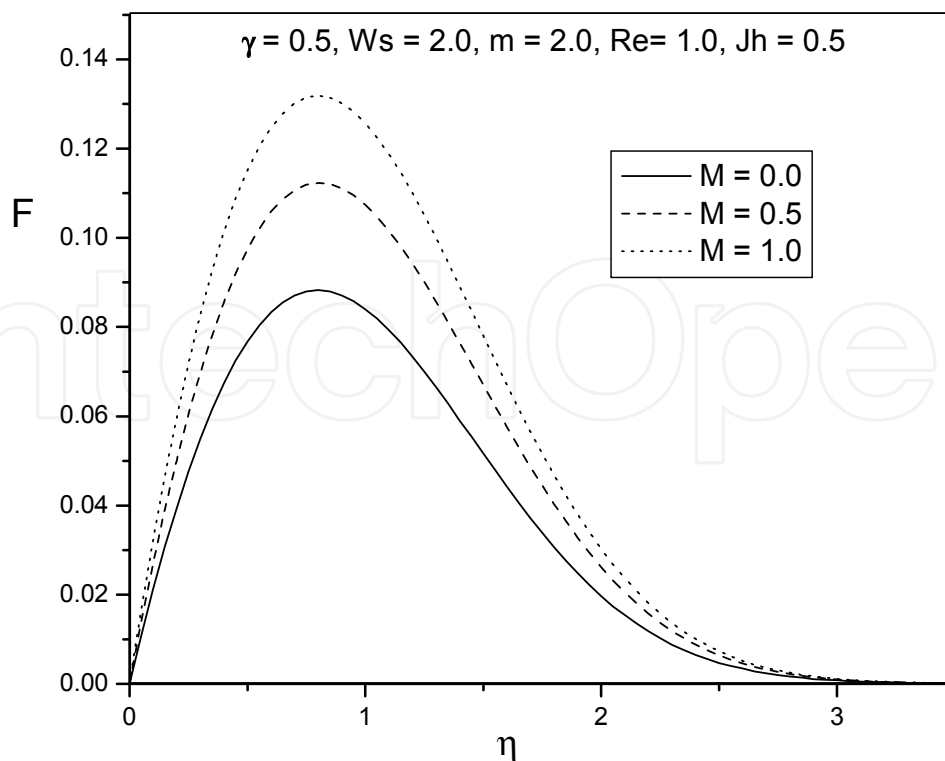


Figure 9. The dimensional radial velocity profiles against η for different values of M .

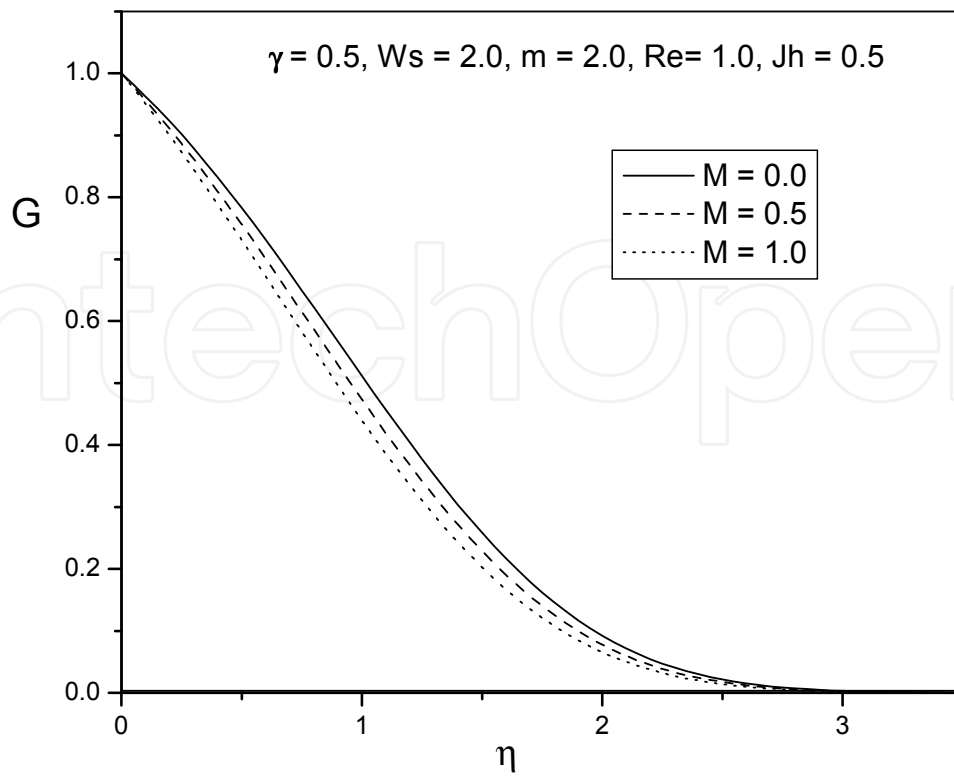


Figure 10. The dimensional tangential velocity profiles against η for different values of M .

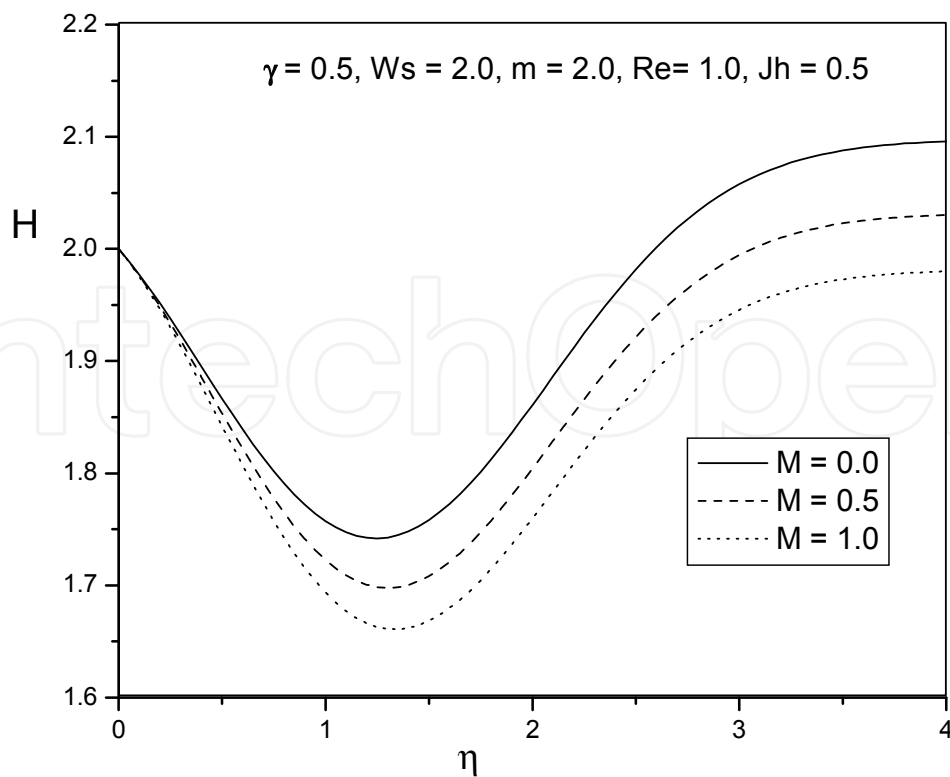


Figure 11. The dimensional axial velocity profiles against η for different values of M .

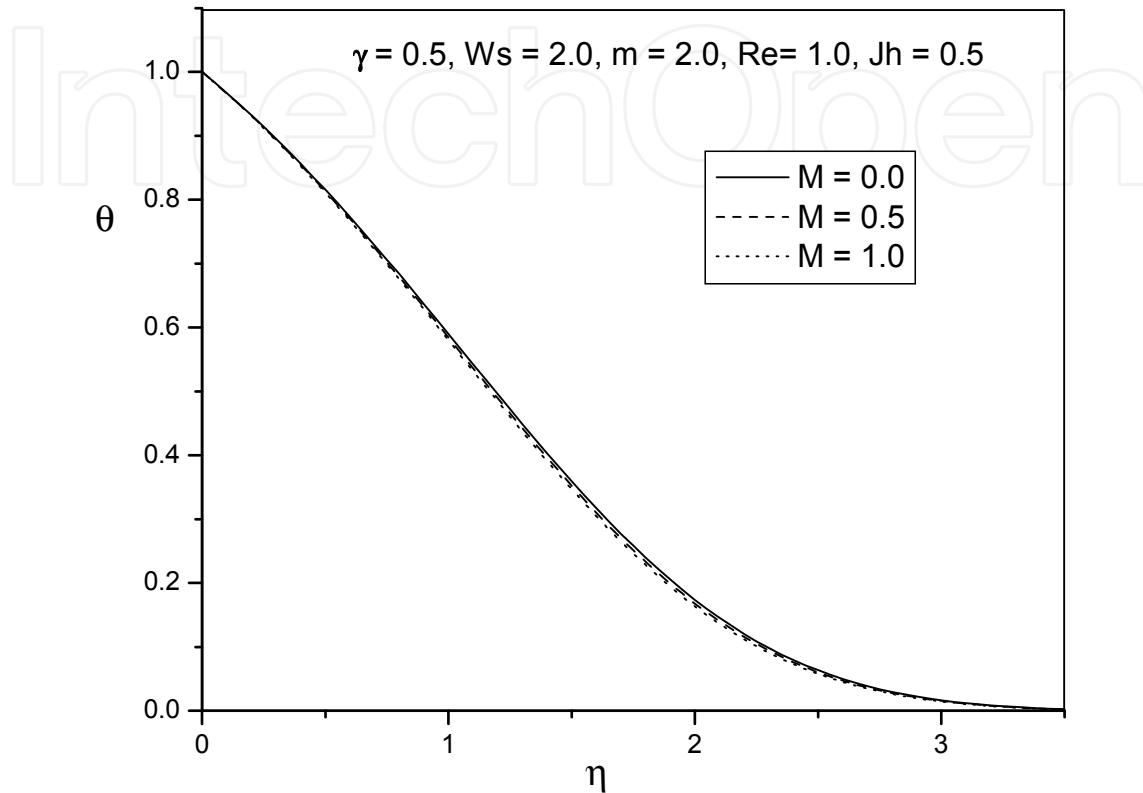


Figure 12. The dimensional temperature profiles against η for different values of M .

The effects of varying the Hall current parameter m on the flow distributions are shown in Figures 13-16. It can be seen that the radial velocity distributions increases with increasing m . This is due to the fact that for large values of m , the term $\frac{1}{1+m^2}$ is very small and hence the resistive effect of the magnetic field is diminished. This phenomenon for small and large values of m has been effectively explained by Hassan and Attia(1997). The maximum velocities are approximately $0.12|_{m=50,\eta=1}, 0.10|_{m=5,\eta=1}, 0.9|_{m=2,\eta=1}, 0.05|_{m=0,\eta=1}$. Thus at $\eta=1$ the boundary layer thickness increases due to the increase in m . Tangential distribution increases with the increase of m . On the other hand, the axial velocity decrease with increasing m and shows local minima indicating that the boundary layer thickness decreases. Like the magnetic parameter M the effect of m is also found to be not much significant in case of temperature distributions.

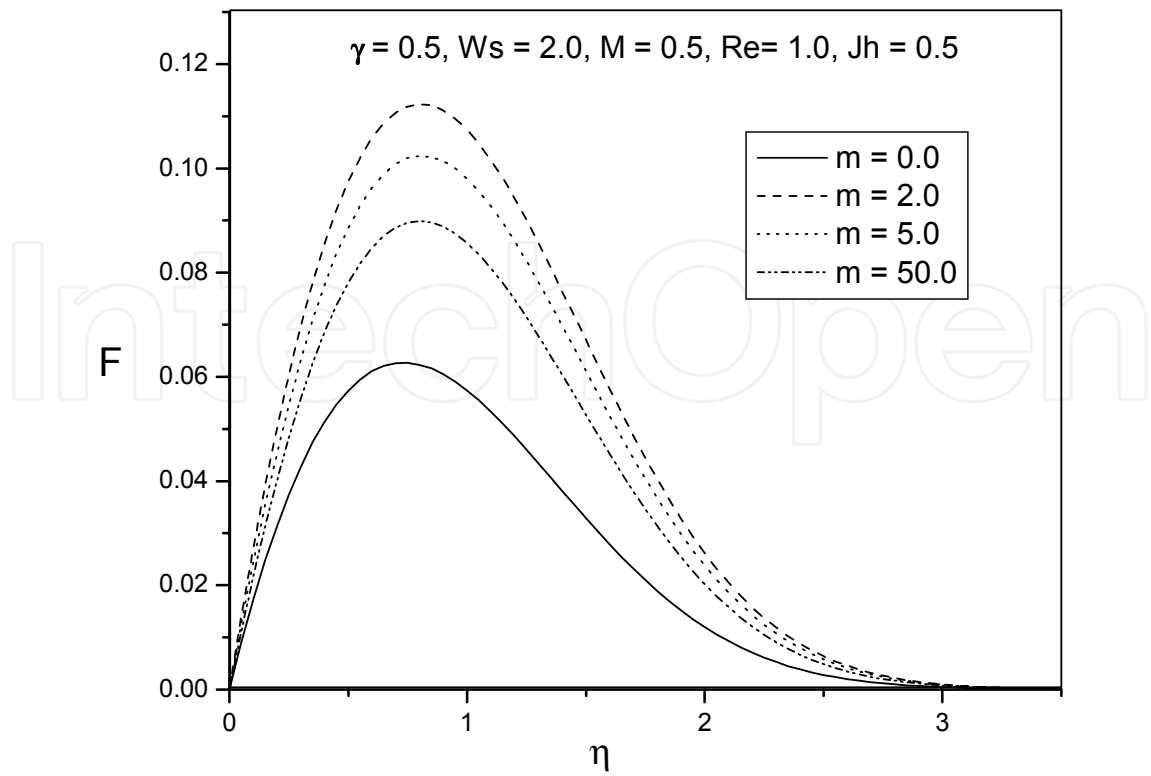


Figure 13. The dimensional radial velocity profiles against η for different values of m .

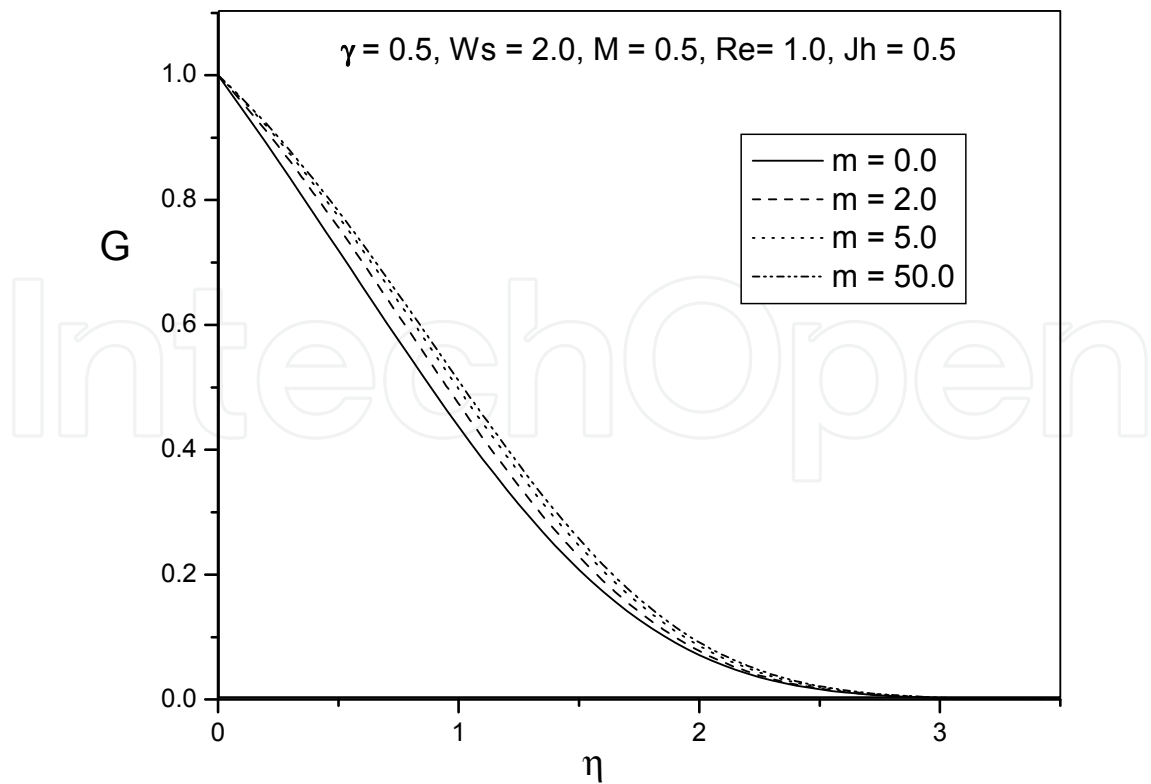


Figure 14. The dimensional tangential velocity profiles against η for different values of m .

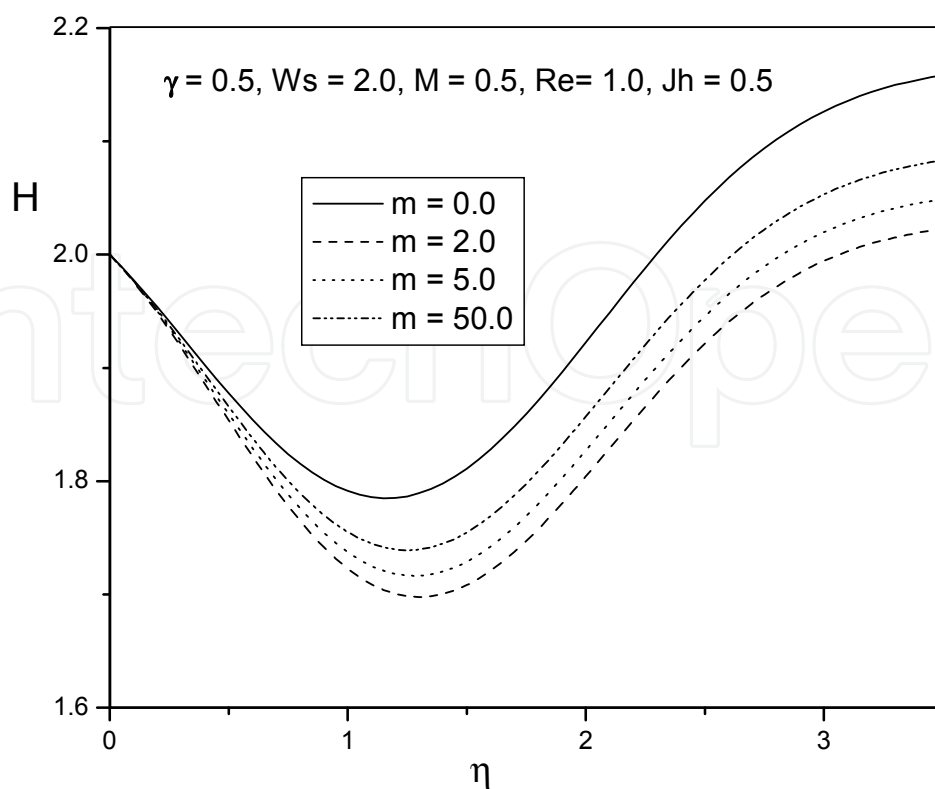


Figure 15. The dimensional axial velocity profiles against η for different values of m .

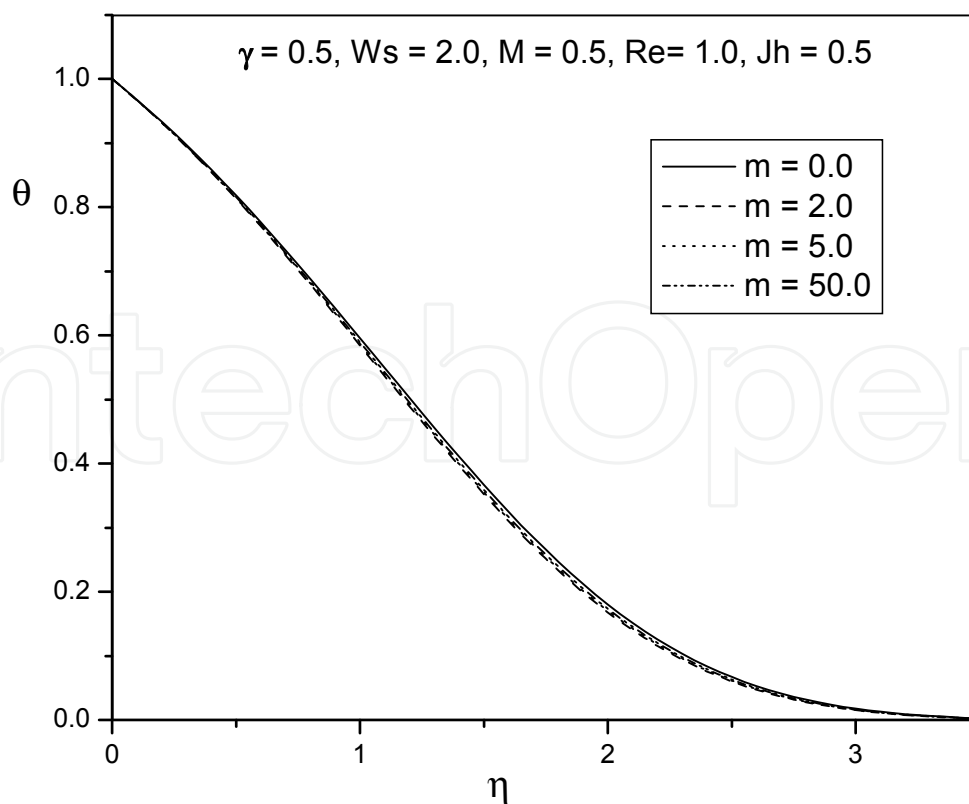


Figure 16. The dimensional temperature profiles against η for different values of m .

Figures 17-20 present the effects of the rotational Reynolds number, R_e , on the flow behavior. It is seen that Reynolds number accelerates the fluid motion in radial and tangential velocity profiles and temperature profiles. However the behavior of the axial velocity profiles decreases with increasing R_e .

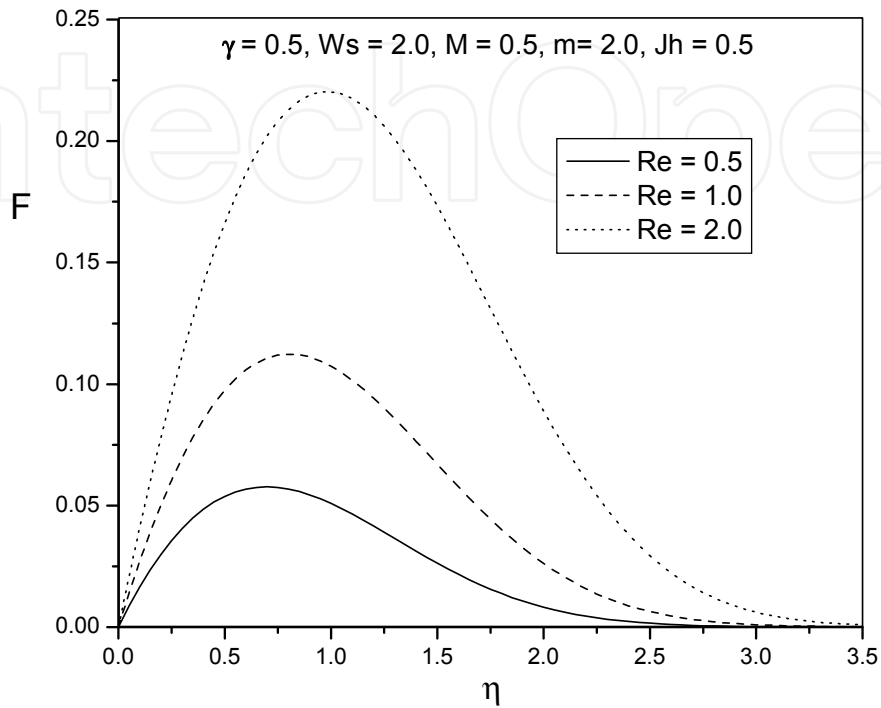


Figure 17. The dimensional radial velocity profiles against η for different values of R_e .

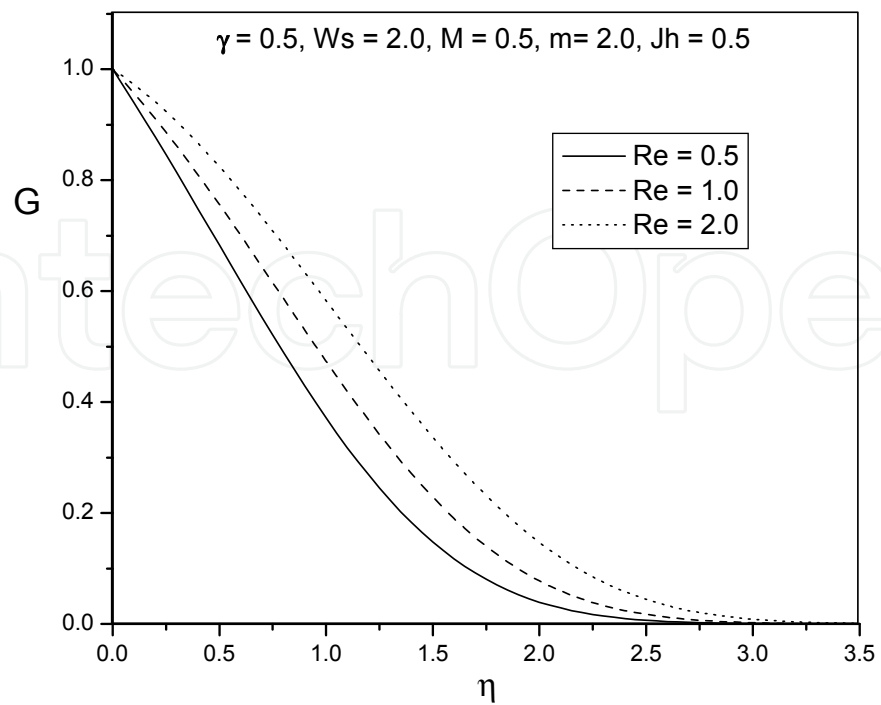


Figure 18. The dimensional tangential velocity profiles against η for different values of R_e .

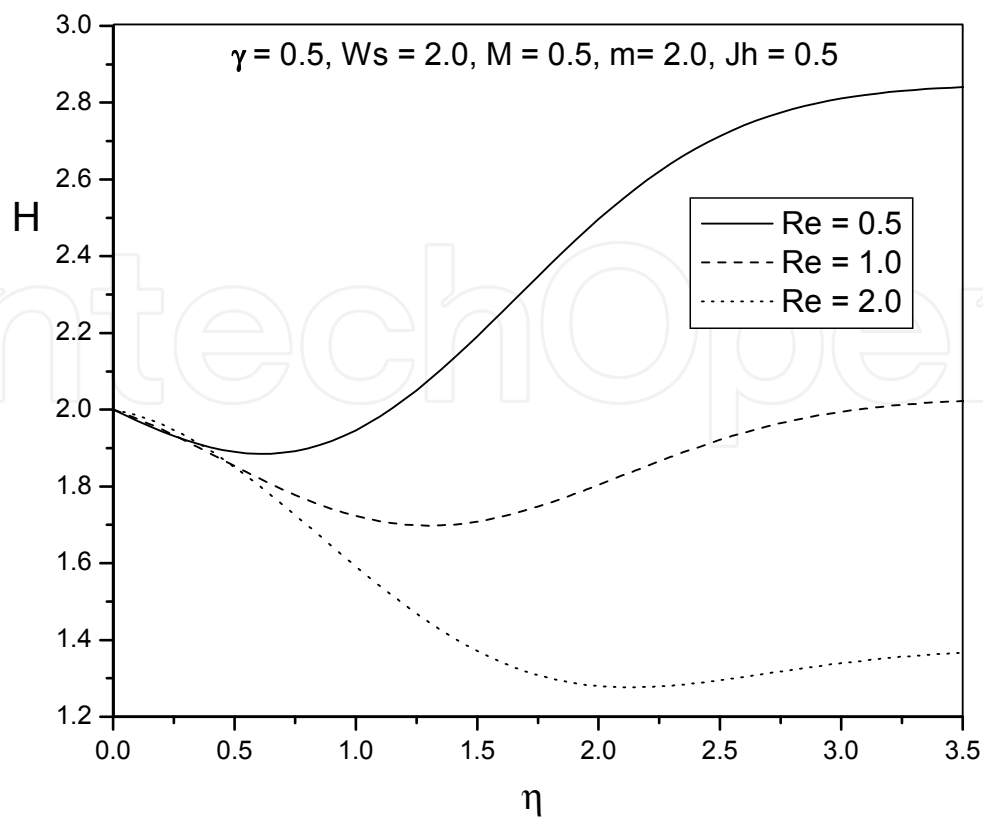


Figure 19. The dimensional axial velocity profiles against η for different values of R_e .

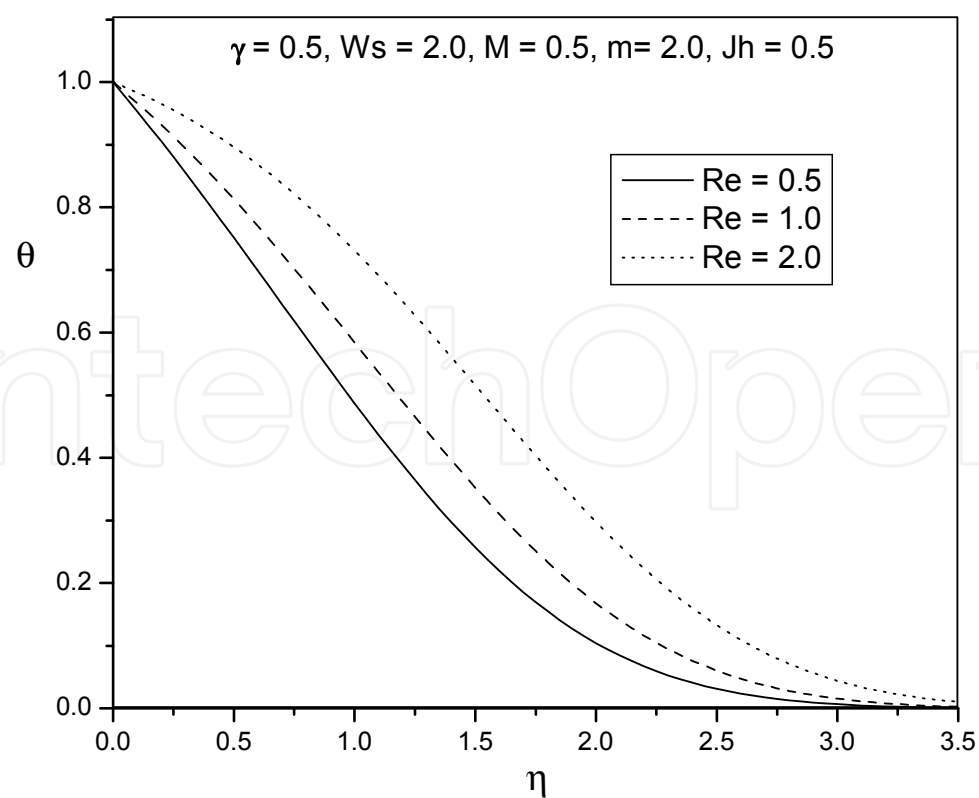


Figure 20. The dimensional temperature profiles against η for different values of R_e .

Figures 21-24 present the effects of the Joule heating parameter, J_h , on the flow behavior. It is seen that the axial velocity profiles and temperature profiles increase with increasing J_h while velocity profiles is generally much smaller between 0.1 and 0.5 except for large values of J_h , when it increases above 0.5, which is expectable on physical basis. The radial and tangential velocity profiles have no significant impact.

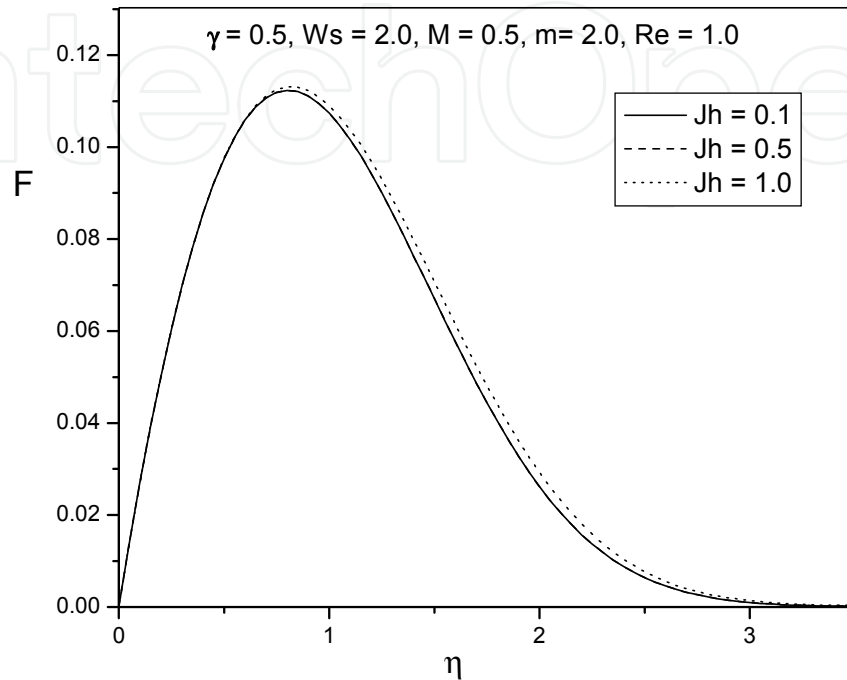


Figure 21. The dimensional radial velocity profiles against η for different values of J_h .

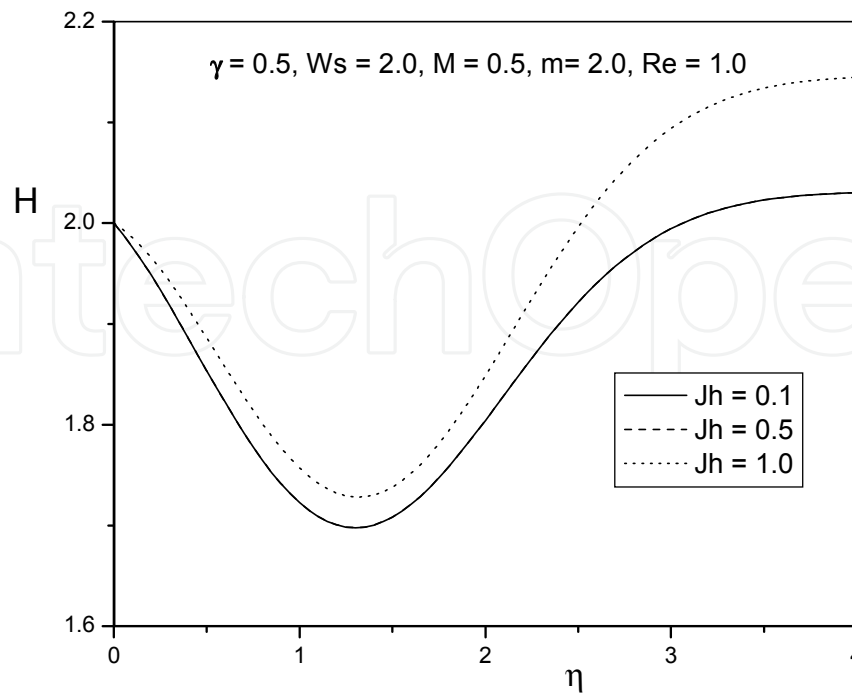


Figure 22. The dimensional tangential velocity profiles against η for different values of J_h .

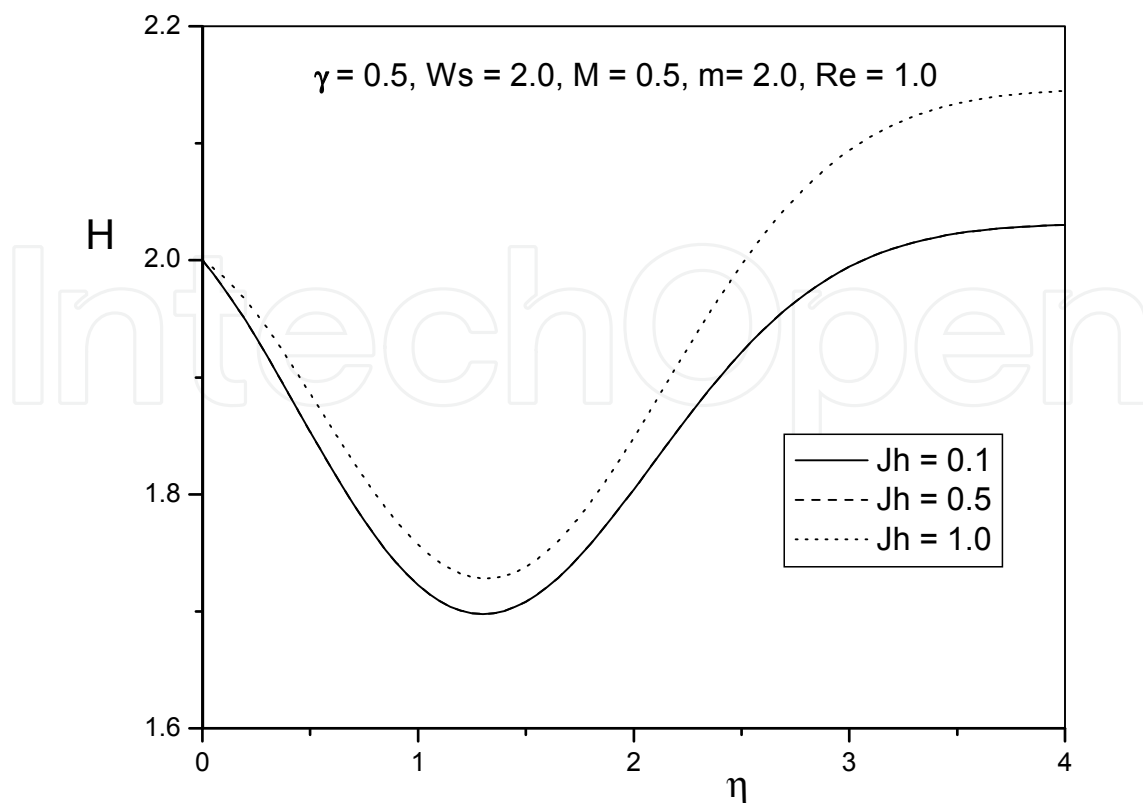


Figure 23. The dimensionless axial velocity profiles against η for different values of J_h .

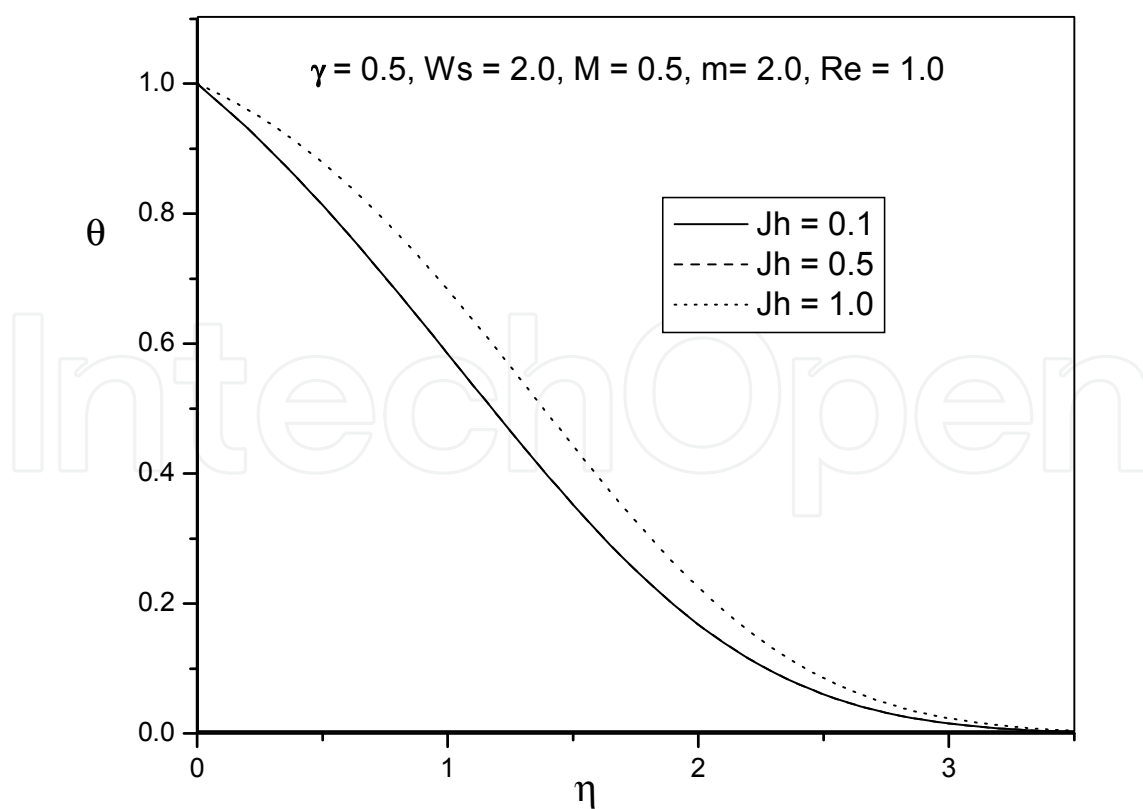


Figure 24. The dimensional temperature profiles against η for different values of J_h .

8. Concluding remarks

In this study, we have investigated numerically the heat transfer phenomenon along with the effects of variable properties for a 2-D unsteady hydrodynamic flow past a rotating disk taking into account viscous dissipation, Joule and hall currents. Using a new class of similarity transformation close to von-Karman, the governing equations have been transformed into non-linear ordinary differential equations that are locally similar. These equations have been solved using the Nachtsheim-Swigert shooting iteration technique along with a sixth-order Runge-Kutta integration scheme. Based on the resulting solutions the following conclusions can be drawn:

1. Similarity approach adopted in the analysis has the advantage that one can separately obtain the steady and unsteady solutions.
2. A comparison of the steady results for the radial and tangential stresses and the rate of heat transfer with those from the available literature leads credence to the numerical code used and hence to the results obtained in the unsteady case.
3. The relative temperature difference parameter γ taken as the variable properties parameter has marked effects on the radial and axial velocity profiles. Close to the surface of the disk tangential velocities and temperature slow down but shortly after they increase with the increasing values of γ .
4. As an influence of the relative temperature difference parameter γ , the thermal boundary layer induces more flow far from the surface of the disk.
5. Separation of flow was detected in different regimes of the momentum and thermal boundary layers.
6. Reduced flows have been observed for increase in injection ($W_s < 0$) while induced Flows were observed for increase in suction ($W_s > 0$).
7. Local maxima and local minima have been observed in the cases of radial and axial velocities for M , m and J_h .
8. As R_e increases, axial and tangential velocity profiles and temperature profiles increase while radial profiles decrease.

Author details

Abdus Sattar

North South University, Bashundhara Baridhara, Dhaka, Bangladesh

Mohammad Ferdows

Department of Mathematics, Dhaka University, Dhaka, Bangladesh

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