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1. Introduction

In the past two decades, a number of novel superconducting materials have been discovered where order parameter symmetries are different from an $s$-wave spin singlet, predicted by the Bardeen-Cooper-Schrieffer (BCS) theory of electron-phonon mediated pairing. From the initial discoveries of unconventional superconductivity in heavy-fermion compounds, the list of examples has now grown to include the high-$T_c$ cuprate superconductors, ruthenates, ferromagnetic superconductors, and possibly organic materials.

In most of these materials, there are strong indications that the pairing is caused by the electron correlations, in contrast to conventional superconductors such as Pb, Nb, etc. Nonphononic mechanisms of pairing are believed to favor a nontrivial spin structure and orbital symmetry of the Cooper pairs. For example, the order parameter in the high-$T_c$ superconductors, where the pairing is thought to be caused by the antiferromagnetic correlations, has the $d$-wave symmetry with lines of zeroes at the Fermi surface. A powerful tool of studying unconventional superconducting states is symmetry analysis, which works even if the pairing mechanism is not known.

In general, the superconducting BCS ground state is formed by Cooper pairs with zero total angular momentum. The electronic states are four-fold degenerate $|k \downarrow\rangle$ and $|-k \uparrow\rangle$ have the same energy $\epsilon(k)$. The states with opposite momenta and opposite spins are transformed to one another under time reversal operation $K|k \uparrow\rangle = |-k \downarrow\rangle$ and states with opposite momenta are transformed to one another under inversion operation $I|k \uparrow\rangle = |-k \uparrow\rangle$.

The four degenerate states are a consequence of space and time inversion symmetries. Parity symmetry is irrelevant for spin-singlet pairing, but is essential for spin-triplet state (Anderson, 1959, 1984).

If this degeneracy is lifted, for example, by a magnetic field or magnetic impurities coupling to the electron spins, then superconductivity is weakened or even suppressed. For spin-triplet pairing, Anderson noticed that additionally inversion symmetry is required to obtain the necessary degenerate electron states. Consequently, it became a widespread view that a material lacking an inversion center would be an unlikely candidate for spin-triplet pairing. For example, the absence of superconductivity in the paramagnetic phase of MnSi close to the quantum critical point to itinerant ferromagnetism was interpreted from this point of view.
view (Mathur, 1998; Saxena, 2000). Near this quantum critical point the most natural spin fluctuation mediated Cooper pairing would occur in the spin-triplet channel. However, MnSi has the so-called B20 structure (P2_1), without an inversion center, inhibiting spin-triplet pairing.

Unusual properties are expected in superconductors whose crystal structure does not possess an inversion center (Edelstein, 1995; Frigeri et al., 2004; Gor’kov & Rashba, 2001; Samokhin et al., 2004; Sergienko & Curnoe, 2004).

Recent discovery of heavy fermion superconductor CePt_3Si has opened up a new field of the study of superconductivity (Bauer et al., 2004). This is because this material does not have inversion center, which has stimulated further studies (Akazawa et al., 2004; Yogi et al., 2005). Because of the broken inversion symmetry, Rashba-type spin-orbit coupling (RSOC) is induced (Edelstein, 1995; Rashba, 1960; Rashba & Bychkov, 1984), and hence different parities, spin-singlet pairing and spin triplet pairing, can be mixed in a superconducting state (Gor’kov & Rashba, 2001).

From a lot of experimental and theoretical studies, it is believed that the most possible candidate of superconducting state in CePt_3Si is s+\(p\)-wave pairing (Frigeri et al., 2004; Hayashi et al., 2006). This mixing of the pairing channels with different parity may result in unusual properties of experimentally observed quantities such as a very high upper critical field \(H_{c2}\) which exceeds the paramagnetic limit (Bauer et al., 2004; Bauer et al., 2005a, 2005b; Yasuda et al., 2004), and the simultaneous appearance of a coherence peak feature in the NMR relaxation rate \(T_1^{-1}\) and low-temperature power-law behavior suggesting line nodes in the quasiparticle gap (Bauer et al., 2005a, 2005b; Yogi et al., 2004). The presence of line nodes in the gap of CePt_3Si is also indicated by measurements of the thermal conductivity (Izawa et al., 2005) and the London penetration depth (Bauer et al., 2005; Bonalde et al., 2005).

It is known that the nonmagnetic as well as the magnetic impurities in the conventional and unconventional superconductors already have been proven to be a useful tool in distinguishing between various symmetries of the superconducting state (Blatsky et al., 2006). For example, in the conventional isotropic s-wave superconductor, the single magnetic impurity induced resonance state is located at the gap edge, which is known as Yu-Shiba-Rusinov state (Shiba, 1968). In the case of unconventional superconductor with \(d_{x^2-y^2}\)-wave symmetry of the superconducting state, the nonmagnetic impurity-induced bound state appears near the Fermi energy as a hallmark of \(d_{x^2-y^2}\)-wave pairing symmetry (Salkalo et al., 1996). The origin of this difference is understood as being due to the nodal structure of two kinds of SC order: in the \(d_{x^2-y^2}\)-wave case, the phase of Cooper pairing wave function changes sign across the nodal line, which yields finite density of states (DOS) below the superconducting gap, while in the isotropic s-wave case, the density of states is gapped up to energies of about \(\Delta_0\) and thus the bound state can appear only at the gap edge. In principle the formation of the impurity resonance states can also occur in unconventional superconductors if the nodal line or point does not exist at the Fermi surface of a superconductor, as it occurs for isotropic nodeless \(p\)-wave and/or \(d_x + id_y\)-wave superconductors for the large value of the potential strength (Wang Q.H. & Wang, Z.D, 2004).
In unconventional superconductors non-magnetic impurities act as pair-breakers, similar to magnetic impurities in s-wave superconductors. A bound state appears near an isolated non-magnetic strong (scattering phase shift $\frac{\pi}{2}$, or unitarity) scatterer, at the energy close to the Fermi level. The broadening of this bound state to an impurity band at finite disorder leads to a finite density of states at zero energy, $N(0)$, that increases with increasing impurity concentration (Borokowski & Hirschfeld, 1994). The impurity scattering changes the temperature dependence of the physical quantities below $T^*$ corresponding to the impurity bandwidth: $\Delta \lambda$ changes the behavior from $T$ to $T^2$, the NMR relaxation rate changes from $T^3$ to $T$, and specific heat $C(T)$ changes from $T^2$ to $T$. In other words, the impurities modify the power laws, especially at low temperatures.

The problem of a magnetic impurity in a superconductor has been extensively studied, but is not completely solved because of the difficulty of treating the dynamical correlations of the coupled impurity-conduction electron system together with pair correlations. Generally, the behavior of the system can be characterized by the ratio of the Kondo energy scale in the normal metal to the superconducting transition temperature $\frac{T_K}{T_c}$. For $\frac{T_K}{T_c} \ll 1$, conduction electrons scatter from classical spins and physics in this regime can be described by the Abrikosov-Gor’kov theory (Abrikosov & Gor’kov, 1961). In the opposite limit, $\frac{T_K}{T_c} \gg 1$, the impurity spin is screened and conduction electrons undergo only potential scattering. In this regime s-wave superconductors are largely unaffected by the presence of Kondo impurities due to Anderson’s theorem. Superconductors with an anisotropic order parameter, e.g. p-wave, d-wave etc., are strongly affected, however and the potential scattering is pair-breaking. The effect of pair breaking is maximal in s-wave superconductors in the intermediate region, $T_K \sim T_c$, while in the anisotropic case it is largest for $\frac{T_K}{T_c} \rightarrow \infty$ (Borkowski & Hirschfeld, 1992).

In the noncentrosymmetric superconductor with the possible coexistence of s-wave and p-wave pairing symmetries, it is very interesting to see what the nature of the impurity state is and whether a low energy resonance state can still occur around the impurity through changing the dominant role played by each of the pairing components. Previously, the effect of nonmagnetic impurity scattering has been studied in the noncentrosymmetric superconductors with respect to the suppression of $T_c$ and the behavior of the upper critical field (Frigeri et al., 2004; Mineev & Samokhin, 2007).

This in turn stimulates me to continue studying more properties. My main goal in this chapter is to find how the superconducting critical temperature, magnetic penetration depth, and spin–lattice relaxation rate of a noncentrosymmetric superconductor depend on the magnetic and nonmagnetic impurity concentration and also discuss the application of our results to a model of superconductivity in CePt$_3$Si. I do these by using the Green’s function method when both s-wave and p-wave Cooper pairings coexist.

The chapter is organized as follows. In Sect. 2, the disorder averaged Green’s functions in the superconducting states are calculated and the effect of impurity is treated via the self-
energies of the system. In Sect. 3, the equations for the superconducting gap functions renormalized by impurities are used to find the critical temperature $T_c$.

In Sect. 4, by using linear response theory I calculate the appropriate correlation function to evaluate the magnetic penetration depth. In this system the low temperature behavior of the magnetic penetration depth is consistent with the presence of line nodes in the energy gap.

In Sect. 5, the spin–lattice relaxation rate of nuclear magnetic resonance (NMR) in a superconductor without inversion symmetry in the presence of impurity effect is investigated. In the last two cases I assume that the superconductivity in CePt$_3$Si is most likely unconventional and our aim is to show how the low temperature power law is affected by nonmagnetic impurities.

Finally sect. 6 contains the discussion and conclusion remarks of my results.

2. Impurity scattering in normal and superconducting state

By using a single band model with electron band energy $\xi_k$, measured from the Fermi energy where electrons with momentum $k$ and spin $s$ are created (annihilated) by operators $C_{k,s}^\dagger$ ($C_{k,s}$), the Hamiltonian including the pairing interaction can be written as

$$H = \sum_{k,s} \xi_k C_{k,s}^\dagger C_{k,s} + \frac{1}{2} \sum_{k,k',s,s'} V_{k,k'} C_{k,s}^\dagger C_{k',s'} C_{k,s'} C_{k',s}$$

(1)

This system possesses time reversal and inversion symmetry ($\xi_k = -\xi_{-k}$) and the pairing interaction does not depend on the spin and favors either even parity (spin-singlet) or odd parity (spin-triplet) pairing as required. The absence of inversion symmetry is incorporated through the antisymmetric Rashba-type spin-orbit coupling

$$H_{so} = \sum_{k,s,s'} \alpha_{s,s'} g_{s,s'} C_{k,s} C_{k,s'}$$

(2)

which removes parity but conserves time-reversal symmetry, i.e., $IH_{so}T^{-1} = -H_{so}$ and $TH_{so}T^{-1} = H_{so}$. In Eq. (2), $\sigma$ denotes the Pauli matrices (this satisfies the above condition $I\sigma T^{-1} = -\sigma$ and $T\sigma T^{-1} = \sigma$), $g_{s,s'}$ is a dimensionless vector $[g_{s,s'} = -g_{s,-s'}]$ to preserve time reversal symmetry], and $\alpha(>0)$ denotes the strength of the spin-orbit coupling. The antisymmetric spin-orbit coupling (ASOC) term $a_{s,s'} g_{s,s'} \sigma$ is different from zero only for crystals without an inversion center and can be derived microscopically by considering the relativistic corrections to the interaction of the electrons with the ionic potential (Frigeri et al., 2004; Dresselhaus, 1995). For qualitative studies, it is sufficient to deduce the structure of the g-vector from symmetry arguments (Frigeri et al., 2004) and to treat $\alpha$ as a parameter. I set $\langle g^2 \rangle_k = 1$, where $\langle \ldots \rangle$ denotes the average over the Fermi surface. The ASOC term lifts the spin degeneracy by generating two bands with different spin structure.

In the normal state the eigenvalues of the total Hamiltonian $(H + H_{so})$ are

$$\xi_{k,s} = \epsilon_k - \mu + \alpha \langle g_{s,s} \rangle$$

(3)
where \( \tilde{\epsilon}_k = \frac{k^2}{2m} \) and and \( \mu \) is the chemical potential.

It is obvious from here that the time reversal symmetry is lost and the shape of the Fermi surfaces does not obey the mirror symmetry.

Due to the big difference between the Fermi momenta we neglected the pairing of electronic states from different bands. The structure of theory is now very similar to the theory of ferromagnetic superconductors with triplet pairing (Mineev, 2004).

Effects of disorder are described by potential scattering of the quasiparticles, which in real-space representation is given by

\[
H_{\text{imp}} = \sum_i \int \psi_i^\dagger(\mathbf{r}) U_{\text{imp}} \psi_i(\mathbf{r}) \, d\mathbf{r}
\]  

where \( U_{\text{imp}} = U_n + U_m \), \( U_n \) is the potential of a non-magnetic impurity, which we consider rather short-ranged such that s-wave scattering is dominant and \( U_m = \mathbf{f}(\mathbf{r}) \hat{S} \cdot \hat{\sigma} \) is the potential interaction between the local spin on the impurity site and conduction electrons, here \( \mathbf{f} \) is the exchange coupling and \( \hat{S} \) is the spin operator.

2.1 Impurity averaging in superconducting state

Let us calculate the impurity-averaged Green’s functions in the superconducting state. The Gor'kov equations with self-energy contributions are formally analogous to those obtained for system with inversion symmetry (Abrikosov et al., 1975).

\[
\left( i \omega_n - \xi_{k \pm} - \Sigma_G(i \omega_n) \right) \tilde{\Delta}_k(k, \omega_n) + \left( \Delta_k + \Sigma_f(i \omega_n) \right) \tilde{F}_k^\dagger(k, \omega_n) = \tilde{\sigma}_0
\]

\[
\left( i \omega_n + \xi_{k \pm} + \Sigma_G(i \omega_n) \right) \tilde{F}_k^\dagger(k, \omega_n) + \left( \Delta_k^\dagger + \Sigma_f(i \omega_n) \right) \tilde{\Delta}_k(k, \omega_n) = 0
\]

where \( \omega_n = (2n + 1) \pi T \) are the Matsubara Fermionic frequencies, \( \tilde{\sigma}_0 \) is the unit matrix in the spin state, and the impurity scattering enters the self-energy of the Green’s function of the normal, \( \Sigma_G \), and the anomalous type, \( \Sigma_f \), their mathematical expressions read

\[
\Sigma_G(i \omega_n) = n_n |U_n|^2 + n_m |U_m|^2 \int \frac{d\mathbf{k}'}{(2\pi)^3} \tilde{F}(\mathbf{k}', i \omega_n)
\]

\[
\Sigma_f(i \omega_n) = n_n |U_n|^2 + n_m |U_m|^2 \int \frac{d\mathbf{k}'}{(2\pi)^3} \tilde{F}(\mathbf{k}', i \omega_n)
\]

where \( n_n \) and \( n_m \) are the concentrations of nonmagnetic and magnetic impurities, respectively.

The equations for each band are only coupled through the order parameters given by the self-consistency equations

\[
\Delta_k = -T \sum_{k', \omega_n} V_{st}(\mathbf{k}, \mathbf{k}') F_n(\mathbf{k}', \omega_n)
\]
where \( \nu = \pm \).
Solving the Gor’kov equations one obtains the following expressions for the disorder-averaged Green’s functions

\[
\hat{G}_\omega(k, \omega_n) = \left( \hat{G}_\omega(k, i\omega_n) - \hat{G}_\omega(-k, -i\omega_n) \right)
\]

where

\[
\hat{G}_\omega(k, \omega_n) = \left( i\omega_n - \sum_{\text{imp}} + \bar{v}_k \tilde{k}_F - \tilde{\xi}_k \right) \left( i\omega_n - \sum_{\text{imp}} + \bar{v}_k \tilde{k}_F + \tilde{\xi}_k \right) - \Delta_k \Delta_k^\dagger
\]

and

\[
\text{F}_\omega(k, \omega_n) = \left( i\omega_n - \sum_{\text{imp}} + \bar{v}_k \tilde{k}_F - \tilde{\xi}_k \right) \left( i\omega_n - \sum_{\text{imp}} + \bar{v}_k \tilde{k}_F + \tilde{\xi}_k \right) - \Delta_k \Delta_k^\dagger
\]

here \( \sum_{\text{imp}} = \sum_{\text{imp}}(\text{g}) + \sum_{\text{imp}}(\text{m}) \) is the self energy due to non magnetic and magnetic impurities.

The energies of elementary excitations are given by

\[
E_{k\pm} = \frac{\tilde{\xi}_k - \tilde{\xi}_{-k} \pm \sqrt{\left( \frac{\tilde{\xi}_k + \tilde{\xi}_{-k}}{2} \right)^2 + \Delta_k \Delta_k^\dagger}}{2}
\]

The presence of the antisymmetric spin-orbit coupling would suppress spin-triplet pairing. However, it has been shown by Frigeri et al., (Frigeri et al., 2004) that the antisymmetric spin-orbit coupling is not destructive to the special spin-triplet state with the \( d \) vector parallel to \( \tilde{g}_k \). Therefore, by choosing \( \tilde{g}_k = \tilde{g}_k = \left( \tilde{d}_k \parallel \tilde{g}_k \right) \). Therefore, by choosing \( \tilde{g}_k \), one adopts the p-wave pairing state with parallel \( \tilde{d} \) vector \( \tilde{d}_k = \Delta \left( -k_y, k_x, 0 \right) \). Here the unit vector \( \tilde{k} = \left( \tilde{k}_x, \tilde{k}_y, \tilde{k}_z \right) = (\cos \varphi \sin \theta, \sin \varphi \sin \theta, \cos \varphi) \).

By considering this parity-mixed pairing state in Eq. (5) and (6) can be expressed as

\[
\Delta(\tilde{r}, \tilde{k}) = \left[ \Lambda_0(\tilde{r}) \tilde{\sigma}_0 + \tilde{d}(\tilde{k}) \tilde{\sigma}_y \right] \tilde{d}^\dagger(\tilde{r}) \left[ \Lambda_0(\tilde{r}) \tilde{\sigma}_0 + \Delta(\tilde{r}) \right]^{-1} \left[ -k_y \tilde{\sigma}_x + k_x \tilde{\sigma}_y \right]
\]

with the spin-singlet s-wave component \( \Lambda_0(\tilde{r}) \) and the \( \tilde{d} \) vector \( \tilde{d}_k = \Delta(\tilde{r}) \left( -\tilde{k}_y, \tilde{k}_x, 0 \right) \), here, the vector \( \tilde{r} \) indicates the real-space coordinates. While this spin-triplet part alone has point nodes (axial state with two point nodes), the pairing state of Eq. (14) can possess line nodes in a gap as a result of the combination with the s-wave component (Hayashi et al., 2006; Sergienko 2004). In the presence of uniform supercurrent the gap function has the \( \tilde{r} \) dependence as

\[
\Delta(\tilde{r}, \tilde{k}) = \Delta_0 e^{2m\tilde{r}/\hbar}
\]

where \( m \) is the bare electron mass.
The particular form of order parameter prevents the existence of interband terms in the Gor'kov equations

\[ (i\omega_n - \xi_{k,+} - \sum_c (i\omega_n)) \mathcal{A}_\alpha (k,i\omega_n) + (\Delta_{k,\pm} + \sum_f (i\omega_n)) F^\pm (k,i\omega_n) = 1 \]  
(16)

\[ (i\omega_n + \xi_{k,+} + \sum_c (i\omega_n)) F^+ (k,i\omega_n) + \left( \Delta_{k,\pm}^+ + \sum_f (i\omega_n) \right) \mathcal{A}_\pm (k,i\omega_n) = 0 \]  
(17)

where in this case

\[ \sum_c (i\omega_n) = \left( n_n |U_n|^2 + n_m |U_m|^2 \right) \int \frac{d\tilde{k}'}{(2\pi)^3} \left[ \mathcal{A}_{\tilde{k}',i\omega_n} + \mathcal{A}'_{\tilde{k}',i\omega_n} \right] \]  
(18)

\[ \sum_f (i\omega_n) = \left( n_n |U_n|^2 + n_m |U_m|^2 \right) \int \frac{d\tilde{k}'}{(2\pi)^3} \left[ F_{\tilde{k}',i\omega_n} + F'_{\tilde{k}',i\omega_n} \right] \]  
(19)

and

\[ \Delta_{\pm} = \Delta_0 \pm d[S_k] \]  
(20)

I consider the superconducting gaps \(|\Delta_0 + \Delta \sin \theta|\) and \(|\Delta_0 - \Delta \sin \theta|\) on the Fermi surfaces I and II, respectively (such as superconductor CePtSi). Such a gap structure can lead to line nodes on either Fermi surface I or II (Hayashi et al., 2006). These nodes are the result of the superposition of spin-singlet and spin-triplet contributions (each separately would not produce line nodes). On the Fermi surface I, the gap is \(|\Delta_0 + \Delta \sin \theta|\) and is nodeless, (not that we choose \(\Delta_0 > 0\) and \(\Delta > 0\)). On the other hand, the form of the gap on the Fermi surface II is \(|\Delta_0 - \Delta \sin \theta|\), where line nodes can appear for \(\Delta_0 < \Delta\) (Hayashi et al., 2006).

3. Effects of impurities on the transition temperature of a noncentrosymmetrical superconductor

In the case of large SO band splitting, the order parameter has only intraband components and the gap equation (Eq. (9)) becomes

\[ \Delta_{k,\pm} = -T \sum_{n_n} \int \frac{d^3k'}{(2\pi)^3} V_{\pm k,k'} \left( \left| i\omega_n - \sum_{\text{imp}} - \xi_{k,\pm} \right| \left| i\omega_n - \sum_{\text{imp}} + \xi_{k,\pm} \right| - \Delta_{k,\pm} \right) \]  
(21)

The coupling constants \(V_{\pm k,k'}\). I have used in previous considerations can be expressed through the real physical interactions between the electrons naturally introduced in the initial spinor basis where BCS type Hamiltonian has the following form

\[ H_{\text{int}} = \frac{1}{4\Omega} \sum_{k,q,q',\alpha\beta\mu\nu} \left[ V_{\alpha\beta}^{k,q,q'} \left( \tilde{k},\tilde{k}' \right) + V_{\alpha\beta}^{k,q,q'} \left( \tilde{k},\tilde{k}' \right) + V_{\alpha\beta}^{k,q,q'} \left( \tilde{k},\tilde{k}' \right) \right] \]
\[ \times c_{k+q,q',\alpha}^{\dagger} c_{k,q',\beta} - c_{k-q,q',\alpha}^{\dagger} c_{k-q,q',\beta} \]
(22)
where the pairing interaction is represented as a sum of the k-even, k-odd, and mixed-parity terms: $V = V^s + V^t + V^m$. The even contribution is

$$V^s_{a/b} (\mathbf{k}, \mathbf{k}') = V^s (\mathbf{k}, \mathbf{k}') (i\sigma_2)_{a\beta} (i\sigma_2)_{\mu\delta}^\dagger \tag{23}$$

The odd contribution is

$$V^t_{a/b} (\mathbf{k}, \mathbf{k}') = V^t (\mathbf{k}, \mathbf{k}') (i\sigma_2)_{a\beta} (i\sigma_2)_{\mu\delta}^\dagger \tag{24}$$

here the amplitudes $V^s (\mathbf{k}, \mathbf{k}')$ and $V^t (\mathbf{k}, \mathbf{k}')$ are even and odd with respect to their arguments correspondingly.

Finally, the mixed-parity contribution is

$$V^m_{a/b} (\mathbf{k}, \mathbf{k}') = V^m (\mathbf{k}, \mathbf{k}') (i\sigma_2)_{a\beta} (i\sigma_2)_{\mu\delta}^\dagger + V^m (\mathbf{k}, \mathbf{k}') (i\sigma_2)_{\mu\delta} (i\sigma_2)_{a\beta}^\dagger \tag{25}$$

The first term on the right-hand side of Eq. (25) is odd in $k$ and even in $k'$, while the second term is even in $k$ and odd in $k'$.

The pairing interaction leading to the gap function [Eq. (14)] is characterized by three coupling constants, $V_s$, $V_t$, and $V_m$. Here, $V_s$, $V_t$, and $V_m$ result from the pairing interaction within each spin channel ($s$: singlet, $t$: triplet). $V_m$ is the scattering of Cooper pairs between those two parity channels, present in systems without inversion symmetry. The linearized gap equations acquire simple algebraic form

$$\Delta_0 = V_s \pi T \sum_n \langle E_n \rangle + V_m \pi T \sum_n \langle \sin \theta E_n \rangle \tag{26}$$

$$\Delta = V_t \pi T \sum_n \langle \sin \theta E_n \rangle + V_m \pi T \sum_n \langle E_n \rangle \tag{27}$$

where the angular brackets denote the average over the Fermi surface, assuming the spherical Fermi surface for simplicity, $E_n = \frac{E_{i,n} + E_{i,n}'}{2}$, $E_{i,n} = \frac{\Delta_0 \pm \Delta \sin \theta}{B_{i,n}}$, and

$$B_{i,n} = \left[ (\omega_n + i \Sigma_{\text{imp}})^2 + |\Delta_0 \pm \Delta \sin \theta|^2 \right]^{1/2} \tag{28}$$

From Eqs. (26) and (27) one obtains then the following expression for the critical temperature

$$\ln \left( \frac{T_{c,0}}{T_c} \right) = (1 - X) \left[ \Psi \left( \frac{1}{2} + \frac{1}{4\pi T_m} \right) - \Psi \left( \frac{1}{2} \right) \right] + X \left[ \Psi \left( \frac{1}{2} + \frac{1}{4\pi T_c} \left( \frac{1}{\tau_n} + \frac{1}{\tau_m} \right) \right) - \Psi \left( \frac{1}{2} \right) \right] \tag{29}$$

where

$$\frac{1}{\tau_n} = 2\pi n_s N_0 |I_n|^2, \quad \frac{1}{\tau_m} = 2\pi n_m N_0 |I_m|^2 \tag{30}$$

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\[ \Psi(x) \] is the digamma function, \( N_0 = (N_+ + N_-)/2 \), \( N_\pm \) are the densities of state (DOS) of the two bands at the Fermi level, and \( T_{c0} \) is the critical temperature of the clean superconductor.

The coefficient \( X = 1 - \frac{(\Delta(p))^2_{FS}}{(\Delta^2(p))_{FS}} \) quantifies the degree of anisotropy of the order parameter on the Fermi surface (FS), where the angular brackets \( \langle \cdots \rangle_{FS} \) stand for a FS average.

For isotropic \( s \)-wave pairing \( \langle \Delta(p) \rangle_{FS}^2 = \langle \Delta^2(p) \rangle_{FS} \) \( (X = 0) \) and for any pairing state with angular momentum \( l > 1 \), e.g., for \( p \)-wave and \( d \)-wave states \( (l = 1, 2) \), \( X = 1, \frac{1}{r_m} = 0 \) Eq. (29) reduces to the well-known expressions (Abrikosov, 1993; Abrikosov, A. A. & Gor’kov, 1959).

\[
\ln \frac{T_{c0}}{T_c} = \Psi \left( \frac{1}{2} + \frac{1}{4 \pi T_c r_m} \right) - \Psi \left( \frac{1}{2} \right)
\]

\[
\ln \frac{T_{c0}}{T_c} = \Psi \left( \frac{1}{2} + \frac{1}{4 \pi T_c r_n} \right) - \Psi \left( \frac{1}{2} \right)
\]

For mixing of \( s \)-wave state with some higher angular harmonic state, e.g., for example \( s + p \) and \( s + d \), \( \left\{ 0 < X < 1, \frac{1}{r_m} = 0 \right\} \), Eq. (29) becomes

\[
\ln \frac{T_{c0}}{T_c} = X \left[ \Psi \left( \frac{1}{2} + \frac{1}{4 \pi T_c r_n} \right) - \Psi \left( \frac{1}{2} \right) \right]
\]

At \( \tau_m T_{c0} \gg 1 \) and \( \tau_m T_{c0} \gg 1 \) (weak scattering) one has from Eq. (29):

\[
T_{c0} - T_c \approx \frac{\pi}{4} \left[ \frac{X}{2 \tau_n} + \frac{1 - X/2}{\tau_m} \right]
\]

In two particular cases of (i) both nonmagnetic and magnetic scattering in an isotropic \( s \)-wave superconductor \( (X = 0) \) and (ii) nonmagnetic scattering only in a superconductor with arbitrary anisotropy of \( \Delta(p) \) \( \left( \frac{1}{r_m} = 0, 0 < X < 1 \right) \), the Eq. (34) reduces to well-known expressions

\[
T_{c0} - T_c \approx \frac{\pi}{4 \tau_m}
\]

\[
T_{c0} - T_c \approx \frac{\alpha X}{8 \tau_n}
\]
In the strong scattering limit \( (\tau_n T_c \ll 1, \tau_m T_c \ll 1) \), by using

\[
\Psi\left(\frac{1}{2} + \frac{1}{4\pi T_c \tau}\right) - \Psi\left(\frac{1}{2}\right) \approx \ln\left(\frac{\gamma}{\pi \tau T_c}\right) + \frac{2\pi^2}{3} (\tau T)^2 + O(\tau T)^3
\]

(37)

From Eq. (29) one finds

\[
\left(\frac{1}{\tau_m}\right)^{1-X} \left(\frac{1}{\tau_n} + \frac{1}{\tau_m}\right)^X = \frac{\pi T_c \gamma}{2^{X-1}}
\]

(38)

One can see that the left hand side of Eq. (38) increases monotonically with both \( \frac{1}{\tau_n} \) and \( \frac{1}{\tau_m} \) for any value of \( X \), with the exception of the case \( X = 0 \) which does not depend on magnetic impurities.

For strongly anisotropic gap parameter \( (X \neq 1) \), Eq. (38) reduces to

\[
\frac{1}{\tau_n} + \frac{1}{\tau_m} = \frac{\pi T_c \gamma}{2^{X-1}}
\]

(39)

i.e., the contribution of magnetic and nonmagnetic impurities to pairing breaking is about the same.

For strongly isotropic case \( (X \ll 1) \), one has

\[
\frac{1}{\tau_m} = \frac{\pi T_c \gamma}{2^{X-1}}
\]

(40)

and \( T_c \) is determined primarily by magnetic impurities.

For the case of \( s + p \) wave pairing in the absence of magnetic impurities, one has

\[
\left(\frac{1}{\tau_n}\right)^X = \frac{\pi T_c \gamma}{2^{X-1}}
\]

(41)

In this case the value of \( T_c \) asymptotically goes to zero as \( \tau_n^{-1} \) increase, whereas \( T_c \) of a d or p wave superconductor with \( X = 1 \) vanishes at a critical value \( \frac{1}{\tau_n} = \frac{\pi T_c \gamma}{2^{X-1}} \).

In the absence of nonmagnetic impurities one obtains

\[
\left(\frac{1}{\tau_m}\right)^X = \frac{\pi T_c \gamma}{2^{X-1}}
\]

(42)

And for the s-wave superconductor with \( X = 0 \) one has \( \frac{1}{\tau_n} = \frac{\pi T_c \gamma}{2^{X-1}} \).
Application of these results to real noncentrosymmetric materials is complicated by the lack of definite information about the superconducting gap symmetry and the distribution of the pairing strength between the bands.

As far as the pairing symmetry is concerned, there is strong experimental evidence that the superconducting order parameter in CePt$_3$Si has lines of gap nodes (Yasuda et al., 2004; Izawa et al., 2005; Bonalde et al., 2005). The lines of nodes are required by symmetry for all nontrivial one-dimensional representations of $C_4v$ ($A_2$, $B_1$, and $B_2$), so that the superconductivity in CePt$_3$Si is most likely unconventional. This can be verified using the measurements of the dependence of $T_c$ on the impurity concentration: For all types of unconventional pairing, the suppression of the critical temperature is described by the universal Abrikosov-Gor’kov function, see Eq. (32).

It should be mentioned that the lines of gap nodes can exist also for conventional pairing ($A_1$ representation), in which case they are purely accidental. While the accidental nodes would be consistent with the power-law behavior of physical properties observed experimentally, the impurity effect on $T_c$ in this case is qualitatively different from the unconventional case. In this case in the absence of magnetic impurities one obtains the following equation for the critical temperature:

$$\ln \frac{T_c}{T_c} = X \left[ \Psi \left( \frac{1}{2} \right) \frac{1}{4\pi T_c \tau_n} \right]$$  \hspace{1cm} (43)

In the low ($\tau_n T_c \gg 1$) and dirty ($\tau_n T_c \ll 1$) limit of impurity concentration one has

$$T_{c0} - T_c \approx \frac{X \pi}{8\tau_n} \tau_{T_{c0}} \gg 1$$  \hspace{1cm} (44)

$$T_c = T_{c0} \left( \frac{\tau_{T_{c0}}}{\tau_c} \right)^{1-X} \tau_{T_{c0}} \ll 1$$  \hspace{1cm} (45)

This means that anisotropy of the conventional order parameter increases the rate at which $T_c$ is suppressed by impurities. Unlike the unconventional case, however, the superconductivity is never completely destroyed, even at strong disorder.

4. Low temperature magnetic penetration depth of a superconductor without inversion symmetry

To determine the penetration depth or superfluid density in a superconductor without inversion symmetry one calculates the electromagnetic response tensor $K(\vec{q}, \vec{v}, T)$, relating the current density $\vec{J}$ to an applied vector potential $\vec{A}$

$$\vec{J}(\vec{q}) = -K(\vec{q}, \vec{v}, T)\vec{A}(\vec{q})$$  \hspace{1cm} (46)

The expression for the response function can be obtained as

$$K(\vec{q}, \vec{v}, T, \omega_m) = \frac{me^2}{mc} \left( 1 + \frac{2\pi}{m} \sum_{\vec{n}, \vec{k}} \frac{\gamma_2}{\gamma_1} \left( \vec{K}, \omega_m \right) \left( \vec{K}, \omega_n \right) \right)$$  \hspace{1cm} (47)

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where \( k_x = k \pm \frac{q}{2} \), \( k_0^2 \) is the direction of the supercurrent and \( \langle .... \rangle \) represents a Fermi surface average. By using the expression of Green’s function into Eq. (47) one obtains

\[
K(q, \tilde{v}, T, \omega_n) = \frac{mc^2}{nmc} \left[ 1 + \frac{2\pi T}{m} \sum_n \frac{d^2k}{(2\pi)^2} k_x^2 \right] \left\{ \begin{array}{l}
(\omega_n - \Sigma_{imp} + \tilde{v}_i \tilde{K}_f)^2 + \tilde{\Delta}_q \tilde{\Delta}^*_q + \Delta_i \Delta_k
\end{array} \right\} \left( \begin{array}{l}
(\omega_n - \Sigma_{imp} + \tilde{v}_i \tilde{K}_f)^2 - E_{K, \pm}^2
\end{array} \right) \left\{ \begin{array}{l}
(\omega_n - \Sigma_{imp} + \tilde{v}_i \tilde{K}_f)^2 - E_{K, \pm}^2
\end{array} \right\}
\]

(48)

Now we separate out the response function as

\[
K(q, \tilde{v}, T) = K(0, 0, 0) + \delta K(q, \tilde{v}, T)
\]

(49)

where \( K(0, 0, 0) = \frac{c}{4\pi^2} \left( \lambda(0) = \frac{mc^2}{4\pi mc^2} \right)^{\frac{1}{2}} \) is the zero temperature London penetration depth.

Doing the summation over Matsubara frequencies for each band one gets

\[
\delta K(q, \tilde{v}, T) = -\frac{2\pi c^2}{mc} \left\{ \begin{array}{l}
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right] \left( \frac{\phi_0}{2m} \right) \left( \frac{\phi_0}{2m} \right)
\end{array} \right\}
\]

(50)

\[
\frac{L}{dR} \left[ f(q + \tilde{v}, \tilde{K}_f) - f(q - \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q + \tilde{v}, \tilde{K}_f) - f(q - \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

\[
\frac{L}{dR} \left[ f(q - \tilde{v}, \tilde{K}_f) - f(q + \tilde{v}, \tilde{K}_f) \right]
\]

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The factor $\alpha g_k$ characterizes and quantifies the absence of an inversion center in a crystal lattice. This is the main result of my work i.e. nonlocality, nonlinearity, and noncentrosymmetry are involved in the response function. The first two terms in Eq. (50) represent the nonlocal correction to the London penetration depth and the third represents the nonlocal and impure renormalization of the response while the forth combined nonlocal, nonlinear, and impure corrections to the temperature dependence.

I consider a system in which a uniform supercurrent flows with the velocity $\vec{v}_s$, so all quasiparticles Matsubara energies modified by the semiclassical Doppler shift $\vec{v}_s \cdot \vec{k}_F$. The specular boundary scattering in terms of response function can be written as (Kosztin & Leggett, 1997)

$$\frac{\Delta \lambda_{\text{spec}}(T)}{\lambda_0} = \frac{2}{\pi} \int d\theta \frac{\delta K(q,v_s T)}{\theta^2 + 1}$$

In the pure case there are four relevant energy scales in the low energy sector in the Meissner state: $T$, $E_{\text{nlin}}$, $E_{\text{nonlin}}$, and $\alpha g_k$. The first two are experimentally controlled parameters while the last two are intrinsic one.

In low temperatures limit the contribution of the fully gap ($|\Delta_0 + \Delta \sin \theta|$) Fermi surface I decrease and the effect of the gap $|\Delta_0 - \Delta \sin \theta|$ Fermi surface II is enhanced. I consider geometry where the magnetic field is parallel to c axis and thus $\vec{B}$ and the penetration direction $q$ are in the ab plane, and in general, $\vec{v}_s$ makes an angle $\phi$ with the axis. There are two effective nonlinear energy scales $E_{\text{nlin}} = v_s k_F u_{\parallel 1}$ and $E_{\text{nonlin}} = v_s k_F u_{\parallel 2}$, where $u_{\parallel 1} = |\cos \phi + i \sin \phi|$ and $l_1, l_2 = \pm 1$.

In the nonlocal ($q \neq 0$), linear ($v_s \to 0$) limit, i.e., in the range of temperature where $E_{\text{nlin}} \ll T \ll E_{\text{nonlin}}$ one gets

$$\delta K(q,0,T) = \frac{-c}{4\pi^2} \int (2\ln 2) \frac{T}{\Delta_0}$$

$$\frac{\alpha' w_{\parallel 1} \ll T}{4\pi^2} \sum u_{\parallel 1}^2 \left( 4 \frac{\pi T}{4} \frac{\alpha' w_{\parallel 1}}{\Delta_0} + \frac{3}{2} \left( \frac{T}{\Delta_0} \right)^3 \frac{T_3}{\Delta_0^2 w_{\parallel 1}^2} \right) \alpha' w_{\parallel 1} \gg T$$

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where \( w_{\theta l} = |\sin \theta - l \cos \theta| \), \( w_{\theta l} = |\cos \theta + l \sin \theta| \), and \( \alpha' = \frac{qF}{2\sqrt{2}} - \sqrt{2} \alpha g_k \).

Depending on the effective nonlocal energy scales
\[
E_{\text{nonloc}}^+ = \frac{v_F \theta_{\theta l}}{\lambda_0}, E_{\text{nonloc}}^- = \frac{v_F \theta_{\theta l}}{\lambda_0}, l_1, l_2 = \pm 1
\]
one obtains
\[
\Delta_{\text{spec}}^+(T) \propto \left( \frac{T}{T_c} \right) \left( \frac{T}{T_c} - 1 \right)
\]
\[
\Delta_{\text{spec}}^-(T) \propto \left( \frac{T}{T_c} \right) \left( \frac{T}{T_c} - 1 \right)
\]
\[
\Delta_{\text{spec}}^+(T) \propto \left( \frac{T}{T_c} \right) \left( \frac{T}{T_c} - 1 \right)
\]
\[
\Delta_{\text{spec}}^-(T) \propto \left( \frac{T}{T_c} \right) \left( \frac{T}{T_c} - 1 \right)
\]

For CePtSi superconductor with \( T_c = 0.75 \text{K} \), the linear temperature dependence would crossover to a quadratic dependence below \( T_{\text{nonloc}} \sim 0.015 \text{K} \).

Magnetic penetration depth measurements in CePtSi did not find a \( T^2 \) law as expected for line nodes. I argue that it may be due to the fact that such measurements were performed above 0.015K. On the other hand, it is note that CePtSi is an extreme type-II superconductor with the Ginzburg-Landau parameter, \( K = 140 \), and the nonlocal effect can be safely neglected, and because this system is a clean superconductor, neglect the impurity effect can be neglected (Bauer et al., 2004; Bauer et al., 2005).

In the local, clean, and nonlinear limit \( (q \to 0, \nu \neq 0) \) the penetration depth is given by
\[
\lambda_{\text{spec}}(T) = \left( \frac{c}{4\pi \delta K(q \to 0, \nu, T)} \right)^{1/2}
\]

Where
\[
\delta K(q, \nu, T) = \ln \left( \frac{mc}{e^2} \right)
\]
\[
\begin{aligned}
&= \frac{1}{2} \left[ \sinh^{-1} \left( \frac{\tilde{q}}{2\Delta_{\nu, +}} \right) \right] + 2 \left( \frac{\tilde{q} \tilde{k}_F}{2 \Delta_{\nu, +}} \right) + 2 \left( \frac{\tilde{q} \tilde{k}_F}{2 \Delta_{\nu, -}} \right) + 2 \left( \frac{\tilde{q} \tilde{k}_F}{2 \Delta_{\nu, -}} \right) \\
&- \frac{1}{2} \left[ \sinh^{-1} \left( \frac{\tilde{q}}{2\Delta_{\nu, -}} \right) \right] + 2 \left( \frac{\tilde{q} \tilde{k}_F}{2 \Delta_{\nu, -}} \right) + 2 \left( \frac{\tilde{q} \tilde{k}_F}{2 \Delta_{\nu, +}} \right) + 2 \left( \frac{\tilde{q} \tilde{k}_F}{2 \Delta_{\nu, +}} \right)
\end{aligned}
\]

Thus by considering only the second term in the right hand side of Eq. (55) into Eq. (51) one gets
The linear temperature dependence of penetration depth is in agreement with Bonalde et al.'s result (Bonalde et al., 2005).

Thus the $T$ behavior at low temperatures of the penetration depth in Eq. (56) is due to nonlinearity indicating the existence of line nodes in the gap parameter in CePt$_3$Si compound. A $T$ linear dependence of the penetration depth in the low temperature region is expected for clean, local and nonlinear superconductors with line nodes in the gap function.

Now the effect of impurities when both $s$-wave and $p$-wave Cooper pairings coexist is considered.

I assume that the superconductivity in CePt$_3$Si is unconventional and is affected only by nonmagnetic impurities. The equation of motion for self-energy can be written as

$$
\Delta_{\text{spec}}^{\text{imp}} = \left\{ \frac{1}{2} \ln 2 \sum_{i=1}^{n} \frac{u_{0}^2}{\Delta_0} \left[ \frac{T}{\Delta_0} + \frac{i u_{0}^2}{2 \sqrt{2}} \left[ \nu_{i} k_{F} + \frac{2 \alpha g_{s}}{\Delta_0} \right] \right] + \left( \frac{-i \nu_{i} k_{F} + 4 \alpha g_{s}}{\sqrt{2} \Delta} \right) \right\}
$$

(57)

where the $T$ matrix is given by

$$
T(p, p', i \omega_n) = n_{c} T(p, p', i \omega_n)
$$

(58)

where $\sigma_3$ is the third Pauli-spin operator.

By using the expression of the Green’s function in Eq. (58) one can write

$$
T(p, p', i \omega_n) = \frac{\pi N_{0} u_{0}^2 I}{1 + (\pi N_{0} u_{0} I)^2}
$$

(59)

where

$$
I = \int_{0}^{2 \pi} \frac{d \Omega}{4 \pi} \frac{-i \omega + i \Sigma_{\text{imp}}(n)}{(\omega + i \Sigma_{\text{imp}}(n))^{2} + \Delta_{s} \Delta_{s}}^{1/2}
$$

(60)

and $u_{0}$ is a single $s$-wave matrix element of scattering potential $u$. Small $u_{0}$ puts us in the limit where the Born approximation is valid, where large $u_{0} (u_{0} \rightarrow \infty)$, puts us in the unitarity limit.
Theoretically it is known that the nodal gap structure is very sensitive to the impurities. If the spin-singlet and triplet components are mixed, the latter might be suppressed by the impurity scattering and the system would behave like a BCS superconductor. For p-wave gap function the polar and axial states have angular structures,

\[ \Delta_{p}(T) = \Delta_{0}(T) \cos \theta, \quad \Delta_{a}(T) = \Delta_{0}(T) \sin \theta, \]

respectively. The electromagnetic response now depends on the mutual orientation of the vector potential \( \mathbf{A} \) and \( \hat{I} \) (unit vector of gap symmetry), which itself may be oriented by surfaces, fields and superflow. A detailed experimental and theoretical study for the axial and polar states was presented in Ref. (Einzel, 1986). In the clean limit and in the absence of Fermi-Liquid effects the following low-temperature asymptotic were obtained for axial and polar states

\[ \frac{\Delta \lambda(T)_{\parallel,\perp}}{\lambda(0)} = d_{\parallel,\perp} \left( \frac{k_{B}T}{\Delta_{0}} \right)^{n_{\parallel,\perp}}, \]

where in the axial state \( n = 2(4) \) and \( a = \pi^{2} \left( \frac{7\pi^{4}}{15} \right) \), and in the polar state \( n = 3(1) \) and

\[ a = \frac{27\pi\xi(3)}{4} \left( \frac{3\pi\ln 2}{2} \right), \]

for the orientations \( \parallel (\perp) \).

The influence of nonmagnetic impurities on the penetration depth of a p-wave superconductor was discussed in detail in Ref. (Gross et al., 1986). At very low temperatures, the main contribution will originated from the eigenvalue with the lower temperature exponent \( n \), i.e., for the axial state (point nodes) with \( T^{2} \) low, and for the polar state (line nodes) the dominating contribution with a linear \( T \). The quadratic dependence in axial state may arise from nonlocality.

The low temperature dependence of penetration depth in polar and axial states used by Einzel et al., (Einzel et al. 1986) to analyze the \( \lambda(T) \sim T^{2} \) behavior of \( Ube_{13} \) at low temperatures. The axial \( \hat{A} \parallel \hat{I} \) case seems to be the proper state to analyze the experiment because it was favored by orientation effects and was the only one with \( T^{2} \) dependence. Meanwhile, it has turned out that \( T^{2} \) behavior is introduced immediately by T-matrix impurity scattering and also by weak scattering in the polar case. The axial state, and according to the Andersons theorem the s-wave value of the London penetration depth are not at all affected by small concentration of nonmagnetic impurities.

Thus, for the polar state, Eq. (60) can be written as

\[ I = \frac{2\pi}{d_{0}} \int_{0}^{2\pi} \frac{d\theta}{2\pi} \frac{\omega + i \Sigma_{imp(n)}}{\left( \omega + i \Sigma_{imp(n)} \right)^{2} + \Delta_{0}^{2} \cos^{2}\theta} \]

Doing the angular integration in Eq. (62) and using Eqs. (57) and (59) one obtains...
Effects of Impurities on a Noncentrosymmetric Superconductor - Application to CePt$_3$Si

\[
\sum_{\text{imp}(n)} = -\left[ 2\tilde{\omega}N(0)u_0^2 \sqrt{\Delta^2 + \Delta_0^2} \right] K\left( \frac{\Delta^2}{\sqrt{\Delta^2 + \Delta_0^2}} \right)
+ \left[ 4N(0)^2u_0^2\tilde{\omega}^2 \left/ \left( \tilde{\omega}^2 + \Delta_0^2 \right) \right. \right] K^2\left( \frac{\Delta_0^2}{\sqrt{\tilde{\omega}^2 + \Delta_0^2}} \right)
\]  

(63)

Here $K$ is the elliptic integral and $\tilde{\omega} = \omega + i\sum_{\text{imp}(n)}$. We note that in the impurity dominated gapless regime, the normalized frequency $\tilde{\omega}$ takes the limiting form $\tilde{\omega} \to \omega + i\gamma$, where $\gamma$ is a constant depending on impurity concentration and scattering strength. In the low temperature limit we can replace the normalized frequency $\tilde{\omega}$ everywhere by its low frequency limiting form and after integration over frequency one gets

\[
\delta K(\tilde{\eta}, \tilde{\vartheta}, T) = -\frac{N_e e^2}{mc}\left\{ \frac{4\pi^2 T^2}{3} \left[ \delta^2 + \frac{\Delta_0^2}{(\Delta_0^2 + \gamma^2)^2} \right] \right\}
\]

(64)

As in the case of d-wave order parameter, from Eqs. (64) and (51) one finds

\[
\frac{\delta\lambda(T)}{\lambda(0)} = \frac{\gamma}{4\pi\Delta_0} \ln \left( \frac{4\Delta_0}{\gamma} \right) + \frac{\pi}{24\gamma\Delta_0} T^2
\]

(65)

In $p$-wave cuprates, scattering fills in electronic states at the gap nodes, thereby suppressing the penetration depth at low temperatures and changing $T^2$ -linear to $T^2$ behavior.

5. Effect of impurities on the low temperature NMR relaxation rate of a noncentrosymmetric superconductor

I consider the NMR spin-lattice relaxation due to the interaction between the nuclear spin magnetic moment $\gamma_n I$ ($\gamma_n$ is the nuclear gyromagnetic ratio) and the hyperfine field $h$, created at the nucleus by the conduction electrons. Thus the system Hamiltonian is

\[
H = H_0 + H_{\text{in}} + H_n + H_{\text{int}}
\]

(66)

where $H_0$ and $H_{\text{in}}$ are defined by Eqs. (1) and (2), $H_n = -\gamma_n H$ is the Zeeman coupling of the nuclear spin with the external field $H$, and $H_{\text{int}} = -\gamma_n h$ is the hyperfine interaction. The spin-lattice relaxation rate due to the hyperfine contact interaction of the nucleus with the band electron is given by

\[
R = \frac{1}{T_1 T} = \frac{j^2}{2\pi} \lim_{\omega \to 0} \frac{\text{Im} K^R(\omega)}{\omega}
\]

(67)

where $\omega$ is the NMR frequency, $J = \frac{8\pi}{3} \gamma_n \gamma_e$ ($\gamma_e$ is the electron gemoagnetic ratio) is the hyperfine coupling constant, and $K^R(\tau)$, the Fourier transform of the retarded correlation

\[
R = \frac{1}{T_1 T} = \frac{j^2}{2\pi} \lim_{\omega \to 0} \frac{\text{Im} K^R(\omega)}{\omega}
\]

(67)

where $\omega$ is the NMR frequency, $J = \frac{8\pi}{3} \gamma_n \gamma_e$ ($\gamma_e$ is the electron gemoagnetic ratio) is the hyperfine coupling constant, and $K^R(\tau)$, the Fourier transform of the retarded correlation
function of the electron spin densities at the nuclear site, in the Matsubara formalism is given by (in our units $\hbar = 1$)

$$K^{R}_{+-}(\tau) = -\left\langle T_{\tau} S_{+}(\tau) S_{-}(0) \right\rangle$$  \hspace{1cm} (68) $$

here $T_{\tau}$ is the time order operator, $\tau$ is the imaginary time, $S_{\pm}(\tau) = e^{H_{c} \tau} S_{\pm} e^{-H_{c} \tau}$, and

$$S_{\pm}(\tau) = \psi^{\dagger}_{\pm}(\vec{r}) \psi_{\pm}(\vec{r})$$  \hspace{1cm} (69) $$

with $\psi^{\dagger}_{\pm}(\vec{r})$ and $\psi_{\pm}(\vec{r})$ being the electron field operators.

The Fourier transform of the correlation function is given by

$$K^{R}_{+-}(i\omega_{n}) = \int_{0}^{\beta} d\tau e^{i\omega_{n} \tau} K^{R}_{+-}(\tau)$$  \hspace{1cm} (70) $$

The retarded correlation function is obtained by analytical continuation of the Matsubara correlation function $K^{R}_{+-}(\omega) = K^{R}_{+-}(i\omega_{n})\big|_{\omega_{n} \rightarrow \omega + i\delta}$. From Eqs. (66)- (70), one gets

$$\frac{1}{T_{1}^{T}} = \frac{\hbar^{2}}{2 \pi} \lim_{\omega \rightarrow 0} \frac{1}{\omega} \times \text{Im} \left\{ T \sum_{p,p',\omega_{n}} \left[ Tr \left( \mathcal{G}_{\pm}(\vec{p},i\omega_{n}) \mathcal{G}_{\pm}^{\dagger}(\vec{p'},i\omega_{n}) \right) - Tr \left( \mathcal{F}_{\pm}(\vec{p},i\omega_{n}) \mathcal{F}_{\pm}^{\dagger}(\vec{p'},i\omega_{n}) \right) \right] \right\}$$  \hspace{1cm} (71) $$

where $\Omega_{m} = 2m \pi T$ are the bosonic Matsubara frequencies. By using Eqs. (11) and (12) into Eq. (71), the final result for the relaxation rate is

$$\frac{1}{T_{1}^{T}} = \frac{\hbar^{2}}{2 \pi} \times \text{Im} \left\{ N_{\sigma}(\omega) N_{-\sigma}(\omega) + M_{\sigma}(\omega) M_{-\sigma}(\omega) \right\}$$  \hspace{1cm} (72) $$

where $f(\omega) = \left( e^{\beta \omega} + 1 \right)^{-1}$ is the Fermi Function, $N_{\sigma}(\omega)$ and $M_{\sigma}(\omega)$ defined by the retarded Green’s functions as

$$N_{\sigma}(\omega) = -\sum_{p,\nu = \pm} \text{Im} \mathcal{G}_{\nu}^{R}(\vec{p},\omega)$$  \hspace{1cm} (73) $$

$$M_{\sigma}(\omega) = -\sum_{p,\nu = \pm} \text{Im} \mathcal{F}_{\nu}^{R}(\vec{p},\omega)$$  \hspace{1cm} (74) $$

In low temperatures limit the contribution of the fully gap $(\Delta_{1} + \Delta \sin \theta)$ Fermi surface I decrease and the effect of the gap $(\Delta_{0} - \Delta \sin \theta)$ Fermi surface II is enhanced.
As I mentioned above, the experimental data for CePt$_3$Si at low temperature seem to point to the presence of lines of the gap nodes in gap parameter (In our gap model for $\Delta_0 < \Delta$, $|\Delta_0 - \Delta \sin \theta|$ has line nodes). Symmetry imposed gap nodes exist only for the order parameters which transform according to one of the nonunity representations of the point group. For all such order parameters $M_\sigma = 0$. Thus, Eq. (72) can be written as

$$\frac{1}{T_i} = \frac{\pi^2 \chi}{4T_0} \int_0^\infty \frac{d\omega}{\cosh^2 \left( \frac{\omega}{2T} \right)} \{N_+ (\omega)N_- (\omega)\} (75)$$

In the clean limit the density of state can be calculated from BCS expression

$$N_\sigma (\omega) = N_0 \text{Re} \left( \frac{\omega}{\sqrt{\omega^2 - \Delta_0^2}} \right) (76)$$

For the gap parameter with line nodes from Eq. (76) one gets

$$N(\omega) = N_0 \frac{\pi \omega}{2 \Delta_0} (77)$$

Thus from Eq. (75) one has

$$\frac{1}{T_i} = \frac{\pi^2 N_0^2 q}{2\Delta_0^2} (78)$$

Therefore, line nodes on the Fermi surface II lead to the low-temperature $T^3$ law in $T_i$, which is in qualitative agreement with the experimental results.

In the dirty limit the density of state can be written as

$$N_{\text{imp}}(\omega) = \int d\Omega \frac{N_{\text{BCS}}(\omega, \theta)}{1 + N_{\text{BCS}}^2(\omega, \theta)} (79)$$

In the limit, $\Gamma \ll \Delta_0$ where $\Gamma = \frac{\hbar \Sigma}{\pi N_0 N} \left( \frac{\hbar}{m} \right)$ is the electron density the density of state is

$$N_{\text{imp}}(\omega) \approx N(0) + a c^2 \omega (80)$$

where $c = \cot g \delta_0$ ($\delta_0$ is the s-wave scattering phase shift), $a$ is a constant, and $N(0)$ the zero energy ($\omega = 0$) quasi-particle density of state is given by

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\[
N(0) = N_0 \left( \frac{\xi}{\sqrt{1 + \frac{1}{4} \xi^2 + \frac{1}{2} \xi^4}} \right)^{\frac{1}{2}}
\]

(81)

where \( \xi = \frac{\Gamma}{\Delta} \).

In the unitary limit \( (u_0 \to \infty) \), \( \xi = 0 \) \( (\delta_0 = \pi/2) \), from Eqs. (75) and (80) one obtains

\[
\frac{1}{T_1} = j^2 N(0)^2 T
\]

(82)

Thus the power-low temperature dependence of \( T_1^{-1} \) is affected by impurities and it changes to linear temperature dependence characteristic of the normal state Koringa relation again in agreement with the experimental results.

6. Conclusion

In this chapter I have studied theoretically the effect of both magnetic and nonmagnetic impurities on the superconducting properties of a non-centrosymmetric superconductor and also I have discussed the application of my results to a model of superconductivity in CePt\(_3\)Si.

First, the critical temperature is obtained for a superconductor with an arbitrary of impurity concentration (magnetic and nonmagnetic) and an arbitrary degree of anisotropy of the superconducting order parameter, ranging from isotropic s wave to p wave and mixed (s+p) wave as particular cases.

The critical temperature is found to be suppressed by disorder, both for conventional and unconventional pairings, in the latter case according to the universal Abrikosov-Gor’kov function.

In the case of nonsentrosymmetrical superconductor CePt\(_3\)Si with conventional pairing ( \( A_1 \) representation with purely accidental line nodes), I have found that the anisotropy of the conventional order parameter increases the rate at which \( T_1 \) is suppressed by impurities.

Unlike the unconventional case, however, the superconductivity is never completely destroyed, even at strong disorder.

In section 4, I have calculated the appropriate correlation function to evaluate the magnetic penetration depth. Besides nonlinearity and nonlocality, the effect of impurities in the magnetic penetration depth when both s-wave and p-wave Cooper pairings coexist, has been considered.

For superconductor CePt\(_3\)Si, I have shown that such a model with different symmetries describes the data rather well. In this system the low temperature behavior of the magnetic penetration depth is consistence with the presence of line nodes in the energy gap and a quadratic dependence due to nonlocality may accrue below \( T_{\text{node}}^* = 0.015 \text{K} \). In a dirty superconductor the quadratic temperature dependence of the magnetic penetration depth may come from either impurity scattering or nonlocality, but the nonlocality and nodal behavior may be hidden by the impurity effects.
Finally, I have calculated the nuclear spin-lattice relaxation of CePt₃Si superconductor. In the clean limit the line nodes which can occur due to the superposition of the two spin channels lead to the low temperature $T^3$ law in $T^{-1}$. In a dirty superconductor the linear temperature dependence of the spin-lattice relaxation rate characteristic of the normal state Koringa relation.

7. Acknowledgment
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Effects of Impurities on a Noncentrosymmetric Superconductor - Application to CePtSi


Yogi, M; Kitaoka,Y; Hashimoto,S; Yasuda, T; Settai,R; Matsuda, T. D; Haga,Y; Onuki, Y; Rogl, P. & Bauer,E. (2004). Evidence for a Novel State of Superconductivity in

Superconductivity was discovered in 1911 by Kamerlingh Onnes. Since the discovery of an oxide superconductor with critical temperature (Tc) approximately equal to 35 K (by Bednorz and Müller 1986), there are a great number of laboratories all over the world involved in research of superconductors with high Tc values, the so-called high-Tc superconductors. This book contains 15 chapters reporting about interesting research about theoretical and experimental aspects of superconductivity. You will find here a great number of works about theories and properties of High-Tc superconductors (materials with Tc > 30 K). In a few chapters there are also discussions concerning low-Tc superconductors (Tc < 30 K). This book will certainly encourage further experimental and theoretical research in new theories and new superconducting materials.

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