Adaptive PID Control for Asymptotic Tracking Problem of MIMO Systems

Kenichi Tamura[1] and Hiromitsu Ohmori[2]
[1]*Tokyo Metropolitan University*
[2]*Keio University*
JAPAN

1. Introduction

PID control, which is usually known as a classical output feedback control for SISO systems, has been widely used in the industrial world(Åström & Hägglund, 1995; Suda, 1992). The tuning methods of PID control are adjusting the proportional, the integral and the derivative gains to make an output of a controlled system track a target value properly. There exist much more researches on tuning methods of PID control for SISO systems than MIMO systems although more MIMO systems actually exist than SISO systems. The tuning methods for SISO systems are difficult to apply to PID control for MIMO systems since the gains usually become matrices in such case.

MIMO systems usually tend to have more complexities and uncertainties than SISO systems. Several tuning methods of PID control for such MIMO system are investigated as follows. From off-line approach, there are progressed classical loop shaping based methods (Ho et al., 2000; Hara et al., 2006) and H_∞ control theory based methods (Mattei, 2001; Saeki, 2006; Zheng et al., 2002). From on-line approach, there are methods from self-tuning control such as the generalized predictive control based method (Gomma, 2004), the generalized minimum variance control based method (Yusof et al., 1994), the model matching based method (Yamamoto et al., 1992) and the method using neural network (Chang et al., 2003).

These conventional methods often require that the MIMO system is stable and are usually used for a regulator problem for a constant target value but a tracking problem for a time-varying target value, which restrictions narrow their application. So trying these problems is significant from a scientific standpoint how there is possibility of PID control and from a practical standpoint of expanding applications. In MIMO case, there is possibility to solve these problems because PID control has more freedoms in tuning of PID gain matrices.

On the other hand, adaptive servo control is known for a problem of the asymptotic output tracking and/or disturbances rejection to unknown systems under guaranteeing stability. There are researches for SISO systems (Hu & Tomizuka, 1993; Miyasato, 1998; Ortega & Kelly, 1985) and for MIMO systems (Chang & Davison, 1995; Dang & Owens, 2006; Johansson, 1987). Their controllers generally depend on structures of the controlled system and the reference system, which features are undesirable from standpoint of utility (Saeki, 2006; Miyamoto, 1999). So it is important to develop the fixed controller like PID controller to solve the servo problem and to show that conditions. But they are difficult to apply to the tuning of PID controller because of differences of their construction.

In this paper, we consider adaptive PID control for the asymptotic output tracking problem of MIMO systems with unknown system parameters under existence of unknown disturbances.

The proposed PID controller has constant gain matrices and adjustable gain matrices. The proposed adaptive tuning laws of the gain matrices are derived by using Lyapunov theorem. That is a Lyapunov function based on characteristics of the proposed PID controller is constructed. This method guarantees the asymptotic output tracking even if the controlled MIMO system is unstable and has uncertainties and unknown constant disturbances. Finally, the effectiveness of the proposed method is confirmed with simulation results for the 8-state, 2-input and 2-output missile control system and the 4-state, 2-input and 2-output unstable system.

2. Problem statement

Consider the MIMO system:

$$\dot{x}(t) = Ax(t) + Bu(t) + d_i, \tag{1}$$
$$y(t) = Cx(t) + d_o, \tag{2}$$

where $x(t) \in \mathcal{R}^n, u(t) \in \mathcal{R}^m, y(t) \in \mathcal{R}^m$ are the state vector, the input vector and the output vector respectively, $d_i \in \mathcal{R}^n, d_o \in \mathcal{R}^m$ are unknown constant disturbances, and A, B, C are unknown system matrices.
The target signal of the output is $y_M(t) \in \mathcal{R}^m$ generated by the reference system:

$$\dot{x}_M(t) = A_M x_M(t) + B_M u_M, \tag{3}$$
$$y_M(t) = C_M x_M(t), \tag{4}$$

where $x_M(t) \in \mathcal{R}^{n_M}$ and $u_M \in \mathcal{R}^{r_M}$ are the state vector and the constant input vector, respectively. Note that A_M, B_M, C_M are allowed to be unknown matrices.
In this article, we propose the new adaptive PID controller:

$$u(t) = K_{I0} \int_0^t e_y(\tau)\mathrm{d}\tau + (K_{P0} + K_{P1}(t))e_y(t) + K_{D1}(t)\dot{e}_y(t) + K_{P2}(t)y_M(t) + K_{D2}(t)\dot{y}_M(t) \tag{5}$$

which has the adjustable gain matrices $K_{P1}(t)$, $K_{P2}(t), K_{D1}(t), K_{D2}(t) \in \mathcal{R}^{m\times m}$ and the constant gain matrices $K_{I0}, K_{P0} \in \mathcal{R}^{m\times m}$, and

$$e_y(t) = y_M(t) - y(t) \tag{6}$$

denotes the error of the output from the target signal $y_M(t)$. The diagram of the proposed PID controller is shown in Fig. 1.
The objective is to design the constant gain matrices K_{I0}, K_{P0} and the adaptive tuning laws of the adjustable gain matrices $K_{P1}(t)$, $K_{P2}(t), K_{D1}(t), K_{D2}(t)$ to solve the asymptotic output tracking, i.e. $e_y(t) \to 0$ as $t \to \infty$.
Here we assume the following conditions:
Assumption 1: rank $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = n + m$, and $\lambda_i(M_{11})\lambda_j(A_M) \neq 1, i = 1, 2, \cdots, n, j = 1, 2, \cdots, n_M$, where $\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} := \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1}$, $M_{11} \in \mathcal{R}^{n\times n}$ and $\lambda(\cdot)$ denotes eigenvalues of a matrix.
Assumption 2: rank $\begin{bmatrix} C_M \\ C_M A_M \end{bmatrix} = n_M$.
Assumption 3: The zero-dynamics of $\{A, B, C\}$ is asymptotically stable.
Assumption 4: (a) $CB = 0,\ CAB > 0$ or (b) $CB > 0$.

Let us explain these assumptions. Assumption 1 is well known condition for a servo problem. Assumption 2 means the output of the reference system and its derivative contain the information of its state. Assumption 3 equals to the minimum phase property of the controlled system. Assumption 4 contains the condition that the relative degrees are ≤ 2. It is inevitable that these conditions seem a little severe because these are conditions for the PID controller that has the structural constraint. But also there is an advantage that the controlled system's stability property, which is often assumed in other PID control's methods, is not assumed.

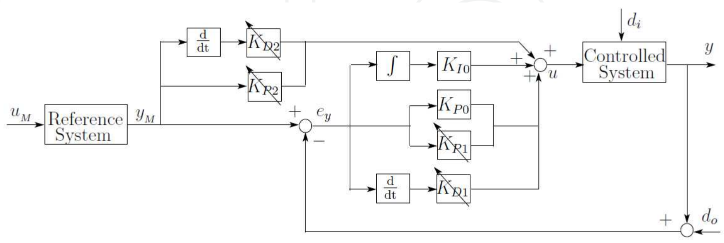


Fig. 1. Proposed Adaptive PID Controller

3. Error system with proposed adaptive PID controller

In this section, we derive the error system with the adaptive PID controller. When the perfect output tracking occurs (i.e. $y(t) = y_M(t),\ \forall t \geq 0$), we can define the corresponding state and input trajectories as $x^*(t), u^*(t)$, respectively. That is $x^*(t), u^*(t)$ are trajectories satisfying the following relation:

$$\dot{x}^*(t) = Ax^*(t) + Bu^*(t) + d_i, \tag{7}$$

$$y_M(t) = Cx^*(t) + d_o,\ \forall t \geq 0. \tag{8}$$

From Appendix A inspired by (Kaufman et al., 1994), there exist matrices $M_{ij}, T_{ij},\ i, j = 1, 2$ under Assumption 1, and the ideal trajectories $x^*(t), u^*(t)$ satisfying relations (7), (8) can be expressed as

$$x^*(t) = T_{11}x_M(t) + T_{12}u_M - M_{11}d_i - M_{12}d_o, \tag{9}$$

$$u^*(t) = T_{21}x_M(t) + T_{22}u_M - M_{21}d_i - M_{22}d_o. \tag{10}$$

Introducing these ideal trajectories, we can define the following state error

$$e_x(t) = x^*(t) - x(t). \tag{11}$$

Then, the output tracking error (6) can be described as

$$e_y(t) = y_M(t) - y(t) = (Cx^*(t) + d_o) - (Cx(t) + d_o) = Ce_x(t), \tag{12}$$

which means that if the error system obtained by differentiating (11):

$$\dot{e}_x(t) = Ae_x(t) + B(u^*(t) - u(t)) \tag{13}$$

can be asymptotically stabilized i.e. $e_x(t) \rightarrow 0$, then the asymptotic output tracking can be achieved i.e. $e_y(t) \rightarrow 0$.
Now, substituting (5) and (10) into (13), we get the following closed loop error system:

$$\begin{aligned}\dot{e}_x(t) = Ae_x(t) - B\Big[&- T_{21}x_M(t) - T_{22}u_M + M_{21}d_i + M_{22}d_o + K_{I0}\int_0^t e_y(\tau)\mathrm{d}\tau \\ &+ K_{P0}e_y(t) + K_{P1}(t)e_y(t) + K_{D1}(t)\dot{e}_y(t) + K_{P2}(t)y_M(t) + K_{D2}(t)\dot{y}_M(t)\Big].\end{aligned} \tag{14}$$

From Appendix B, there exist matrices $S_1, S_2 \in \mathcal{R}^{m\times m}$ under Assumption 2, and $T_{21}x_M(t)$ in (14) can be decomposed as

$$T_{21}x_M(t) = S_1 y_M(t) + S_2(\dot{y}_M(t) - C_M B_M u_M). \tag{15}$$

Hence, (14) can be expressed as

$$\begin{aligned}\dot{e}_x(t) = Ae_x(t) - B\Big[&(S_2C_MB_M - T_{22})u_M + M_{21}d_i + M_{22}d_o + K_{I0}\int_0^t e_y(\tau)\mathrm{d}\tau + K_{P0}e_y(t)\\ &+ K_{P1}(t)e_y(t) + K_{D1}(t)\dot{e}_y(t) + (K_{P2}(t) - S_1)y_M(t) + (K_{D2}(t) - S_2)\dot{y}_M(t)\Big].\end{aligned} \tag{16}$$

Here put the constant term of the above equation as

$$\widetilde{d} := (S_2C_MB_M - T_{22})u_M + M_{21}d_i + M_{22}d_o$$

to represent (16) simply as

$$\begin{aligned}\dot{e}_x(t) = Ae_x(t) - B\Big[&\widetilde{d} + K_{I0}\int_0^t e_y(\tau)\mathrm{d}\tau + K_{P0}e_y(t) + K_{P1}(t)e_y(t) + K_{D1}(t)\dot{e}_y(t)\\ &+ (K_{P2}(t) - S_1)y_M(t) + (K_{D2}(t) - S_2)\dot{y}_M(t)\Big].\end{aligned} \tag{17}$$

Therefore, if the origin of this close-loop error system is asymptotically stabilized i.e. $e_x(t) \to 0$, the asymptotic output tracking i.e. $e_y(t) \to 0$ is achieved. We derive the constant gain matrices and the adaptive tuning laws of adjustable gain matrices to accomplish $e_x(t) \to 0$ in the next section.

4. Adaptive tuning laws of PID gain matrices

In this section, we show the constant gain matrices K_{I0}, K_{P0} and the adaptive tuning law of the adjustable gain matrices $K_{P1}(t), K_{P2}(t), K_{D1}(t), K_{D2}(t)$ to asymptotically stabilize the error dynamics (17) (i.e. $e_x \to 0$ as $t \to \infty$) at *Case A* when Assumption 4(a) is hold or at *Case B* when Assumption 4(b) is hold.

4.1 Case A

Theorem 1: Suppose Assumption 3 and Assumption 4(a). Give the constant gain matrices K_{I0}, K_{P0} as

$$K_{I0} = \gamma_I H_1,\ \ K_{P0} = \gamma_I H_2, \tag{18}$$

and the adaptive tuning laws of the adjustable gain matrices $K_{Pi}(t), K_{Di}(t), i = 1, 2$ as

$$\dot{K}_{P1}(t) = \Gamma_{P1}\big(H_1e_y(t) + H_2\dot{e}_y(t)\big)e_y(t)^{\mathrm{T}}, \tag{19a}$$

$$\dot{K}_{D1}(t) = \Gamma_{D1}\big(H_1e_y(t) + H_2\dot{e}_y(t)\big)\dot{e}_y(t)^{\mathrm{T}}, \tag{19b}$$

$$\dot{K}_{P2}(t) = \Gamma_{P2}\big(H_1e_y(t) + H_2\dot{e}_y(t)\big)y_M(t)^{\mathrm{T}}, \tag{19c}$$

$$\dot{K}_{D2}(t) = \Gamma_{D2}\big(H_1e_y(t) + H_2\dot{e}_y(t)\big)\dot{y}_M(t)^{\mathrm{T}} \tag{19d}$$

where

$$H_1 = \mathrm{diag}\{h_{11}, \cdots, h_{1m}\},\ H_2 = \mathrm{diag}\{h_{21}, \cdots, h_{2m}\},\ h_{1j}, h_{2j} > 0,\ j = 1, \cdots, m, \tag{20}$$

then the origin of (17) is asymptotically stable ($e_x(t) \to 0$ as $t \to \infty$) and the adjustable gain matrices are bounded. Here $\Gamma_{P1}, \Gamma_{P2}, \Gamma_{D1}, \Gamma_{D2} \in \mathcal{R}^{m\times m}$ are arbitrary positive definite matrices and γ_I is arbitrary positive scalar.

Proof: From Assumption 4(a), the error dynamics (17) is transformed into the normal form (see e.g. (Isidori, 1995)):

$$\begin{bmatrix}\dot{\xi}_1(t)\\ \dot{\xi}_2(t)\\ \dot{\eta}(t)\end{bmatrix} = \begin{bmatrix}0 & I_m & 0\\ Q_{21} & Q_{22} & Q_{23}\\ Q_{31} & Q_{32} & A_\eta\end{bmatrix}\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\\ \eta(t)\end{bmatrix} - \begin{bmatrix}0\\ CAB\\ 0\end{bmatrix}\Big[K_{I0}\int_0^t \xi_1(\tau)\mathrm{d}\tau + K_{P0}\xi_1(t) + \widetilde{d}$$
$$+ K_{P1}(t)\xi_1(t) + K_{D1}(t)\xi_2(t) + (K_{P2}(t) - S_1)y_M(t) + (K_{D2}(t) - S_2)\dot{y}_M(t)\Big], \quad (21)$$

which Q_{ij} are unknown matrices, by transformation

$$\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\\ \eta(t)\end{bmatrix} = \begin{bmatrix}C\\ CA\\ T\end{bmatrix} e_x(t) \quad (22)$$

where $TB = 0,\ T \in \mathcal{R}^{(n-2m)\times n}$ and

$$\xi_1(t) = Ce_x(t) = e_y(t),\ \xi_2(t) = CAe_x(t) = \dot{e}_y(t). \quad (23)$$

Note that when $\xi_1(t), \xi_2(t) \equiv 0$,

$$\dot{\eta}(t) = A_\eta \eta(t), \quad (24)$$

which is called zero-dynamics, is asymptotic stable from Assumption 3.
Thus (21) is can be rewritten as

$$\begin{bmatrix}\dot{\xi}_1(t)\\ \dot{\xi}_2(t)\\ \dot{\eta}(t)\end{bmatrix} = \begin{bmatrix}0 & I_m & 0\\ -K_{\xi 1} & -K_{\xi 2} & Q_{23}\\ Q_{31} & Q_{32} & A_\eta\end{bmatrix}\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\\ \eta(t)\end{bmatrix} - \begin{bmatrix}0\\ I_m\\ 0\end{bmatrix}\Big[CAB(K_{I0}\int_0^t \xi_1(\tau)\mathrm{d}\tau + K_{P0}\xi_1(t) + \widetilde{d})$$
$$+ (CABK_{P1}(t) - Q_{21} - K_{\xi 1})\xi_1(t) + (CABK_{D1}(t) - Q_{22} - K_{\xi 2})\xi_2(t)$$
$$+ CAB(K_{P2}(t) - S_1)y_M(t) + CAB(K_{D2}(t) - S_2)\dot{y}_M(t)\Big] \quad (25)$$

where $K_{\xi 1}, K_{\xi 2} \in \mathcal{R}^{m\times m}$ are the constant matrices only used in the proof.

For simplicity, put

$$\xi(t) := \begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}, A_\xi := \begin{bmatrix}0 & I_m\\ 0 & 0\end{bmatrix}, B_\xi := \begin{bmatrix}0\\ I_m\end{bmatrix}, \quad (26)$$

$$K_\xi := \begin{bmatrix}K_{\xi 1} & K_{\xi 2}\end{bmatrix}, \overline{Q}_1 := \begin{bmatrix}0\\ Q_{23}\end{bmatrix},\ \overline{Q}_2 := [Q_{31}\ Q_{32}], \quad (27)$$

$$\psi_I(t) := CAB(K_{I0}\int_0^t \xi_1(\tau)\mathrm{d}\tau + K_{P0}\xi_1(t) + \widetilde{d}), \quad (28)$$

$$\Psi_{P1}(t) := CABK_{P1}(t) - Q_{21} - K_{\xi 1}, \quad (29a)$$
$$\Psi_{D1}(t) := CABK_{D1}(t) - Q_{22} - K_{\xi 2}, \quad (29b)$$
$$\Psi_{P2}(t) := CAB(K_{P2}(t) - S_1), \quad (29c)$$
$$\Psi_{D2}(t) := CAB(K_{D2}(t) - S_2) \quad (29d)$$

to describe (25) as

$$\begin{bmatrix}\dot{\xi}(t)\\ \dot{\eta}(t)\end{bmatrix} = \begin{bmatrix}A_\xi - B_\xi K_\xi & \overline{Q}_1\\ \overline{Q}_2 & A_\eta\end{bmatrix}\begin{bmatrix}\xi(t)\\ \eta(t)\end{bmatrix} - \begin{bmatrix}B_\xi\\ 0\end{bmatrix}\Big[\psi_I(t) + \Psi_{P1}(t)\xi_1(t) + \Psi_{D1}(t)\xi_2(t) + \Psi_{P2}(t)y_M(t) + \Psi_{D2}(t)\dot{y}_M(t)\Big], \tag{30}$$

where

$$\dot{\psi}_I(t) = CAB(K_{I0}\xi_1(t) + K_{P0}\xi_2(t)), \tag{31}$$
$$\dot{\Psi}_{P1}(t) = CAB\dot{K}_{P1}(t), \tag{32a}$$
$$\dot{\Psi}_{D1}(t) = CAB\dot{K}_{D1}(t), \tag{32b}$$
$$\dot{\Psi}_{P2}(t) = CAB\dot{K}_{P2}(t), \tag{32c}$$
$$\dot{\Psi}_{D2}(t) = CAB\dot{K}_{D2}(t). \tag{32d}$$

Meanwhile because $\{A_\xi, B_\xi\}$ is controllable pair from (26), there exist K_ξ such that Lyapunov equation

$$P_\xi(A_\xi - B_\xi K_\xi) + (A_\xi - B_\xi K_\xi)^{\mathrm{T}} P_\xi = -Q,\;\; Q > 0$$

has an unique positive solution $P_\xi > 0$. So here we set $Q = 2\varepsilon I_{2m},\ \varepsilon > 0$ and select K_ξ as

$$K_{\xi_1} = \varepsilon H_1^{-1},\;\; K_{\xi_2} = \varepsilon H_2^{-1}(I_m + (1/\varepsilon)H_1), \tag{33}$$
$$H_i = \mathrm{diag}\{h_{i1}, \cdots, h_{im}\}, h_{ij} > 0, i = 1, 2,\; j = 1, \cdots, m,$$

such that

$$P_\xi(A_\xi - B_\xi K_\xi) + (A_\xi - B_\xi K_\xi)^{\mathrm{T}} P_\xi = -2\varepsilon I_{2m},\ \varepsilon > 0, \tag{34}$$

has the unique positive solution

$$P_\xi = \begin{bmatrix}P_{\xi 1} & \overline{P}\\ \overline{P}^{\mathrm{T}} & P_{\xi 2}\end{bmatrix} \in \mathcal{R}^{2m\times 2m}, \tag{35}$$
$$\overline{P} = H_1,\;\; P_{\xi 2} = H_2,\;\; P_{\xi 1} = \varepsilon(H_1H_2^{-1} + H_1^{-1}H_2) + H_1H_2^{-1}H_1.$$

It is clear P_ξ of (35) is a positive matrix on $\varepsilon > 0$ from Schur complement (see e.g. (Iwasaki, 1997)) because $P_{\xi 2} = H_2 > 0,\ P_{\xi 1} - \overline{P}P_{\xi 2}^{-1}\overline{P}^{\mathrm{T}} = \varepsilon(H_1H_2^{-1} + H_1^{-1}H_2) > 0$.

Furthermore since A_η of (24) is asymptotic stable matrix from Assumption 3, there exists an unique solution $P_\eta \in \mathcal{R}^{(n-2m)\times(n-2m)} > 0$ satisfying

$$P_\eta A_\eta + A_\eta^{\mathrm{T}} P_\eta = -I_{n-2m}. \tag{36}$$

Now, by using P_ξ of (35) and P_η of (36), we consider the following Lyapunov function candidate:

$$\begin{aligned}&V(\xi(t), \eta(t), \psi_I(t), \Psi_{P1}(t), \Psi_{P2}(t), \Psi_{D1}(t), \Psi_{D2}(t))\\ &= \begin{bmatrix}\xi(t)\\ \eta(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}P_\xi & 0\\ 0 & P_\eta\end{bmatrix}\begin{bmatrix}\xi(t)\\ \eta(t)\end{bmatrix} + \psi_I(t)^{\mathrm{T}}\gamma_I^{-1}(CAB)^{-1}\psi_I(t)\\ &\quad + \mathrm{Tr}\big[\Psi_{P1}(t)^{\mathrm{T}}\Gamma_{P1}^{-1}(CAB)^{-1}\Psi_{P1}(t)\big] + \mathrm{Tr}\big[\Psi_{D1}(t)^{\mathrm{T}}\Gamma_{D1}^{-1}(CAB)^{-1}\Psi_{D1}(t)\big]\\ &\quad + \mathrm{Tr}\big[\Psi_{P2}(t)^{\mathrm{T}}\Gamma_{P2}^{-1}(CAB)^{-1}\Psi_{P2}(t)\big] + \mathrm{Tr}\big[\Psi_{D2}(t)^{\mathrm{T}}\Gamma_{D2}^{-1}(CAB)^{-1}\Psi_{D2}(t)\big]\end{aligned} \tag{37}$$

where $\Gamma_{P1}, \Gamma_{D1}, \Gamma_{P2}, \Gamma_{D2} \in \mathcal{R}^{m\times m}$ are arbitrary positive definite matrices, γ_I is positive scalar. $\mathrm{Tr}[\cdot]$ denotes trace of a square matrix. Here put $V(t) := V(\xi(t), \eta(t), \psi_I(t), \Psi_{P1}(t), \Psi_{P2}(t), \Psi_{D1}(t), \Psi_{D2}(t))$ for simplicity. The derivative of (37) along the trajectories of the error system (30) $\sim$ (32d) can be calculated as

$$
\begin{aligned}
\dot{V}(t) &= 2\begin{bmatrix}\dot{\xi}(t)\\ \dot{\eta}(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}P_\xi & 0\\ 0 & P_\eta\end{bmatrix}\begin{bmatrix}\xi(t)\\ \eta(t)\end{bmatrix} + 2\psi_I(t)^{\mathrm{T}}\gamma_I^{-1}(CAB)^{-1}\dot{\psi}_I(t)\\
&\quad + 2\mathrm{Tr}\left[\Psi_{P1}(t)^{\mathrm{T}}\Gamma_{P1}^{-1}(CAB)^{-1}\dot{\Psi}_{P1}(t)\right] + 2\mathrm{Tr}\left[\Psi_{D1}(t)^{\mathrm{T}}\Gamma_{D1}^{-1}(CAB)^{-1}\dot{\Psi}_{D1}(t)\right]\\
&\quad + 2\mathrm{Tr}\left[\Psi_{P2}(t)^{\mathrm{T}}\Gamma_{P2}^{-1}(CAB)^{-1}\dot{\Psi}_{P2}(t)\right] + 2\mathrm{Tr}\left[\Psi_{D2}(t)^{\mathrm{T}}\Gamma_{D2}^{-1}(CAB)^{-1}\dot{\Psi}_{D2}(t)\right]\\
&= \begin{bmatrix}\xi(t)\\ \eta(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}P_\xi(A_\xi - B_\xi K_\xi) + (A_\xi - B_\xi K_\xi)^{\mathrm{T}}P_\xi & P_\xi\overline{Q}_1 + \overline{Q}_2^{\mathrm{T}}P_\eta\\ (P_\xi\overline{Q}_1 + \overline{Q}_2^{\mathrm{T}}P_\eta)^{\mathrm{T}} & P_\eta A_\eta + A_\eta^{\mathrm{T}}P_\eta\end{bmatrix}\begin{bmatrix}\xi(t)\\ \eta(t)\end{bmatrix}\\
&\quad + 2\psi_I(t)^{\mathrm{T}}\Big[-B_\xi^{\mathrm{T}}P_\xi\xi(t) + \gamma_I^{-1}(CAB)^{-1}\dot{\psi}_I(t)\Big]\\
&\quad + 2\mathrm{Tr}\Big[\Psi_{P1}(t)^{\mathrm{T}}\Big(-B_\xi^{\mathrm{T}}P_\xi\xi(t)\xi_1(t)^{\mathrm{T}} + \Gamma_{P1}^{-1}(CAB)^{-1}\dot{\Psi}_{P1}(t)\Big)\Big]\\
&\quad + 2\mathrm{Tr}\Big[\Psi_{D1}(t)^{\mathrm{T}}\Big(-B_\xi^{\mathrm{T}}P_\xi\xi(t)\xi_2(t)^{\mathrm{T}} + \Gamma_{D1}^{-1}(CAB)^{-1}\dot{\Psi}_{D1}(t)\Big)\Big]\\
&\quad + 2\mathrm{Tr}\Big[\Psi_{P2}(t)^{\mathrm{T}}\Big(-B_\xi^{\mathrm{T}}P_\xi\xi(t)y_M(t)^{\mathrm{T}} + \Gamma_{P2}^{-1}(CAB)^{-1}\dot{\Psi}_{P2}(t)\Big)\Big]\\
&\quad + 2\mathrm{Tr}\Big[\Psi_{D2}(t)^{\mathrm{T}}\Big(-B_\xi^{\mathrm{T}}P_\xi\xi(t)\dot{y}_M(t)^{\mathrm{T}} + \Gamma_{D2}^{-1}(CAB)^{-1}\dot{\Psi}_{D2}(t)\Big)\Big]\\
&= \begin{bmatrix}\xi(t)\\ \eta(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}P_\xi(A_\xi - B_\xi K_\xi) + (A_\xi - B_\xi K_\xi)^{\mathrm{T}}P_\xi & P_\xi\overline{Q}_1 + \overline{Q}_2^{\mathrm{T}}P_\eta\\ (P_\xi\overline{Q}_1 + \overline{Q}_2^{\mathrm{T}}P_\eta)^{\mathrm{T}} & P_\eta A_\eta + A_\eta^{\mathrm{T}}P_\eta\end{bmatrix}\begin{bmatrix}\xi(t)\\ \eta(t)\end{bmatrix}\\
&\quad + 2\psi_I(t)^{\mathrm{T}}\Big[-B_\xi^{\mathrm{T}}P_\xi\xi(t) + \gamma_I^{-1}\big(K_{I0}\xi_1(t) + K_{P0}\xi_2(t)\big)\Big]\\
&\quad + 2\mathrm{Tr}\Big[\Psi_{P1}(t)^{\mathrm{T}}\Big(-B_\xi^{\mathrm{T}}P_\xi\xi(t)\xi_1(t)^{\mathrm{T}} + \Gamma_{P1}^{-1}\dot{K}_{P1}(t)\Big)\Big]\\
&\quad + 2\mathrm{Tr}\Big[\Psi_{D1}(t)^{\mathrm{T}}\Big(-B_\xi^{\mathrm{T}}P_\xi\xi(t)\xi_2(t)^{\mathrm{T}} + \Gamma_{D1}^{-1}\dot{K}_{D1}(t)\Big)\Big]\\
&\quad + 2\mathrm{Tr}\Big[\Psi_{P2}(t)^{\mathrm{T}}\Big(-B_\xi^{\mathrm{T}}P_\xi\xi(t)y_M(t)^{\mathrm{T}} + \Gamma_{P2}^{-1}\dot{K}_{P2}(t)\Big)\Big]\\
&\quad + 2\mathrm{Tr}\Big[\Psi_{D2}(t)^{\mathrm{T}}\Big(-B_\xi^{\mathrm{T}}P_\xi\xi(t)\dot{y}_M(t)^{\mathrm{T}} + \Gamma_{D2}^{-1}\dot{K}_{D2}(t)\Big)\Big]. \qquad (38)
\end{aligned}
$$

Therefore from $\xi(t) = [\xi_1^{\mathrm{T}},\ \xi_2^{\mathrm{T}}]^{\mathrm{T}} = [e_y^{\mathrm{T}},\ \dot{e}_y^{\mathrm{T}}]^{\mathrm{T}}$ and $B_\xi^{\mathrm{T}}P_\xi = [H_1\ H_2]$, giving the constant gain matrices K_{I0}, K_{P0} as (18), (20) and the adaptive tuning laws of $K_{Pi}(t)$, $K_{Di}(t)$, $i = 1, 2$ as (19a) $\sim$ (19d), (20) , we can get (38) be

$$
\dot{V}(t) = \begin{bmatrix}\xi(t)\\ \eta(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}P_\xi(A_\xi - B_\xi K_\xi) + (A_\xi - B_\xi K_\xi)^{\mathrm{T}}P_\xi & P_\xi\overline{Q}_1 + \overline{Q}_2^{\mathrm{T}}P_\eta\\ (P_\xi\overline{Q}_1 + \overline{Q}_2^{\mathrm{T}}P_\eta)^{\mathrm{T}} & P_\eta A_\eta + A_\eta^{\mathrm{T}}P_\eta\end{bmatrix}\begin{bmatrix}\xi(t)\\ \eta(t)\end{bmatrix}. \qquad (39)
$$

Here the symmetric matrix of (39) can be expressed as

$$
\begin{bmatrix}-2\varepsilon I_{2m} & P_\xi\overline{Q}_1 + \overline{Q}_2^{\mathrm{T}}P_\eta\\ (P_\xi\overline{Q}_1 + \overline{Q}_2^{\mathrm{T}}P_\eta)^{\mathrm{T}} & -I_{n-2m}\end{bmatrix} \qquad (40)
$$

from (36), (34). Using Schur complement, we have the following necessary and sufficient conditions such that (40) is negative definite:

$$-2\varepsilon I_{2m} < 0, \tag{41}$$

$$-I_{n-2m} + (P_\xi \overline{Q}_1 + \overline{Q}_2^{\mathrm{T}} P_\eta)^{\mathrm{T}} \frac{1}{2\varepsilon} (P_\xi \overline{Q}_1 + \overline{Q}_2^{\mathrm{T}} P_\eta) < 0 \tag{42}$$

where

$$P_\xi \overline{Q}_1 = \begin{bmatrix} \overline{P} \\ P_{\xi 2} \end{bmatrix} Q_{23} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} Q_{23} \tag{43}$$

from (27), (35). Obviously, the first inequality (41) is hold. The second inequality (42) is also achieved under large $\varepsilon > 0$ (because $\overline{Q}_2^{\mathrm{T}} P_\eta$ and $P_\xi \overline{Q}_1$ are independent of ε). At this time, (40) becomes negative definite matrix and (39) is

$$\dot{V}(t) = \begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -2\varepsilon I_{2m} & P_\xi \overline{Q}_1 + \overline{Q}_2^{\mathrm{T}} P_\eta \\ (P_\xi \overline{Q}_1 + \overline{Q}_2^{\mathrm{T}} P_\eta)^{\mathrm{T}} & -I_{n-2m} \end{bmatrix} \begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix} \leq 0. \tag{44}$$

Hence, giving the constant gain matrices K_{I0},K_{P0} as (18), (20) and the adaptive law of $K_{Pi}(t)$, $K_{Di}(t)$, $i = 1, 2$ as (19a) $\sim$ (19d), (20), we have shown that there exists the Lyapunov function which derivative is (44). Therefore, all variables in $V(\cdot)$ is bounded, that is $\xi(t), \eta(t), \psi_I(t)$, $\Psi_{P1}(t)$, $\Psi_{P2}(t)$, $\Psi_{D1}(t)$, $\Psi_{D2}(t) \in \mathcal{L}_\infty$. Furthermore, $\dot{\xi}(t), \dot{\eta}(t)$ are bounded from (30) and $\xi(t), \eta(t) \in \mathcal{L}_2$ from (44). Accordingly, since $\xi(t), \eta(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, $\dot{\xi}(t), \dot{\eta}(t) \in \mathcal{L}_\infty$, the origin of the error system $(\xi, \eta) = (0, 0)$, namely $e_x = 0$ is asymptotically stable from Barbalat's lemma, and $K_{Pi}(t)$, $K_{Di}(t)$, $i = 1, 2$ are bounded from $\Psi_{P1}(t)$, $\Psi_{P2}(t)$, $\Psi_{D1}(t)$, $\Psi_{D2}(t) \in \mathcal{L}_\infty$. □

Remark 1: In proposed method, it is important how to select H_1, H_2, $h_{ij} > 0$ which always guarantee the asymptotic stability because they also affect the transient response. Especially, taking large h_{ij} causes the large over shoot of inputs at first time range because of the proportional gain matrix K_{P0} with h_{ij}. So it seems to be appropriate to adjust h_{ij} from small values slowly such that better response is gotten although it is difficult to show concrete guide because system's parameters are unknown. But it is also one of the characteristic in our proposed method that the designer can adjust transient response manually under guaranteeing stability.

4.2 Case B

Corollary 1: Suppose Assumption 3 and Assumption 4(b). Give the constant gain matrices K_{I0}, K_{P0} as (18) and the adaptive tuning law of the adjustable gain matrices $K_{Pi}(t), K_{Di}(t), i = 1, 2$ as (19a) $\sim$ (19d) where $H_1 = \mathrm{diag}\{h_{1j}, \cdots, h_{1m}\}$, $H_2 = 0, h_{1j} > 0$, $j = 1, \cdots, m$, then (17) is asymptotically stable and the adjustable gain matrices are bounded. Here Γ_{P1}, Γ_{P2}, $\Gamma_{D1}, \Gamma_{D2} \in \mathcal{R}^{m \times m}$ are arbitrary positive definite matrices and γ_I is arbitrary positive scalar.

Proof : After transforming the error system (17) into the normal form (see e.g. (Isidori, 1995)) based on Assumption 4(b), do the procedure like Theorem 1, it can be proved more easily than Theorem 1. □

5. Simulations

Example 1

Consider the missile control system (Bar-Kana & Kaufman, 1985):

$$\dot{x}(t) = \begin{bmatrix} 3.23 & 12.5 & -476 & 0 & 228 & 0 & 0 & 0 \\ -12.5 & -3.23 & 0 & 476.0 & 0 & -228 & 0 & 0 \\ 0.39 & 0 & -1.93 & -10 & -415 & 0 & 0 & 0 \\ 0 & -0.39 & 10 & -1.93 & 0 & -415 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -75 \\ 0 & 0 & 22.4 & 0 & -300 & 0 & -150 & 0 \\ 0 & 0 & 0 & -22.4 & 0 & 300 & 0 & -150 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} u(t) + d_i.$$

$$y(t) = \begin{bmatrix} -2.99 & 0 & -1.19 & 1.5375 & -27.64 & 0 & 0 & 0 \\ 0 & -2.99 & 1.5375 & 1.19 & 0 & 27.64 & 0 & 0 \end{bmatrix} x(t) + d_o.$$

Let the reference system be

$$\dot{x}_M(t) = \begin{bmatrix} 0 & q_{M1} & 0 & 0 \\ -q_{M1} & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{M2} \\ 0 & 0 & -q_{M2} & 0 \end{bmatrix} x_M(t), \;\; y_M(t) = \begin{bmatrix} 0 & q_{M3} & 0 & 0 \\ 0 & 0 & q_{M4} & 0 \end{bmatrix} x_M(t).$$

which means $y_M(t) = \begin{bmatrix} q_{M3} \cos q_{M1} t & q_{M4} \sin q_{M2} t \end{bmatrix}^{\mathrm{T}}$ at $x_M(0) = \begin{bmatrix} 0 \, 1 \, 0 \, 1 \end{bmatrix}^{\mathrm{T}}$.
Set disturbances d_i, d_o and parameters of the reference system q_M as follows:

$$q_{M1} = 1, \; q_{M2} = 2.0, \; q_{M3} = 0.5, \; q_{M4} = 1, \;\; d_i = \begin{bmatrix} 0\,0\,0\,0\,0\,0\,1\,2 \end{bmatrix}^{\mathrm{T}}, \; d_o = \begin{bmatrix} 0.5 & -1 \end{bmatrix}^{\mathrm{T}}.$$

Select arbitrary H_1, H_2 as $H_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, H_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ based on Remark 1. Set the $\Gamma_{P1} = \Gamma_{P2} = \Gamma_{D1} = \Gamma_{D2} = I_2$ and $\gamma_I = 1$. Put the initial values $x(0) = 0$, $K_{Pi}(0) = K_{Di}(0) = 0$, $i = 1, 2$. It is observed from simulation results at Fig. 2 that $K_{P1}(t), K_{P2}(t), K_{D1}(t), K_{D2}(t)$ are on-line adjusted and the asymptotic output tracking is achieved.

Example 2
Consider the following unstable system:

$$\dot{x}(t) = \begin{bmatrix} 1 & 1 & 4 & 3 \\ 1 & 4 & -3 & 1 \\ -1 & 1 & -5 & -1 \\ 1 & 0 & -1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u(t) + d_{\mathrm{i}},$$

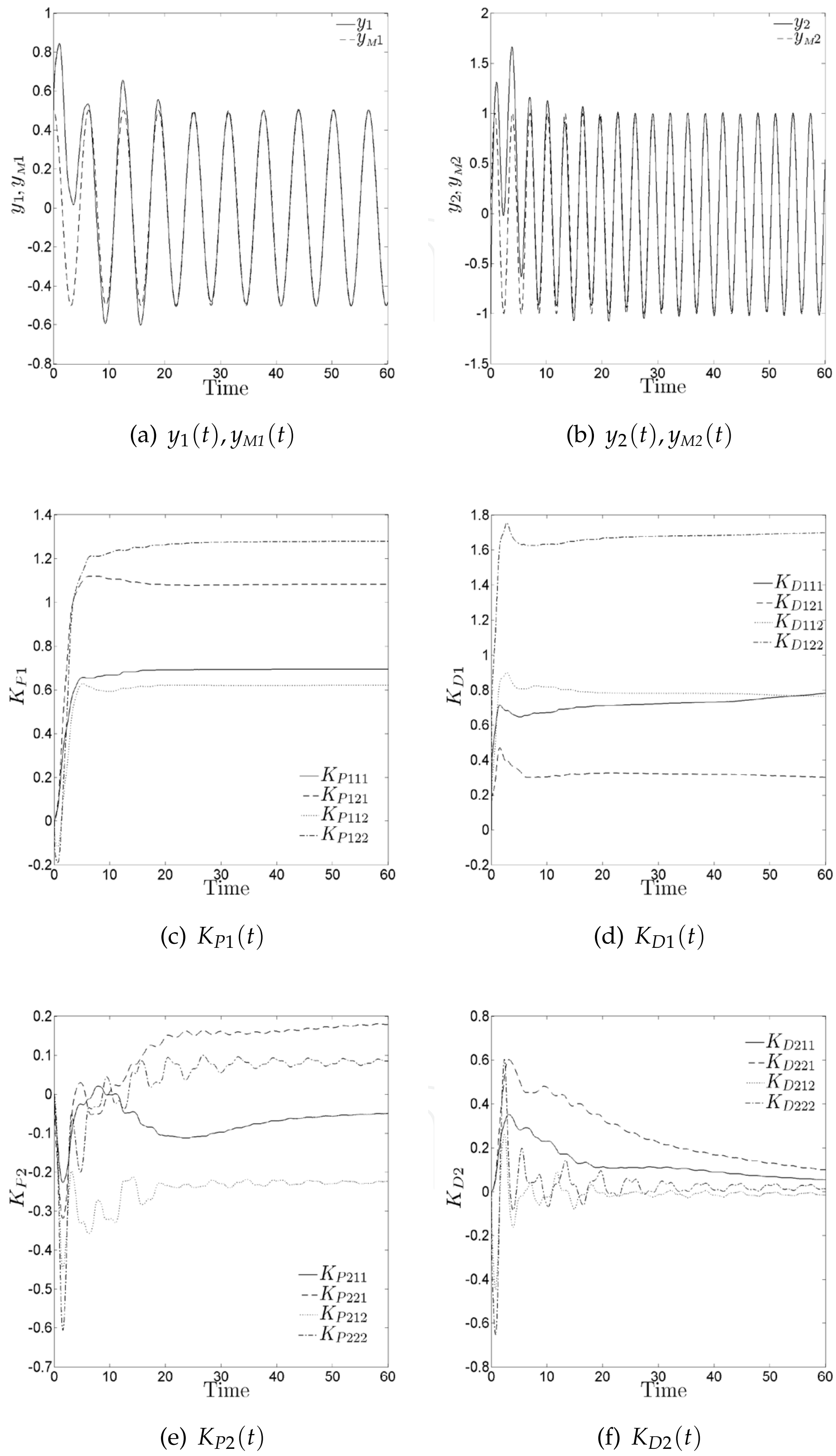
$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x(t) + d_{\mathrm{o}}.$$

Set the reference system be

$$\dot{x}_M(t) = \begin{bmatrix} 0 & q_{M1} & 0 & 0 \\ -q_{M1} & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{M2} \\ 0 & 0 & -q_{M2} & 0 \end{bmatrix} x_M(t) + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & 0 \end{bmatrix} u_M,$$

$$y_M(t) = \begin{bmatrix} 0 & q_{M3} & 0 & 0 \\ 0 & 0 & q_{M4} & 0 \end{bmatrix} x_M(t),$$

which generates $y_M(t) = \begin{bmatrix} q_{M3} \cos q_{M1} t & q_{M4} \sin q_{M2} t \end{bmatrix}^{\mathrm{T}}$ at $x_M(0) = \begin{bmatrix} 0 \, 1 \, 0 \, 1 \end{bmatrix}^{\mathrm{T}}$ when $u_M = 0$.

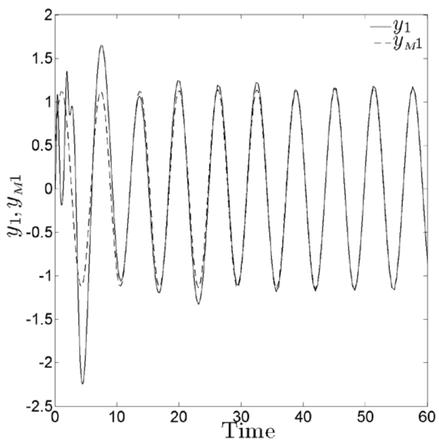


(a) $y_1(t), y_{M1}(t)$ (b) $y_2(t), y_{M2}(t)$

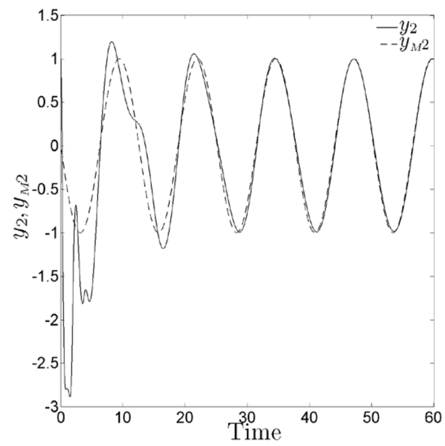
(c) $K_{P1}(t)$ (d) $K_{D1}(t)$

(e) $K_{P2}(t)$ (f) $K_{D2}(t)$

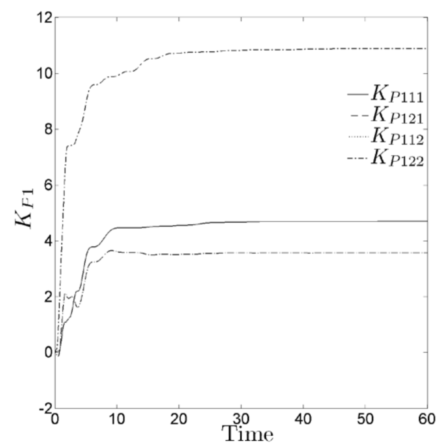
Fig. 2. Simulation Results of Example 1



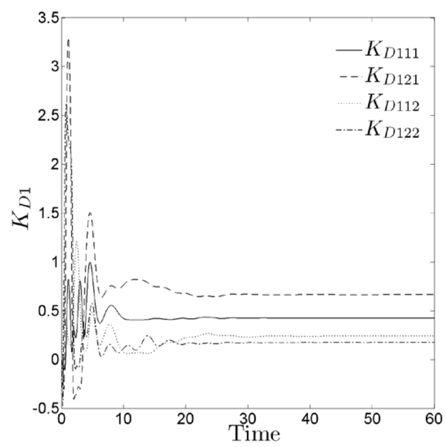
(a) $y_1(t), y_{M1}(t)$



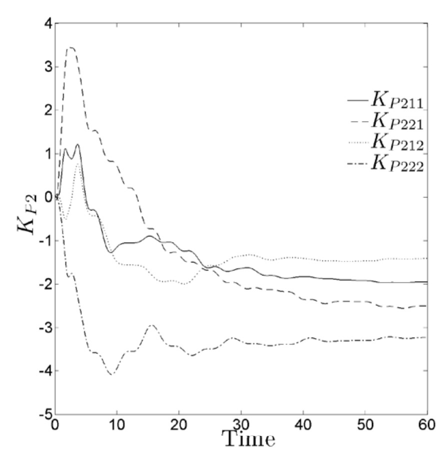
(b) $y_2(t), y_{M2}(t)$



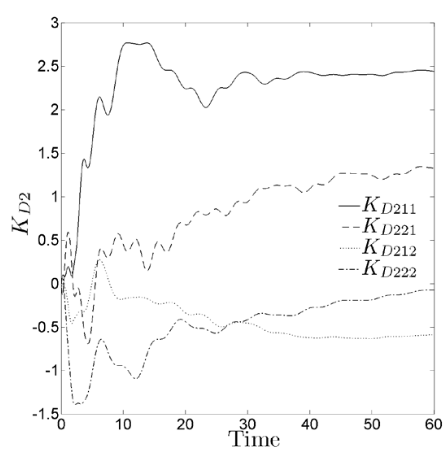
(c) $K_{P1}(t)$



(d) $K_{D1}(t)$



(e) $K_{P2}(t)$



(f) $K_{D2}(t)$

Fig. 3. Simulation Results of Example 2

Disturbances d_i, d_o and parameters of the reference system q_M are set as follows:

$$q_{M1} = 1.0,\ q_{M2} = 0.5,\ q_{M3} = 0.5,\ q_{M4} = 1,\ u_M = \begin{bmatrix} 1\ 2 \end{bmatrix}^{\mathrm{T}},$$
$$d_{\mathrm{i}} = \begin{bmatrix} 1 & -2 & 0 & 0 \end{bmatrix}^{\mathrm{T}},\ d_{\mathrm{o}} = \begin{bmatrix} 0 & 1 \end{bmatrix}^{\mathrm{T}}$$

From Colloraly 1, select arbitrary H_1, H_2 as $H_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, H_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.
Set the $\Gamma_{P1} = \Gamma_{P2} = \Gamma_{D1} = \Gamma_{D2} = I_2$ and $\gamma_I = 1$. Put the initial values $x(0) = 0$, $K_{Pi}(0) = K_{Di}(0) = 0$, $i = 1, 2$. We can observe that $K_{P1}(t), K_{P2}(t), K_{D1}(t), K_{D2}(t)$ are on-line adjusted and the asymptotic output tracking is achieved from simulation results at Fig. 3 .

6. Conclusions

We have proposed the new adaptive PID control and its parameter tuning method for the MIMO system. In our method, the asymptotic output tracking can be guaranteed even if the MIMO system is unstable and has unknown system parameters and unknown constant disturbances. The effectiveness of the method is confirmed by numerical simulations. Our future task is extending the controlled system to the nonlinear one.

7. References

Åström, K.J. & Hägglund, T. (1995). *PID Controllers: Theory, Design, and Tuning, 2nd Edition*, ISA, ISBN 978-1-55617-516-9, North Carolina.

Suda, N. (1992). *PID Control*, Asakura Publishing Co., Ltd., ISBN 978-4-254-20966-2, Tokyo.

Ho, W.K.; Lee, T.H.; Xu, W.; Zhou, J.R. & Tay, E.B. (2000). The direct Nyquist array design of PID Controllers. *IEEE Trans. Indust. Elect.*, Vol.47, No.1, 175-185, ISSN 0278-0046

Hara, S.; Iwasaki, T. & Shiokata, D. (2006). Robust PID Control Using Generalized KYP Synthesis. *IEEE Control Systems Magazine*, Vol.26, No.1, 80-91, ISSN 0272-1708

Mattei, M. (2001). Robust multivariable PID control for linear parameter varying systems. *Automatica*, Vol.37, No.12, 1997-2003, ISSN 0005-1098

Saeki, M. (2006). Fixed structure PID controller design for standard H_∞ control problem. *Automatica*, Vol.42, No.1, 93-100, ISSN 0005-1098

Zheng, F.; Wang, Q.G. & Lee, T.H. (2002). On the design of multivariable PID controllers via LMI approach. *Automatica*, Vol.38, No.3, 517-526, ISSN 0005-1098

Gomma, H.W. (2004). Adaptive PID Control Design Based on Generalized Predictive Control (GPC). *Proc. of 2004 IEEE CCA*, 1685-1690, ISBN 0-7803-8633-7, Taipei, Taiwan, Sept. 2004

Yusof, R.; Omatu, S. & Khalid, M. (1994). Self-Tuning PID Control; A Multivariable Derivation and Application. *Automatica*, Vol.30, No.12, 1975-1981, ISSN 0005-1098

Yamamoto, T.; Ishihara, H.; Omatu, S. & Kitamori, T. (1992). A Construction of Multivariable Self-Tuning Controller with Two-Degree-of-Freedom PID Structure. *Transactions of the SICE*, Vol.28, No.4, 484-491, ISSN 0453-4654

Chang, W.D.; Hwang, R.C. & Hsieh, J.G. (2003). A multivariable on-line adaptive PID controllers using auto-tuning neurons. *Engineering Applications of Artificial Intelligence*, Vol.16, No.1, 57-63, ISSN 0952-1976

Hu, J. & Tomizuka, M. (1993). Adaptive asymptotic tracking of repetitive signals – a frequency domain approach. *IEEE Trans. Automatic Control*, Vol.38, No.10, 1752-1578, ISSN 0018-9286

Miyasato, Y. (1998). A design method of universal adaptive servo controller, *Proc. the 37th IEEE CDC*, 2294-2299, ISBN 0-7803-4394-8, Tampa, Florida, Dec. 1998

Ortega, R. & Kelly, R. (1985). On Stability-Instability Conditions of a Simple Robust Adaptive Servo Controller, *Proc. the 24th IEEE CDC*, 502 - 507, Fort Lauderdale, Florida, Dec. 1985
Chang, M. & Davison, E.J. (1995). The Adaptive Servomechanism Problem for MIMO Systems, *Proc. 34th IEEE CDC*, 1738-1743, ISBN 0-7803-2685-7, New Orleans, LA, Dec. 1995
Dang, H & Owens, D.H. (2006). MIMO multi-periodic control system: universal adaptive control schemes. *Int. J. Adaptive Control and Signal Processing*, Vol.20, No.9, 409-429, ISSN 0890-6327
Johansson, R. (1987). Parametric models of linear multivariable systems for adaptive control, *IEEE Trans. Automatic Control*, Vol.32, No.4, 303-313, ISSN 0018-9286
Miyamoto, M. (1999). Design of Fixed-Structure H_∞ Controllers Based on Coprime Factorization and LMI Optimization. *Journal of ISCIE: Systems, Control and Information*, Vol.42, No.2, 80-86, ISSN 0916-1600
Kaufman, H.; Bar-Kana, I. & Sobel, S. (1994). *Direct Adaptive Control Algorithms – Theory and Applications, 2nd edition*, Springer-Verlag, ISBN 0-387-94884-8, New York.
Isidori, A. (1995). *Nonlinear Control Systems, 3rd edition*, Springer-Verlag, ISBN 978-3-540-19916-8, New York.
Iwasaki, T. (1997). *LMI and Control*, SHOKODO Co.,Ltd., ISBN 978-4-7856-9053-3, Tokyo.
Bar-Kana, I. & Kaufman, H. (1985). Global Stability and Performance of a Simplified Adaptive Algorithm. *International Journal of Control*, Vol.42, No.6, 1491-1505, ISSN 0020-7179
Kodama, S. & Suda, N. (1995). *Matrix Theory for System Control*, CORONA PUBLISHING CO.,LTD., ISBN 978-4-339-08330-9, Tokyo.

A. (Proof)

(7), (8) are rewritten as

$$\begin{bmatrix} \dot{x}^*(t) \\ y_M(t) \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x^*(t) \\ u^*(t) \end{bmatrix} + \begin{bmatrix} d_i \\ d_o \end{bmatrix}. \tag{45}$$

Now we prove that the above equation is hold under Assumption 1 by substituting (9), (10). First, we calculate the right side of (45). Since (9), (10) are expressed as

$$\begin{bmatrix} x^*(t) \\ u^*(t) \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} x_M(t) \\ u_M \end{bmatrix} - \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} d_i \\ d_o \end{bmatrix}, \tag{46}$$

substitute (46) into the right side of (45) to get

$$\text{The right side of (45)} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} x_M(t) \\ u_M \end{bmatrix} \tag{47}$$

by using the relation

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1} \tag{48}$$

from Assumption 1.

Then we calculate the left side of (45). Substituting $\dot{x}^*(t) = T_{11}\dot{x}_M(t)$ which is the time derivative of (9) and using the relation of (3), (4), we can get

$$\text{The left side of (45)} = \begin{bmatrix} T_{11}A_M & T_{11}B_M \\ C_M & 0 \end{bmatrix} \begin{bmatrix} x_M(t) \\ u_M \end{bmatrix}. \tag{49}$$

Therefore from (47), (49), the equation obtained from substituting (9), (10) into (45) is

$$\begin{bmatrix} T_{11}A_M & T_{11}B_M \\ C_M & 0 \end{bmatrix} \begin{bmatrix} x_M(t) \\ u_M \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} x_M(t) \\ u_M \end{bmatrix}. \tag{50}$$

This equation is always hold for all $x_M(t)$ and u_M if

$$\begin{bmatrix} T_{11}A_M & T_{11}B_M \\ C_M & 0 \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$$

is hold. This is the matrix linear equation with variables T_{11}.

Now we will show that this matrix equation is solvable. Multiplying both left side of above equation by the nonsingular matrix (48), we have

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} T_{11}A_M & T_{11}B_M \\ C_M & 0 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}.$$

Obviously, T_{11} is the solution to the linear matrix equation

$$T_{11} = M_{11}T_{11}A_M + M_{12}C_M, \tag{51}$$

and there exists unique solution T_{11} under Assumption 1 (see (Kodama & Suda, 1995)). Therefore rests of T_{ij} exist uniquely as

$$T_{12} = M_{11}T_{11}B_M, \; T_{22} = M_{21}T_{11}B_M,$$
$$T_{21} = M_{21}T_{11}A_M + M_{22}C_M.$$

We have proved that (9), (10) satisfy the relation (7), (8) for all d_o, d_i, u_M under Assumption 1. □

B. (Proof)

Using (4), we can calculate (15) as

$$(T_{21} - S_1C_M - S_2C_MA_M)x_M(t) = 0.$$

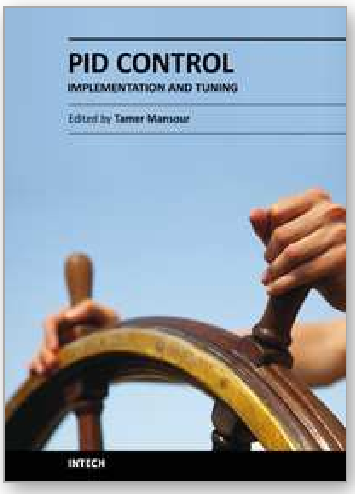
This equation is always hold for all $x_M(t)$ if

$$T_{21} - S_1C_M - S_2C_MA_M = 0$$

is satisfied, that is if

$$[S_1 \; S_2] \begin{bmatrix} C_M \\ C_MA_M \end{bmatrix} = T_{21}$$

is solvable on S_1, S_2. In fact this equation is solvable from Assumption 2 (see (Kodama & Suda, 1995)), so there exist S_1, S_2 satisfying (15). □



PID Control, Implementation and Tuning

Edited by Dr. Tamer Mansour

ISBN 978-953-307-166-4

Hard cover, 238 pages

Publisher InTech

Published online 19, April, 2011

Published in print edition April, 2011

The PID controller is considered the most widely used controller. It has numerous applications varying from industrial to home appliances. This book is an outcome of contributions and inspirations from many researchers in the field of PID control. The book consists of two parts; the first is related to the implementation of PID control in various applications whilst the second part concentrates on the tuning of PID control to get best performance. We hope that this book can be a valuable aid for new research in the field of PID control in addition to stimulating the research in the area of PID control toward better utilization in our life.