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# Discrete Wavelet Multitone Modulation for ADSL & Equalization Techniques

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## 1. Introduction

The reliable delivery of information over severe fading wireless or wired channels is a major challenge in communication systems. At the heart of every communication system is the physical layer, consisting of a transmitter, a channel and a receiver. A transmitter maps the input digital information into a waveform suitable for transmission over the channel. The communication channel distorts the transmitted waveform. One of the many sources of signal distortion is the presence of multipath in the communication channel. Due to the effect of the multipath signal propagation, inter-symbol interference (ISI) occurs in the received waveform. Moreover, the transmitted signal gets distorted due to the effect of various kinds of interference and noise, as it propagates through the channel. ISI and the channel noise distort the amplitude and phase of the transmitted signal, which lead to erroneous bit detection at the receiver. It is desirable for a good communication system that its receiver is able to retrieve the digital information from the received waveform, even in the presence of channel impairments such as, multipath effect and noise.

Orthogonal Frequency Division Multiplexing (OFDM) is a Multi-Carrier Modulation (MCM) technique that enables high data rate transmission and is robust against ISI (Saltzberg, 1967), (Weinstein and Ebert, 1971), (Hirosaki, 1981). It is a form of frequency division multiplexing (FDM), where data is transmitted in several narrowband streams at various carrier frequencies. The sub-carriers in an OFDM system are orthogonal under ideal propagation conditions. By dividing the input bit-stream into multiple and parallel bit-streams, the objective is to lower the data rate in each sub-channel as compared to the total data rate and also to make sub-channel bandwidth lower than the coherence bandwidth of the communication channel. Therefore, each sub-channel will experience flat-fading and will have small ISI. Hence an OFDM system requires simplified equalization techniques, to mitigate the inter-symbol interference. The ISI can be completely eliminated in OFDM transceivers by utilizing the principle of cyclic prefixing (CP). Therefore, high data rate communication systems prefer to apply multicarrier modulation techniques. OFDM has been standardized for many digital communication systems, including ADSL, the 802.11a and 802.11g Wireless LAN standards, Digital audio broadcasting including EUREKA 147

and Digital Radio Mondiale, Digital Video Broadcasting (DVB), some Ultra Wide Band (UWB) systems, WiMax, and Power Line Communication (PLC) (Sari, et al., 1995) (Frederiksen and Prasad, 2002), (Baig and Gohar, 2003).

Over the years, OFDM has evolved into variants, such as Discrete Multitone (DMT), and hybrid modulation techniques, such as multi-carrier code division multiple access (MC-CDMA), Wavelet OFDM and Discrete Wavelet Multitone (DWMT). Several factors are responsible for the development of these variants, especially Wavelet based OFDM techniques, which target several disadvantages associated with Multicarrier modulation (MCM) techniques. Some of these drawbacks are:

- the spectral inefficiency associated with the guard interval insertion, which includes the cyclic prefix
- the high degree of spectral leakage due to high magnitude side lobes of pulse shape of sinusoidal carriers
- OFDM based communication system's sensitivity to inter-carrier interference (ICI) and narrowband interference (NBI)

Therefore, a Discrete Wavelet Transform (DWT) based MCM system was developed as an alternative to DFT based MCM scheme (Lindsey, 1995). DWT based MCM techniques came to be known as Wavelet-OFDM in wireless communications and as Discrete Wavelet Multitone (DWMT) for harsh and noisy wireline communication channels such as Digital Subscriber Line (DSL) or Power Line Communications (PLC) (Baig and Mughal, 2009).

This chapter describes the application of DWT in Discrete Multitone (DMT) transceivers and its performance analysis in Digital Subscriber Line (DSL) channel, in the presence of background noise, crosstalk etc. Time domain equalization techniques proposed for DWT based multitone that is DWMT are discussed, along with the simulation results. The pros and cons of adopting DWT instead of DFT in DMT transceivers will also be discussed, highlighting the open areas of research.

## 2. Basics of wavelet filter banks & multirate signal processing systems

Wavelets and filter banks play an important role in signal decomposition into various subbands, signal analysis, modeling and reconstruction. Some areas of DSP, such as audio and video compression, signal denoising, digital audio processing and adaptive filtering are based on wavelets and multirate DSP systems. Digital communication is a relatively new area for multirate DSP applications. The wavelets are implemented by utilizing multirate filter banks (Fliege, 1994). The discovery of Quadrature Mirror Filter banks (QMF) led to the idea of Perfect Reconstruction (PR), and thus to subband decomposition. Mallat came up with the idea of implementing wavelets by filter banks for subband coding and multiresolution decomposition (Mallat, 1999). DWT gives time-scale representation of a digital signal using digital filtering techniques. The DWT analyzes the signal at different frequency bands with different resolutions by decomposing the signal into approximation and detail coefficients. The decomposition of the signal into different frequency bands is obtained simply by successive high pass and low pass filtering of the time domain signal.

### 2.1 Analysis and synthesis filter banks

Analysis filter banks decomposes input signal into frequency subbands. A two channel analysis filter bank, as shown in Fig. 1, splits the input signal  $X(z)$  into a high frequency

component  $U_0(z)$  and a low frequency component  $U_1(z)$ . The input signal  $X(z)$  is passed through a low pass filter  $H_0(z)$  and a high pass filter  $H_1(z)$ , yielding the  $U_0(z)$  and  $U_1(z)$  respectively.

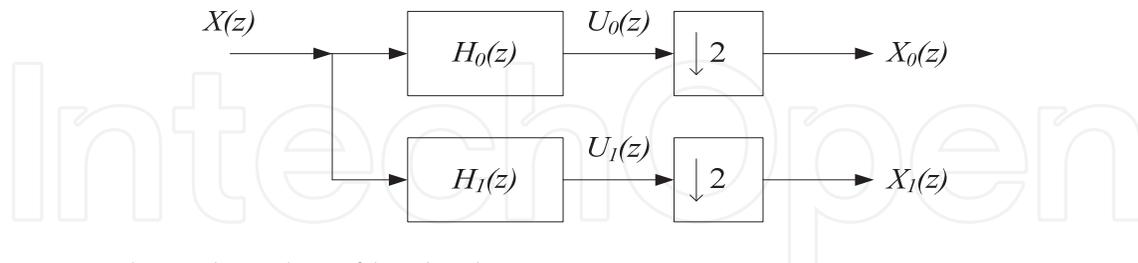


Fig. 1. Two-channel Analysis filter bank.

Consequently, with the sampling frequency,  $F_s = 2\pi$ , the available bandwidth from 0 to  $\pi$ , is divided into two halves,  $0 \leq f \leq F_s/4$  for the lower frequency signal  $U_0(z)$  and  $F_s/4 \leq f \leq F_s/2$  for the high frequency signal  $U_1(z)$ . Therefore, the filtered signals  $U_0(z)$  and  $U_1(z)$  have half the bandwidth of the input signal after being convolved with the low pass filter and high pass filter respectively. The filtered and downsampled signal spectra are shown in Fig. 2. In matrix form the sub-band signals are represented as (Fliege, 1994),

$$\begin{bmatrix} X_0(z) \\ X_1(z) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} H_0(z^{1/2}) & H_0(-z^{1/2}) \\ H_0(z^{1/2}) & H_0(-z^{1/2}) \end{bmatrix} \begin{bmatrix} X(z^{1/2}) \\ X(-z^{1/2}) \end{bmatrix} \tag{1}$$

The two signal spectra overlap. The downsampling will produce aliased components of the signals, that are functions of  $X(-z^{1/2})$  in Eq. 1, since the filtered signals are not bandlimited to  $\pi$ . Two-channel synthesis filter bank is the dual of analysis filter bank, as shown in Fig. 3.  $G_0(z)$  and  $G_1(z)$  denote the lowpass and highpass filters, which recombine the upsampled signals  $U_0(z)$  and  $U_1(z)$  into  $X(z)$ , the reconstructed version of the input signal. The aliased images are removed by the filter  $G_0(z)$  in the frequency range  $F_s/4 \leq f \leq F_s/2$ , while the filter  $G_1(z)$  eliminates the images in the upsampled signal  $U_1(z)$  in the frequency range  $0 \leq f \leq F_s/4$ . Therefore, the signal  $X(z)$ , output from the synthesis filter bank is (Fliege, 1994),

$$X(z) = \begin{bmatrix} G_0(z) & G_1(z) \end{bmatrix} \begin{bmatrix} X_0(z^2) \\ X_1(z^2) \end{bmatrix} \tag{2}$$

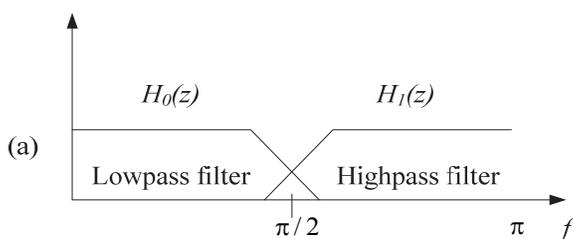


Fig. 2. (Continued)

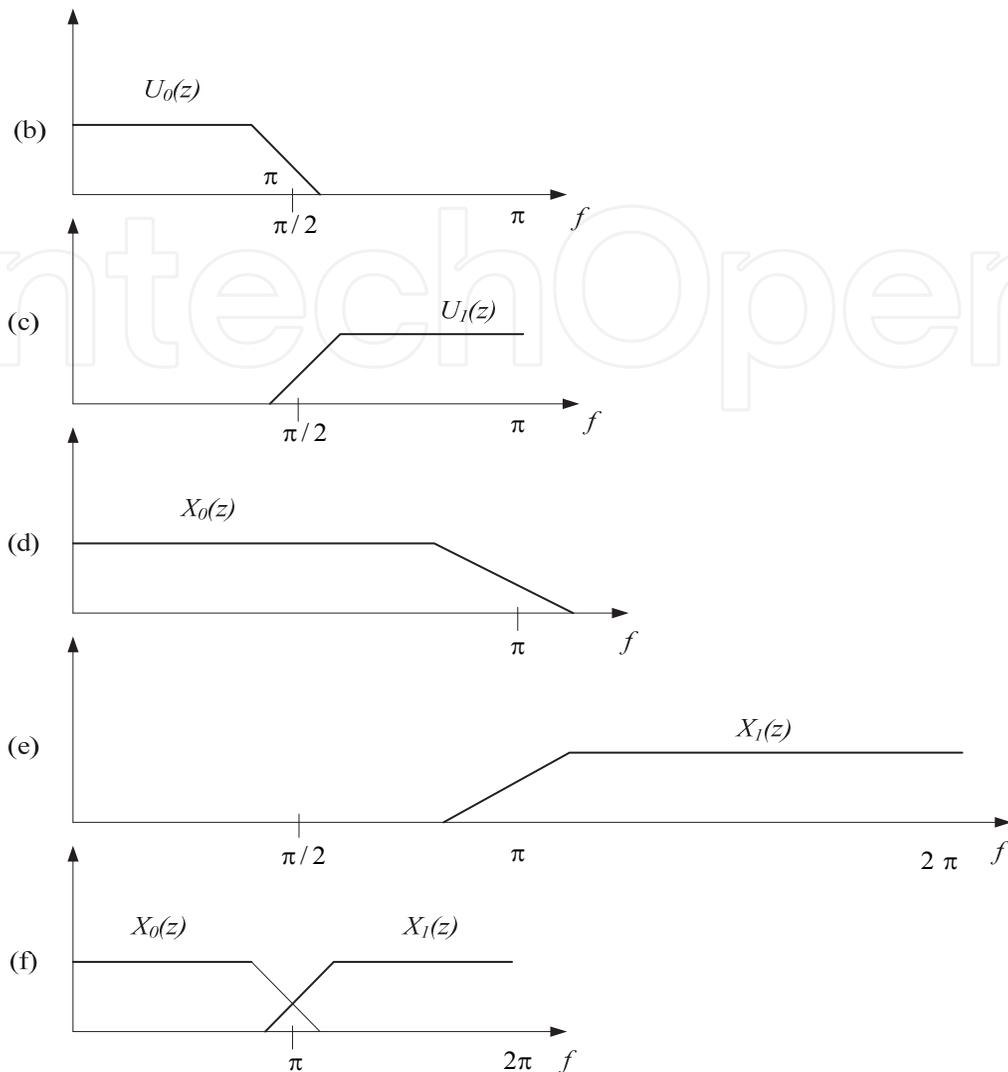


Fig. 2. Signal spectra in two-channel analysis filter bank. (a) Low pass & high pass filter transfer functions. (b) low pass filtered signal spectrum  $U_0(z)$ . (c) high pass filtered signal spectrum  $U_1(z)$ . (d) downsampled signal  $X_0(z)$  spectrum. (e) downsampled signal  $X(z)$  spectrum (f) output signal spectra.

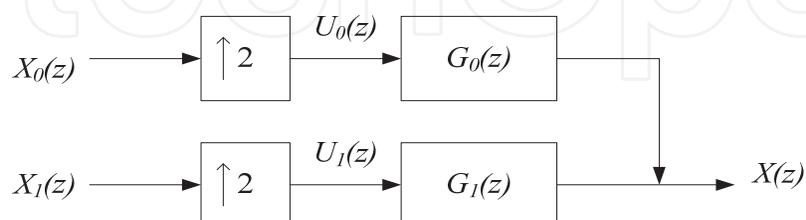


Fig. 3. Two-channel Synthesis filter bank.

## 2.2 Quadrature mirror filter bank

The analysis and the synthesis filter banks combine to form a structure commonly known as the two-channel quadrature mirror filter (QMF) bank. QMF bank serves as the basic

building block in many multirate systems. A two-channel QMF bank is shown in Fig. 4. The constituent analysis and synthesis filter banks have power complementary frequency responses. The low pass and high pass filters in the analysis filter bank decompose the input signal into sub-bands, and the decimation introduces a certain amount of aliasing, due to the non-ideal frequency response of the analysis filters. However, the synthesis filters characteristics are chosen with such frequency response, that the aliasing introduced by the analysis filter bank is canceled out in the reconstruction process. The output signal  $\hat{X}(z)$  is the recovered version of the input signal  $X(z)$ . Therefore, the output signal  $\hat{X}(z)$  is expressed as,

$$\hat{X}(z) = [G_0(z) \quad G_1(z)] \frac{1}{2} \begin{bmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{bmatrix} \begin{bmatrix} X(z) \\ X(-z) \end{bmatrix} \quad (3)$$

$$\hat{X}(z) = F_0(z)X(z) + F_1(z)X(-z) \quad (4)$$

The reconstructed signal  $\hat{X}(z)$  consists of two terms, the first term that is the product of the transfer function  $F_0(z)$  and  $X(z)$  is the desired QMF output, while the second term is the product of the transfer function  $F_1(z)$  and  $X(-z)$  is the aliasing term  $F_1(z)$  denotes the aliasing components produced by the overlapping frequency responses of the analysis and synthesis filter banks. For an alias-free filter bank,  $F_1(z)$  must be equal to zero. This condition is mathematically expressed as (Vaidyanthan, 1993),

$$F_1(z) = \frac{1}{2} [G_0(z)H_0(-z) + G_1(z)H_1(-z)] = 0 \quad (5)$$

This condition may be satisfied by choosing  $G_0(z) = H_1(-z)$  and  $G_1(z) = H_0(-z)$ , then the desired QMF output is represented as (Fliege, 1994),

$$F_1(z) = \frac{1}{2} [H_0(z)H_1(-z) - H_1(z)H_0(-z)] \quad (6)$$

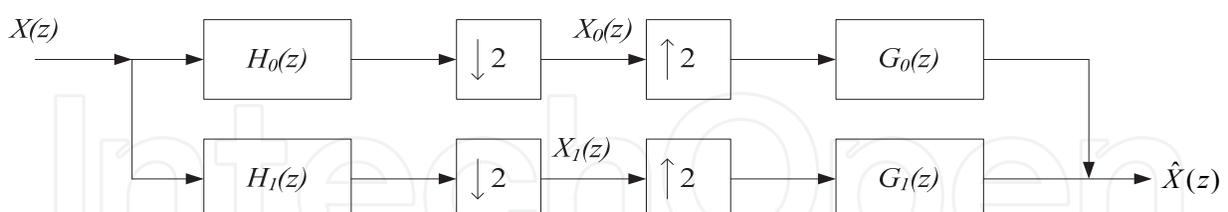


Fig. 4. Two-channel QMF bank.

The filter banks, which are able to perfectly reconstruct the input signal are the perfect reconstruction filter banks, that satisfy the perfect reconstruction condition. The desired QMF output includes the function  $F_0(z)$  which gives perfect reconstruction of the input signal if it is a mere delay, that is  $F_0(z) = z^{-K}$  (Fliege, 1994). Two-channel filter bank, shown in Fig. 4, can be utilized to construct an octave-spaced wavelet filter bank with the help of a tree type structure. Octave filter bank is constructed by the successive decomposition of the low pass signal into constituent sub-bands, every time using the two-channel filter bank (Qian, 2002). A three-level octave-spaced analysis filter bank is shown in Fig. 5 (a) and a three-level octave-spaced synthesis filter bank is shown in Fig. 5 (b).

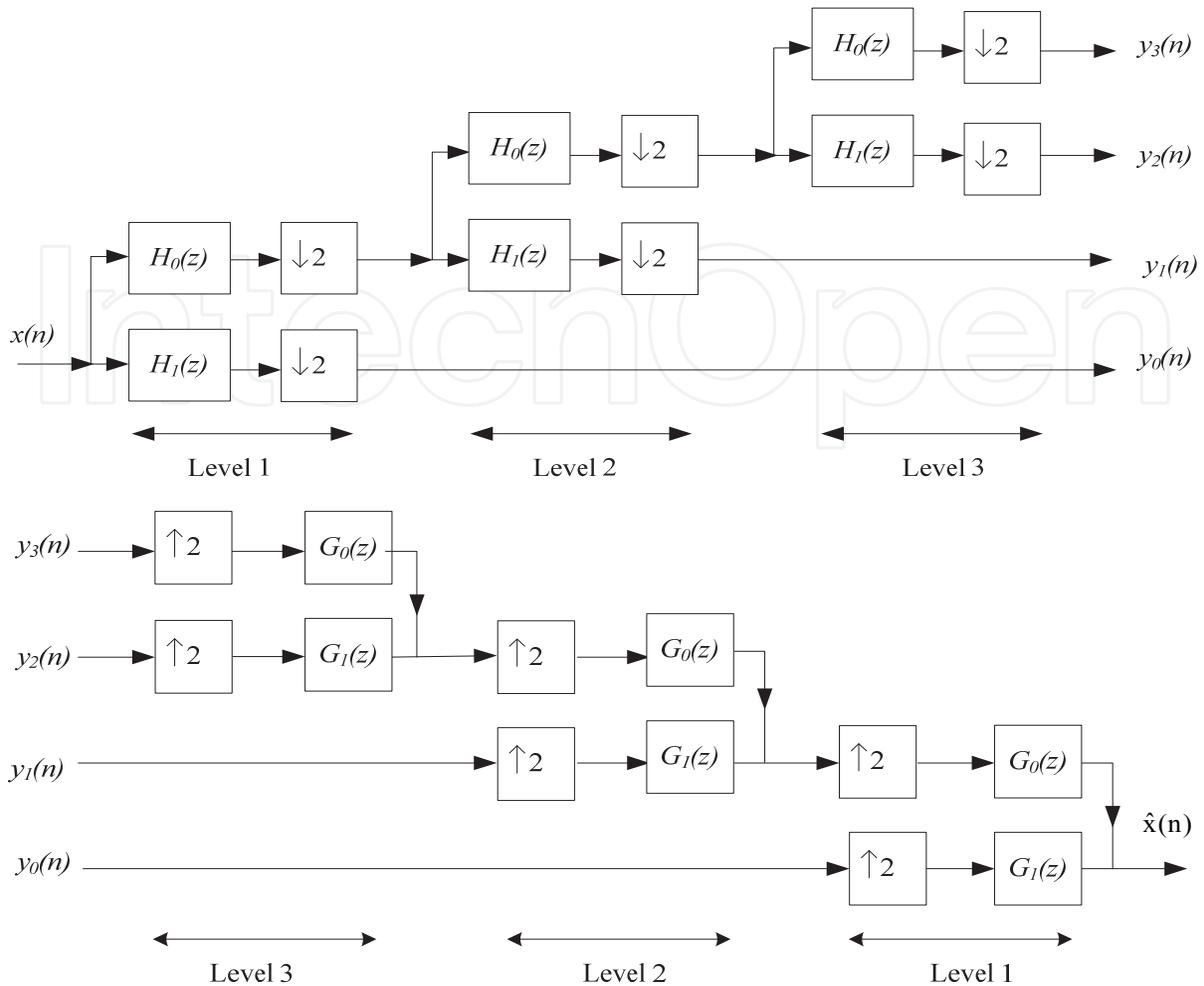


Fig. 5. (a) Three-level analysis filter bank (b) Three-level synthesis filter bank.

### 2.3 Transmultiplexer

Transmultiplexers form an integral part of modems and transceivers based on filter banks that work on the principle of perfect reconstruction. A simple two-channel filter bank can be utilized to illustrate the perfect reconstruction condition. A transmultiplexer is the dual of Sub-band coder (SBC) in structure. Fig. 6 shows a two-channel transmultiplexer filter bank, which converts a time-interleaved signal at its input to a FDM signal, having separate bands of spectrum multiplexed together and then converts it back into TDM signal at its output. Transmultiplexers find application in modems and transceivers for digital communication (Vaidyanathan, 1993).

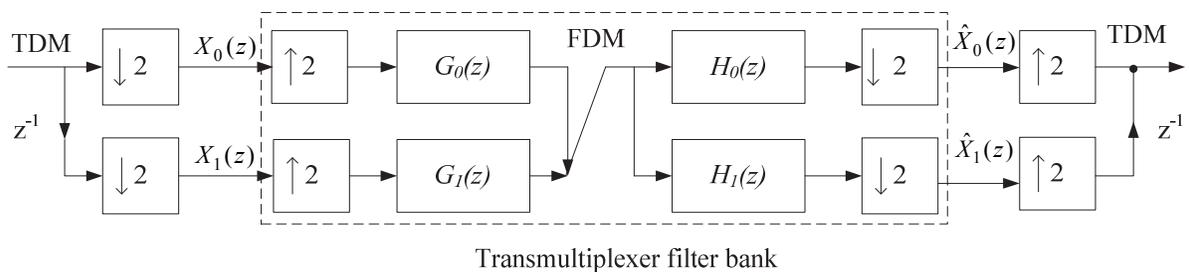


Fig. 6. Two-channel Transmultiplexer.

### 3. Discrete multitone modulation technique

Discrete Multitone (DMT) modulation is a variant of OFDM associated with various loading algorithms, so as to optimize a transceiver's performance in wireline channels like Asymmetrical Digital Subscriber Line (ADSL) and power line (Chow, et al., 1991). In literature, several loading algorithms have been developed; allocating resources such as data bits, or power in order to optimize high data rate, low average transmitting power, or low bit error rate. Typically two of these parameters are kept constant and third is the goal of optimization.

A conventional DFT based DMT transceiver block diagram is shown in Fig. 7. The channel bandwidth is divided into  $N$  sub-channels. The input serial bit-stream is also split into  $N$  parallel sub-streams. The data bits assigned to sub-channels are according to a loading algorithm. For water-filling bit-loading algorithm, greater number of bits is assigned to higher SNR sub-channels. If the value of SNR of a sub-channel is below a pre-assigned threshold, then no bits are allocated to that sub-channel. The assigned bits are mapped onto Quadrature amplitude modulation (QAM) constellation forming a complex symbol. The QAM symbols are then modulated onto orthogonal sub-carriers using Inverse Fast Fourier Transform (IFFT). The  $N$  QAM symbols are duplicated with their conjugate symmetric counterparts and subjected to  $2N$  point IFFT, in order to generate real samples for transmission through the channel. A DMT symbol is thus formulated.

A guard band consisting of a few samples of the DMT symbol is pre-appended to the symbol. This is the cyclic prefix, which consists of the last  $v$  samples of the DMT symbol, circularly wrapped to the  $2N$  DMT symbol. The length of cyclic prefix  $v$  is chosen such that it will be longer than the length of the channel response. The cyclic prefix added to a DMT symbol lengthens the symbol period, making it longer than the worst possible delay spread, which is caused by time delayed reflections of the original symbol arriving at the receiver. Consequently, the cyclic prefix serves the purpose of absorbing any multipath interference. Due to this cyclic extended symbol, the samples required for performing the FFT can be taken anywhere over the length of the symbol, without degradation by the neighboring symbols. However, the information sent in the cyclic prefix is redundant and reduces the transceiver throughput by  $(2N + v)/2N$ . Between the transmitter and receiver lies the communication channel, which introduces both noise and distortion (mainly due to multipath propagation) to the composite transmit signal. The channel can be modeled as a finite impulse response (FIR) filter that possesses a frequency-selective fading characteristic. The cyclic prefixed signal is transmitted through the channel, the output of which gives the product of the channel impulse response and the transmitted symbols in frequency domain. DMT receiver is basically the dual of the DMT transmitter, with the exception of the equalization part. The equalization block consists of two parts, the time-domain equalizer (TEQ) and the frequency-domain equalizer (FEQ). The purpose of TEQ is channel-shortening and it immediately follows the channel, as shown in the Fig. 7. It serves to shorten the channel impulse response, so that the equalized channel impulse response is less than the length of the cyclic prefix  $v$ . At the receiver the cyclic prefix samples are discarded and remaining samples are subject to Fast Fourier transform (FFT). The frequency domain equalizer divides the received sub-symbols by the FFT coefficients of the shortened channel impulse response. The resulting signal is demodulated to recover the original data bits and converted into a serial bit stream.

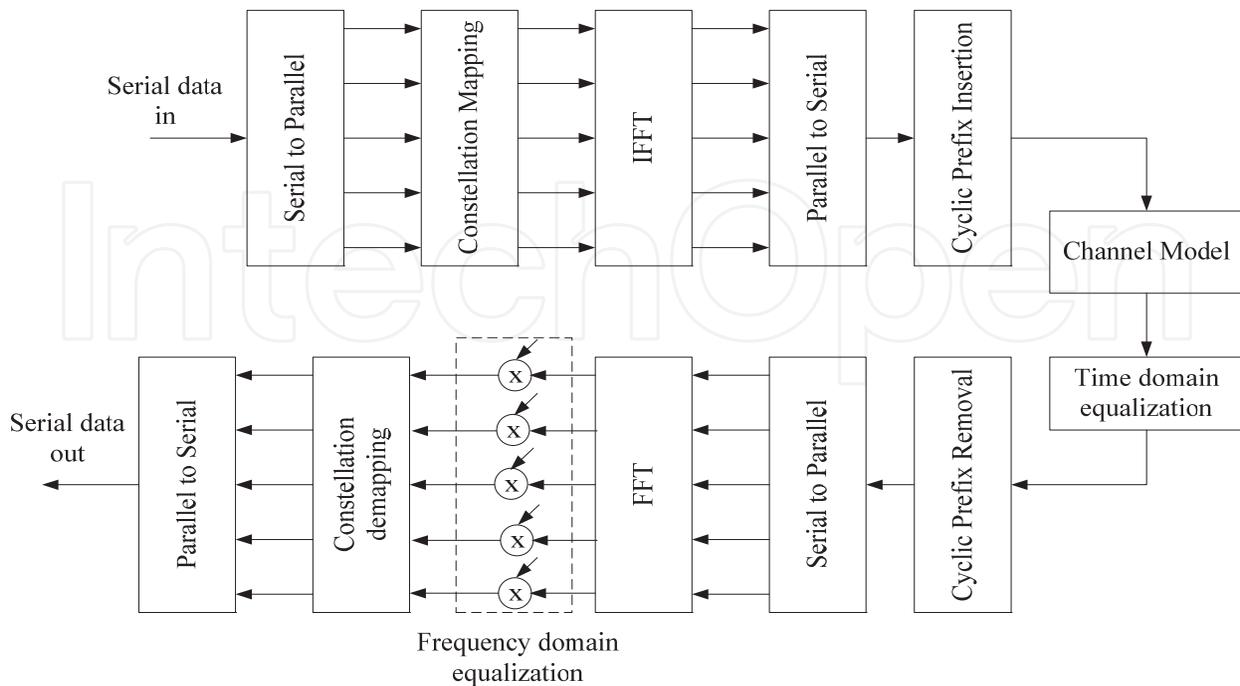


Fig. 7. Functional block diagram of a DMT transceiver.

### 3.1 Evolution of discrete wavelet multitone modulation

A major drawback of DFT-DMT is that the rectangular low-pass prototype filter results in *sinc* shaped sub-band spectral response, the first side lobes of which are only 13dB down, as pointed out by Sandzberg (Sandzberg, 1995). A dispersive channel will thus introduce Inter-Carrier Interference (ICI) at significant levels. To mitigate this we can increase filter bank genus, and design sub-channels with greater spectral isolation. We call this a lapped transform, and much work has been done on the particular case of the Lapped Orthogonal Transform (LOT) (Malvar, 1992). General Extended Lapped Transform (ELT) design is computationally prohibitive, however Cosine Modulated Filter Banks (CMFB) can be efficiently implemented utilizing Discrete Cosine Transform (DCT). In this way the design procedure is simplified if we allow the transmultiplexer filters to be modulated versions of a low pass, linear-phase prototype. Therefore, instead of designing  $N$  filters, we now only design one prototype filter. Modulated filter banks implementing lapped transforms with applications to communications are generally referred to as Discrete Wavelet Multitone (DWMT), to distinguish from DMT which uses a rectangular prototype.

Many contributions in literature have emphasized the need for DWMT in specific channel conditions. Tzannes and Proakis have proposed DWMT in (Tzannes, et al., 1994), and shown it to be superior to DFT-DMT. Authors suggest implementing DWMT in DSL channel for improved performance (Doux et al., 2003). Studies have compared DMT and DWMT performance in DSL channel (Akansu and Xueming 1998).

DWT exhibits better spectral shaping compared to the rectangular shaped subcarriers of OFDM. Therefore, it offers much lower side lobes in transmitted signal, which reduces its sensitivity to narrowband interference (NBI) and inter-carrier interference (ICI). However, it

cannot utilize CP to mitigate ISI created by the frequency-selective channel, as various DWT symbols overlap in time domain (Vaidyanathan, 1993). Nevertheless, such MCM systems based on DWT require an efficient equalization technique to counter the ISI created by the channel.

#### 4. Discrete Wavelet Multitone (DWMT) in Digital Subscriber Line (DSL)

A system based on Discrete Wavelet Multitone (DWMT) for modulating and demodulating the required signal using Discrete Wavelet Transform as a basis function has been suggested in wireless applications (Jamin and Mähönen, 2005). The importance of DWMT in wireless communication is a recognized area of research and on similar lines a DWMT system can be implemented in wireline communication. It can be used as a maximally decimated filter bank with its overlapping symbols in time-domain. Therefore, this structure does not require the addition of CP which is an overhead in DMT and DWMT based wireline systems (Vaidyanathan, 1993). On the other hand, the wavelet filters also possess the advantages of having greater side-lobe attenuation and requires no CP (Bingham, 1990). Therefore, the DWMT systems are bandwidth efficient by not using the CP which creates the problem of bandwidth containment in DMT based systems. However, application of the DWMT systems in a dispersive channel like ADSL necessitates a robust channel equalization technique (Sandberg and Tzannes, 1995). In literature some equalization techniques for DMT based multicarrier systems have been suggested by many authors (Pollet and Peeters, 2000); (Acker et al., 2001); (Acker et al., 2004); (Karp et al., 2003) and DWMT based multicarrier systems (Viholainen et al., 1999). Equalization is a key factor in the design of modems based on DWMT modulation technique and till date, it remains an open research area. When using the Discrete Wavelet Packet Transform (DWPT) as a basis function in DWMT systems, it is difficult to equalize the overlapped symbols in time domain. We emphasize on the design of equalizer for DWPT based DWMT multicarrier systems. The proposed system is based on DWPT for DWMT wireline systems and time-domain equalization is suggested for the equalization process of overlapped symbols.

In this chapter, the time-domain equalization through a linear transversal filter is applied. The equalization algorithms are based on Zero-Forcing (Z-F) and minimum mean squared error (MMSE) criterion to a discrete wavelet-packet transform based DWMT transceiver for a wireline ADSL channel. It is then compared with the system's performance of a DMT based ADSL system. For a fair comparison between the two systems, the DMT system also utilizes the same time-domain equalization. The performance of the proposed wavelet-packet based transceiver is also evaluated in the presence of near-end crosstalk (NEXT) and far-end crosstalk (FEXT) for downstream ADSL. It is shown that the DWMT system conserves precious bandwidth by not utilizing any CP, and gives improvement in bit error rate (BER) performance over the DMT system with time-domain equalization (TEQ).

##### 4.1 System model of DWMT

The DWMT system model's block diagram is shown in Fig. 8. It divides the input data bit-stream into multiple and parallel bit-streams. The proposed DWMT transceiver is based on discrete wavelet packet transform (DWPT). The DWPT is implemented through a reverse order perfect reconstruction filter bank transmultiplexer. Wavelet packets can be

implemented as a set of FIR filters, which leads to the filter bank realization of wavelet transform, according to Mallat's algorithm (Mallat, 1998). The blocked version of the input signal  $x_k(n)$  is mapped to a variable QAM constellation according to the number of bits loaded. This is interpolated and filtered by the  $k^{th}$  branch synthesis filter  $F_k(z)$ . The combined signal is sent through the channel, and the received signal is filtered by an equalizer filter. The equalized signal is passed through the corresponding analysis filter  $H_k(z)$  and decimated to retrieve the QAM encoded version of the transmitted signal. The transmitted signal is recovered after QAM decoding.

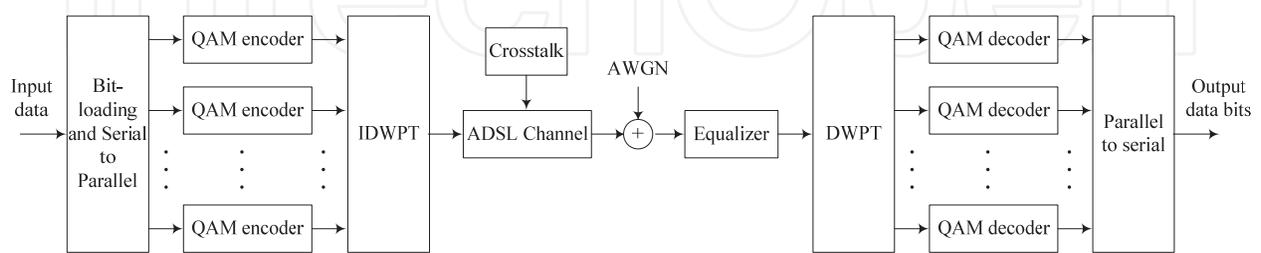


Fig. 8. Functional Block diagram of DWMT system.

#### 4.1.1 Water filling bit loading

Bit loading is usually applied to DMT modulated systems applied to wireline channels, by first estimating the signal-to-noise ratio (SNR) of each sub-channel through channel estimation techniques, which is followed by the distribution of bits to these sub-channels according to their respective SNR. Water-Filling bit loading algorithm applied in the proposed system is rate adaptive and it is suitable for achieving maximum bit rate and also useful when considering the large number of sub-channels and variable QAM constellation (Leke and Cioffi, 1997);(Yu and Cioffi, 2001). A discrete version of this algorithm is applied, in which the bit-loading procedure initiates by determining the sub-channels that should be turned off, due to very low SNR. The bits are assigned to channels according to their capacity, expressed mathematically as (Thomas et al., 2002),

$$b = \frac{1}{2} \log_2 \left[ 1 + \frac{SNR}{\Gamma \cdot \gamma_m} \right] \quad (7)$$

where  $SNR = \epsilon_n \cdot g_n$  is the SNR of each sub-channel,  $\epsilon_n$  is the sub-channel energy and  $g_n$  is the sub-channel SNR and it can be calculated as,

$$g_n = \frac{|H_n|^2}{\sigma^2} \quad (8)$$

where  $H_n$  is the ADSL channel impulse response and  $\sigma^2$  is the noise power,  $\Gamma$  is the SNR gap and  $\gamma_m$  is the performance margin, which is the amount by which SNR can be reduced (Yu and Cioffi, 2001). The water filling bit-loading for the proposed system is shown in Fig. 9. While considering the DWMT based communication system for the ADSL channel, it is necessary to consider its frequency response and the effect of crosstalk, near-end crosstalk (NEXT), and far-end crosstalk (FEXT) in system simulation. The ADSL channel impairments and crosstalk is briefly discussed in the following section.

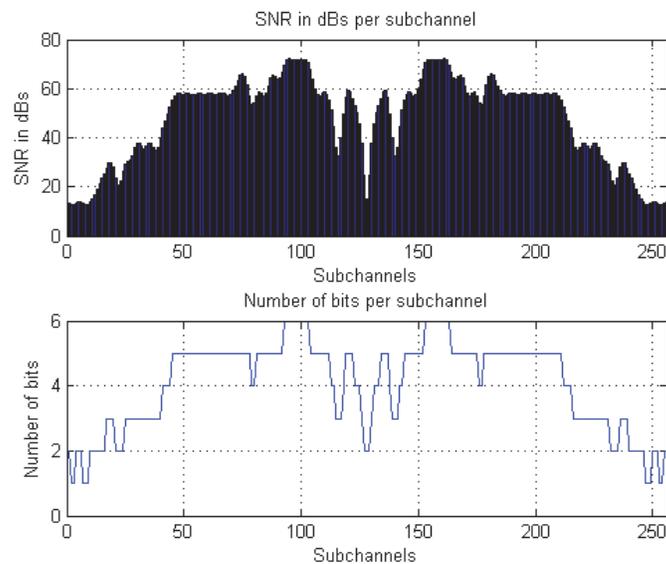


Fig. 9. ADSL channel frequency response & number of bits loaded according to discrete water-filling algorithm.

#### 4.2 ADSL channel

Digital Subscriber Line, commonly known as DSL is the most popular and ubiquitously available wireline medium which provides high-speed Internet access over the twisted pair telephone network. Fig. 10 shows a typical DSL network, which consists of copper lines extending all the way from the central office (CO) to the customer's premises. Current and future applications such as Interactive Personalized TV, high definition TV (HDTV) and video-on-demand through high-speed Internet access, will require more bandwidth. Researchers are exploring cost-effective ways to exploit the existing copper infrastructure to deliver greater bandwidth.

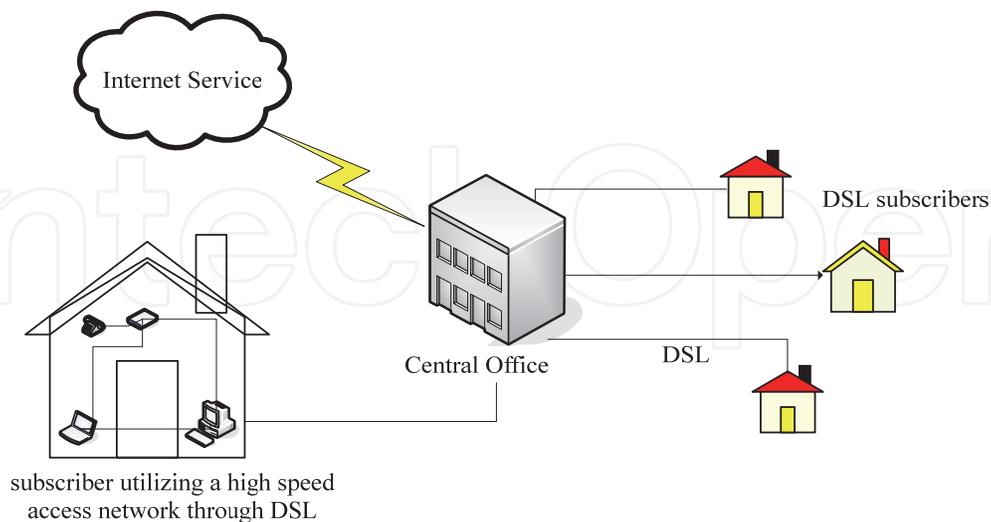


Fig. 10. A typical DSL network connecting subscribers to internet services through DSL to the Central Office.

Although the DSL channel offers the advantage of utilizing the already in place telephone lines to carry digital data, however there are different channel impairments that pose

difficulties in achieving the objective of high-speed and reliable communication (Cook, et al.,1999). These channel impairments include different types of noise and interference. The noise sources include crosstalk, impulse noise and narrow band noise (Thomas Starr, et al., 2002). Also, interference in the communication signal may occur due to the electromagnetic conduction (EMC) in the unshielded twisted pair (UTP) and DSL operating in the vicinity of transmitters may pick up radio frequency interference (RFI) (Cook, et al.,1999). Moreover, signal reflection may be induced due to bridge tabs, unterminated lines and load mismatching in the telephone network. This leads to multipath signal propagation, due to ISI occurs (Bingham, 2000). BER deterioration, due to ISI is a significant problem in the communication systems utilizing the DSL channel. A typical telephone line frequency response and its impulse response are shown in Fig. 11 and Fig. 12 respectively. Multicarrier modulation is a possible solution to the ISI problem in DSL, which is already standardized in Asymmetric digital subscriber line (ADSL), in the form of DMT modulation, as G.DMT and G.lite ADSL.

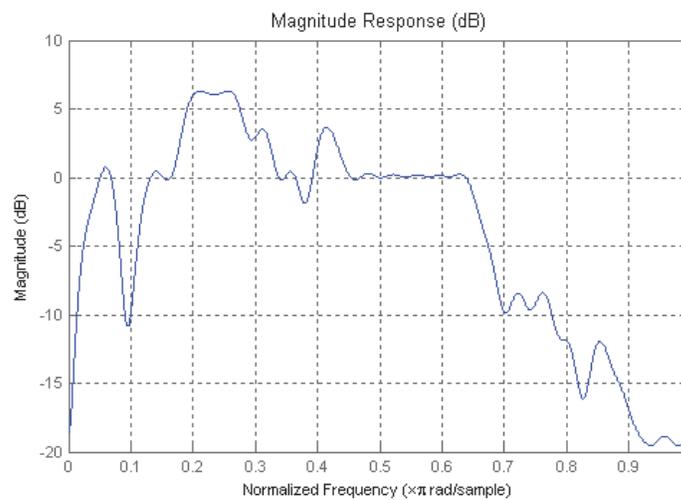


Fig. 11. Frequency response of telephone line FIR channel.

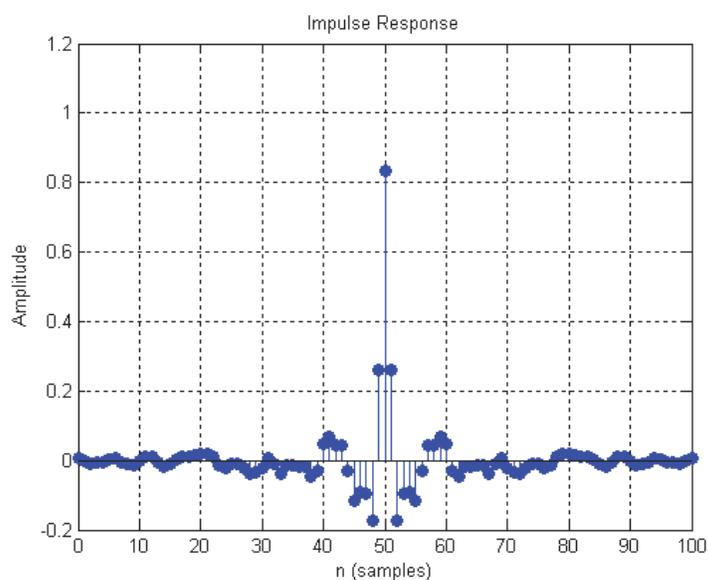


Fig. 12. DSL channel impulse response.

**4.2.1 Crosstalk**

In a telephone network, each subscriber is connected to the CO through a twisted pair, however, hundreds of such pairs are bound together in a cable. The twisting in the wires keeps the electromagnetic coupling between them to a minimum, however, when the pairs are numerous, all crosstalk between the pairs cannot be completely removed. Therefore, this crosstalk constitutes a dominant impairment, where DSL channel is concerned. The DSL crosstalk types, namely near end crosstalk (NEXT) and far-end crosstalk (FEXT) are illustrated in Fig. 13 (Thomas Starr, et al., 2002). NEXT is the crosstalk due to the neighboring transmitter on a different twisted pair line and its power increases with increase in frequency. FEXT is the noise detected by the receiver located at the far end of the cable from the transmitter. FEXT is typically less severe than NEXT, because FEXT is attenuated as the cable length increases.

In this chapter, the performance of DWMT transceiver is evaluated for the downstream ADSL channel. For this purpose, the NEXT and FEXT are modeled using the ADSL standard G.992.1/G.992.2(ITU-T, 2003).

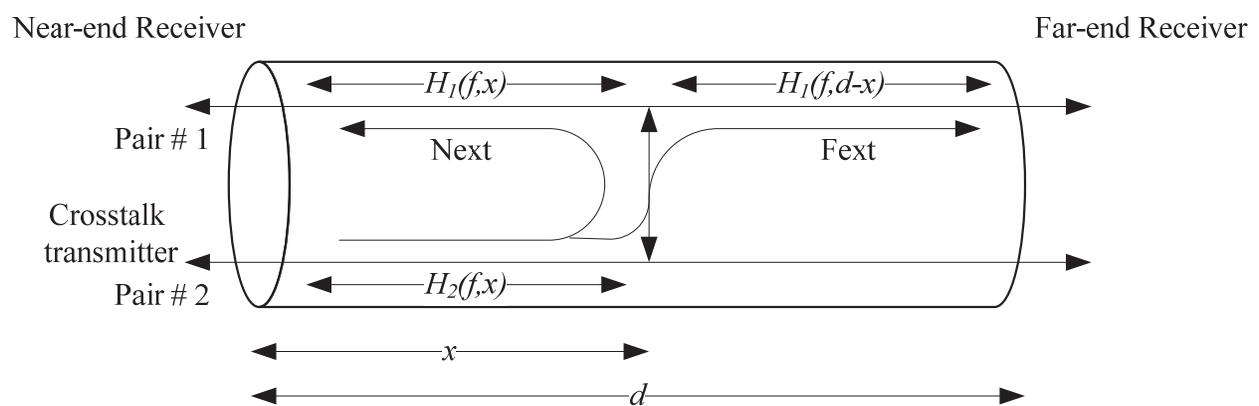


Fig. 13. NEXT and FEXT, the DSL crosstalks illustrated (Thomas Starr, et al., 2002).

The PSD of the ADSL transceiver disturbers for downstream is given by (ITU-T, 2003),

$$PSD_{ADSL,ds-Disturber} = K_{ADSL,ds} \times \frac{2}{f_0} \times \frac{\left[ \sin\left(\pi \frac{f}{f_0}\right) \right]^2}{\left(\pi \frac{f}{f_0}\right)^2} \times \frac{1}{1 + \left(\frac{f}{f_{HP3dB}}\right)^{12}} \times \frac{1}{1 + \left(\frac{f_{HP3dB}}{f}\right)^{16}},$$

$$(0 \leq f < \infty) \tag{9}$$

where  $f$  is in Hz and the remaining parameters are defined in Table 1. The PSD of the ADSL transceiver downstream NEXT is given by (ITU-T, 2003),

$$PSD_{ADSL,ds-NEXT} = PSD_{ADSL,ds-Disturber} \times \left[ 10^{-\frac{NPSL_n}{10}} \times f_{NXT}^{-1.5} \right] \times f^{1.5}, (0 \leq f < \infty) \tag{10}$$

where  $f$  is in Hz and the remaining parameters are also given in Table 1. The PSD of the ADSL transceiver downstream FEXT is given by (ITU-T, 2003),

$$PSD_{ADSL,ds-FEXT} = PSD_{ADSL,ds-Disturber} \times |H_{channel}(f)|^2 \times \left[ 10^{-\frac{FPSL_n}{10}} \times d_{FXT}^{-1} \times d_{FXT}^{-2} \right] \times d \times f^2, \quad (0 \leq f < \infty) \quad (11)$$

where  $f$  is in Hz, and  $H_{channel}(f)$  is the channel transfer function and the remaining parameters are given in Table 1.

PSD of disturbers and NEXT is shown in Fig. 14(a) and Fig. 14 (b) displays the FEXT PSD for downstream ADSL (ITU-T, 2003). The NEXT and FEXT for upstream can be computed in a similar manner (ITU-T, 2003).

| Parameter            | WPT-DWMT     | DFT-DMT      |
|----------------------|--------------|--------------|
| Number of disturbers | 24           | 24           |
| $f_{LP3dB}$          | $f_s/2$      | $f_s/2$      |
| $f_{HP3dB}$          | 138 kHz      | 138 kHz      |
| $K_{ADSL}$           | 0.1104 watts | 0.1104 watts |
| $f_{NXT}$            | 160 kHz      | 160 kHz      |
| NPSL                 | 47.0 dB      | 47.0 dB      |
| $f_{FXT}$            | 160 kHz      | 160 kHz      |
| $d_{FXT}$            | 1.0 km       | 1.0 km       |
| FPSL                 | 45.0 dB      | 45.0 dB      |

Table 1. NEXT & FEXT Simulation Parameters.

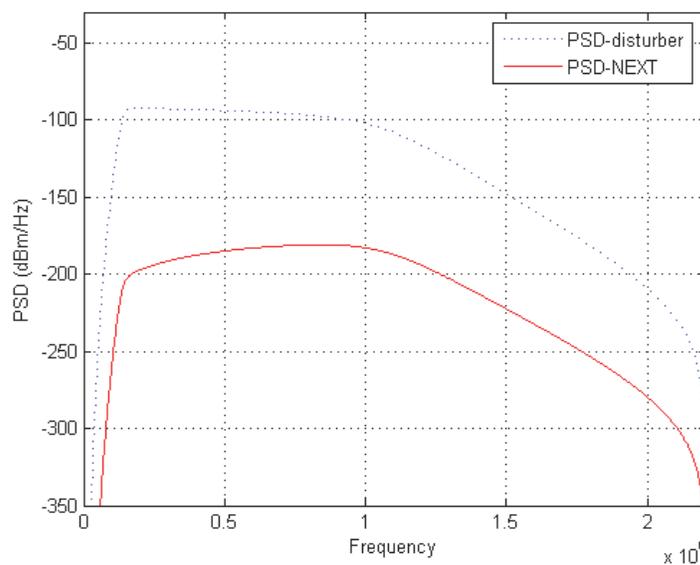


Fig. 14. (a) PSD-disturber & PSD-NEXT for downstream ADSL in G.992.1/G.992.2 standard.

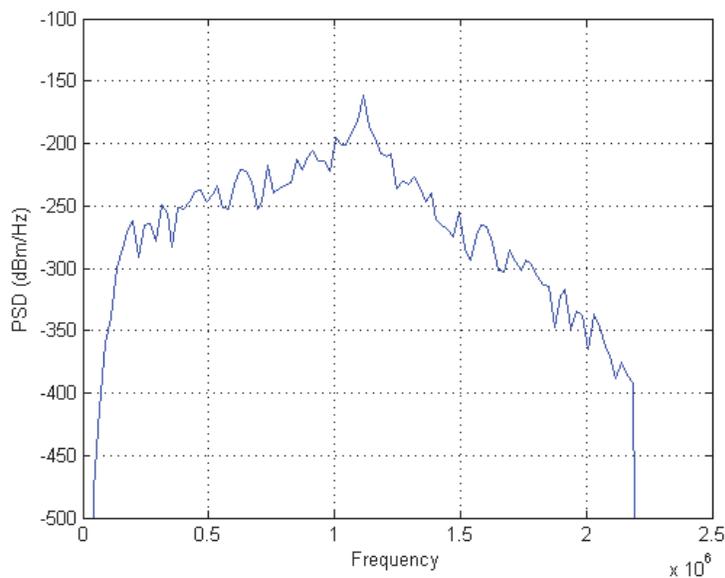


Fig. 14. (b) PSD-FEXT for downstream ADSL in G.992.1/G.992.2 standard.

The wavelet packet transform (WPT) transmultiplexer in the proposed DWMT transceiver gives perfect reconstruction of the transmitted signal, if ideal channel conditions are assumed. However, an actual channel like ADSL is far from ideal, and therefore requires some form of equalization to reliably retrieve the transmitted signal. Time domain equalization is proposed here for DWMT based transceiver for ADSL. There are some equalization techniques for ADSL proposed in literature (Acker et al., 2004);(SMÉKAL et al., 2003);(Trautmann and Fliege, 2002); (Yap and McCanny, 2002).

### 4.3 Time domain equalization

In order to equalize the signal after it has been dispersed by the ADSL channel, time domain equalization is proposed, and it is implemented through a linear transversal filter. The equalizer filter is a linear function of the channel length  $L$ , and the filter coefficients are optimized using the zero-forcing (ZF) and mean squared error (MSE) criterion (Farrukh et al., 2007); (Farrukh et al., 2009).

#### 4.3.1 ZF finite length equalizer

In ZF algorithm it cancels out the channel effect completely by multiplying the received signal with the inverse of the channel impulse response, as shown in Fig. 15. With an infinite length equalizer filter, it is possible to force the system impulse response to zero at all sampling points (Proakis, 1995). However, since an infinite length filter is unrealizable. Therefore, a finite length filter is considered that approximates the infinite length filter (Proakis, 1995). The received signal  $\mathbf{y}$  is the distorted version of the transmitted signal  $\mathbf{x}$  after convolution with the channel  $\mathbf{c}_h$  plus the channel noise  $\mathbf{r}$ . The received signal can be expressed in vector notation as,

$$\mathbf{y} = \mathbf{x}\mathbf{c}_h + \mathbf{r} \quad (12)$$

The equalizer output vector  $\mathbf{z}$  can be found by convolving a set of a training sequence input samples  $\mathbf{h}$  and equalizer tap weights  $\mathbf{c}$  (Sklar, 2001),

$$\mathbf{z} = \mathbf{h}\mathbf{c} \quad (13)$$

However, we continue with the assumption that channel state information is entirely known at the receiver. Therefore, a square matrix  $\mathbf{h}$ , consisting of channel coefficients is formulated with the help of ZF criterion. The ZF algorithm defines that in order to minimize the peak ISI distortion by selecting the equalizer filter weights  $\mathbf{c}$  such that the equalizer output is enforced to zero at sample points other than at the desired pulse. The weights are chosen such that (Sklar, 2001)

$$z(k) = \begin{cases} 1 & \text{for } k = 0 \\ 0 & \text{for } k = \pm 1, \pm 2, \dots, N \end{cases} \quad (14)$$

The equalizing filter has  $L=2N+1$  taps. Equalizer filter coefficients are computed by (Sklar, 2001)

$$\mathbf{c} = \mathbf{h}^{-1}\mathbf{z} \quad (15)$$

The job of equalizing filter is to recover the transmitted signal  $\hat{\mathbf{x}}$  from the received channel-distorted signal  $\mathbf{y}$ , as follows,

$$\begin{aligned} \hat{\mathbf{x}} &= \mathbf{y}\mathbf{c} \\ &= \mathbf{x}\mathbf{c}_h\mathbf{c} + \mathbf{r}\mathbf{c} \end{aligned} \quad (16)$$

where  $\hat{\mathbf{x}}$  is the distorted received signal which was transmitted through ADSL channel and recovered after ZF equalization.

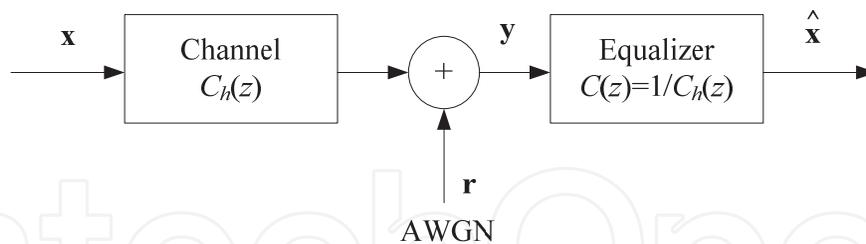


Fig. 15. A Linear transversal equalizer with coefficients optimized by Zero-Forcing criterion.

#### 4.3.2 MMSE criterion

The MMSE criterion represents a more robust solution compared to the ZF since it considers the effect of additive channel noise (Proakis, 1995);(Sklar, 2001). The MMSE criterion of transversal equalizer filter coefficients optimizes the mean squared error of all the ISI terms plus the noise at the equalizer output. A set of over determined equations is formed, in order to derive a minimum MSE solution of the equalizer filter (Sklar, 2001). Therefore, for a  $2N+1$  tap filter, the matrix  $\mathbf{h}$  will have dimensions of  $4N+1$  by  $2N+1$ . Multiplying Eq. (13) by  $\mathbf{h}^T$  (Sklar, 2001),

$$\mathbf{h}^T\mathbf{z} = \mathbf{h}^T\mathbf{h}\mathbf{c} \quad (17)$$

$$\mathbf{R}_{hz} = \mathbf{R}_{hh}\mathbf{c} \quad (18)$$

where  $\mathbf{R}_{hz}$  is the cross correlation matrix and  $\mathbf{R}_{hh} = \mathbf{h}^T\mathbf{h}$  is the autocorrelation matrix of the input noisy signal, which are used to determine the equalizer coefficients  $\mathbf{c}$ ,

$$\mathbf{c} = \mathbf{R}_{hh}^{-1}\mathbf{R}_{hz} \quad (19)$$

For the MMSE solution of the equalizing filter, an over sampled non-square matrix  $\mathbf{h}$  is formed which is transformed to a square autocorrelation matrix  $\mathbf{R}_{hh}$ , yielding the optimized filter coefficients.

#### 4.4 Simulation results

An ADSL system is investigated which is based on DWPT transmultiplexer. The system utilizes  $M = 256$  sub-channels and rate adaptive bit-loading algorithm is applied for bit allocation to each sub-channel in channel environment which is based on ADSL along with the crosstalk noise standards G.992.1/G.992.2 (ITU-T, 2003). For fair comparison, two systems are simulated, which are based on DWMT and DMT transceiver using time-domain equalization (TEQ) techniques for ADSL channel in the presence of AWGN and crosstalk noise. The channel is considered to be stationary during symbol duration. MatLab is used for all this simulation purpose and the parameters for simulation are specified in Table 2.

| Parameter                  | WPT-DWMT                       | DFT-DMT                        |
|----------------------------|--------------------------------|--------------------------------|
| Data rate                  | 1 Mbps                         | 1 Mbps                         |
| Sampling Frequency         | 2.208 MHz                      | 2.208 MHz                      |
| Modulation                 | $M$ -QAM (2, 4, 8, 16, 32, 64) | $M$ -QAM (2, 4, 8, 16, 32, 64) |
| Cyclic Prefix              | None                           | 20%                            |
| FFT size (N)               | -                              | 512                            |
| Wavelet-level              | 2                              | -                              |
| Number of bits/sub-channel | 1 to 6                         | 1 to 6                         |

Table 2. DWMT & DMT System Simulation Parameters.

This corresponds to a system bandwidth of 2 MHz with data rate of 1 Mbps with discrete wavelet packet filter which is used for transmitter and receiver end. The channel equalization is performed by applying a linear equalizing filter in time-domain. The filter coefficients for equalization are optimized by ZF algorithm and MMSE criterion. The ADSL channel is simulated by an FIR filter of 100 taps.

The prototype filter for the synthesis and analysis part of the transmultiplexer is a discrete wavelet filter using 2-level wavelet packet. The input symbols  $x_k(n)$  are  $M$ -QAM modulated. The equalizer frequency response of ZF equalizer FIR filter is shown in Fig. 16. Initially DWMT transceiver and DMT systems are compared regarding the bit error rate (BER) performance in AWGN channel, having identical time-domain zero-forcing channel equalization. Although, the conventional DMT system equalization is a combination of time-domain equalization (TEQ) and frequency-domain equalization (FEQ) techniques, in this case DMT is equalized with a time-domain Zero-Forcing for fair comparison between the two systems. The DWPT transform is applied utilizing Haar wavelet. Fig. 17 shows the comparative performance of two systems in the presence of AWGN without crosstalk. The

BER curve, shown in Fig. 17, presents the fact that the two systems give almost identical performance for lower SNR, and at higher SNR, the DWMT system exhibits an improvement of 1 dB in  $E_b/N_o$  over the DMT system for an AWGN channel, at a BER of  $1E-6$ . It shows that both techniques using DMT and DWPT based ADSL without crosstalk perform identically except at higher SNR. In the next step, the simulation is performed according to the ADSL standard with crosstalk from G.992.1/G.992.2 (ITU-T, 2003).

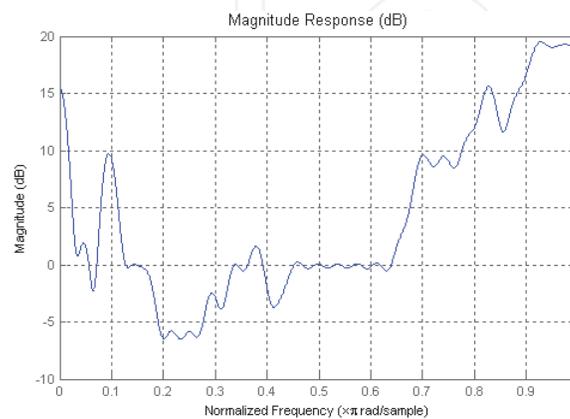


Fig. 16. Equalizing Zero-Forcing filter frequency response.

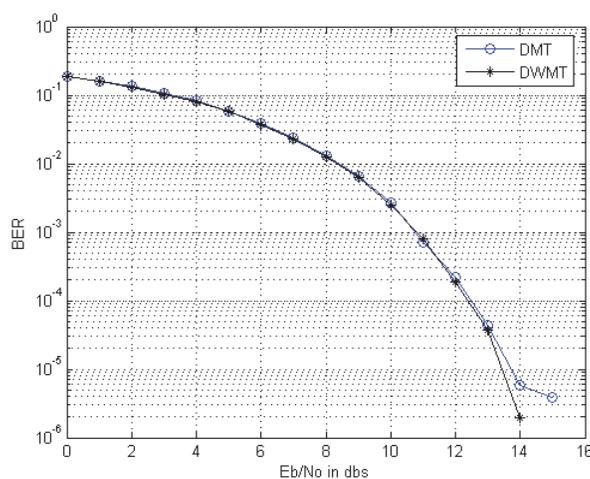


Fig. 17. BER Comparison of DWMT & DMT systems in AWGN with ZF Equalization techniques.

Fig. 18 shows the performance of DWMT and DMT systems in ADSL channel with AWGN, NEXT and FEXT (crosstalk), utilizing time-domain equalization (TEQ) techniques. The NEXT & FEXT represent the downstream crosstalk in ADSL channel according to the G.992.1/G.992.2 standard (ITU-T, 2003), with the simulation parameters as described in Table 1. DMT system is still equalized by ZF-TEQ, while the DWMT transceiver is equalized by ZF-TEQ, time-domain MMSE (MMSE-TEQ). The BER curves shown in Fig. 18 validate the fact that the wavelet packet transmultiplexer improves the performance of DWMT transceiver, having ZF-TEQ by  $E_b/N_o$  margin of 1.0 db for BER of  $1E-4$ , over a DMT transceiver, having an identical equalizer. Moreover the MMSE-TEQ technique for DWMT system shows an improvement of 2 dBs in  $E_b/N_o$  over ZF-TEQ technique for DWMT and a 3 dB gain over the ZF-TEQ equalized DMT system, at a BER of  $1E-4$ .

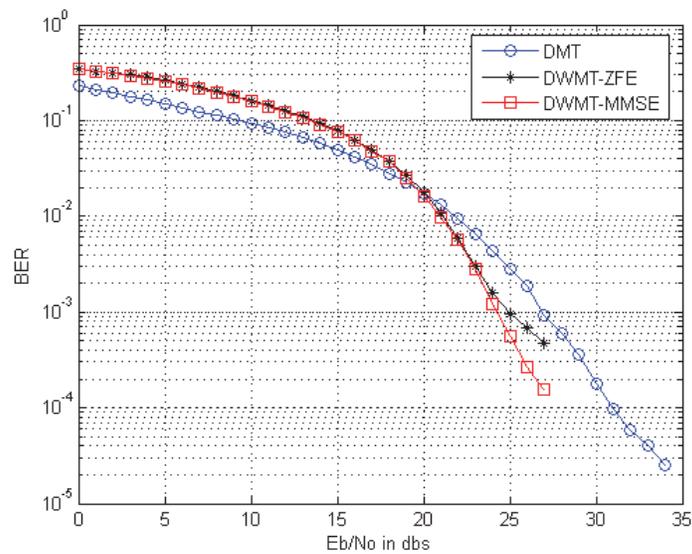


Fig. 18. BER Comparison of DWTM & DMT systems for ADSL channel with AWGN, NEXT & FEXT.

## 5. Pros & cons of applying DWT in multicarrier modulation techniques

DWTM modulation based transceiver, appears to be an interesting choice, when utilizing multi-carrier modulation techniques in wireline systems. It not only recommends the unique time-frequency localization advantage over the conventional frequency localized DMT systems, but also preserves precious bandwidth, which is wasted in DMT based systems in the form of cyclic prefix. However, when utilized in time dispersive channel like ADSL, DWTM transceiver cannot do without an equalization technique because of the time overlapped symbols. In this chapter DWTM based transceiver is discussed and its performance analyzed for the ADSL channel, in comparison with a conventional DMT modulation with ZF and MMSE algorithms using the time-domain equalization. DWTM system based on WPT performs well in the presence of AWGN and crosstalk in comparison with the DMT system for ADSL. ZF equalization algorithm does not consider noise, while the MMSE criterion of optimizing the equalizer coefficients takes into account the effect of channel noise. Therefore MMSE algorithm based DWTM transceiver gives better BER performance in comparison with ZF criterion, since ZF is known to enhance channel noise. The time-domain equalization is computationally complex in comparison to frequency domain equalization, however it offers improved bit error rate.

## 6. Conclusion

The multirate digital signal processing techniques, including wavelets and filter banks are part of new emerging technologies, which are finding applications in the field of digital communications. DWT based Multicarrier modulation techniques have opened new avenues for researchers, to avoid the spectral leakage and spectral inefficiency associated with Fourier Transform based MCM techniques. Time domain equalizers based on ZF and MMSE algorithms are utilized for DSL channel equalization in DWTM transceivers. MMSE based equalizers outperform the ZF equalizers in terms of BER. The equalization techniques adopted

for DWMT transceiver is a topic of active research. Moreover, simulation results found in literature have shown that DWT based MCM systems exhibit higher immunity to narrowband interference (NBI). Therefore, WOFDM/DWMT can be considered as a viable alternative to spectrally inefficient OFDM/DMT, however at the cost of higher computational complexity of equalization.

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## **Discrete Wavelet Transforms - Algorithms and Applications**

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The discrete wavelet transform (DWT) algorithms have a firm position in processing of signals in several areas of research and industry. As DWT provides both octave-scale frequency and spatial timing of the analyzed signal, it is constantly used to solve and treat more and more advanced problems. The present book: Discrete Wavelet Transforms: Algorithms and Applications reviews the recent progress in discrete wavelet transform algorithms and applications. The book covers a wide range of methods (e.g. lifting, shift invariance, multi-scale analysis) for constructing DWTs. The book chapters are organized into four major parts. Part I describes the progress in hardware implementations of the DWT algorithms. Applications include multitone modulation for ADSL and equalization techniques, a scalable architecture for FPGA-implementation, lifting based algorithm for VLSI implementation, comparison between DWT and FFT based OFDM and modified SPIHT codec. Part II addresses image processing algorithms such as multiresolution approach for edge detection, low bit rate image compression, low complexity implementation of CQF wavelets and compression of multi-component images. Part III focuses watermarking DWT algorithms. Finally, Part IV describes shift invariant DWTs, DC lossless property, DWT based analysis and estimation of colored noise and an application of the wavelet Galerkin method. The chapters of the present book consist of both tutorial and highly advanced material. Therefore, the book is intended to be a reference text for graduate students and researchers to obtain state-of-the-art knowledge on specific applications.

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