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Reduction of Reflection from Conducting Surfaces using Plasma Shielding
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1. Introduction

Plasma mediums have taken considerable interest in recent studies due to their tunable characteristics offering some advantages in radio communications, radio astronomy and military stealth applications. Special plasma mediums have been used as electromagnetic wave reflectors, absorbers and scatterers. Reflection, absorption and transmission of electromagnetic waves by a magnetized nonuniform plasma slab are analysed by different authors using different methods in literature. It is known that plasma parameters such as length, collision frequency and electron density distribution function considerably affect plasma response. Among those, especially the electron density distribution considerably affects the frequency selectivity of the plasma (Gurel & Oncu, 2009a, 2009b, 2009c). Conducting plane covered with plasma layer has been considered and analysed in literature for some specific density distribution functions such as exponential and hyperbolic distributions (Shi et al., 2001; Su et al., 2003; J. Zhang & Z. Liu, 2007). The effects of external magnetic field applied in different directions to the plasma are also important and considered in those studies.

In order to analyze the characteristics of electromagnetic wave propagation in plasma, many theoretical methods have been developed. Gregoire et al. have used W.K.B approximate method to analyze the electromagnetic wave propagation in unmagnetized plasmas (Gregoire et al., 1992) and Cao et al. used the same method to find out the absorption characteristics of conductive targets coated with plasma (Cao et al., 2002). Hu et al. analyzed reflection, absorption, and transmission characteristics from nonuniform magnetized plasma slab by using scattering matrix method (SMM) (Hu et al., 1999). Zhang et al. and Yang et al. used the recursion formula for generalized reflection coefficient to find out electromagnetic wave reflection characteristics from nonuniform plasma (Yang et al., 2001; J. Zhang & Z. Liu, 2007). Liu et al. used the finite difference time domain method (FDTD) to analyze the electromagnetic reflection by conductive plane covered with magnetized inhomogeneous plasma (Liu et al., 2002).

The aim of this study is to determine the effect of plasma covering on the reflection characteristics of conducting plane as the function of special electron density distributions and plasma parameters. Plasma covered conducting plane is taken to model general stealth application and normally incident electromagnetic wave propagation through the
plasma medium is assumed. Special distribution functions are chosen as linearly varying electron density distribution having positive or negative slopes and purely sinusoidal distribution which have shown to provide wideband frequency selectivity characteristics in plasma shielding applications in recent studies (Gurel & Oncu, 2009a, 2009b, 2009c). It is shown that linearly varying profile with positive and negative slopes can provide adjustable reflection or absorption performances in different frequency bands due to proper selection of operational parameters. Sinusoidally-varying electron distribution with adjustable phase shift is also important to provide tunable plasma response. The positions of maximums and minimums of the electron number density along the slab can be changed by adjusting the phase of the sinusoid as well as the other plasma parameters. Thus plasma layer can be tuned to behave as a good reflector or as a good absorber. In this study, plasma is taken as cold, weakly ionized, steady state, collisional, nonuniform while background magnetic field is assumed to be uniform and parallel to the magnetized slab.

2. Physical model and basic theory

There are several theoretical methods as mentioned in the previous section for the analysis of electromagnetic wave propagation through the plasma which will be summarized in this part.

2.1 Generalized reflection coefficient formula

Firstly two successive subslabs of plasma layer are considered as shown in Fig. 1.

![Fig. 1. Two successive plasma subslabs.](image)

The incident and reflected field equations for the $m$th subslab can be written as

$$
E_i(m, z) = e_i(m) \exp\{-jk(m)(z-Z_m)\} \quad (1)
$$

$$
E_r(m, z) = e_r(m) \exp\{jk(m)(z-Z_m)\} \quad (2)
$$

where $E_i$ is the incident field and $E_r$ is the reflected field. Then, incident and reflected field equations for the $(m+1)$th subslab can be similarly given as
\[ E_i(m+1,z) = e_i(m+1) \exp\left\{-jk(m+1)(z-Z_{m+1})\right\} \]  
\[ E_r(m+1,z) = e_r(m+1) \exp\left\{-jk(m+1)(z-Z_{m+1})\right\} \]  

Then total electric field in the \( m^{th} \) subslab is

\[ E_y(m,z) = e_i(m) \exp\left\{-jk(m)(z-Z_m)\right\} + e_r(m) \exp\left\{jk(m)(z-Z_m)\right\} \]  
and for \( (m+1)^{th} \) sublab

\[ E_y(m+1,z) = e_i(m+1) \exp\left\{-jk(m+1)(z-Z_{m+1})\right\} + e_r(m+1) \exp\left\{jk(m+1)(z-Z_{m+1})\right\} \]  

After getting the electric field equations, magnetic field equations are obtained as follows

\[ H_x(m,z) = \frac{1}{\eta_m} \left[ -e_i(m) \exp\left\{-jk(m)(z-Z_m)\right\} + e_r(m) \exp\left\{jk(m)(z-Z_m)\right\} \right] \]  
\[ H_x(m+1,z) = \frac{1}{\eta_{m+1}} \left[ -e_i(m+1) \exp\left\{-jk(m+1)(z-Z_{m+1})\right\} + e_r(m+1) \exp\left\{jk(m+1)(z-Z_{m+1})\right\} \right] \]  
where \( \eta_m \) and \( \eta_{m+1} \) are the intrinsic impedances of \( m^{th} \) and \( (m+1)^{th} \) subslabs respectively. The intrinsic impedance for the \( m^{th} \) subslab is

\[ \eta_m = \sqrt{\frac{\mu_0 \epsilon_0}{\epsilon_{r_m}}} \]  

To match the boundary conditions at \( z = Z_m \), following two equations can be written

\[ E_y(m,z) = E_y(m+1,z) \]  
\[ H_x(m,z) = H_x(m+1,z) \]  

Then (5) and (6) are inserted into equation (10), and it is obtained that

\[ e_i(m) \exp\left\{-jk(m)(z-Z_m)\right\} + e_r(m) \exp\left\{jk(m)(z-Z_m)\right\} = e_i(m+1) \]  
\[ \cdot \exp\left\{-jk(m+1)(z-Z_{m+1})\right\} + e_r(m+1) \exp\left\{jk(m+1)(z-Z_{m+1})\right\} \]  

Since \( z = Z_m \) at the boundary, equation (12) can be arranged as below

\[ e_i(m) + e_r(m) = \exp\left\{jk(m+1)d_{m+1}\right\} e_i(m+1) \exp\left\{-2jk(m+1)d_{m+1}\right\} + e_r(m+1) \]  

where \( d_{m+1} \) is the thickness of the \( (m+1)^{th} \) sublab. 
By using the following equalities,
\[
\exp\left\{ jk(m+1)d_{m+1}\right\} = A
\]  (14)

\[
\exp\left\{-2jk(m+1)d_{m+1}\right\} = B
\]  (15)

Equation (13) becomes

\[
e_i(m) + e_r(m) = A\left[ e_i(m+1)B + e_r(m+1)\right]
\]  (16)

By inserting equations (7) and (8) into (11), it is obtained that

\[
\frac{1}{\eta_m}\left[-e_i(m)\exp\left\{-jk(m)(z-Z_m)\right\} + e_r(m)\exp\left\{jk(m)(z-Z_m)\right\}\right] = \frac{1}{\eta_{m+1}}e_r(m+1)
\]

\[
\exp\left\{-jk(m+1)(z-Z_{m+1})\right\} + \frac{1}{\eta_{m+1}}e_r(m+1)\exp\left\{jk(m+1)(z-Z_{m+1})\right\}
\]  (17)

Then by replacing \( z = Z_m \) in (17) it is obtained that

\[
\frac{1}{\eta_m}\left[-e_i(m) + e_r(m)\right] = \frac{1}{\eta(m+1)}\left[-e_i(m+1)\exp\left\{-jk(m+1)d_{m+1} + e_r(m+1)\right\}\right]
\]

\[
\exp\left\{jk(m+1)d_{m+1}\right\}
\]  (18)

By relating the intrinsic impedance to permittivity and arranging the equation (18),

\[
-e_i(m) + e_r(m) = \sqrt{\frac{e_r(m+1)}{e_r(m)}}\exp\left\{jk(m+1)d_{m+1}\left[e_i(m+1)\exp\left\{-2jk(m+1)d_{m+1}\right\} + e_r(m+1)\right]\right\}
\]  (19)

Now, the final equation is obtained as

\[
-e_i(m) + e_r(m) = A\left[-e_i(m+1)B + e_r(m+1)\right]C
\]  (20)

where

\[
C = \sqrt{\frac{e_r(m+1)}{e_r(m)}}
\]  (21)

Then equations (16) and (20) are combined and expressed in matrix form as

\[
\begin{bmatrix}
    e_i(m) \\
    e_r(m)
\end{bmatrix} = \frac{1}{2}A
\begin{bmatrix}
    B(1+C) & (1-C) \\
    B(1-C) & (1+C)
\end{bmatrix}
\begin{bmatrix}
    e_i(m+1) \\
    e_r(m+1)
\end{bmatrix}
\]  (22)

\[
S_m = \frac{1}{2}A
\begin{bmatrix}
    B(1+C) & (1-C) \\
    B(1-C) & (1+C)
\end{bmatrix}
\]  (23)

For \( m = n - 1 \)

\[
\begin{bmatrix}
    e_i(n-1) \\
    e_r(n-1)
\end{bmatrix} = S_{n-1}\begin{bmatrix}
    e_i(n) \\
    e_r(n)
\end{bmatrix}
\]  (25)
where \( n \) is the last boundary of the plasma slab which is located before conductive target. For \( m = n - 2 \)

\[
\begin{bmatrix}
e_r(n-2) \\
e_i(n-2)
\end{bmatrix}
= S_{n-2}
\begin{bmatrix}
e_r(n-1) \\
e_i(n-1)
\end{bmatrix}
\tag{25}
\]

When we continue to write the field equations iteratively until \( m = 0 \) which means the boundary between free space and the first subslab of plasma, we have

\[
\begin{bmatrix}
e_r(0) \\
e_i(0)
\end{bmatrix}
= -S_0S_1S_2S_3\ldots S_{n-1}
\begin{bmatrix}
e_r(n) \\
e_i(n)
\end{bmatrix}
\tag{26}
\]

This can be written in the following compact form,

\[
\begin{bmatrix}
e_r(0) \\
e_i(0)
\end{bmatrix}
= \prod_{m=0}^{n-1} S_m
\begin{bmatrix}
e_r(n) \\
e_i(n)
\end{bmatrix}
\tag{27}
\]

Letting

\[
\prod_{m=0}^{n-1} S_m
= \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} = M
\tag{28}
\]

Then by inserting equation (28) into equation (27)

\[
\begin{bmatrix}
e_r(0) \\
e_i(0)
\end{bmatrix}
= \prod_{m=0}^{n-1} S_m
\begin{bmatrix}
e_r(n) \\
e_i(n)
\end{bmatrix}
\tag{29}
\]

Hence

\[
e_r(0) = M_1 e_r(n) + M_2 e_i(n)
\tag{30}
\]

\[
e_i(0) = M_3 e_r(n) + M_4 e_i(n)
\tag{31}
\]

By dividing the both sides of the equations by \( e_i(n) \), the following two equations are obtained

\[
\frac{e_r(0)}{e_i(n)} = M_1 \frac{e_r(n)}{e_i(n)} + M_2
\tag{32}
\]

\[
\frac{e_i(0)}{e_i(n)} = M_3 \frac{e_r(n)}{e_i(n)} + M_4
\tag{33}
\]

Then by dividing these two equations side by side, we get (J. Zhang & Z. Liu, 2007)

\[
\frac{e_r(0)}{e_i(0)} = \frac{M_1 \Gamma(n) + M_2}{M_3 \Gamma(n) + M_4}
\tag{34}
\]
where $\Gamma(n)$ is the reflection coefficient of the conductive target. In order to calculate the total reflection coefficient, the matrix $M$ is needed to be computed.

### 2.2 Wentzel-Kramers-Brillouin (WKB) approximate method

It is known that WKB method is used for finding the approximate solutions to linear partial differential equations that have spatially varying coefficients. This mathematical approximate method can be used to solve the wave equation that defines the electromagnetic wave propagation in a dielectric plasma medium.

Let us write the wave equation as

$$\frac{d^2E}{dz^2} + k_z^2E_z = 0 \quad (35)$$

Then WKB method can be applied to derive an approximate solution (Gregoire et al., 1992),

$$E_z = E_0 \exp \left( \int_0^z k(z') dz' \right) \quad (36)$$

This solution is valid in any region where

$$\frac{1}{k^2} \frac{dk}{dz} \ll 1 \quad (37)$$

The physical meaning of (37) is that the wavenumber of the propagating electromagnetic wave changes very little over a distance of one wavelength.

It is assumed that the electromagnetic wave enters the plasma at $z = z_0$ and reflects back at $z = z_1$. Then total reflected power can be found by WKB approximation (Gregoire et al., 1992) as

$$\overline{P}_r = \exp \left( -4 \int_{z_0}^{z_1} \text{Im}(k(z')) dz' \right) \quad (38)$$

where $\overline{P}_r$ is the normalized total reflected power.

### 2.3 Finite-difference time-domain analysis

Finite-difference time-domain analysis have been extensively used in literature to solve the electromagnetic wave propagation in various media (Hunsberger et al., 1992; Young, 1994, 1996; Cummer, 1997; Lee et al., 2000; M. Liu et al., 2007). When the electromagnetic wave propagates in a thin plasma layer, the W.K.B method may not accurately investigate the wave propagation (X.W. Hu, 2004; S. Zhang et al., 2006). The reason is the plasma thickness is near or less than the wavelength of the plasma exceeds the wavelength of the incident wave, the variation of the wave vector with distance cannot be considered as weak (M. Liu et al., 2007).

In the analysis electric field is considered in the x direction and propagation vector is in z direction and the electromagnetic wave enters normally into the plasma layer.
Lorentz equation (electron momentum equation) and the Maxwell's equations can be written as

\[ n_e m_e \frac{\partial \vec{v}_e}{\partial t} = -e n_e \vec{E} - n_e m_e \nu_d \vec{\nu}_e \]  

(39)

\[ \frac{\partial \vec{H}}{\partial t} = -\frac{1}{\mu_0} \nabla \times \vec{E} \]  

(40)

\[ \frac{\partial \vec{E}}{\partial t} = \frac{1}{\varepsilon_0} (\nabla \times \vec{H} - \vec{j}) \]  

(41)

\[ \vec{j} = -e n_e \vec{v}_e \]  

(42)

where \( \vec{E} \) is the electric field vector, \( \vec{H} \) is the magnetic field vector, \( \varepsilon_0 \) is the permittivity, \( \mu_0 \) is the magnetic permeability of free space, \( \vec{j} \) is the current density, \( n_e \) is the density of electron, \( m_e \) and \( \vec{v}_e \) are the mass and velocity vector of the electron, respectively and \( \nu_d \) is collision frequency. Then FDTD algorithm of equations (39), (40), (41) and (42) can be written as (Chen et al., 1999; Jiang et al., 2006; Kousaka & Ono, 2002; M.H. Liu et al., 2006)

\[ H_{n+\frac{1}{2},y+\frac{1}{2},j+\frac{1}{2}} = H_{n,\frac{1}{2},y,\frac{1}{2},j,\frac{1}{2}} \Delta t \frac{E^n_{n,\frac{1}{2},y,\frac{1}{2},j,k}-E^n_{n,\frac{1}{2},y,\frac{1}{2},j,k-\frac{1}{2}}}{\Delta z} \]  

(43)

\[ E_{n+\frac{1}{2},x,\frac{1}{2},j,k} = E_{n,\frac{1}{2},x,\frac{1}{2},j,k} - \frac{\Delta t}{\varepsilon} \left( H_{n,\frac{1}{2},y,\frac{1}{2},j,k+\frac{1}{2}} - H_{n,\frac{1}{2},y,\frac{1}{2},j,k-\frac{1}{2}} \right) \frac{f_{n+\frac{1}{2},x,\frac{1}{2},j,k}}{\Delta z} \]  

(44)

\[ f_{n+\frac{1}{2},x,\frac{1}{2},j,k} = -e n_e \frac{\Delta t}{\Delta z} \frac{\vec{v}_e \cdot \vec{v}_e}{2 + \nu_d \Delta t} \left[ 2 - \nu_d \Delta t \right] \]  

(45)

\[ \vec{v}_e_{n+\frac{1}{2},x,\frac{1}{2},j,k} = \frac{1}{2 + \nu_d \Delta t} \left[ 2 - \nu_d \Delta t \right] \frac{2e \Delta t}{m_e} E^n_{n,\frac{1}{2},y,\frac{1}{2},j,k} \]  

(46)

where \( \Delta z \) is the spatial discretization and \( \Delta t \) is the time step. By using equations (43) to (46), the electromagnetic wave propagation in a plasma slab can be simulated in time domain (M. Liu et al., 2007).

2.4 Scattering matrix method (SMM) analysis

This analytical technique is the manipulation of the 2x2 matrix approach which was presented by Kong (Kong, 1986). SMM analysis gives the partial reflection and transmission.

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coefficients in the subslabs. This makes it easy to analyze the partial absorbed power in each subslab of the plasma (Hu et al., 1999).

Let us write the incident and reflected fields as follows

\[ E_y^i = E_0 \exp\left(-jk_z^0 z\right) \]  \hspace{1cm} (47)

\[ E_y^r = AE_0 \exp\left(jk_z^0 z\right) \]  \hspace{1cm} (48)

where \( k_z^0 \) is the z component of the free space wave number and \( A \) is the reflection coefficient for the first subslab.

The total electric field in incidence region is

\[ E_y^0 = E_0 \left(\exp\left(-jk_z^0 z\right) + A \exp\left(jk_z^0 z\right)\right) \]  \hspace{1cm} (49)

In the same manner, we can write the total electric field in \( m^{th} \) layer as

\[ E_y^m = E_0 \left(B_m \exp\left(-jk_z^m z\right) + C_m \exp\left(jk_z^m z\right)\right) \]  \hspace{1cm} (50)

where \( B_m \) and \( C_m \) are the unknown coefficients.

After the last subslab there is only transmitted wave that travels in free space. The electric field for this region is

\[ E_y^f = DE_0 \exp\left(-jk_z^f z\right) \]  \hspace{1cm} (51)

where \( D \) is the unknown coefficient. After writing the total electric fields in each subslab, boundary conditions can be applied.

For the first boundary

\[ \begin{pmatrix} B_1 \\ C_1 \end{pmatrix} = S_1 \begin{pmatrix} A \\ 1 \end{pmatrix} \]  \hspace{1cm} (52)

where

\[ S_1 = \frac{1}{2k_z^1} \begin{pmatrix} k_z^1 - k_z^0 & k_z^1 + k_z^0 \\ k_z^1 + k_z^0 & k_z^1 - k_z^0 \end{pmatrix} \]  \hspace{1cm} (53)

For the \( m^{th} \) boundary,

\[ \begin{pmatrix} B_m \\ C_m \end{pmatrix} = S_m \begin{pmatrix} B_{m-1} \\ C_{m-1} \end{pmatrix} \]  \hspace{1cm} (54)

where

\[ S_m = \begin{pmatrix} \exp\left(-jk_z^m d_m\right) & \exp\left(jk_z^m d_m\right) \\ k_z^m \exp\left(-jk_z^m d_m\right) & -k_z^m \exp\left(jk_z^m d_m\right) \end{pmatrix}^{-1} \times \begin{pmatrix} \exp\left(-jk_z^{m-1} d_m\right) & \exp\left(jk_z^{m-1} d_m\right) \\ k_z^{m-1} \exp\left(-jk_z^{m-1} d_m\right) & -k_z^{m-1} \exp\left(jk_z^{m-1} d_m\right) \end{pmatrix} \]  \hspace{1cm} (55)
Lastly, for the last boundary

\[
\begin{pmatrix} B_n \\ C_n \end{pmatrix} = V_p D
\]  

(56)

where

\[
V_p = \frac{1}{2k_0^p} \begin{pmatrix} (k_0^p + k_0^n) \exp \left( j \left( k_0^p - k_0^n \right) d_p \right) \\ (k_0^p - k_0^n) \exp \left( - j \left( k_0^p + k_0^n \right) d_p \right) \end{pmatrix}
\]  

(57)

By using equations (52) and (54), equation (56) can be written as,

\[
S_g \begin{pmatrix} A \\ 1 \end{pmatrix} = V_p D
\]  

(58)

where \( S_g \) is the global scattering matrix and can be written as,

\[
S_g = (S_{g1}, S_{g2})
\]  

(59)

where \( S_{g1} \) represents the first column vector and \( S_{g2} \) represents the last column vector of the global scattering matrix. Then equation (58) can be written (Hu et al., 1999) as

\[
\begin{pmatrix} A \\ D \end{pmatrix} = -\left( S_{g1} - V_p \right)^{-1} S_{g2}
\]  

(60)

By using equation (60), A and D coefficients can be computed. The coefficient A represents total reflection coefficient and the coefficient B represents total transmission coefficient. Absorbed power values for each subslab and the total absorbed power inside the plasma can be obtained by the help of equations (52), (54) and (56).

2.5 Formulation of reflection from plasma covered conducting plane

In this chapter another method is presented to analyze the characteristics of electromagnetic wave propagation in a plasma slab. This method is simple, accurate and provides less computational time as compared to other methods mentioned in previous sections. Normally incident electromagnetic wave propagation through a plasma slab is assumed as shown in Fig. 2. In the analysis, inhomogeneous plasma is divided into sufficiently thin, adjacent subslabs, in each of which plasma parameters are constant. Then starting with Maxwell’s equations, reflected, absorbed and transmitted power expressions are derived. Here, plasma layer is taken as cold, weakly ionized, steady state and collisional. Background magnetic field is assumed to be uniform and parallel to the magnetized slab. For a magnetized and source free plasma medium, plasma permittivity is in tensor form. This tensor form permittivity can be approximated by a scalar permittivity. Let us give the details of this approximation.

The equation of motion for an electron of mass \( m \) is

\[
-m \omega^2 \vec{r} + m \mathbf{v} \mathbf{a} = e \mathbf{E} + e \omega \mathbf{v} \times \mathbf{B}
\]  

(61)
where \( w \) is the angular frequency, \( \vec{r} \) is the distance vector, \( \nu_\alpha \) is the collision frequency and \( \vec{B} \) is the magnetic field vector.

Then we insert the polarization vector \( \vec{P} \) into equation (61) and we get,

\[
-\omega^2 m\vec{P} + j\nu m\omega \vec{P} = N\varepsilon^2 \vec{E} + j\omega e \vec{P} \times \vec{B}
\]

(62)

Now inserting the following terms into equation (62)

\[
X = \omega / \omega^2 = N \varepsilon^2 / (\varepsilon_0 m\omega^2)
\]

(63)

\[
\hat{Y} = \frac{e\vec{B}}{m\omega}
\]

(64)

\[
Z = \frac{\nu}{\omega}
\]

(65)

Then, it is obtained that

\[
\varepsilon X \vec{E} = \vec{P}(1 - jZ) - j\hat{Y} \times \vec{P}
\]

(66)

In cartesian coordinates, equation (66) can be written as

\[
-\varepsilon X \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} U & jY & -jX \\ -jZ & U & jX \\ jY & -jX & U \end{bmatrix} \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}
\]

(67)

where

\[
U = 1 - jZ
\]

(68)
In equation (69), \( l_x \), \( l_y \) and \( l_z \) are the direction cosines of \( \vec{Y} \). We can take the polarization matrix by using equation (67)

\[
\vec{P} = \varepsilon_o \left[ M \right] \vec{E}
\]

(69)

where \( [M] \) is the susceptibility tensor. Then dielectric tensor is obtained as

\[
\varepsilon = \varepsilon_0 \{ 1 + [M] \}
\]

(70)

If coordinate system is oriented such that \( \vec{Y} \) is parallel to the xz plane, equation (67) becomes

\[
\begin{bmatrix}
    E_X \\
    E_Y \\
    E_Z
\end{bmatrix} =
\begin{bmatrix}
    U & jY_t & 0 \\
    -jY_t & U & jY_t \\
    0 & -jY_t & U
\end{bmatrix}
\begin{bmatrix}
    P_X \\
    P_Y \\
    P_Z
\end{bmatrix}
\]

(71)

where \( Y_t \) is the longitudinal component and \( Y_t \) is the transverse component of \( \vec{Y} \). In the direction of the electromagnetic wave we have,

\[
D_Z = \varepsilon_0 E_Z + P_Z = 0
\]

(72)

where \( D_z \) is the z component of electric flux density vector.

By using equation (71)

\[
-\varepsilon_0 X E_Z = -jY_t P_Y + UP_Z
\]

(73)

If \( E_Z \) is eliminated by using equations (72) and (73), it is obtained that

\[
(U - X)P_Z = jY_t P_Y
\]

(74)

By using equation (69) and the following equations

\[
\bar{D} = \varepsilon_0 \bar{E} + \bar{P}
\]

(75)

\[
\rho = \frac{E_Y}{E_X} = \frac{-H_X}{H_Y} = \frac{D_Y}{D_X}
\]

(76)

\[
P_Y = \rho P_X
\]

(77)

where \( \rho \) is the polarization ratio, we have

\[
-\varepsilon_0 X E_X = (U + j\rho Y_t)P_X
\]

(78)

\[
-\varepsilon_0 X E_Y = \left[ \rho U - jY_t - \rho Y^2, (U - X)^{-1} \right] P_X
\]

(79)

When equations (78) and (79) are divided side by side, it is obtained that

\[
\rho^2 - j\rho Y^2 \left[ (U - X)Y^2 \right]^{-1} + 1 = 0
\]

(80)
The solution of the equation (80) is given by

\[ \rho = \pm \frac{jY_i^2}{2(U - X)Y_t} \pm j \left[ 1 + \frac{Y_t^4}{4(U - X)^2Y_i^2} \right]^{1/2} \]  

(81)

By using equations (72) and (74)

\[ E_Z = \frac{1}{\varepsilon_0} \frac{jY_t}{U - X} \rho P_X \]  

(82)

By the help of equation (72)

\[ P_X = D_X - \varepsilon_0 E_X = \varepsilon_0 (\mu^2 - 1) E_X \]  

(83)

Then if it is inserted into equation (78)

\[ \mu^2 - 1 = \frac{-X}{U + jY_t \rho} \]  

(84)

By using equation (81)

\[ \mu^2 = 1 - \frac{X(U - X)}{U(U - X) - 1/2Y_t^2 \pm \left\{ 1/4Y_t^4 + Y_t^2(U - X)^2 \right\}^{1/2}} \]  

(85)

By inserting the followings into equation (83)

\[ X = \frac{w_p^2}{w^2} \]  

(86)

\[ Z = \frac{\nu}{w} \]  

(87)

\[ U = 1 - jZ \]  

(88)

\[ Y_t = \frac{w_p}{w} \sin \theta \]  

(89)

\[ Y_i = \frac{w_p}{w} \cos \theta \]  

(90)

Finally Appleton’s formula (Heald & Wharton, 1978) results as

\[ \dot{E}_r = 1 - \left[ \frac{(\omega_p / \omega)^2}{1 - j \frac{v_m}{\omega}} \left( \frac{\omega_p / \omega)^2 \sin^2 \theta}{2 \left( 1 - \frac{\omega_e}{\omega} j \frac{v_m}{\omega} \right)} + \frac{(\omega_p / \omega)^4 \sin^4 \theta}{4 \left( 1 - \frac{\omega_e}{\omega} - j \frac{v_m}{\omega} \right)} \right]^{1/2} \]  

(91)
where
\[ \varepsilon^* \] is the complex permittivity of the plasma,
\[ \omega_p \] is the plasma frequency,
\[ \omega \] is the angular frequency of the electromagnetic wave,
\[ \nu_{cm} \] is the collision frequency,
\[ \omega_{ce} \] is the electron gyrofrequency,
\[ \theta \] is the incident angle of the electromagnetic wave.

Plasma frequency \( \omega_p \) and electron-cyclotron frequency \( \omega_{ce} \) are given (Ginzburg, 1970) as
\[
\omega_p^2 = \frac{e^2 N}{m \varepsilon_0} \\
\omega_{ce}^2 = \frac{eB}{m}
\]  
(92) \hspace{1cm} (93)

where \( e \) is the charge of an electron, \( N \) is the electron number density, \( m \) is the mass of an electron, \( \varepsilon_0 \) is the permittivity of free space and \( B \) is the external magnetic field strength.

The presence of the ± sign in the Appleton’s formula is due to two separate solutions for the refractive index. In the case of propagation parallel to the magnetic field, the ‘+’ sign represents a left-hand circularly polarized mode, and the ‘-’ sign represents a right-hand circularly polarized mode.

For an incident electromagnetic wave (that is \( \theta = 0^\circ \)), complex dielectric constant of the plasma can be simply determined from equation (91) as
\[
\varepsilon_r = 1 - \frac{(\omega_p / \omega)^2}{1 - j \left( \nu_{cm} / \omega \right) - \left( \omega_{ce} / \omega \right)}
\]  
(94)

Now, in order to analyze electromagnetic wave propagation in a plasma slab in Fig. 2, multiple reflections are taken into consideration as shown in Fig. 3.

Reflection coefficient \( \Gamma(j, z) \) at the \( j^{th} \) interface and total reflection coefficient at \( z = -d \) interface for normal incidence case can be obtained as (Balanis, 1989)
\[
\Gamma_{in}(j = 1, z = -d) = \Gamma_{12} + \frac{T_{12} T_{21} \Gamma_{23} e^{2j \pi d}}{1 - \Gamma_{21} \Gamma_{23} e^{2j \pi d}}
\]  
(95)

The relations between the reflection and transmission coefficients are given as
\[
\Gamma_{21} = -\Gamma_{12} \\
T_{12} = 1 + \Gamma_{21} = 1 - \Gamma_{12} \\
T_{21} = 1 + \Gamma_{12}
\]  
(96) \hspace{1cm} (97) \hspace{1cm} (98)

When multiple reflections are ignored due to highly lossy plasma, by taking \( |\Gamma_{12}| << 1 \) and \( |\Gamma_{23}| << 1 \), reflection coefficient on the \( (j + 1)^{th} \) interface is obtained as (Balanis, 1989)
\[
\Gamma_{in}(j+1) = \frac{\sqrt{E_r(j)} - \sqrt{E_r(j+1)}}{\sqrt{E_r(j)} + \sqrt{E_r(j+1)}}
\]  

(99)

Fig. 3. Representation of multiple reflection in a subslab of the plasma layer (Balanis, 1989).

While the electromagnetic wave propagates through the plasma slab as seen in Fig. 2, reflection occurs at each interface of subslabs. The reflected electromagnetic wave from the first interface propagates in free space. Hence, there is no attenuation, reflected wave is given as

\[
P_{t1} = P_{i} \times \Gamma(1)^2
\]  

(100)

The power of the transmitted wave can be computed as

\[
P_{t} = \Gamma(1)^2 \times P_{i} + P_{t1} \Rightarrow P_{t1} = P_{i} \left(1 - \Gamma(1)^2\right)
\]  

(101)

While reaching the second interface, the wave attenuates inside the slab

\[
P_{t2} = e^{-2\pi(1)d}P_{i} \left(1 - \Gamma(1)^2\right)
\]  

(102)

Some portion of the electromagnetic wave reflects back and some portion continues to propagate inside the plasma.

\[
P_{t2} = P_{i}(1 - \Gamma(1)^2)\Gamma(2)^2 e^{-2\pi(1)d}
\]  

(103)
The reflected portion of the wave attenuates until it reaches the free space. Hence, the power of the reflected wave is,

\[ P_r = P_0 e^{-4\alpha(1)d} (1 - \Gamma(1)^2) \Gamma(2)^2 \]  

(104)

The transmitted portion also attenuates inside the plasma until it reaches the third interface

\[ P_t = P_0 e^{-2\alpha(1)d} (1 - \Gamma(1)^2) e^{-2\alpha(2)d} (1 - \Gamma(2)^2) \]  

(105)

Some portion reflects from the third interface as

\[ P_{r_3} = P_0 e^{-2\alpha(1)d} (1 - \Gamma(1)^2) e^{-2\alpha(2)d} (1 - \Gamma(2)^2) \Gamma(3)^2 \]  

(106)

The reflected wave attenuates until it leaves the slab and reaches to the free space.

\[ P_{r_3} = P_0 e^{-4\alpha(1)d} (1 - \Gamma(1)^2) e^{-4\alpha(2)d} (1 - \Gamma(2)^2) \Gamma(3)^2 \]  

(107)

The reflected waves from other interfaces can be written in the same manner. Hence, total reflected power is written as

\[ P_r = P_0 \Gamma(1)^2 + P_0 e^{-4\alpha(1)d} (1 - \Gamma(1)^2) \Gamma(2)^2 + P_0 e^{-4\alpha(1)d} (1 - \Gamma(1)^2) (1 - \Gamma(2)^2) e^{-4\alpha(2)d} \Gamma(3)^2 + \ldots \]  

(108)

Equation (108) can be arranged as follows (Tang et al., 2003)

\[ P_r = P_0 \left[ \Gamma(1)^2 + \sum_{j=2}^{M} \left( \Gamma(j)^2 \prod_{i=1}^{j-1} (\exp[-4\alpha(i)d](1 - |\Gamma(i)|^2)) \right) \right] \]  

(109)

where \( d \) is the thickness of each subslab.

In order to obtain total transmitted wave power, attenuation inside the plasma slab must be considered. The total transmitted power can be computed from

\[ P_t = e^{-2\alpha(1)d} (1 - \Gamma(1)^2) e^{-2\alpha(2)d} (1 - \Gamma(2)^2) e^{-2\alpha(3)d} \ldots e^{-2\alpha(M)d} (1 - \Gamma(M)^2) \]  

(110)

From equation (110), we get (Tang et al., 2003),

\[ P_t = P_0 \prod_{i=1}^{M} \left( \exp[-2\alpha(i)d](1 - |\Gamma(i)|^2) \right) \]  

(111)

The absorbed power by the plasma slab can be computed from

\[ P_a = P_t - P_r - P_i \]  

(112)

Due to perfect electric conductor after the plasma slab, \( |\Gamma(M)| = 1 \) and thus equation (112) becomes

\[ P_a = P_r - P_i \]  

(113)

This mentioned model is acceptable as the first approximation under the assumption that the plasma properties vary slowly along the wave propagation path (Heald & Wharton, 1978).
3. Results and discussion

In this part, reflection of electromagnetic wave power from plasma coated conducting plane is analysed by considering three different electron density distribution functions. These functions are selected as linear distribution function with positive slope, linear distribution function with negative slope and sinusoidal distribution function. Linearly varying electron density distribution function with positive and negative slopes is defined as

$$\begin{align*}
N &= \frac{N_m^2}{L}, \\
N &= \frac{N_m(L - z)}{L}
\end{align*}$$

respectively. Sinusoidal electron density distribution function is written as

$$N = N_m(0.505 + 0.5\cos(pz / \pi + \phi))$$

where $N_m$ is the maximum electron number density value, $L$ is the thickness of the plasma, $p$ is the sinusoidal frequency parameter taken as 2 and $\phi$ is the phase shift introduced to the sinusoidal distribution. In this part, the plasma length $L$ is taken as 12 cm. In Fig. 4, it is shown that linearly increasing distribution reduces the reflected power much more than the other two distributions in 1-18 GHz range.

![Fig. 4. Reflected power for $N_m = 1 \times 10^{18}$ m$^{-3}$, $\nu_m = 1$ GHz, $B = 0.25$ T.](image)

After 20 GHz, the reflected power increases as the frequency increases due to mismatch between the free space and the plasma slab. After 35 GHz the reflected power for linearly increasing case is below -10 dB which means that plasma slab behaves as a transparent medium for incident electromagnetic wave propagation. For other distribution functions, reflection coefficient is nearly equal to 1 for 7.5 - 13 GHz range thus all wave power is reflected back. But as the frequency increases, plasma absorbs more wave power. Fig. 5. shows the results when maximum electron number density is increased to $5 \times 10^{18}$ m$^{-3}$ while the other parameters are remained same.

In this case, linearly increasing distribution reduces the reflected power much more than the other distribution functions up to 33 GHz. For 35-50 GHz range, sinusoidal and linearly decreasing functions are more useful in terms of small reflection.
Fig. 6. shows the results when maximum electron number density is decreased to $5 \times 10^{17} \text{ m}^{-3}$. It is observed that linearly increasing distribution considerably minimizes the reflected power in 1-20 GHz range. The other two distributions show nearly no attenuation for 7.5-11.5 GHz range. It can be seen that the zero attenuation region for sinusoidal and linearly decreasing distribution functions become narrower as the maximum electron number density decreases.

As shown in Fig. 7, when maximum electron number density is decreased to $1 \times 10^{17} \text{ m}^{-3}$ small reflection band of the plasma slab becomes narrower for all distribution functions. Up to 15 GHz, reflected power is below -10 dB for all three cases.

In Fig. 8. the reflected power characteristics when effective collision frequency is increased to 60 GHz while the other parameters are the same with Fig. 4. Considerably reduced reflection is observed for all cases with respect to the previous cases. Thus it can be concluded that by increasing the effective collision frequency, reflection from plasma covered conducting plane can be considerably reduced in critical applications.

Fig. 5. Reflected power for $N_m = 5 \times 10^{18} \text{ m}^{-3}, \nu_m = 1 \text{ GHz}, B = 0.25 T$.

Fig. 6. Reflected power for $N_m = 5 \times 10^{17} \text{ m}^{-3}, \nu_m = 1 \text{ GHz}, B = 0.25 T$.
The following two graphs are given to determine the effect of magnetic field on the wave propagation in the plasma slab. The magnetic field strength is taken as 0.5 T and 0.75 T in Fig. 9 and in Fig. 10, respectively. Other parameters are the same with Fig. 4. It is seen from Fig. 9 and Fig. 10 that the reflected power characteristics shift to right as the magnetic field increases. Hence, magnetic field can be used for tuning of the reflection and the passbands of the power reflectivity characteristics of the plasma slab. It must also be noted that adjustment of magnetic field slightly affects the bandwidth and amplitudes of the power reflection characteristics while providing frequency shift.

In the last figure, Fig. 11, the effect of plasma slab thickness on the reflected wave power is shown. In this figure, thickness of the plasma slab is doubled and taken as 24 cm. The other parameters are the same with Fig. 4. The reflected power decreases due to increased plasma thickness as seen from the figure. This is an expected result since the absorption of the electromagnetic wave power increases due to increased propagation path along the plasma.

Fig. 7. Reflected power for $N_m = 1 \times 10^{17}$ m$^{-3}$, $\nu_m = 1$ GHz, $B = 0.25T$.

Fig. 8. Reflected power for $N_m = 1 \times 10^{18}$ m$^{-3}$, $\nu_m = 60$ GHz, $B = 0.25T$. 
Fig. 9. Reflected power for \( N_m = 1 \times 10^{18} \text{ m}^{-3}, \nu_c = 1 \text{ GHz}, B = 0.5 \text{T} \).

Fig. 10. Reflected power for \( N_m = 1 \times 10^{18} \text{ m}^{-3}, \nu_c = 1 \text{ GHz}, B = 0.75 \text{T} \).

Fig. 11. Reflected power for \( L=24 \text{ cm}, N_m = 1 \times 10^{18} \text{ m}^{-3}, \nu_c = 1 \text{ GHz}, B = 0.25 \text{T} \).
4. Conclusion

In this chapter, some of the methods presented in literature for the analysis of electromagnetic wave propagation through the plasma slab is explained briefly and the stealth characteristics of the plasma covered conducting plane is analysed for three different electron density distribution functions. The selected distributions show tunable stealth characteristics in different frequency ranges depending on the adjustment of the plasma parameters. It is seen that especially the linearly increasing density distribution function shows better stealth characteristics considerably reducing the reflected power for 1-20 GHz range. Above 20 GHz, other two functions show better characteristics up to 50 GHz due to adjustment of plasma parameters. It must be noted that the maximum value of electron density function, effective collision frequency, the length of the plasma slab and the external magnetic field strength considerably effect the stealth characteristics of the plasma covered conducting plane and must be carefully adjusted in special applications. In the following studies other distribution functions for electron density will be analysed to obtain an improved performance.

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6. References


This book is dedicated to various aspects of electromagnetic wave theory and its applications in science and technology. The covered topics include the fundamental physics of electromagnetic waves, theory of electromagnetic wave propagation and scattering, methods of computational analysis, material characterization, electromagnetic properties of plasma, analysis and applications of periodic structures and waveguide components, and finally, the biological effects and medical applications of electromagnetic fields.

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