We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

4,500
Open access books available

118,000
International authors and editors

130M
Downloads

154
Countries delivered to

Our authors are among the
TOP 1%
most cited scientists

12.2%
Contributors from top 500 universities

WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com

14

Design Principles of Active Robust Fault Tolerant Control Systems

Anna Filasová and Dušan Krokavec
Technical University of Košice
Slovakia

1. Introduction

The complexity of control systems requires the fault tolerance schemes to provide control of the faulty system. The fault tolerant systems are that one of the more fruitful applications with potential significance for those domains in which control must proceed while the controlled system is operative and testing opportunities are limited by given operational considerations. The real problem is usually to fix the system with faults so that it can continue its mission for some time with some limitations of functionality. These large problems are known as the fault detection, identification and reconfiguration (FDIR) systems. The practical benefits of the integrated approach to FDIR seem to be considerable, especially when knowledge of the available fault isolations and the system reconfigurations is used to reduce the cost and to increase the control reliability and utility. Reconfiguration can be viewed as the task to select these elements whose reconfiguration is sufficient to do the acceptable behavior of the system. If an FDIR system is designed properly, it will be able to deal with the specified faults and maintain the system stability and acceptable level of performance in the presence of faults.

The essential aspect for the design of fault-tolerant control requires the conception of diagnosis procedures that can solve the fault detection and isolation problem. The fault detection is understood as a problem of making a binary decision either that something has gone wrong or that everything is in order. The procedure composes residual signal generation (signals that contain information about the failures or defects) followed by their evaluation within decision functions, and it is usually achieved designing a system which, by processing input/output data, is able generating the residual signals, detect the presence of an incipient fault and isolate it.

In principle, in order to achieve fault tolerance, some redundancy is necessary. So far direct redundancy is realized by redundancy in multiple hardware channels, fault-tolerant control involve functional redundancy. Functional (analytical) redundancy is usually achieved by design of such subsystems, which functionality is derived from system model and can be realized using algorithmic (software) redundancy. Thus, analytical redundancy most often means the use of functional relations between system variables and residuals are derived from implicit information in functional or analytical relationships, which exist between measurements taken from the process, and a process model. In this sense a residual is a fault indicator, based on a deviation between measurements and model-equation-based computation and model based diagnosis use models to obtain residual signals that are as a rule zero in the fault free case and non-zero otherwise.

A fault in the fault diagnosis systems can be detected and isolated when has to cause a residual change and subsequent analyze of residuals have to provide information about faulty component localization. From this point of view the fault decision information is capable in a suitable format to specify possible control structure class to facilitate the appropriate adaptation of the control feedback laws. Whereas diagnosis is the problem of identifying elements whose abnormality is sufficient to explain an observed malfunction, reconfiguration can be viewed as a problem of identifying elements whose in a new structure are sufficient to restore acceptable behavior of the system.

1.1 Fault tolerant control

Main task to be tackled in achieving fault-tolerance is design a controller with suitable reconfigurable structure to guarantee stability, satisfactory performance and plant operation economy in nominal operational conditions, but also in some components malfunction. Generally, fault-tolerant control is a strategy for reliable and highly efficient control law design, and includes fault-tolerant system requirements analysis, analytical redundancy design (fault isolation principles) and fault accommodation design (fault control requirements and reconfigurable control strategy). The benefits result from this characterization give a unified framework that should facilitate the development of an integrated theory of FDIR and control (fault-tolerant control systems (FTCS)) to design systems having the ability to accommodate component failures automatically.

FTCS can be classified into two types: passive and active. In passive FTCS, fix controllers are used and designed in such way to be robust against a class of presumed faults. To ensure this a closed-loop system remains insensitive to certain faults using constant controller parameters and without use of on-line fault information. Because a passive FTCS has to maintain the system stability under various component failures, from the performance viewpoint, the designed controller has to be very conservative. From typical relationships between the optimality and the robustness, it is very difficult for a passive FTCS to be optimal from the performance point of view alone.

Active FTCS react to the system component failures actively by reconfiguring control actions so that the stability and acceptable (possibly partially degraded, graceful) performance of the entire system can be maintained. To achieve a successful control system reconfiguration, this approach relies heavily on a real-time fault detection scheme for the most up-to-date information about the status of the system and the operating conditions of its components. To reschedule controller function a fixed structure is modified to account for uncontrollable changes in the system and unanticipated faults. Even though, an active FTCS has the potential to produce less conservative performance.

The critical issue facing any active FTCS is that there is only a limited amount of reaction time available to perform fault detection and control system reconfiguration. Given the fact of limited amount of time and information, it is highly desirable to design a FTCS that possesses the guaranteed stability property as in a passive FTCS, but also with the performance optimization attribute as in an active FTCS.

Selected useful publications, especially interesting books on this topic (Blanke et al.,2003), (Chen and Patton,1999), (Chiang et al.,2001), (Ding,2008), (Ducard,2009), (Simani et al.,2003) are presented in References.

1.2 Motivation

A number of problems that arise in state control can be reduced to a handful of standard convex and quasi-convex problems that involve matrix inequalities. It is known that the optimal solution can be computed by using interior point methods (Nesterov and Nemirovsky,1994) which converge in polynomial time with respect to the problem size and efficient interior point algorithms have recently been developed for and further development of algorithms for these standard problems is an area of active research. For this approach, the stability conditions may be expressed in terms of linear matrix inequalities (LMI), which have a notable practical interest due to the existence of powerful numerical solvers. Some progres review in this field can be found e.g. in (Boyd et al.,1994), (Herrmann et al.,2007), (Skelton et al.,1998), and the references therein.

In contradiction to the standard pole placement methods application in active FTCS design there don't exist so much structures to solve this problem using LMI approach (e.g. see (Chen et al.,1999), (Filasova and Krokavec,2009), (Liao et al.,2002), (Noura et al.,2009)). To generalize properties of non-expansive systems formulated as H_∞ problems in the bounded real lemma (BRL) form, the main motivation of this chapter is to present reformulated design method for virtual sensor control design in FTCS structures, as well as the state estimator based active control structures for single actuator faults in the continuous-time linear MIMO systems. To start work with this formalism structure residual generators are designed at first to demonstrate the application suitability of the unified algebraic approach in these design tasks. LMI based design conditions are outlined generally to posse the sufficient conditions for a solution. The used structure is motivated by the standard ones (Dong et al.,2009), and in this presented form enables to design systems with the reconfigurable controller structures.

2. Problem description

Through this chapter the task is concerned with the computation of reconfigurable feedback $\mathbf{u}(t)$, which control the observable and controllable faulty linear dynamic system given by the set of equations

$$\dot{\mathbf{q}}(t) = \mathbf{A}\mathbf{q}(t) + \mathbf{B}_u\mathbf{u}(t) + \mathbf{B}_f\mathbf{f}(t) \tag{1}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{q}(t) + \mathbf{D}_u\mathbf{u}(t) + \mathbf{D}_f\mathbf{f}(t) \tag{2}$$

where $\mathbf{q}(t) \in \mathbb{R}^n$, $\mathbf{u}(t) \in \mathbb{R}^r$, $\mathbf{y}(t) \in \mathbb{R}^m$, and $\mathbf{f}(t) \in \mathbb{R}^l$ are vectors of the state, input, output and fault variables, respectively, matrices $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{B}_u \in \mathbb{R}^{n\times r}$, $\mathbf{C} \in \mathbb{R}^{m\times n}$, $\mathbf{D}_u \in \mathbb{R}^{m\times r}$, $\mathbf{B}_f \in \mathbb{R}^{n\times l}$, $\mathbf{D}_f \in \mathbb{R}^{m\times l}$ are real matrices. Problem of the interest is to design the asymptotically stable closed-loop systems with the linear memoryless state feedback controllers of the form

$$\mathbf{u}(t) = -\mathbf{K}_o\mathbf{y}_e(t) \tag{3}$$

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{q}_e(t) - \mathbf{L}\mathbf{f}_e(t) \tag{4}$$

respectively. Here $\mathbf{K}_o \in \mathbb{R}^{r\times m}$ is the output controller gain matrix, $\mathbf{K} \in \mathbb{R}^{r\times n}$ is the nominal state controller gain matrix, $\mathbf{L} \in \mathbb{R}^{r\times l}$ is the compensate controller gain matrix, $\mathbf{y}_e(t)$ is by virtual sensor estimated output of the system, $\mathbf{q}_e(t) \in \mathbb{R}^n$ is the system state estimate vector, and $\mathbf{f}_e(t) \in \mathbb{R}^l$ is the fault estimate vector. Active compensate method can be applied for such systems, where

$$\begin{bmatrix} \mathbf{B}_f \\ \mathbf{D}_f \end{bmatrix} = \begin{bmatrix} \mathbf{B}_u \\ \mathbf{D}_u \end{bmatrix} \mathbf{L} \tag{5}$$

and the additive term $\mathbf{B}_f\mathbf{f}(t)$ is compensated by the term

$$-\mathbf{B}_f\mathbf{f}_e(t) = -\mathbf{B}_u\mathbf{L}\mathbf{f}_e(t) \tag{6}$$

which implies (4). The estimators are then given by the set of the state equations

$$\dot{\mathbf{q}}_e(t) = \mathbf{A}\mathbf{q}_e(t) + \mathbf{B}_u\mathbf{u}(t) + \mathbf{B}_f\mathbf{f}_e(t) + \mathbf{J}(\mathbf{y}(t) - \mathbf{y}_e(t)) \tag{7}$$

$$\dot{\mathbf{f}}_e(t) = \mathbf{M}\mathbf{f}_e(t) + \mathbf{N}(\mathbf{y}(t) - \mathbf{y}_e(t)) \tag{8}$$

$$\mathbf{y}_e(t) = \mathbf{C}\mathbf{q}_e(t) + \mathbf{D}_u\mathbf{u}(t) + \mathbf{D}_f\mathbf{f}_e(t) \tag{9}$$

where $\mathbf{J} \in \mathbb{R}^{n\times m}$ is the state estimator gain matrix, and $\mathbf{M} \in \mathbb{R}^{l\times l}$, $\mathbf{N} \in \mathbb{R}^{l\times m}$ are the system and input matrices of the fault estimator, respectively or by the set of equation

$$\dot{\mathbf{q}}_{fe}(t) = \mathbf{A}\mathbf{q}_{fe}(t) + \mathbf{B}_u\mathbf{u}_f(t) + \mathbf{J}(\mathbf{y}_f(t) - \mathbf{D}_u\mathbf{u}_f(t) - \mathbf{C}_f\mathbf{q}_{fe}(t)) \tag{10}$$

$$\mathbf{y}_e(t) = \mathbf{E}(\mathbf{y}_f(t) + (\mathbf{C} - \mathbf{E}\mathbf{C}_f)\mathbf{q}_{fe}(t) \tag{11}$$

where $\mathbf{E} \in \mathbb{R}^{m\times m}$ is a switching matrix, generally used in such a way that $\mathbf{E} = \mathbf{0}$, or $\mathbf{E} = \mathbf{I}_m$.

3. Basic preliminaries

Definition 1 (Null space) *Let* $\mathbf{E}$, $\mathbf{E} \in \mathbb{R}^{h\times h}$, *rank*$(\mathbf{E}) = k < h$ *be a rank deficient matrix. Then the null space* $\mathcal{N}_{\mathbf{E}}$ *of* $\mathbf{E}$ *is the orthogonal complement of the row space of* $\mathbf{E}$.
Proposition 1 (Orthogonal complement) *Let* $\mathbf{E}$, $\mathbf{E} \in \mathbb{R}^{h\times h}$, *rank*$(\mathbf{E}) = k < h$ *be a rank deficient matrix. Then an orthogonal complement* $\mathbf{E}^{\perp}$ *of* $\mathbf{E}$ *is*

$$\mathbf{E}^{\perp} = \mathbf{E}^{\circ}\mathbf{U}_2^T \tag{12}$$

where $\mathbf{U}_2^T$ *is the null space of* $\mathbf{E}$ *and* $\mathbf{E}^{\circ}$ *is an arbitrary matrix of appropriate dimension.*
Proof. The singular value decomposition (SVD) of $\mathbf{E}$, $\mathbf{E} \in \mathbb{R}^{h\times h}$, rank$(\mathbf{E}) = k < h$ gives

$$\mathbf{U}^T\mathbf{E}\mathbf{V} = \begin{bmatrix}\mathbf{U}_1^T\\ \mathbf{U}_2^T\end{bmatrix}\mathbf{E}\begin{bmatrix}\mathbf{V}_1 & \mathbf{V}_2\end{bmatrix} = \begin{bmatrix}\mathbf{\Sigma}_1 & \mathbf{0}_{12}\\ \mathbf{0}_{21} & \mathbf{0}_{22}\end{bmatrix} \tag{13}$$

where $\mathbf{U}^T \in \mathbb{R}^{h\times h}$ is the orthogonal matrix of the left singular vectors, $\mathbf{V} \in \mathbb{R}^{h\times h}$ is the orthogonal matrix of the right singular vectors of $\mathbf{E}$ and $\mathbf{\Sigma}_1 \in \mathbb{R}^{k\times k}$ is the diagonal positive definite matrix of the form

$$\mathbf{\Sigma}_1 = \operatorname{diag}\begin{bmatrix}\sigma_1 \cdots \sigma_k\end{bmatrix}, \quad \sigma_1 \geq \cdots \geq \sigma_k > 0 \tag{14}$$

which diagonal elements are the singular values of $\mathbf{E}$. Using orthogonal properties of $\mathbf{U}$ and $\mathbf{V}$, i.e. $\mathbf{U}^T\mathbf{U} = \mathbf{I}_h$, as well as $\mathbf{V}^T\mathbf{V} = \mathbf{I}_h$, and

$$\begin{bmatrix}\mathbf{U}_1^T\\ \mathbf{U}_2^T\end{bmatrix}\begin{bmatrix}\mathbf{U}_1 & \mathbf{U}_2\end{bmatrix} = \begin{bmatrix}\mathbf{I}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_2\end{bmatrix}, \quad \mathbf{U}_2^T\mathbf{U}_1 = \mathbf{0} \tag{15}$$

respectively, where $\mathbf{I}_h \in \mathbb{R}^{h\times h}$ is the identity matrix, then $\mathbf{E}$ can be written as

$$\mathbf{E} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = \begin{bmatrix}\mathbf{U}_1 \, \mathbf{U}_2\end{bmatrix}\begin{bmatrix}\mathbf{\Sigma}_1 & \mathbf{0}_{12}\\ \mathbf{0}_{21} & \mathbf{0}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1^T\\ \mathbf{V}_2^T\end{bmatrix} = \begin{bmatrix}\mathbf{U}_1 \, \mathbf{U}_2\end{bmatrix}\begin{bmatrix}\mathbf{S}_1\\ \mathbf{0}_2\end{bmatrix} = \mathbf{U}_1\mathbf{S}_1 \tag{16}$$

where $\mathbf{S}_1 = \boldsymbol{\Sigma}_1 \mathbf{V}_1^T$. Thus, (15) and (16) implies

$$\mathbf{U}_2^T \mathbf{E} = \mathbf{U}_2^T \begin{bmatrix} \mathbf{U}_1 \ \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{0}_2 \end{bmatrix} = \mathbf{0} \tag{17}$$

It is evident that for an arbitrary matrix $\mathbf{E}^\circ$ is

$$\mathbf{E}^\circ \mathbf{U}_2^T \mathbf{E} = \mathbf{E}^\perp \mathbf{E} = \mathbf{0} \tag{18}$$

$$\mathbf{E}^\perp = \mathbf{E}^\circ \mathbf{U}_2^T \tag{19}$$

respectively, which implies (12). This concludes the proof. ■

Proposition 2. (Schur Complement) *Let* $\mathbf{Q} > \mathbf{0}$, $\mathbf{R} > 0$, $\mathbf{S}$ *are real matrices of appropriate dimensions, then the next inequalities are equivalent*

$$\begin{bmatrix} \mathbf{Q} & \mathbf{S} \\ \mathbf{S}^T & -\mathbf{R} \end{bmatrix} < 0 \ \Leftrightarrow \ \begin{bmatrix} \mathbf{Q} + \mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T & \mathbf{0} \\ \mathbf{0} & -\mathbf{R} \end{bmatrix} < 0 \ \Leftrightarrow \ \mathbf{Q} + \mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T < 0, \ \mathbf{R} > 0 \tag{20}$$

Proof. Let the linear matrix inequality takes form

$$\begin{bmatrix} \mathbf{Q} & \mathbf{S} \\ \mathbf{S}^T & -\mathbf{R} \end{bmatrix} < 0 \tag{21}$$

then using Gauss elimination principle it yields

$$\begin{bmatrix} \mathbf{I} & \mathbf{S}\mathbf{R}^{-1} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{Q} & \mathbf{S} \\ \mathbf{S}^T & -\mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{R}^{-1}\mathbf{S}^T & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{Q} + \mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T & \mathbf{0} \\ \mathbf{0} & -\mathbf{R} \end{bmatrix} \tag{22}$$

Since

$$\det \begin{bmatrix} \mathbf{I} & \mathbf{S}\mathbf{R}^{-1} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} = 1 \tag{23}$$

and it is evident that this transform doesn't change negativity of (21), and so (22) implies (20). This concludes the proof. ■

Note that in the next the matrix notations $\mathbf{E}$, $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{S}$, $\mathbf{U}$, and $\mathbf{V}$ be used in another context, too.

Proposition 3 (Bounded real lemma) *For given* $\gamma \in \mathbb{R}$ *and the linear system (1), (2) with* $\mathbf{f}(t) = \mathbf{0}$ *if there exists symmetric positive definite matrix* $\mathbf{P} > 0$ *such that*

$$\begin{bmatrix} \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} & \mathbf{P}\mathbf{B}_u & \mathbf{C}^T \\ * & -\gamma^2 \mathbf{I}_r & \mathbf{D}_u^T \\ * & * & -\mathbf{I}_m \end{bmatrix} < 0 \tag{24}$$

where $\mathbf{I}_r \in \mathbb{R}^{r \times r}$, $\mathbf{I}_m \in \mathbb{R}^{m \times m}$ *are the identity matrices, respectively then given system is asymptotically stable.*

Hereafter, $*$ denotes the symmetric item in a symmetric matrix.

Proof. Defining Lyapunov function as follows

$$v(\mathbf{q}(t)) = \mathbf{q}^T(t)\mathbf{P}\mathbf{q}(t) + \int_0^t \left(\mathbf{y}^T(r)\mathbf{y}(r) - \gamma^2 \mathbf{u}^T(r)\mathbf{u}(r) \right) \mathrm{d}r > 0 \tag{25}$$

where $\mathbf{P} = \mathbf{P}^T > 0$, $\mathbf{P} \in \mathbb{R}^{n \times n}$, $\gamma \in \mathbb{R}$, and evaluating the derivative of $v(\mathbf{q}(t))$ with respect to t then it yields

$$\dot{v}(\mathbf{q}(t)) = \dot{\mathbf{q}}^T(t)\mathbf{P}\mathbf{q}(t) + \mathbf{q}^T(t)\mathbf{P}\dot{\mathbf{q}}(t) + \mathbf{y}^T(t)\mathbf{y}(t) - \gamma^2 \mathbf{u}^T(t)\mathbf{u}(t) < 0 \tag{26}$$

Thus, substituting (1), (2) with $\mathbf{f}(t) = \mathbf{0}$ it can be written

$$\begin{aligned}\dot{v}(\mathbf{q}(t)) = (\mathbf{A}\mathbf{q}(t) + \mathbf{B}_u\mathbf{u}(t))^T\mathbf{P}\mathbf{q}(t) + \mathbf{q}^T(t)\mathbf{P}(\mathbf{A}\mathbf{q}(t) + \mathbf{B}_u\mathbf{u}(t)) + \\ + (\mathbf{C}\mathbf{q}(t) + \mathbf{D}_u\mathbf{u}(t))^T(\mathbf{C}\mathbf{q}(t) + \mathbf{D}_u\mathbf{u}(t)) - \gamma^2\mathbf{u}^T(t)\mathbf{u}(t) < 0\end{aligned} \tag{27}$$

and with notation

$$\mathbf{q}_c^T(t) = \begin{bmatrix} \mathbf{q}^T(t) \; \mathbf{u}^T(t) \end{bmatrix} \tag{28}$$

it is obtained

$$\dot{v}(\mathbf{q}(t)) = \mathbf{q}_c^T(t)\mathbf{P}_c\mathbf{q}_c(t) < 0 \tag{29}$$

where

$$\mathbf{P}_c = \begin{bmatrix} \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} & \mathbf{P}\mathbf{B}_u \\ * & -\gamma^2\mathbf{I}_r \end{bmatrix} + \begin{bmatrix} \mathbf{C}^T\mathbf{C} & \mathbf{C}^T\mathbf{D}_u \\ * & \mathbf{D}_u^T\mathbf{D}_u \end{bmatrix} < 0 \tag{30}$$

Since

$$\begin{bmatrix} \mathbf{C}^T\mathbf{C} & \mathbf{C}^T\mathbf{D}_u \\ * & \mathbf{D}_u^T\mathbf{D}_u \end{bmatrix} = \begin{bmatrix} \mathbf{C}^T \\ \mathbf{D}_u^T \end{bmatrix} \begin{bmatrix} \mathbf{C} \; \mathbf{D}_u \end{bmatrix} \geq 0 \tag{31}$$

Schur complement property implies

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{C}^T \\ * & \mathbf{0} & \mathbf{D}_u^T \\ * & * & -\mathbf{I}_m \end{bmatrix} \geq 0 \tag{32}$$

then using (32) the LMI (30) can now be written compactly as (24). This concludes the proof. ■

Remark 1 (Lyapunov inequality) Considering Lyapunov function of the form

$$v(\mathbf{q}(t)) = \mathbf{q}^T(t)\mathbf{P}\mathbf{q}(t) > 0 \tag{33}$$

where $\mathbf{P} = \mathbf{P}^T > 0$, $\mathbf{P} \in \mathbb{R}^{n \times n}$, and the control law

$$\mathbf{u}(t) = -\mathbf{K}_o\big(\mathbf{y}(t) - \mathbf{D}_u\mathbf{u}(t)\big) = -\mathbf{K}_o\mathbf{C}\mathbf{q}(t) \tag{34}$$

where $\mathbf{K}_o \in \mathbb{R}^{r \times m}$ is a gain matrix. Because in this case (27) gives

$$\dot{v}(\mathbf{q}(t)) = (\mathbf{A}\mathbf{q}(t) + \mathbf{B}_u\mathbf{u}(t))^T\mathbf{P}\mathbf{q}(t) + \mathbf{q}^T(t)\mathbf{P}(\mathbf{A}\mathbf{q}(t) + \mathbf{B}_u\mathbf{u}(t)) < 0 \tag{35}$$

then inserting (34) into (35) it can be obtained

$$\dot{v}(\mathbf{q}(t)) = \mathbf{q}^T(t)\mathbf{P}_{cb}\mathbf{q}(t) < 0 \tag{36}$$

where

$$\mathbf{P}_{cb} = \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{B}_u\mathbf{K}_o\mathbf{C} - (\mathbf{P}\mathbf{B}_u\mathbf{K}_o\mathbf{C})^T < 0 \tag{37}$$

Especially, if all system state variables are measurable the control policy can be defined as follows

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{q}(t) \tag{38}$$

and (37) can be written as

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{B}_u\mathbf{K} - (\mathbf{P}\mathbf{B}_u\mathbf{K})^T < 0 \tag{39}$$

Note that in a real physical dynamic plant model usually $\mathbf{D}_u = \mathbf{0}$. ■

Proposition 4 *Let for given real matrices* $\mathbf{F}$, $\mathbf{G}$ *and* $\boldsymbol{\Theta} = \boldsymbol{\Theta}^T > 0$ *of appropriate dimension a matrix* $\boldsymbol{\Lambda}$ *has to satisfy the inequality*

$$\mathbf{F}\boldsymbol{\Lambda}\mathbf{G}^T + \mathbf{G}\boldsymbol{\Lambda}^T\mathbf{F}^T - \boldsymbol{\Theta} < 0 \tag{40}$$

then any solution of $\boldsymbol{\Lambda}$ *can be generated using a solution of inequality*

$$\begin{bmatrix} -\mathbf{F}\mathbf{H}\mathbf{F}^T - \boldsymbol{\Theta} & \mathbf{F}\mathbf{H} + \mathbf{G}\boldsymbol{\Lambda}^T \\ * & -\mathbf{H} \end{bmatrix} < 0 \tag{41}$$

where $\mathbf{H} = \mathbf{H}^T > 0$ *is a free design parameter.*

Proof. If (40) yields then there exists a matrix $\mathbf{H}^{-1} = \mathbf{H}^{-T} > 0$ such that

$$\mathbf{F}\boldsymbol{\Lambda}\mathbf{G}^T + \mathbf{G}\boldsymbol{\Lambda}^T\mathbf{F}^T - \boldsymbol{\Theta} + \mathbf{G}\boldsymbol{\Lambda}^T\mathbf{H}^{-1}\boldsymbol{\Lambda}\mathbf{G}^T < 0 \tag{42}$$

Completing the square in (42) it can be obtained

$$(\mathbf{F}\mathbf{H} + \mathbf{G}\boldsymbol{\Lambda}^T)\mathbf{H}^{-1}(\mathbf{F}\mathbf{H} + \mathbf{G}\boldsymbol{\Lambda}^T)^T - \mathbf{F}\mathbf{H}\mathbf{F}^T - \boldsymbol{\Theta} < 0 \tag{43}$$

and using Schur complement (43) implies (41). ■

4. Fault isolation

4.1 Structured residual generators of sensor faults

4.1.1 Set of the state estimators

To design structured residual generators of sensor faults based on the state estimators, all actuators are assumed to be fault-free and each estimator is driven by all system inputs and all but one system outputs. In that sense it is possible according with given nominal fault-free system model (1), (2) to define the set of structured estimators for $k = 1, 2, \ldots, m$ as follows

$$\dot{\mathbf{q}}_{ke}(t) = \mathbf{A}_{ke}\mathbf{q}_{ke}(t) + \mathbf{B}_{uke}\mathbf{u}(t) + \mathbf{J}_{sk}\mathbf{T}_{sk}\big(\mathbf{y}(t) - \mathbf{D}_u\mathbf{u}(t)\big) \tag{44}$$

$$\mathbf{y}_{ke}(t) = \mathbf{C}\mathbf{q}_{ke}(t) + \mathbf{D}_u\mathbf{u}(t) \tag{45}$$

where $\mathbf{A}_{ke} \in \mathbb{R}^{n\times n}$, $\mathbf{B}_{uke} \in \mathbb{R}^{n\times r}$, $\mathbf{J}_{sk} \in \mathbb{R}^{n\times(m-1)}$, and $\mathbf{T}_{sk} \in \mathbb{R}^{(m-1)\times m}$ takes the next form

$$\mathbf{T}_{sk} = \mathbf{I}_{m\oslash k} = \begin{bmatrix} 1\,0 \cdots 0\,0\,0\,0\,0 \cdots 0\,0 \\ \vdots \qquad\qquad \vdots \\ 0\,0 \cdots 0\,1\,0\,0\,0 \cdots 0\,0 \\ 0\,0 \cdots 0\,0\,0\,1\,0 \cdots 0\,0 \\ \vdots \qquad\qquad \vdots \\ 0\,0 \cdots 0\,0\,0\,0\,0 \cdots 0\,1 \end{bmatrix} \tag{46}$$

Note that $\mathbf{T}_{sk}$ can be obtained by deleting the k-th row in identity matrix $\mathbf{I}_m$.
Since the state estimate error is defined as $\mathbf{e}_k(t) = \mathbf{q}(t) - \mathbf{q}_{ke}(t)$ then

$$\begin{aligned}\dot{\mathbf{e}}_k(t) &= \mathbf{A}\mathbf{q}(t) + \mathbf{B}_u\mathbf{u}(t) - \mathbf{A}_{ke}\mathbf{q}_{ke}(t) - \mathbf{B}_{uke}\mathbf{u}(t) - \mathbf{J}_{sk}\mathbf{T}_{sk}\big(\mathbf{y}(t) - \mathbf{D}_u\mathbf{u}(t)\big) = \\ &= (\mathbf{A} - \mathbf{A}_{ke} - \mathbf{J}_{sk}\mathbf{T}_{sk}\mathbf{C})\mathbf{q}(t) + (\mathbf{B}_u - \mathbf{B}_{uke})\mathbf{u}(t) + \mathbf{A}_{ke}\mathbf{e}_k(t)\end{aligned} \tag{47}$$

To obtain the state estimate error autonomous it can be set

$$\mathbf{A}_{ke} = \mathbf{A} - \mathbf{J}_{sk}\mathbf{T}_{sk}\mathbf{C}, \qquad \mathbf{B}_{uke} = \mathbf{B}_u \tag{48}$$

It is obvious that (48) implies

$$\dot{\mathbf{e}}_k(t) = \mathbf{A}_{ke}\mathbf{e}_k(t) = (\mathbf{A} - \mathbf{J}_{sk}\mathbf{T}_{sk}\mathbf{C})\mathbf{e}_k(t) \tag{49}$$

(44) can be rewritten as

$$\begin{aligned} \dot{\mathbf{q}}_{ke}(t) &= (\mathbf{A} - \mathbf{J}_{sk}\mathbf{T}_{sk}\mathbf{C})\mathbf{q}_{ke}(t) + \mathbf{B}_u\mathbf{u}(t) + \mathbf{J}_{sk}\mathbf{T}_{sk}\big(\mathbf{y}(t) - \mathbf{D}_u\mathbf{u}(t)\big) = \\ &= \mathbf{A}\mathbf{q}_{ke}(t) + \mathbf{B}_u\mathbf{u}(t) + \mathbf{J}_{sk}\mathbf{T}_{sk}\big(\mathbf{y}(t) - (\mathbf{C}\mathbf{q}_{ke}(t) + \mathbf{D}_u\mathbf{u}(t))\big) \end{aligned} \tag{50}$$

and (44), (45) can be rewritten equivalently as

$$\dot{\mathbf{q}}_{ke}(t) = \mathbf{A}\mathbf{q}_{ke}(t) + \mathbf{B}_u\mathbf{u}(t) + \mathbf{J}_{sk}\mathbf{T}_{sk}\big(\mathbf{y}(t) - \mathbf{y}_{ke}(t)\big) \tag{51}$$

$$\mathbf{y}_{ke}(t) = \mathbf{C}\mathbf{q}_{ke}(t) + \mathbf{D}_u\mathbf{u}(t) \tag{52}$$

Theorem 1 *The k-th state-space estimator (52), (53) is stable if there exist a positive definite symmetric matrix* $\mathbf{P}_{sk} > 0$, $\mathbf{P}_{sk} \in \mathbb{R}^{n\times n}$ *and a matrix* $\mathbf{Z}_{sk} \in \mathbb{R}^{n\times(m-1)}$ *such that*

$$\mathbf{P}_{sk} = \mathbf{P}_{sk}^T > 0 \tag{53}$$

$$\mathbf{A}^T\mathbf{P}_{sk} + \mathbf{P}_{sk}\mathbf{A} - \mathbf{Z}_{sk}\mathbf{T}_{sk}\mathbf{C} - \mathbf{C}^T\mathbf{T}_{sk}^T\mathbf{Z}_{sk}^T < 0 \tag{54}$$

Then $\mathbf{J}_{sk}$ *can be computed as*

$$\mathbf{J}_{sk} = \mathbf{P}_{sk}^{-1}\mathbf{Z}_{sk} \tag{55}$$

Proof. Since the estimate error is autonomous Lyapunov function of the form

$$v(\mathbf{e}_k(t)) = \mathbf{e}_k^T(t)\mathbf{P}_{sk}\mathbf{e}_k(t) > 0 \tag{56}$$

where $\mathbf{P}_{sk} = \mathbf{P}_{sk}^T > 0$, $\mathbf{P}_{sk} \in \mathbb{R}^{n\times n}$ can be considered. Thus,

$$\dot{v}(\mathbf{e}_k(t)) = \mathbf{e}_k^T(t)(\mathbf{A} - \mathbf{J}_{sk}\mathbf{T}_{sk}\mathbf{C})^T\mathbf{P}_{sk}\mathbf{e}_k(t) + \mathbf{e}_k^T(t)\mathbf{P}_{sk}(\mathbf{A} - \mathbf{J}_{sk}\mathbf{T}_{sk}\mathbf{C})\mathbf{e}_k(t) < 0 \tag{57}$$

$$\dot{v}(\mathbf{e}_k(t)) = \mathbf{e}_k^T(t)\mathbf{P}_{skc}\mathbf{e}_k(t) < 0 \tag{58}$$

respectively, where

$$\mathbf{P}_{skc} = \mathbf{A}^T\mathbf{P}_{sk} + \mathbf{P}_{sk}\mathbf{A} - \mathbf{P}_{sk}\mathbf{J}_{sk}\mathbf{T}_{sk}\mathbf{C} - (\mathbf{P}_{sk}\mathbf{J}_{sk}\mathbf{T}_{sk}\mathbf{C})^T < 0 \tag{59}$$

Using notation $\mathbf{P}_{sk}\mathbf{J}_{sk} = \mathbf{Z}_{sk}$ (59) implies (54). This concludes the proof. ■

4.1.2 Set of the residual generators

Exploiting the model-based properties of state estimators the set of residual generators can be considered as

$$\mathbf{r}_{sk}(t) = \mathbf{X}_{sk}\mathbf{q}_{ke}(t) + \mathbf{Y}_{sk}(\mathbf{y}(t) - \mathbf{D}_u\mathbf{u}(t)), \quad k = 1, 2, \ldots, m \tag{60}$$

Subsequently

$$\mathbf{r}_{sk}(t) = \mathbf{X}_{sk}\big(\mathbf{q}(t) - \mathbf{e}_k(t)\big) + \mathbf{Y}_{sk}\mathbf{C}\mathbf{q}(t) = (\mathbf{X}_{sk} + \mathbf{Y}_{sk}\mathbf{C})\mathbf{q}(t) - \mathbf{X}_{sk}\mathbf{e}_k(t) \tag{61}$$

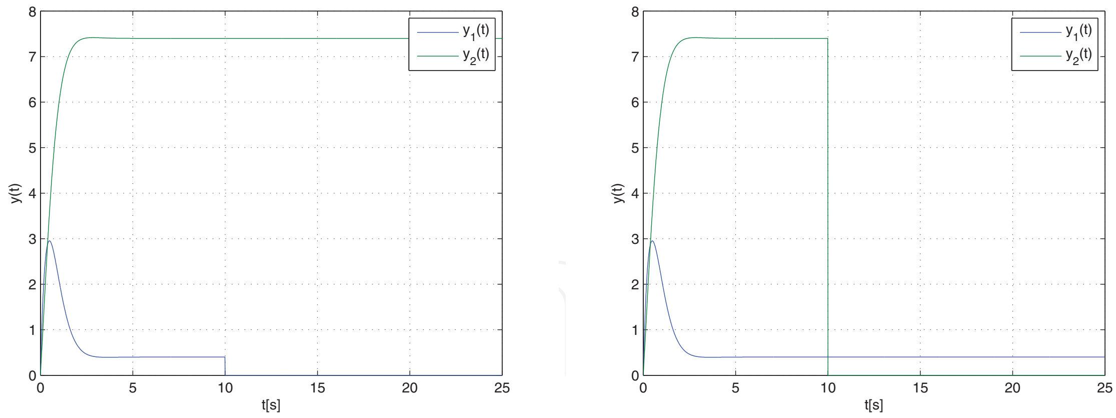

Fig. 1. Measurable outputs for single sensor faults

To eliminate influences of the state variable vector it is necessary in (61) to consider

$$\mathbf{X}_{sk} + \mathbf{Y}_{sk}\mathbf{C} = \mathbf{0} \tag{62}$$

Choosing $\mathbf{X}_{sk} = -\mathbf{T}_{sk}\mathbf{C}$ (62) implies

$$\mathbf{X}_{sk} = -\mathbf{T}_{sk}\mathbf{C}, \qquad \mathbf{Y}_{sk} = \mathbf{T}_{sk} \tag{63}$$

Thus, the set of residuals (60) takes the form

$$\mathbf{r}_{sk}(t) = \mathbf{T}_{sk}\big(\mathbf{y}(t) - \mathbf{D}_u\mathbf{u}(t) - \mathbf{C}\mathbf{q}_{ke}(t)\big), \quad k = 1, 2, \ldots, m \tag{64}$$

When all actuators are fault-free and a fault occurs in the l-th sensor the residuals will satisfy the isolation logic

$$\|\mathbf{r}_{sk}(t)\| \le h_{sk},\ k = l, \qquad \|\mathbf{r}_{sk}(t)\| > h_{sk},\ k \ne l \tag{65}$$

This residual set can only isolate a single sensor fault at the same time. The principle can be generalized based on a regrouping of faults in such way that each residual will be designed to be sensitive to one group of sensor faults and insensitive to others.

Illustrative example

To demonstrate algorithm properties it was assumed that the system is given by (1), (2) where the nominal system parameters are given as

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -9 & -5 \end{bmatrix}, \quad \mathbf{B}_u = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 5 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \quad \mathbf{D}_u = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

and it is obvious that

$$\mathbf{T}_{s1} = \mathbf{I}_{2\oslash 1} = \begin{bmatrix} 0 & 1 \end{bmatrix}, \mathbf{T}_{s2} = \mathbf{I}_{2\oslash 2} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \mathbf{T}_{s1}\mathbf{C} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}, \mathbf{T}_{s2}\mathbf{C} = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Solving (53), (54) with respect to the LMI matrix variables $\mathbf{P}_{sk}$, and $\mathbf{Z}_{sk}$ using Self-Dual-Minimization (SeDuMi) package for Matlab, the estimator gain matrix design problem was feasible with the results

$$\mathbf{P}_{s1} = \begin{bmatrix} 0.8258 & -0.0656 & 0.0032 \\ -0.0656 & 0.8541 & 0.0563 \\ 0.0032 & 0.0563 & 0.2199 \end{bmatrix}, \quad \mathbf{Z}_{s1} = \begin{bmatrix} 0.6343 \\ 0.2242 \\ -0.8595 \end{bmatrix}, \quad \mathbf{J}_{s1} = \begin{bmatrix} 0.8312 \\ 0.5950 \\ -4.0738 \end{bmatrix}$$

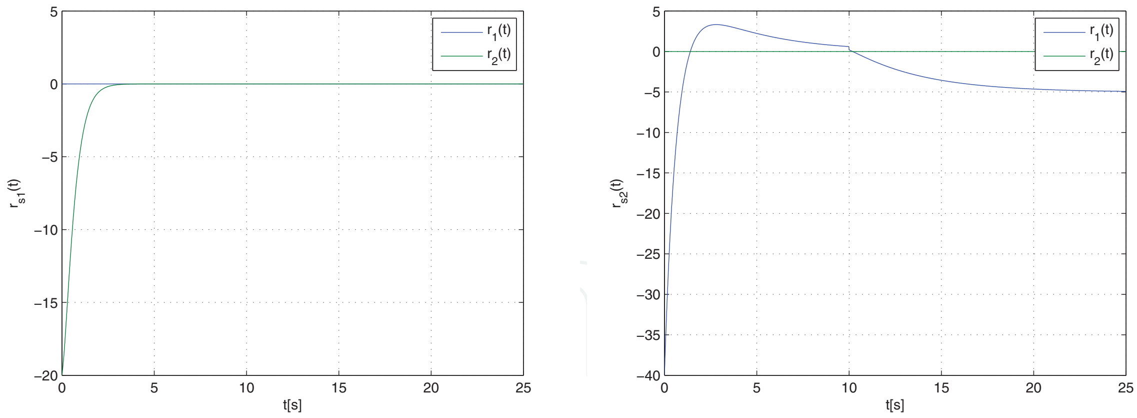

Fig. 2. Residuals for the 1st sensor fault

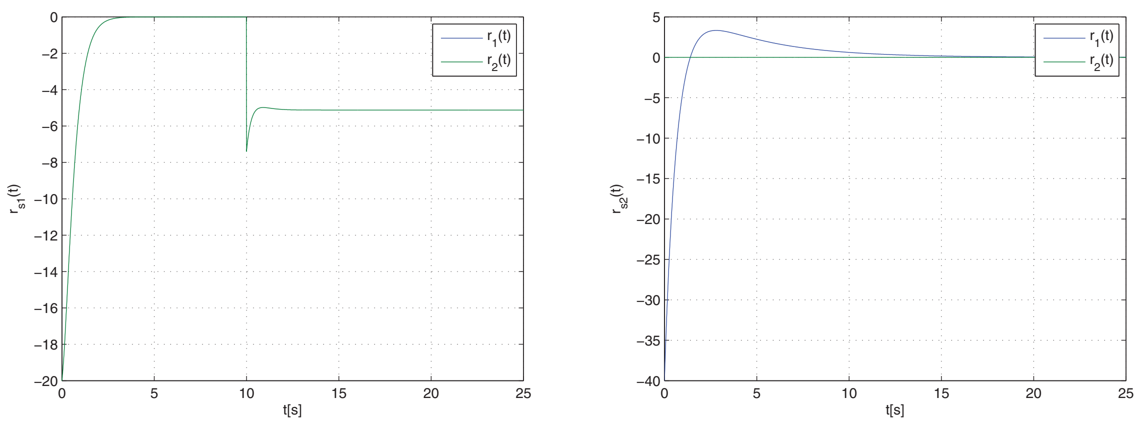

Fig. 3. Residuals for the 2nd sensor fault

$$\mathbf{P}_{s2} = \begin{bmatrix} 0.8258 & -0.0656 & 0.0032 \\ -0.0656 & 0.8541 & 0.0563 \\ 0.0032 & 0.0563 & 0.2199 \end{bmatrix}, \quad \mathbf{Z}_{s2} = \begin{bmatrix} 0.0335 \\ 0.6344 \\ -0.9214 \end{bmatrix}, \quad \mathbf{J}_{s2} = \begin{bmatrix} 0.1412 \\ 1.0479 \\ -4.4614 \end{bmatrix}$$

respectively. It is easily verified that the system matrices of state estimators are stable with the eigenvalue spectra

$$\rho(\mathbf{A} - \mathbf{J}_{s1}\mathbf{T}_{s1}\mathbf{C}) = \{-1.0000 \;\; -2.3459 \;\; -3.0804\}$$

$$\rho(\mathbf{A} - \mathbf{J}_{s2}\mathbf{T}_{s2}\mathbf{C}) = \{-1.5130 \;\; -1.0000 \;\; -0.2626\}$$

respectively, and the set of residuals takes the form

$$\mathbf{r}_{s1}(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \left(\mathbf{y}(t) - \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \mathbf{q}_{ke}(t) \right)$$

$$\mathbf{r}_{s2}(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \left(\mathbf{y}(t) - \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \mathbf{q}_{ke}(t) \right)$$

Fig. 1-3 plot the residuals variable trajectories over the duration of the system run. The results show that one residual profile remain about the same through the entire run while the second shows step changes, which can be used in the fault isolation stage.

IntechOpen

4.2 Structured residual generators of actuator faults

4.2.1 Set of the state estimators

To design structured residual generators of actuator faults based on the state estimators, all sensors are assumed to be fault-free and each estimator is driven by all system outputs and all but one system inputs. To obtain this a congruence transform matrix $\mathbf{T}_{ak} \in \mathbb{R}^{n \times n}$, $k = 1, 2, \ldots, r$ be introduced, and so it is natural to write

$$\mathbf{T}_{ak}\dot{\mathbf{q}}(t) = \mathbf{T}_{ak}\mathbf{A}\mathbf{q}(t) + \mathbf{T}_{ak}\mathbf{B}_u\mathbf{u}(t) \tag{66}$$

$$\dot{\mathbf{q}}_k(t) = \mathbf{A}_k\mathbf{q}(t) + \mathbf{B}_{uk}\mathbf{u}(t) \tag{67}$$

respectively, where

$$\mathbf{A}_k = \mathbf{T}_{ak}\mathbf{A}, \qquad \mathbf{B}_{uk} = \mathbf{T}_{ak}\mathbf{B}_u \tag{68}$$

as well as

$$\mathbf{y}_k(t) = \mathbf{C}\mathbf{T}_{ak}\mathbf{q}(t) = \mathbf{C}\mathbf{q}_k(t) \tag{69}$$

The set of state estimators associated with (67), (69) for $k = 1, 2, \ldots, r$ can be defined in the next form

$$\dot{\mathbf{q}}_{ke}(t) = \mathbf{A}_k\mathbf{q}_{ke}(t) + \mathbf{B}_{uke}\mathbf{u}(t) + \mathbf{L}_k\mathbf{y}(t) - \mathbf{J}_k\mathbf{y}_{ke}(t) \tag{70}$$

$$\mathbf{y}_{ke}(t) = \mathbf{C}\mathbf{q}_{ke}(t) \tag{71}$$

$\mathbf{A}_{ke} \in \mathbb{R}^{n \times n}$, $\mathbf{B}_{uke} \in \mathbb{R}^{n \times r}$, $\mathbf{J}_k, \mathbf{L}_k \in \mathbb{R}^{n \times m}$. Denoting the estimate error as $\mathbf{e}_k(t) = \mathbf{q}_k(t) - \mathbf{q}_{ke}(t)$ the next differential equations can be written

$$\begin{aligned}
\dot{\mathbf{e}}_k(t) &= \dot{\mathbf{q}}_k(t) - \dot{\mathbf{q}}_{ke}(t) = \\
&= \mathbf{A}_k\mathbf{q}(t) + \mathbf{B}_{uk}\mathbf{u}(t) - \mathbf{A}_k\mathbf{q}_{ke}(t) - \mathbf{B}_{uke}\mathbf{u}(t) - \mathbf{L}_k\mathbf{y}(t) + \mathbf{J}_k\mathbf{y}_{ke}(t) = \\
&= \mathbf{A}_k\mathbf{q}(t) + \mathbf{B}_{uk}\mathbf{u}(t) - \mathbf{A}_k\big(\mathbf{q}_k(t) - \mathbf{e}_k(t)\big) - \mathbf{B}_{uke}\mathbf{u}(t) - \\
&\quad -\mathbf{L}_k\mathbf{C}\mathbf{q}(t) + \mathbf{J}_k\mathbf{C}\big(\mathbf{q}_k(t) - \mathbf{e}_k(t)\big) = \\
&= (\mathbf{A}_k - \mathbf{A}_k\mathbf{T}_{ak} + \mathbf{J}_k\mathbf{C}\mathbf{T}_{ak} - \mathbf{L}_k\mathbf{C})\mathbf{q}(t) + (\mathbf{B}_{uk} - \mathbf{B}_{uke})\mathbf{u}(t) + (\mathbf{A}_k - \mathbf{J}_k\mathbf{C})\mathbf{e}_k(t)
\end{aligned} \tag{72}$$

$$\dot{\mathbf{e}}_k(t) = (\mathbf{T}_{ak}\mathbf{A} - \mathbf{A}_{ke}\mathbf{T}_{ak} - \mathbf{L}_k\mathbf{C})\mathbf{q}(t) + (\mathbf{B}_{uk} - \mathbf{B}_{uke})\mathbf{u}(t) + \mathbf{A}_{ke}\mathbf{e}_k(t) \tag{73}$$

respectively, where

$$\mathbf{A}_{ke} = \mathbf{A}_k - \mathbf{J}_k\mathbf{C} = \mathbf{T}_{ak}\mathbf{A} - \mathbf{J}_k\mathbf{C}, \qquad k = 1, 2, \ldots, r \tag{74}$$

are elements of the set of estimators system matrices. It is evident, to make estimate error autonomous that it have to be satisfied

$$\mathbf{L}_k\mathbf{C} = \mathbf{T}_{ak}\mathbf{A} - \mathbf{A}_{ke}\mathbf{T}_{ak}, \qquad \mathbf{B}_{uke} = \mathbf{B}_{uk} = \mathbf{T}_{ak}\mathbf{B}_u \tag{75}$$

Using (75) the equation (73) can be rewritten as

$$\dot{\mathbf{e}}_k(t) = \mathbf{A}_{ke}\mathbf{e}_k(t) = (\mathbf{A}_k - \mathbf{J}_k\mathbf{C})\mathbf{e}_k(t) = (\mathbf{T}_{ak}\mathbf{A} - \mathbf{J}_k\mathbf{C})\mathbf{e}_k(t) \tag{76}$$

and the state equation of estimators are then

$$\dot{\mathbf{q}}_{ke}(t) = (\mathbf{T}_{ak}\mathbf{A} - \mathbf{J}_k\mathbf{C})\mathbf{q}_{ke}(t) + \mathbf{B}_{uk}\mathbf{u}(t) + \mathbf{L}_k\mathbf{y}(t) - \mathbf{J}_k\mathbf{y}_{ke}(t) \tag{77}$$

$$\mathbf{y}_{ke}(t) = \mathbf{C}\mathbf{q}_{ke}(t) \tag{78}$$

4.2.2 Congruence transform matrices

Generally, the fault-free system equations (1), (2) can be rewritten as

$$\dot{\mathbf{q}}(t) = \mathbf{A}\mathbf{q}(t) + \mathbf{b}_{uk}\mathbf{u}_k(t) + \sum_{h=1,h\neq k}^{r} \mathbf{b}_{uh}\mathbf{u}_h(t) \tag{79}$$

$$\dot{\mathbf{y}}(t) = \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{D}_u\dot{\mathbf{u}}(t) = \mathbf{C}\mathbf{A}\mathbf{q}(t) + \mathbf{C}\mathbf{b}_{uk}\mathbf{u}_k(t) + \mathbf{D}_u\dot{\mathbf{u}}(t) + \sum_{h=1,h\neq k}^{r} \mathbf{C}\mathbf{b}_{uh}\mathbf{u}_h(t) \tag{80}$$

$$\mathbf{C}\mathbf{b}_{uk}\mathbf{u}_k(t) = \dot{\mathbf{y}}(t) - \mathbf{C}\mathbf{A}\mathbf{q}(t) - \mathbf{D}_u\dot{\mathbf{u}}(t) - \sum_{h=1,h\neq k}^{r} \mathbf{C}\mathbf{b}_{uh}\mathbf{u}_h(t) \tag{81}$$

respectively. Thus, using matrix pseudoinverse it yields

$$\mathbf{u}_k(t) \doteq (\mathbf{C}\mathbf{b}_{uk})^{\ominus 1}\Big(\dot{\mathbf{y}}(t) - \mathbf{C}\mathbf{A}\mathbf{q}(t) - \mathbf{D}_u\dot{\mathbf{u}}(t) - \sum_{h=1,h\neq k}^{r} \mathbf{C}\mathbf{b}_{uh}\mathbf{u}_h(t)\Big) \tag{82}$$

and substituting (81)

$$\mathbf{b}_{uk}\mathbf{u}_k(t) \doteq \mathbf{b}_{uk}(\mathbf{C}\mathbf{b}_{uk})^{\ominus 1}\mathbf{C}\mathbf{b}_{uk}\mathbf{u}_k(t) \tag{83}$$

$$\big(\mathbf{I}_n - \mathbf{b}_{uk}(\mathbf{C}\mathbf{b}_{uk})^{\ominus 1}\mathbf{C}\big)\mathbf{b}_{uk}\mathbf{u}_k(t) \doteq \mathbf{0} \tag{84}$$

respectively. It is evident that if

$$\mathbf{T}_{ak} = \mathbf{I}_n - \mathbf{b}_{uk}(\mathbf{C}\mathbf{b}_{uk})^{\ominus 1}\mathbf{C}\,, \qquad k = 1, 2, \ldots, r \tag{85}$$

influence of $\mathbf{u}_k(t)$ in (77) be suppressed (the k-th column in $\mathbf{B}_{uk} = \mathbf{T}_{ak}\mathbf{B}_u$ is the null column, approximatively).

4.2.3 Estimator stability

Theorem 2 *The k-th state-space estimator (77), (78) is stable if there exist a positive definite symmetric matrix $\mathbf{P}_{ak} > 0$, $\mathbf{P}_{ak} \in \mathbb{R}^{n\times n}$ and a matrix $\mathbf{Z}_{ak} \in \mathbb{R}^{n\times m}$ such that*

$$\mathbf{P}_{ak} = \mathbf{P}_{ak}^T > 0 \tag{86}$$

$$\mathbf{A}^T\mathbf{T}_{ak}\mathbf{P}_{ak} + \mathbf{P}_{ak}\mathbf{T}_{ak}\mathbf{A} - \mathbf{Z}_{ak}\mathbf{C} - \mathbf{C}^T\mathbf{Z}_{ak}^T < 0 \tag{87}$$

Then $\mathbf{J}_k$ can be computed as

$$\mathbf{J}_k = \mathbf{P}_{ak}^{-1}\mathbf{Z}_{ak} \tag{88}$$

Proof. Since the estimate error is autonomous Lyapunov function of the form

$$v(\mathbf{e}_k(t)) = \mathbf{e}_k^T(t)\mathbf{P}_{ak}\mathbf{e}_k(t) > 0 \tag{89}$$

where $\mathbf{P}_{ak} = \mathbf{P}_{ak}^T > 0$, $\mathbf{P}_{ak} \in \mathbb{R}^{n\times n}$ can be considered. Thus,

$$\dot{v}(\mathbf{e}_k(t)) = \mathbf{e}_k^T(t)(\mathbf{T}_{ak}\mathbf{A} - \mathbf{J}_k\mathbf{C})^T\mathbf{P}_{ak}\mathbf{e}_k(t) + \mathbf{e}_k^T(t)\mathbf{P}_{ak}(\mathbf{T}_{ak}\mathbf{A} - \mathbf{J}_k\mathbf{C})\mathbf{e}_k(t) < 0 \tag{90}$$

$$\dot{v}(\mathbf{e}_k(t)) = \mathbf{e}_k^T(t)\mathbf{P}_{akc}\mathbf{e}_k(t) < 0 \tag{91}$$

respectively, where

$$\mathbf{P}_{akc} = \mathbf{A}^T\mathbf{T}_{ak}^T\mathbf{P}_{ak} + \mathbf{P}_{ak}\mathbf{T}_{ak}\mathbf{A} - \mathbf{P}_{ak}\mathbf{J}_k\mathbf{C} - (\mathbf{P}_{ak}\mathbf{J}_k\mathbf{C})^T < 0 \tag{92}$$

Using notation $\mathbf{P}_{ak}\mathbf{J}_k = \mathbf{Z}_{ak}$ (92) implies (87). This concludes the proof. ■

4.2.4 Estimator gain matrices

Knowing $\mathbf{J}_k,\ k = 1,2,\ \dots\ ,r$ elements of this set can be inserted into (75). Thus

$$\begin{aligned}\mathbf{L}_k\mathbf{C} &= \mathbf{A}_k - \mathbf{A}_{ke}\mathbf{T}_{ak} = \mathbf{A}_k - \big(\mathbf{A}_k - \mathbf{J}_k\mathbf{C}\big)\big(\mathbf{I} - \mathbf{b}_{uk}(\mathbf{C}\mathbf{b}_{uk})^{\ominus 1}\mathbf{C}\big) = \\ &= \Big(\mathbf{J}_k + (\mathbf{A}_k - \mathbf{J}_k\mathbf{C})\mathbf{b}_{uk}(\mathbf{C}\mathbf{b}_{uk})^{\ominus 1}\Big)\mathbf{C} = \Big(\mathbf{J}_k + \mathbf{A}_{ke}\mathbf{b}_{uk}(\mathbf{C}\mathbf{b}_{uk})^{\ominus 1}\Big)\mathbf{C}\end{aligned} \tag{93}$$

and

$$\mathbf{L}_k = \mathbf{J}_k + \mathbf{A}_{ke}\mathbf{b}_{uk}(\mathbf{C}\mathbf{b}_{uk})^{\ominus 1}, \quad k = 1,2,\dots,r \tag{94}$$

4.2.5 Set of the residual generators

Exploiting the model-based properties of state estimators the set of residual generators can be considered as

$$\mathbf{r}_{ak}(t) = \mathbf{X}_{ak}\mathbf{q}_{ke}(t) + \mathbf{Y}_{ak}(\mathbf{y}(t) - \mathbf{D}_u\mathbf{u}(t)), \quad k = 1,2,\ \dots,\ m \tag{95}$$

Subsequently

$$\mathbf{r}_{ak}(t) = \mathbf{X}_{ak}\big(\mathbf{T}_{ak}\mathbf{q}(t) - \mathbf{e}_k(t)\big) + \mathbf{Y}_{ak}\mathbf{C}\mathbf{q}(t) = (\mathbf{X}_{ak}\mathbf{T}_{ak} + \mathbf{Y}_{ak}\mathbf{C})\mathbf{q}(t) - \mathbf{X}_{ak}\mathbf{e}_k(t) \tag{96}$$

To eliminate influences of the state variable vector it is necessary to consider

$$\mathbf{X}_{ak}\mathbf{T}_{ak} + \mathbf{Y}_{ak}\mathbf{C} = \mathbf{0} \tag{97}$$

$$\mathbf{X}_{ak}\big(\mathbf{I}_n - \mathbf{b}_{uk}(\mathbf{C}\mathbf{b}_{uk})^{\ominus 1}\mathbf{C}\big) + \mathbf{Y}_{ak}\mathbf{C} = \mathbf{0} \tag{98}$$

respectively. Choosing $\mathbf{X}_{ak} = -\mathbf{C}$ (98) gives

$$-\big(\mathbf{C} - \mathbf{C}\mathbf{b}_{uk}(\mathbf{C}\mathbf{b}_{uk})^{\ominus 1}\mathbf{C}\big) + \mathbf{Y}_{ak}\mathbf{C} = -(\mathbf{I}_m - \mathbf{C}\mathbf{b}_{uk}(\mathbf{C}\mathbf{b}_{uk})^{\ominus 1})\mathbf{C} + \mathbf{Y}_{ak}\mathbf{C} = \mathbf{0} \tag{99}$$

i.e.

$$\mathbf{Y}_{ak} = \mathbf{I}_m - \mathbf{C}\mathbf{b}_{uk}(\mathbf{C}\mathbf{b}_{uk})^{\ominus 1} \tag{100}$$

Thus, the set of residuals (95) takes the form

$$\mathbf{r}_{ak}(t) = (\mathbf{I}_m - \mathbf{C}\mathbf{b}_{uk}(\mathbf{C}\mathbf{b}_{uk})^{\ominus 1})\mathbf{y}(t) - \mathbf{C}\mathbf{q}_{ke}(t) \tag{101}$$

When all sensors are fault-free and a fault occurs in the l-th actuator the residuals will satisfy the isolation logic

$$\|\mathbf{r}_{sk}(t)\| \le h_{sk},\ k = l, \qquad \|\mathbf{r}_{sk}(t)\| > h_{sk},\ k \ne l \tag{102}$$

This residual set can only isolate a single actuator fault at the same time. The principle can be generalized based on a regrouping of faults in such way that each residual will be designed to be sensitive to one group of actuator faults and insensitive to others.

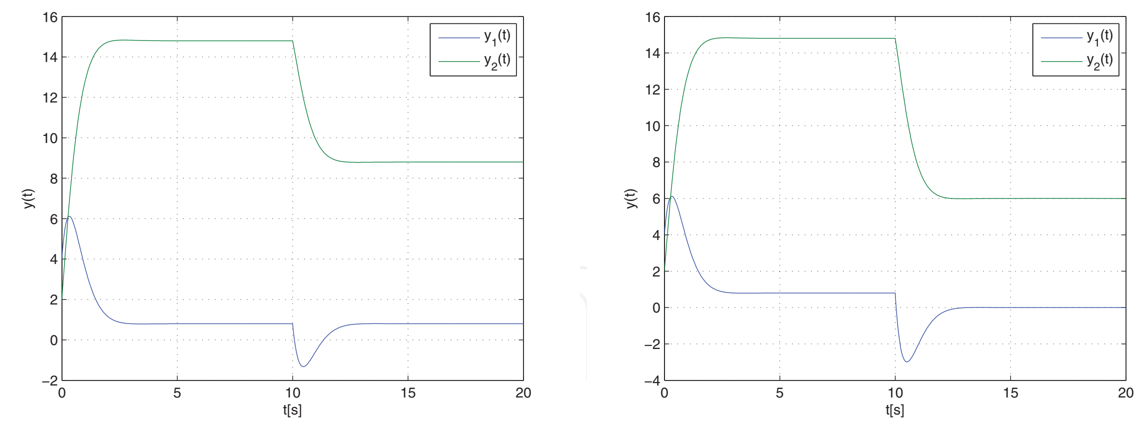

Fig. 4. System outputs for single actuator faults

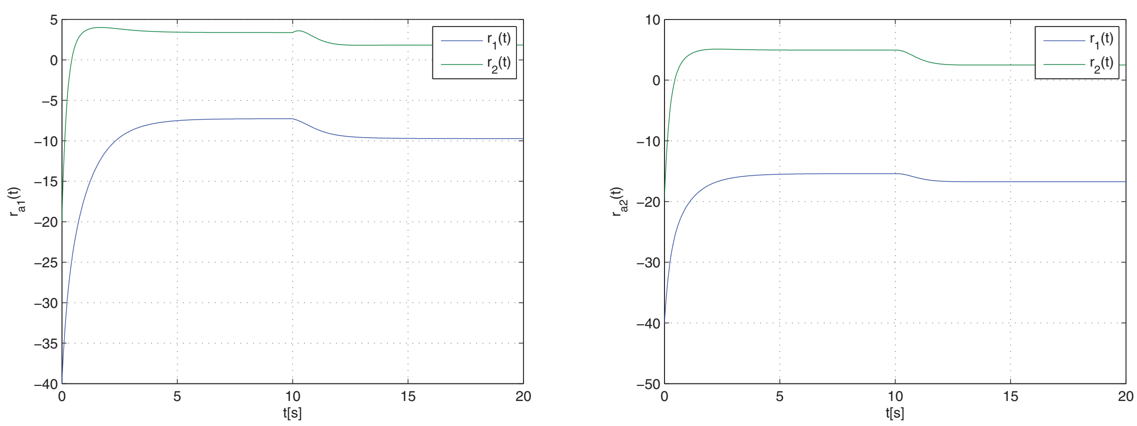

Fig. 5. Residuals for the 1st actuator fault

Illustrative example

Using the same system parameters as that given in the example in Subsection 4.1.2, the next design parameters be computed

$$\mathbf{b}_{u1} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \ (\mathbf{C}\mathbf{b}_{u1})^{\ominus 1} = \begin{bmatrix} 0.1333 & 0.0667 \end{bmatrix}, \ \mathbf{T}_{a1} = \begin{bmatrix} 0.8000 & -0.3333 & -0.1333 \\ -0.4000 & 0.3333 & -0.2667 \\ -0.2000 & -0.3333 & 0.8667 \end{bmatrix}$$

$$\mathbf{b}_{u2} = \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}, \ (\mathbf{C}\mathbf{b}_{u2})^{\ominus 1} = \begin{bmatrix} 0.0862 & 0.0345 \end{bmatrix}, \ \mathbf{T}_{a2} = \begin{bmatrix} 0.6379 & -0.6207 & -0.2586 \\ -0.1207 & 0.7931 & -0.0862 \\ -0.6034 & -1.0345 & 0.5690 \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} 0.6667 & 2.0000 & 0.3333 \\ 1.3333 & 2.0000 & 1.6667 \\ -4.3333 & -8.0000 & -4.6667 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 1.2931 & 2.9655 & 0.6724 \\ 0.4310 & 0.6552 & 1.2241 \\ -2.8448 & -5.7241 & -3.8793 \end{bmatrix}$$

$$\mathbf{Y}_{a1} = \begin{bmatrix} 0.2 & -0.4 \\ -0.4 & 0.8 \end{bmatrix}, \quad \mathbf{Y}_{a2} = \begin{bmatrix} 0.1379 & -0.3448 \\ -0.3448 & 0.8621 \end{bmatrix}$$

Solving (86), (87) with respect to the LMI matrix variables $\mathbf{P}_{ak}$, and $\mathbf{Z}_{ak}$ using

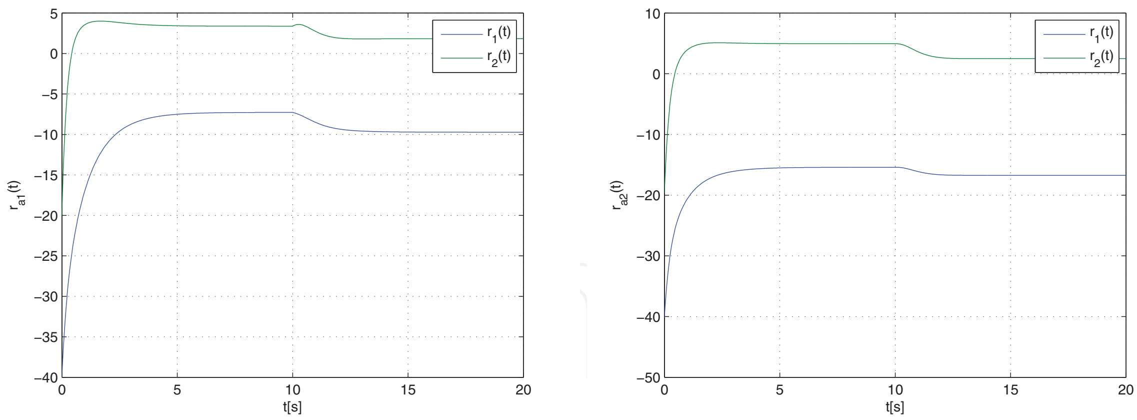

Fig. 6. Residuals for the 2nd actuator fault

Self-Dual-Minimization (SeDuMi) package for Matlab, the estimator gain matrix design problem was feasible with the results

$$\mathbf{P}_{a1} = \begin{bmatrix} 0.7555 & -0.0993 & 0.0619 \\ -0.0993 & 0.7464 & 0.1223 \\ 0.0619 & 0.1223 & 0.3920 \end{bmatrix}$$

$$\mathbf{Z}_{a1} = \begin{bmatrix} 0.0257 & 0.7321 \\ 0.4346 & 0.2392 \\ -0.7413 & -0.7469 \end{bmatrix}, \mathbf{J}_1 = \begin{bmatrix} 0.3504 & 1.2802 \\ 0.9987 & 0.8810 \\ -2.2579 & -2.3825 \end{bmatrix}, \mathbf{L}_1 = \begin{bmatrix} 0.2247 & 1.2173 \\ 0.7807 & 0.7720 \\ -2.8319 & -2.6695 \end{bmatrix}$$

$$\mathbf{P}_{a2} = \begin{bmatrix} 0.6768 & -0.0702 & 0.0853 \\ -0.0702 & 0.7617 & 0.0685 \\ 0.0853 & 0.0685 & 0.4637 \end{bmatrix}$$

$$\mathbf{Z}_{a2} = \begin{bmatrix} 0.2127 & 0.9808 \\ 0.3382 & 0.0349 \\ -0.6686 & -0.4957 \end{bmatrix}, \mathbf{J}_2 = \begin{bmatrix} 0.5888 & 1.6625 \\ 0.6462 & 0.3270 \\ -1.6457 & -1.4233 \end{bmatrix}, \mathbf{L}_2 = \begin{bmatrix} 0.3878 & 1.5821 \\ 0.6720 & 0.3373 \\ -2.6375 & -1.8200 \end{bmatrix}$$

respectively. It is easily verified that the system matrices of state estimators are stable with the eigenvalue spectra

$$\rho(\mathbf{T}_{a1}\mathbf{A} - \mathbf{J}_1\mathbf{C}) = \{-1.0000 \;\; -1.6256 \pm 0.3775\,\mathrm{i}\}$$

$$\rho(\mathbf{T}_{a2}\mathbf{A} - \mathbf{J}_2\mathbf{C}) = \{-1.0000 \;\; -1.5780 \pm 0.4521\,\mathrm{i}\}$$

respectively, and the set of residuals takes the form

$$\mathbf{r}_{a1}(t) = \begin{bmatrix} 0.2 & -0.4 \\ -0.4 & 0.8 \end{bmatrix} \mathbf{y}(t) - \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \mathbf{q}_{1e}(t)$$

$$\mathbf{r}_{a2}(t) = \begin{bmatrix} 0.1379 & -0.3448 \\ -0.3448 & 0.8621 \end{bmatrix} \mathbf{y}(t) - \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \mathbf{q}_{2e}(t)$$

Fig. 4-6 plot the residuals variable trajectories over the duration of the system run. The results show that both residual profile show changes through the entire run, therefore a fault isolation has to be more sophisticated.

5. Control with virtual sensors

5.1 Stability of the system

Considering a sensor fault then (1), (2) can be written as

$$\dot{\mathbf{q}}_f(t) = \mathbf{A}\mathbf{q}_f(t) + \mathbf{B}_u\mathbf{u}_f(t) \tag{103}$$

$$\mathbf{y}_f(t) = \mathbf{C}_f\mathbf{q}_f(t) + \mathbf{D}_u\mathbf{u}_f(t) \tag{104}$$

where $\mathbf{q}_f(t) \in \mathbb{R}^n$, $\mathbf{u}_f(t) \in \mathbb{R}^r$ are vectors of the state, and input variables of the faulty system, respectively, $\mathbf{C}_f \in \mathbb{R}^{m\times n}$ is the output matrix of the system with a sensor fault, and $\mathbf{y}_f(t) \in \mathbb{R}^m$ is a faulty measurement vector. This interpretation means that one row of $\mathbf{C}_f$ is null row.

Problem of the interest is to design a stable closed-loop system with the output controller

$$\mathbf{u}_f(t) = -\mathbf{K}_o\mathbf{y}_e(t) \tag{105}$$

where

$$\mathbf{y}_e(t) = \mathbf{E}\mathbf{y}_f(t) + (\mathbf{C} - \mathbf{E}\mathbf{C}_f)\mathbf{q}_{fe}(t) \tag{106}$$

$\mathbf{K}_o \in \mathbb{R}^{r\times m}$ is the controller gain matrix, and $\mathbf{E} \in \mathbb{R}^{m\times m}$ is a switching matrix, generally used in such a way that $\mathbf{E} = \mathbf{0}$, or $\mathbf{E} = \mathbf{I}_m$. If $\mathbf{E} = \mathbf{0}$ full state vector estimation is used for control, if $\mathbf{E} = \mathbf{I}_m$ the outputs of the fault-free sensors are combined with the estimated state variables to substitute a missing output of the faulty sensor.

Generally, the controller input is generated by the virtual sensor realized in the structure

$$\dot{\mathbf{q}}_{fe}(t) = \mathbf{A}\mathbf{q}_{fe}(t) + \mathbf{B}_u\mathbf{u}_f(t) + \mathbf{J}(\mathbf{y}_f(t) - \mathbf{D}_u\mathbf{u}_f(t) - \mathbf{C}_f\mathbf{q}_{fe}(t)) \tag{107}$$

The main idea is, instead of adapting the controller to the faulty system virtually adapt the faulty system to the nominal controller.

Theorem 3 *Control of the faulty system with virtual sensor defined by (103) – (107) is stable in the sense of bounded real lemma if there exist positive definite symmetric matrices* $\mathbf{Q}$, $\mathbf{R} \in \mathbb{R}^{n\times n}$, *and matrices* $\mathbf{K}_o \in \mathbb{R}^{r\times m}$, $\mathbf{J} \in \mathbb{R}^{n\times m}$ *such that*

$$\begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{Q}\mathbf{B}_u\mathbf{K}_o(\mathbf{C}-\mathbf{E}\mathbf{C}_f) & -\mathbf{Q}\mathbf{B}_u\mathbf{K}_o\mathbf{E} & \left(\mathbf{C}_f - \mathbf{D}_u\mathbf{K}_o(\mathbf{C}-\mathbf{E}\mathbf{C}_f)\right)^T \\ * & \mathbf{\Phi}_2 & \mathbf{0} & \left(\mathbf{D}_u\mathbf{K}_o(\mathbf{C}-\mathbf{E}\mathbf{C}_f)\right)^T \\ * & * & -\gamma^2\mathbf{I}_r & -\left(\mathbf{D}_u\mathbf{K}_o\mathbf{E}\right)^T \\ * & * & * & -\mathbf{I}_m \end{bmatrix} < 0 \tag{108}$$

where

$$\mathbf{\Phi}_1 = \mathbf{Q}\left(\mathbf{A} - \mathbf{B}_u\mathbf{K}_o(\mathbf{C}-\mathbf{E}\mathbf{C}_f)\right) + \left(\mathbf{A} - \mathbf{B}_u\mathbf{K}_o(\mathbf{C}-\mathbf{E}\mathbf{C}_f)\right)^T\mathbf{Q} \tag{109}$$

$$\mathbf{\Phi}_2 = \mathbf{R}\left(\mathbf{A} - \mathbf{J}\mathbf{C}_f\right) + \left(\mathbf{A} - \mathbf{J}\mathbf{C}_f\right)^T\mathbf{R} \tag{110}$$

Proof. Assembling (103), (104), and (107) gives

$$\begin{bmatrix} \dot{\mathbf{q}}_f(t) \\ \dot{\mathbf{q}}_{fe}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{J}\mathbf{C}_f & \mathbf{A}-\mathbf{J}\mathbf{C}_f \end{bmatrix} \begin{bmatrix} \mathbf{q}_f(t) \\ \mathbf{q}_{fe}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_u \\ \mathbf{B}_u \end{bmatrix} \mathbf{u}_f(t) \tag{111}$$

$$\mathbf{y}_f(t) = \mathbf{C}_f\mathbf{q}_f(t) + \mathbf{D}_u\mathbf{u}_f(t) \tag{112}$$

Thus, defining the estimation error vector

$$\mathbf{e}_{qf}(t) = \mathbf{q}_f(t) - \mathbf{q}_{fe}(t) \tag{113}$$

as well as the congruence transform matrix

$$\mathbf{T} = \mathbf{T}^{-1} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{I} & -\mathbf{I} \end{bmatrix} \tag{114}$$

and then multiplying left-hand side of (111) by (114) results in

$$\mathbf{T}\begin{bmatrix} \dot{\mathbf{q}}_f(t) \\ \dot{\mathbf{q}}_{fe}(t) \end{bmatrix} = \mathbf{T}\begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{JC}_f & \mathbf{A}-\mathbf{JC}_f \end{bmatrix}\mathbf{T}^{-1}\mathbf{T}\begin{bmatrix} \mathbf{q}_f(t) \\ \mathbf{q}_{fe}(t) \end{bmatrix} + \mathbf{T}\begin{bmatrix} \mathbf{B}_u \\ \mathbf{B}_u \end{bmatrix}\mathbf{u}_f(t) \tag{115}$$

$$\begin{bmatrix} \dot{\mathbf{q}}_f(t) \\ \dot{\mathbf{e}}_{qf}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}-\mathbf{JC}_f \end{bmatrix}\begin{bmatrix} \mathbf{q}_f(t) \\ \mathbf{e}_{qf}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_u \\ \mathbf{0} \end{bmatrix}\mathbf{u}_f(t) \tag{116}$$

respectively. Subsequently, inserting (105), (106) into (116), (112) gives

$$\begin{bmatrix} \dot{\mathbf{q}}_f(t) \\ \dot{\mathbf{e}}_{qf}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}-\mathbf{B}_u\mathbf{K}_o(\mathbf{C}-\mathbf{EC}_f) & \mathbf{B}_u\mathbf{K}_o(\mathbf{C}-\mathbf{EC}_f) \\ \mathbf{0} & \mathbf{A}-\mathbf{JC}_f \end{bmatrix}\begin{bmatrix} \mathbf{q}_f(t) \\ \mathbf{e}_{qf}(t) \end{bmatrix} + \begin{bmatrix} -\mathbf{B}_u\mathbf{K}_o\mathbf{E} \\ \mathbf{0} \end{bmatrix}\mathbf{y}_e(t) \tag{117}$$

together with

$$\mathbf{y}_f(t) = \begin{bmatrix} \mathbf{C}_f - \mathbf{D}_u\mathbf{K}_o(\mathbf{C}-\mathbf{EC}_f) & \mathbf{D}_u\mathbf{K}_o(\mathbf{C}-\mathbf{EC}_f) \end{bmatrix}\begin{bmatrix} \mathbf{q}_f(t) \\ \mathbf{e}_{qf}(t) \end{bmatrix} - \mathbf{D}_u\mathbf{K}_o\mathbf{E}\mathbf{y}_e(t) \tag{118}$$

and it is evident, that the separation principle yields.
Denoting

$$\mathbf{q}_\varepsilon^T(t) = \begin{bmatrix} \mathbf{q}_f^T(t) & \mathbf{e}_{qf}^T(t) \end{bmatrix}, \qquad \mathbf{w}_\varepsilon(t) = \mathbf{y}_e(t) \tag{119}$$

$$\mathbf{A}_\varepsilon = \begin{bmatrix} \mathbf{A}-\mathbf{B}_u\mathbf{K}_o(\mathbf{C}-\mathbf{EC}_f) & \mathbf{B}_u\mathbf{K}_o(\mathbf{C}-\mathbf{EC}_f) \\ \mathbf{0} & \mathbf{A}-\mathbf{JC}_f \end{bmatrix}, \qquad \mathbf{B}_\varepsilon = \begin{bmatrix} -\mathbf{B}_u\mathbf{K}_o\mathbf{E} \\ \mathbf{0} \end{bmatrix} \tag{120}$$

$$\mathbf{C}_\varepsilon = \begin{bmatrix} \mathbf{C}_f - \mathbf{D}_u\mathbf{K}_o(\mathbf{C}-\mathbf{EC}_f) & \mathbf{D}_u\mathbf{K}_o(\mathbf{C}-\mathbf{EC}_f) \end{bmatrix}, \qquad \mathbf{D}_\varepsilon = -\mathbf{D}_u\mathbf{K}_o\mathbf{E} \tag{121}$$

To accept the separation principle a block diagonal symmetric matrix $\mathbf{P}_\varepsilon > 0$ is chosen, i.e.

$$\mathbf{P}_\varepsilon = \text{diag}\begin{bmatrix} \mathbf{Q} & \mathbf{R} \end{bmatrix} \tag{122}$$

where $\mathbf{Q} = \mathbf{Q}^T > 0$, $\mathbf{R} = \mathbf{R}^T > 0$, $\mathbf{Q}, \mathbf{R} \in \mathbb{R}^{n \times n}$ Thus, with (109), (110) it yields

$$\mathbf{P}_\varepsilon\mathbf{A}_\varepsilon + \mathbf{A}_\varepsilon^T\mathbf{P}_\varepsilon = \begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{QB}_u\mathbf{K}_o(\mathbf{C}-\mathbf{EC}_f) \\ * & \mathbf{\Phi}_2 \end{bmatrix}, \qquad \mathbf{P}_\varepsilon\mathbf{B}_\varepsilon = \begin{bmatrix} -\mathbf{QB}_u\mathbf{K}_o\mathbf{E} \\ \mathbf{0} \end{bmatrix} \tag{123}$$

and inserting (121), (123), into (24) gives (108). This concludes the proof. ■
It is evident that there are the cross parameter interactions in the structure of (108). Since the separation principle pre-determines the estimator structure (error vectors are independent on the state as well as on the input variables), the controller, as well as estimator have to be designed independent.

5.2 Output feedback controller design

Theorem 4 (Unified algebraic approach) *A system (103), (104) with control law (105) is stable if there exist positive definite symmetric matrices* $\mathbf{P} > 0$, $\mathbf{\Pi} = \mathbf{P}^{-1} > 0$ *such that*

$$\begin{bmatrix} \mathbf{B}_u^{\perp}(\mathbf{A\Pi}+\mathbf{\Pi A}^T)\mathbf{B}_u^{\perp T} & \mathbf{B}_u^{\perp}\mathbf{\Pi C}_{fi}^T \\ * & -\mathbf{I}_m \end{bmatrix} < 0, \qquad i = 0,1,2,\dots,m \tag{124}$$

$$\begin{bmatrix} \mathbf{C}_{fi}^{\bullet T\perp}\begin{bmatrix} \mathbf{PA}+\mathbf{A}^T\mathbf{P} & \mathbf{0} \\ * & -\gamma^2\mathbf{I}_r \end{bmatrix}\mathbf{C}_{fi}^{\bullet T\perp T} & \mathbf{C}_{fi}^{\bullet T\perp}\begin{bmatrix} \mathbf{C}_{fi}^T \\ \mathbf{0} \end{bmatrix} \\ * & -\mathbf{I}_m \end{bmatrix} < 0 \; i = 1,2,\dots,m, \; \mathbf{E} = \mathbf{I}_m \tag{125}$$

$$\begin{bmatrix} \mathbf{C}^{T\perp}(\mathbf{PA}+\mathbf{A}^T\mathbf{P})\mathbf{C}^{T\perp T} & \mathbf{C}^{T\perp}\mathbf{C}_{fi}^T \\ * & -\mathbf{I}_m \end{bmatrix} < 0 \quad i = 0,1,2,\dots m, \quad \mathbf{E} = 0 \tag{126}$$

where

$$\mathbf{C}_{fi}^{\bullet T\perp} = \begin{bmatrix} (\mathbf{C}-\mathbf{EC}_{fi})^T \\ \mathbf{E} \end{bmatrix}^{\perp} \tag{127}$$

and $\mathbf{B}_u^{\perp}$ *is the orthogonal complement to* $\mathbf{B}_u$. *Then the control law gain matrix* $\mathbf{K}_o$ *exists if for obtained* $\mathbf{P}$ *there exist a symmetric matrices* $\mathbf{H} > 0$ *such that*

$$\begin{bmatrix} -\mathbf{FHF}^T - \mathbf{\Theta}_i & \mathbf{FH}+\mathbf{G}_i\mathbf{K}_o^T \\ * & -\mathbf{H} \end{bmatrix} < 0 \tag{128}$$

where $i = 0,1,2,\dots,m$, *and*

$$\mathbf{\Theta}_i = -\begin{bmatrix} \mathbf{PA}+\mathbf{A}^T\mathbf{P} & \mathbf{0} & \mathbf{C}_{fi}^T \\ * & -\gamma^2\mathbf{I}_r & \mathbf{0} \\ * & * & -\mathbf{I}_m \end{bmatrix} < 0, \quad \mathbf{F} = -\begin{bmatrix} \mathbf{PB}_u \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} (\mathbf{C}-\mathbf{EC}_{fi})^T \\ \mathbf{E} \\ \mathbf{0} \end{bmatrix} \tag{129}$$

Proof. Considering $\mathbf{e}_q(t) = \mathbf{0}$ then inserting $\mathbf{Q} = \mathbf{P}$ (108) implies

$$\begin{bmatrix} \mathbf{\Phi}_1 & -\mathbf{PB}_u\mathbf{K}_o\mathbf{E} & \big(\mathbf{C}_f - \mathbf{D}_u\mathbf{K}_o(\mathbf{C}-\mathbf{EC}_f)\big)^T \\ * & -\gamma^2\mathbf{I}_r & -\big(\mathbf{D}_u\mathbf{K}_o\mathbf{E}\big)^T \\ * & * & -\mathbf{I}_m \end{bmatrix} < 0 \tag{130}$$

where

$$\mathbf{\Phi}_1 = \mathbf{P}\big(\mathbf{A}-\mathbf{B}_u\mathbf{K}_o(\mathbf{C}-\mathbf{EC}_f)\big) + \big(\mathbf{A}-\mathbf{B}_u\mathbf{K}_o(\mathbf{C}-\mathbf{EC}_f)\big)^T\mathbf{P} \tag{131}$$

For the simplicity it is considered in the next that $\mathbf{D}_u = \mathbf{0}$ (in real physical systems this condition is satisfied) and subsequently (130), (131) can now be rewritten as

$$\begin{bmatrix} \mathbf{PA}+\mathbf{A}^T\mathbf{P} & \mathbf{0} & \mathbf{C}_f^T \\ * & -\gamma^2\mathbf{I}_r & \mathbf{0} \\ * & * & -\mathbf{I}_m \end{bmatrix} - \\ -\begin{bmatrix} \mathbf{PB}_u \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}\mathbf{K}_o\begin{bmatrix} \mathbf{C}-\mathbf{EC}_f & \mathbf{E} & \mathbf{0} \end{bmatrix} - \begin{bmatrix} (\mathbf{C}-\mathbf{EC}_f)^T \\ \mathbf{E} \\ \mathbf{0} \end{bmatrix}\mathbf{K}_o^T\begin{bmatrix} \mathbf{B}_u^T\mathbf{P} & \mathbf{0} & \mathbf{0} \end{bmatrix} < 0 \tag{132}$$

Defining the congruence transform matrix

$$\mathbf{T}_v = \operatorname{diag}\left[\mathbf{P}^{-1}\ \mathbf{I}_r\ \mathbf{I}_m\right] \tag{133}$$

then pre-multiplying left-hand side and right-hand side of (132) by (133) gives

$$\begin{bmatrix} \mathbf{A}\mathbf{P}^{-1}+\mathbf{P}^{-1}\mathbf{A}^T & \mathbf{0} & \mathbf{P}^{-1}\mathbf{C}_f^T \\ * & -\gamma^2\mathbf{I}_r & \mathbf{0} \\ * & * & -\mathbf{I}_m \end{bmatrix} - \\ -\begin{bmatrix}\mathbf{B}_u\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}\mathbf{K}_o\left[(\mathbf{C}-\mathbf{E}\mathbf{C}_f)\mathbf{P}^{-1}\ \mathbf{E}\ \mathbf{0}\right] - \begin{bmatrix}\mathbf{P}^{-1}(\mathbf{C}-\mathbf{E}\mathbf{C}_f)^T\\ \mathbf{E}\\ \mathbf{0}\end{bmatrix}\mathbf{K}_o^T\left[\mathbf{B}_u^T\ \mathbf{0}\ \mathbf{0}\right] < 0 \tag{134}$$

Since it yields

$$\mathbf{B}_u^{\circ\perp} = \begin{bmatrix}\mathbf{B}_u\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}^{\perp} = \begin{bmatrix}\mathbf{B}_u^{\perp} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_r & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I}_m\end{bmatrix} \tag{135}$$

pre-multiplying left hand side of (134) by (135) as well as right-hand side of (134) by transposition of (135) leads to inequalities

$$\begin{bmatrix}\mathbf{B}_u^{\perp}(\mathbf{A}\mathbf{P}^{-1}+\mathbf{P}^{-1}\mathbf{A}^T)\mathbf{B}_u^{\perp T} & \mathbf{0} & \mathbf{B}_u^{\perp}\mathbf{P}^{-1}\mathbf{C}_f^T\\ * & -\gamma^2\mathbf{I}_r & \mathbf{0}\\ * & * & -\mathbf{I}_m\end{bmatrix} < 0 \tag{136}$$

$$\begin{bmatrix}\mathbf{B}_u^{\perp}(\mathbf{A}\mathbf{P}^{-1}+\mathbf{P}^{-1}\mathbf{A}^T)\mathbf{B}_u^{\perp T} & \mathbf{B}_u^{\perp}\mathbf{P}^{-1}\mathbf{C}_f^T\\ * & -\mathbf{I}_m\end{bmatrix} < 0 \tag{137}$$

respectively. Considering all possible structures $\mathbf{C}_{fi},\ i = 1, 2, \ldots, m$ associated with simple sensor faults, as well as fault-free regime associated with the nominal matrix $\mathbf{C} = \mathbf{C}_{f0}$, then using the substitution $\mathbf{P}^{-1} = \mathbf{\Pi}$ the inequality (136) implies (124).
Analogously, using orthogonal complement

$$\mathbf{C}_f^{\circ T\perp} = \begin{bmatrix}(\mathbf{C}-\mathbf{E}\mathbf{C}_f)^T\\ \mathbf{E}\\ \mathbf{0}\end{bmatrix}^{\perp} = \begin{bmatrix}\begin{bmatrix}(\mathbf{C}-\mathbf{E}\mathbf{C}_f)^T\\ \mathbf{E}\end{bmatrix}^{\perp} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_m\end{bmatrix} = \begin{bmatrix}\mathbf{C}_f^{\bullet T\perp} & \mathbf{0}\\ * & \mathbf{I}_m\end{bmatrix} \tag{138}$$

and pre-multiplying left-hand side of (132) by (138) and its right-hand side by transposition of (138) results in

$$\begin{bmatrix}\mathbf{C}_f^{\bullet T\perp}\begin{bmatrix}\mathbf{P}\mathbf{A}+\mathbf{A}^T\mathbf{P} & \mathbf{0}\\ * & -\gamma^2\mathbf{I}_r\end{bmatrix}\mathbf{C}_f^{\bullet T\perp T} & \mathbf{C}_f^{\bullet T\perp}\begin{bmatrix}\mathbf{C}_f^T\\ \mathbf{0}\end{bmatrix}\\ * & -\mathbf{I}_m\end{bmatrix} < 0 \tag{139}$$

Considering all possible structures $\mathbf{C}_{fi},\ i = 1, 2, \ldots, m$ (139) implies (125).
Inequality (125) takes a simpler form if $\mathbf{E} = \mathbf{0}$. Thus, now

$$\mathbf{C}_f^{\circ T\perp} = \begin{bmatrix}\mathbf{C}^T\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}^{\perp} = \begin{bmatrix}\mathbf{C}^{T\perp} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_r & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I}_m\end{bmatrix} \tag{140}$$

and pre-multiplying left-hand sides of (132) by (140) and its right-hand side by transposition of (140) results in

$$\begin{bmatrix} \mathbf{C}^{T\perp}(\mathbf{PA}+\mathbf{A}^T\mathbf{P})\mathbf{C}^{T\perp T} & \mathbf{0} & \mathbf{C}^{T\perp}\mathbf{C}_f^T \\ * & -\gamma^2\mathbf{I}_r & \mathbf{0} \\ * & * & -\mathbf{I}_m \end{bmatrix} < 0 \tag{141}$$

which implies (126). This concludes the proof. ■

Solving LMI problem (124), (125), (126) with respect to LMI variable $\mathbf{P}$, then it is possible to construct (128), and subsequently to solve (127) defining the feedback control gain $\mathbf{K}_o$, and $\mathbf{H}$ as LMI variables.

Note, (124), (125), (126) have to be solved iteratively to obtain any approximation $\mathbf{P}^{-1} = \mathbf{\Pi}$. This implies that these inequalities together define only the sufficient condition of a solution, and so one from $(\mathbf{P}, \mathbf{\Pi}^{-1})$ can be used in design independently while verifying solution using the latter. Since of an approximative solution the matrix Θ defined in (129) need not be negative definite, and so it is necessary to introduce into (128) a negative definite matrix $\mathbf{\Theta}_{fi}^{\circ}$ as follows

$$\mathbf{\Theta}_{fi}^{\circ} = \mathbf{\Theta}_{fi} - \mathbf{\Delta} < 0 \tag{142}$$

where $\mathbf{\Delta} > 0$.

If (124), (125), (126) is infeasible the principle can be modified based on inequalities regrouping e.g. in such way that solving (124), (125), and (124), (126) separatively and obtaining two virtual sensor structures (one for $\mathbf{E} = \mathbf{0}$ and other for $\mathbf{E} = \mathbf{I}_m$). It is evident that virtual sensor switching be more sophisticated in this case.

5.3 Virtual sensor design

Theorem 5 *Virtual sensor (107) associated with the system (103), (104) is stable if there exist symmetric positive definite matrix* $\mathbf{R} \in \mathbb{R}^{n\times n}$*, and a matrix* $\mathbf{Z} \in \mathbb{R}^{n\times m}$*, such that*

$$\mathbf{R} = \mathbf{R}^T > 0 \tag{143}$$

$$\mathbf{RA} + \mathbf{A}^T\mathbf{R} - \mathbf{ZC}_{fi} + \mathbf{C}_{fi}^T\mathbf{Z}^T < 0, \qquad i = 0, 1, 2, \ldots, m \tag{144}$$

The virtual sensor matrix parameter is then given as

$$\mathbf{J} = \mathbf{R}^{-1}\mathbf{Z} \tag{145}$$

Proof. Supposing that $\mathbf{q}(t) = \mathbf{0}$ and $\mathbf{D}_u = \mathbf{0}$ then (108), (110) is reduced as follows

$$\begin{bmatrix} \mathbf{\Phi}_2 & \mathbf{0} & \mathbf{0} \\ * & -\gamma^2\mathbf{I}_r & \mathbf{0} \\ * & * & -\mathbf{I}_m \end{bmatrix} < 0 \tag{146}$$

$$\mathbf{R}(\mathbf{A} - \mathbf{JC}_f) + (\mathbf{A} - \mathbf{JC}_f)^T\mathbf{R} < 0 \tag{147}$$

respectively. Thus, with the notation

$$\mathbf{Z} = \mathbf{RJ} \tag{148}$$

(147) implies (144). This concludes the proof. ■

Illustrative example

Using for $\mathbf{E} = \mathbf{0}$ the same system parameters as that given in the example in Subsection 4.1.2, then the next design parameters were computed

$$\mathbf{B}_u^{\perp} = \begin{bmatrix} -0.8581 & 0.1907 & 0.4767 \end{bmatrix}, \ \mathbf{C}^{T\perp} = \begin{bmatrix} 0.5774 & -0.5774 & 0.5774 \end{bmatrix}$$

$$\mathbf{C}_{f0} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \ \mathbf{C}_{f1} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \ \mathbf{C}_{f2} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Solving (124) and the set of polytopic inequalities (126) with respect to $\mathbf{P}, \mathbf{\Pi}$ using the SeDuMi package the problem was feasible and the matrices

$$\mathbf{P} = \begin{bmatrix} 0.6836 & 0.0569 & -0.0569 \\ 0.0569 & 0.6836 & 0.0569 \\ -0.0569 & 0.0569 & 0.6836 \end{bmatrix}$$

as well as $\mathbf{H} = 0.1\mathbf{I}_2$ was used to construct the next ones

$$\mathbf{\Theta}_0 = \begin{bmatrix} 0.5688 & 0.9111 & -3.0769 & & & 1 & 1 \\ 0.9111 & -0.9100 & -5.8103 & & & 2 & 1 \\ -3.0769 & -5.8103 & -6.7225 & & & 1 & 0 \\ & & & -0.1 & & & \\ & & & & -0.1 & & \\ 1.0000 & 2.0000 & 1.0000 & & & -1 & \\ 1.0000 & 1.0000 & 0 & & & & -1 \end{bmatrix}, \quad \Theta_1, \quad \Theta_2$$

$$\mathbf{F}^T = -\begin{bmatrix} 0.7405 & 1.4810 & 0.7405 & 0 & 0 & 0 & 0 \\ 1.8234 & 1.1386 & 3.3044 & 0 & 0 & 0 & 0 \end{bmatrix}, \ \mathbf{G}^T = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

To obtain negativity of $\mathbf{\Theta}_{fi}^{\circ}$ the matrix $\mathbf{\Delta} = 4.94\mathbf{I}_7$ was introduced. Solving the set of polytopic inequalities (128) with respect to $\mathbf{K}_o$ the problem was also feasible and it gave the result

$$\mathbf{K}_o = \begin{bmatrix} -0.0734 & -0.0008 \\ -0.1292 & 0.1307 \end{bmatrix}$$

which secure robustness of control stability with respect to all structures of output matrices $\mathbf{C}_{fi}$, $i = 0, 1, 2$. In this sense

$$\rho(\mathbf{A} - \mathbf{B}_u\mathbf{K}_o\mathbf{C}) = \begin{bmatrix} -1.0000 & -1.3941 \pm 2.3919\,\mathrm{i} \end{bmatrix}$$

$$\rho(\mathbf{A} - \mathbf{B}_u\mathbf{K}_o\mathbf{C}_{f1}) = \begin{bmatrix} -1.0000 & -2.2603 \pm 1.6601\,\mathrm{i} \end{bmatrix}$$

$$\rho(\mathbf{A} - \mathbf{B}_u\mathbf{K}_o\mathbf{C}_{f2}) = \begin{bmatrix} -1.0000 & -1.1337 \pm 1.8591\,\mathrm{i} \end{bmatrix}$$

Solving the set of polytopic inequalities (144) with respect to $\mathbf{R}, \mathbf{Z}$ the feasible solution was

$$\mathbf{R} = \begin{bmatrix} 0.7188 & 0.0010 & 0.0016 \\ 0.0010 & 0.7212 & 0.0448 \\ 0.0016 & 0.0448 & 0.1299 \end{bmatrix}, \quad \mathbf{Z} = \begin{bmatrix} -0.0006 & 0.4457 \\ 0.0117 & 0.0701 \\ -0.0629 & -0.5894 \end{bmatrix}$$

Thus, the virtual sensor gain matrix $\mathbf{J}$ was computed as

$$\mathbf{J} = \begin{bmatrix} 0.0002 & 0.6296 \\ 0.0473 & 0.3868 \\ -0.5003 & -4.6799 \end{bmatrix}$$

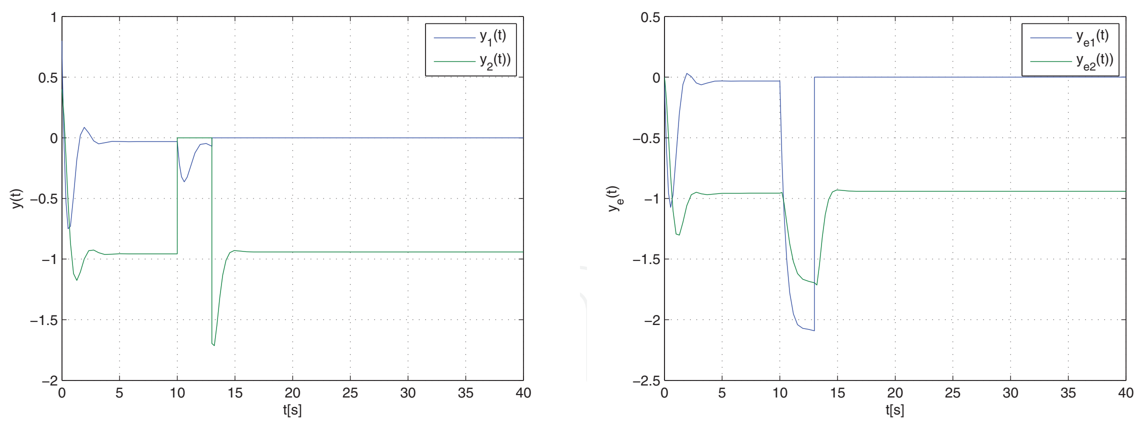

Fig. 7. System output and its estimation

which secure robustness of virtual sensor stability with respect to all structures of output matrices $\mathbf{C}_{fi}$, $i = 0, 1, 2$. In this sense

$$\rho(\mathbf{A} - \mathbf{JC}) = \begin{bmatrix} -1.0000 \\ -1.1656 \\ -3.4455 \end{bmatrix}, \quad \rho(\mathbf{A} - \mathbf{JC}_{f1}) = \begin{bmatrix} -1.0000 \\ -1.2760 \\ -3.7405 \end{bmatrix}$$

$$\rho(\mathbf{A} - \mathbf{B}_u\mathbf{K}_o\mathbf{C}_{f2}) = \begin{bmatrix} -1.0000 \\ -1.1337 + 1.8591\,\mathrm{i} \\ -1.1337 - 1.8591\,\mathrm{i} \end{bmatrix}$$

As was mentioned above the simulation results were obtained by solving the semi-definite programming problem under Matlab with SeDuMi package 1.2, where the initial conditions were set to

$$\mathbf{q}(0) = \begin{bmatrix} 0.2\ 0.2\ 0.2 \end{bmatrix}^T, \quad \mathbf{q}_e(0) = \begin{bmatrix} 0\ 0\ 0 \end{bmatrix}^T$$

respectively, and the control law in forced mode was

$$\mathbf{u}_f(t) = -\mathbf{K}_o\mathbf{y}_e(t) + \mathbf{w}(t), \quad \mathbf{w}(t) = \begin{bmatrix} -0.2\ -0.2 \end{bmatrix}^T$$

Fig. 7 shows the trajectory of the system outputs and the trajectory of the estimate system outputs using virtual sensor structure. It can be seen there a reaction time available to perform fault detection and isolation in the trajectory of the estimate system outputs, as well as a reaction time of control system reconfiguration in the system output trajectory. The results confirm that the true signals and their estimation always reside between limits given by static system error of the closed-loop structure.

6. Active control structures with a single actuator fault

6.1 Stability of the system

Theorem 6 *Fault tolerant control system defined by (1) – (9) is stable in the sense of bounded real lemma if there exist positive definite symmetric matrices* $\mathbf{Q}$, $\mathbf{R} \in \mathbb{R}^{n\times n}$, $\mathbf{S} \in \mathbb{R}^{l\times l}$, *and matrices*

$\mathbf{K} \in \mathbb{R}^{r\times n}$, $\mathbf{L} \in \mathbb{R}^{r\times l}$, $\mathbf{J} \in \mathbb{R}^{n\times m}$, $\mathbf{M} \in \mathbb{R}^{l\times l}$, $\mathbf{N} \in \mathbb{R}^{l\times m}$ *such that*

$$\begin{bmatrix} \mathbf{\Phi}_{11} & \mathbf{QB}_u\mathbf{K} & \mathbf{QB}_u\mathbf{L} & \mathbf{0} & \mathbf{0} & (\mathbf{C}-\mathbf{D}_u\mathbf{K})^T \\ * & \mathbf{\Phi}_{22} & \mathbf{R}(\mathbf{B}_f-\mathbf{JD}_f)-(\mathbf{SNC})^T & \mathbf{0} & \mathbf{0} & (\mathbf{D}_u\mathbf{K})^T \\ * & * & \mathbf{\Phi}_{33} & -\mathbf{SM} & \mathbf{S} & (\mathbf{D}_u\mathbf{L})^T \\ * & * & * & -\gamma^2\mathbf{I}_l & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -\gamma^2\mathbf{I}_l & \mathbf{0} \\ * & * & * & * & * & -\mathbf{I}_m \end{bmatrix} < 0 \qquad (149)$$

where

$$\mathbf{\Phi}_{11} = \mathbf{Q}(\mathbf{A}-\mathbf{B}_u\mathbf{K}) + (\mathbf{A}-\mathbf{B}_u\mathbf{K})^T\mathbf{Q}, \quad \mathbf{\Phi}_{22} = \mathbf{R}(\mathbf{A}-\mathbf{JC}) + (\mathbf{A}-\mathbf{JC})^T\mathbf{R} \qquad (150)$$

$$\mathbf{\Phi}_{33} = \mathbf{S}(\mathbf{M}-\mathbf{ND}_f) + (\mathbf{M}-\mathbf{ND}_f)^T\mathbf{S} \qquad (151)$$

Proof. Considering equality $\dot{\mathbf{f}}(t) = \dot{\mathbf{f}}(t)$ and assembling this equality with (1) – (4), and with (7) – (9) gives the result

$$\begin{bmatrix} \dot{\mathbf{q}}(t) \\ \dot{\mathbf{q}}_e(t) \\ \dot{\mathbf{f}}(t) \\ \dot{\mathbf{f}}_e(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & -\mathbf{B}_u\mathbf{K} & \mathbf{B}_f & -\mathbf{B}_u\mathbf{L} \\ \mathbf{JC} & \mathbf{A}-\mathbf{B}_u\mathbf{K}-\mathbf{JC} & \mathbf{JD}_f & \mathbf{B}_f-\mathbf{JD}_f-\mathbf{B}_u\mathbf{L} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{NC} & -\mathbf{NC} & \mathbf{ND}_f & \mathbf{M}-\mathbf{ND}_f \end{bmatrix} \begin{bmatrix} \mathbf{q}(t) \\ \mathbf{q}_e(t) \\ \mathbf{f}(t) \\ \mathbf{f}_e(t) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \dot{\mathbf{f}}(t) \\ \mathbf{0} \end{bmatrix} \qquad (152)$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{C} & -\mathbf{D}_u\mathbf{K} & \mathbf{D}_f & -\mathbf{D}_u\mathbf{L} \end{bmatrix} \begin{bmatrix} \mathbf{q}(t) \\ \mathbf{q}_e(t) \\ \mathbf{f}(t) \\ \mathbf{f}_e(t) \end{bmatrix} \qquad (153)$$

which can be written in a compact form as

$$\dot{\mathbf{q}}_\alpha(t) = \mathbf{A}_\alpha\mathbf{q}_\alpha(t) + \mathbf{f}_\alpha(t) \qquad (154)$$

$$\mathbf{y} = \mathbf{C}_\alpha\mathbf{q}_\alpha(t) \qquad (155)$$

where

$$\mathbf{q}_\alpha^T(t) = \begin{bmatrix} \mathbf{q}^T(t) & \mathbf{q}_e^T(t) & \mathbf{f}^T(t) & \mathbf{f}_e^T(t) \end{bmatrix}, \quad \mathbf{f}_\alpha^T(t) = \begin{bmatrix} \mathbf{0}^T & \mathbf{0}^T & \dot{\mathbf{f}}^T(t) & \mathbf{0}^T \end{bmatrix} \qquad (156)$$

$$\mathbf{A}_\alpha = \begin{bmatrix} \mathbf{A} & -\mathbf{B}_u\mathbf{K} & \mathbf{B}_f & -\mathbf{B}_u\mathbf{L} \\ \mathbf{JC} & \mathbf{A}-\mathbf{B}_u\mathbf{K}-\mathbf{JC} & \mathbf{JD}_f & \mathbf{B}_f-\mathbf{JD}_f-\mathbf{B}_u\mathbf{L} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{NC} & -\mathbf{NC} & \mathbf{ND}_f & \mathbf{M}-\mathbf{ND}_f \end{bmatrix} \qquad (157)$$

$$\mathbf{C}_\alpha = \begin{bmatrix} \mathbf{C} & -\mathbf{D}_u\mathbf{K} & \mathbf{D}_f & -\mathbf{D}_u\mathbf{L} \end{bmatrix} \qquad (158)$$

Using notations

$$\mathbf{e}_q(t) = \mathbf{q}(t) - \mathbf{q}_e(t), \qquad \mathbf{e}_f(t) = \mathbf{f}(t) - \mathbf{f}_e(t) \qquad (159)$$

where $\mathbf{e}_q(t)$ is the error between the actual state and the estimated state, and $\mathbf{e}_f(t)$ is the error between the actual fault and the estimated fault, respectively then it is possible to define the state transformation

$$\mathbf{q}_\beta(t) = \mathbf{T}\mathbf{q}_\alpha(t) = \begin{bmatrix} \mathbf{q}(t) \\ \mathbf{e}_q(t) \\ \mathbf{f}(t) \\ \mathbf{e}_f(t) \end{bmatrix}, \ \mathbf{f}_\beta(t) = \mathbf{T}\mathbf{f}_\alpha(t) = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \dot{\mathbf{f}}(t) \\ \dot{\mathbf{f}}(t) \end{bmatrix}, \ \mathbf{T} = \mathbf{T}^{-1} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{I} & -\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & -\mathbf{I} \end{bmatrix} \qquad (160)$$

and to rewrite (154), (155) as follows

$$\dot{\mathbf{q}}_\beta(t) = \mathbf{A}_\beta \mathbf{q}_\beta(t) + \mathbf{f}_\beta(t) \tag{161}$$

$$\mathbf{y} = \mathbf{C}_\beta \mathbf{q}_\beta(t) \tag{162}$$

where

$$\mathbf{A}_\beta = \mathbf{T}\mathbf{A}_\alpha\mathbf{T}^{-1} = \begin{bmatrix} \mathbf{A} & \mathbf{B}_u\mathbf{K} & \mathbf{B}_f - \mathbf{B}_u\mathbf{L} & \mathbf{B}_u\mathbf{L} \\ \mathbf{0} & \mathbf{A}-\mathbf{J}\mathbf{C} & \mathbf{0} & \mathbf{B}_f - \mathbf{J}\mathbf{D}_f \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{N}\mathbf{C} & -\mathbf{M} & \mathbf{M}-\mathbf{N}\mathbf{D}_f \end{bmatrix} \tag{163}$$

$$\mathbf{C}_\beta = \mathbf{C}_\alpha\mathbf{T}^{-1} = \begin{bmatrix} \mathbf{C}-\mathbf{D}_u\mathbf{K} & \mathbf{D}_u\mathbf{K} & \mathbf{D}_f - \mathbf{D}_u\mathbf{L} & \mathbf{D}_u\mathbf{L} \end{bmatrix} \tag{164}$$

Since (5) implies

$$\mathbf{B}_f - \mathbf{B}_u\mathbf{L} = \mathbf{0}, \qquad \mathbf{D}_f - \mathbf{D}_u\mathbf{L} = \mathbf{0} \tag{165}$$

it obvious that (163), (165) can be simplified as

$$\mathbf{A}_\beta = \mathbf{T}\mathbf{A}_\alpha\mathbf{T}^{-1} = \begin{bmatrix} \mathbf{A} & \mathbf{B}_u\mathbf{K} & \mathbf{0} & \mathbf{B}_u\mathbf{L} \\ \mathbf{0} & \mathbf{A}-\mathbf{J}\mathbf{C} & \mathbf{0} & \mathbf{B}_f - \mathbf{J}\mathbf{D}_f \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{N}\mathbf{C} & -\mathbf{M} & \mathbf{M}-\mathbf{N}\mathbf{D}_f \end{bmatrix} \tag{166}$$

$$\mathbf{C}_\beta = \mathbf{C}_\alpha\mathbf{T}^{-1} = \begin{bmatrix} \mathbf{C}-\mathbf{D}_u\mathbf{K} & \mathbf{D}_u\mathbf{K} & \mathbf{0} & \mathbf{D}_u\mathbf{L} \end{bmatrix} \tag{167}$$

Eliminating out equality $\dot{\mathbf{f}}(t) = \dot{\mathbf{f}}(t)$ it can be written

$$\dot{\mathbf{q}}_\delta(t) = \mathbf{A}_\delta \mathbf{q}_\delta(t) + \mathbf{B}_\delta \mathbf{w}_\delta(t) \tag{168}$$

$$\mathbf{y} = \mathbf{C}_\delta \mathbf{q}_\delta(t) + \mathbf{D}_\delta \mathbf{w}_\delta(t) \tag{169}$$

where

$$\mathbf{q}_\delta^T(t) = \begin{bmatrix} \mathbf{q}^T(t) & \mathbf{e}_q^T(t) & \mathbf{e}_f^T(t) \end{bmatrix}, \qquad \mathbf{w}_\delta^T(t) = \begin{bmatrix} \mathbf{f}^T(t) & \dot{\mathbf{f}}^T(t) \end{bmatrix} \tag{170}$$

$$\mathbf{A}_\delta = \begin{bmatrix} \mathbf{A} & \mathbf{B}_u\mathbf{K} & \mathbf{B}_u\mathbf{L} \\ \mathbf{0} & \mathbf{A}-\mathbf{J}\mathbf{C} & \mathbf{B}_f - \mathbf{J}\mathbf{D}_f \\ \mathbf{0} & -\mathbf{N}\mathbf{C} & \mathbf{M}-\mathbf{N}\mathbf{D}_f \end{bmatrix}, \qquad \mathbf{B}_\delta = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ -\mathbf{M} & \mathbf{I} \end{bmatrix} \tag{171}$$

$$\mathbf{C}_\delta = \begin{bmatrix} \mathbf{C}-\mathbf{D}_u\mathbf{K} & \mathbf{D}_u\mathbf{K} & \mathbf{D}_u\mathbf{L} \end{bmatrix}, \qquad \mathbf{D}_\delta = \begin{bmatrix} \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{172}$$

To apply the separation principle a block diagonal symmetric matrix $\mathbf{P}_\delta > 0$ has to be chosen, i.e.

$$\mathbf{P}_\delta = \mathrm{diag}\begin{bmatrix} \mathbf{Q} & \mathbf{R} & \mathbf{S} \end{bmatrix} \tag{173}$$

where $\mathbf{Q},\ \mathbf{R} \in \mathbb{R}^{n\times n}$, $\mathbf{S} \in \mathbb{R}^{l\times l}$. Thus, with (150), (151) it yields

$$\mathbf{P}_\delta\mathbf{A}_\delta + \mathbf{A}_\delta^T\mathbf{P}_\delta = \begin{bmatrix} \mathbf{\Phi}_{11} & \mathbf{Q}\mathbf{B}_u\mathbf{K} & \mathbf{Q}\mathbf{B}_u\mathbf{L} \\ * & \mathbf{\Phi}_{22} & \mathbf{R}(\mathbf{B}_f - \mathbf{J}\mathbf{D}_f) - (\mathbf{S}\mathbf{N}\mathbf{C})^T \\ * & * & \mathbf{\Phi}_{33} \end{bmatrix}, \ \mathbf{P}_\delta\mathbf{B}_\delta = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ -\mathbf{S}\mathbf{M} & \mathbf{S} \end{bmatrix} \tag{174}$$

and inserting (171), (172), and (174) into (24) gives (149). This concludes the proof. ∎

6.2 Feedback controller gain matrix design

It is evident that there are the cross parameter interactions in the structure of (149). Since the separation principle pre-determines the estimator structure (error vectors are independent on the state as well as on the input variables), at the first design step can be computed a feedback controller gain matrix $\mathbf{K}$, and at the next step be designed the estimators gain matrices $\mathbf{J} \in \mathbb{R}^{n\times m}$, $\mathbf{M} \in \mathbb{R}^{l\times l}$, $\mathbf{N} \in \mathbb{R}^{l\times m}$, including obtained $\mathbf{K}$.

Theorem 7. *For a fault-free system (1), (2) exists a stable nominal control (4) if there exist a positive definite symmetric matrix $\mathbf{X} > 0$, $\mathbf{X} \in \mathbb{R}^{n\times n}$, a matrix $\mathbf{Y} \in \mathbb{R}^{r\times n}$, and a positive scalar $\gamma > 0$, $\gamma \in \mathbb{R}$ such that*

$$\mathbf{X} = \mathbf{X}^T > 0 \tag{175}$$

$$\begin{bmatrix} \mathbf{AX} + \mathbf{XA}^T - \mathbf{Y}^T\mathbf{B}_u^T - \mathbf{B}_u\mathbf{Y} & \mathbf{B}_u\mathbf{L} & \mathbf{XC}^T - \mathbf{Y}^T\mathbf{D}_u^T \\ * & -\gamma^2\mathbf{I}_l & \mathbf{L}^T\mathbf{D}_u^T \\ * & * & -\mathbf{I}_m \end{bmatrix} < 0 \tag{176}$$

The control law gain matrix is then given as

$$\mathbf{K} = \mathbf{YX}^{-1} \tag{177}$$

Proof. Considering $\mathbf{e}_q(t) = \mathbf{0}$, then separating $\mathbf{q}(t)$ from (168)-(169) gives

$$\dot{\mathbf{q}}(t) = \mathbf{A}^\circ\mathbf{q}(t) + \mathbf{B}^\circ\mathbf{w}^\circ(t) \tag{178}$$

$$\mathbf{y}(t) = \mathbf{C}^\circ\mathbf{q}(t) + \mathbf{D}^\circ\mathbf{w}^\circ(t) \tag{179}$$

where

$$\mathbf{w}^\circ(t) = \mathbf{e}_f(t) \tag{180}$$

$$\mathbf{A}^\circ = \mathbf{A} - \mathbf{B}_u\mathbf{K}, \quad \mathbf{B}^\circ = \mathbf{B}_u\mathbf{L}, \quad \mathbf{C}^\circ = \mathbf{C} - \mathbf{D}_u\mathbf{K}, \quad \mathbf{D}^\circ = \mathbf{D}_u\mathbf{L} \tag{181}$$

and with (181), and $\mathbf{P} = \mathbf{Q}$ inequality (24) can be written as

$$\begin{bmatrix} \mathbf{QA} + \mathbf{A}^T\mathbf{Q} - \mathbf{QB}_u\mathbf{K} - \mathbf{K}^T\mathbf{B}_u^T\mathbf{Q} & \mathbf{QB}_u\mathbf{L} & \mathbf{C}^T - \mathbf{K}^T\mathbf{D}_u^T \\ * & -\gamma^2\mathbf{I}_l & \mathbf{L}^T\mathbf{D}_u^T \\ * & * & -\mathbf{I}_m \end{bmatrix} < 0 \tag{182}$$

Introducing the congruence transform matrix

$$\mathbf{H} = \operatorname{diag}\left[\, \mathbf{Q}^{-1} \ \mathbf{I}_l \ \mathbf{I}_m \,\right] \tag{183}$$

then multiplying left-hand side, as well right-hand side of (182) by (183) gives

$$\begin{bmatrix} \mathbf{AQ}^{-1} + \mathbf{Q}^{-1}\mathbf{A}^T - \mathbf{B}_u\mathbf{KQ}^{-1} - \mathbf{QK}^T\mathbf{B}_u^T & \mathbf{B}_u\mathbf{L} & \mathbf{Q}^{-1}(\mathbf{C}^T - \mathbf{K}^T\mathbf{D}_u^T) \\ * & -\gamma^2\mathbf{I}_l & \mathbf{L}^T\mathbf{D}_u^T \\ * & * & -\mathbf{I}_m \end{bmatrix} < 0 \tag{184}$$

With notation

$$\mathbf{Q}^{-1} = \mathbf{X} > 0, \qquad \mathbf{KQ}^{-1} = \mathbf{Y} \tag{185}$$

(184) implies (176). This concludes the proof. ∎

6.3 Estimator system matrix design

Theorem 8 *For given scalar $\gamma > 0$, $\gamma \in \mathbb{R}$, and matrices $\mathbf{Q} = \mathbf{Q}^T > 0$, $\mathbf{Q} \in \mathbb{R}^{n\times n}$, $\mathbf{K} \in \mathbb{R}^{r\times n}$, $\mathbf{L} \in \mathbb{R}^{r\times l}$ estimators (7) – (9) associated with the system (1), (2) are stable if there exist symmetric positive definite matrices $\mathbf{R} \in \mathbb{R}^{n\times n}$, $\mathbf{S} \in \mathbb{R}^{l\times l}$, and matrices $\mathbf{Z} \in \mathbb{R}^{n\times m}$, $\mathbf{V} \in \mathbb{R}^{l\times l}$, $\mathbf{W} \in \mathbb{R}^{l\times m}$ such that*

$$\mathbf{R} = \mathbf{R}^T > 0 \tag{186}$$

$$\mathbf{S} = \mathbf{S}^T > 0 \tag{187}$$

$$\begin{bmatrix} \mathbf{\Phi}_{22} & \mathbf{R}\mathbf{B}_f - \mathbf{Z}\mathbf{D}_f - (\mathbf{W}\mathbf{C})^T & \mathbf{0} & \mathbf{0} & (\mathbf{D}_u\mathbf{K})^T \\ * & \mathbf{\Phi}_{33} & -\mathbf{V} & \mathbf{S} & (\mathbf{D}_u\mathbf{L})^T \\ * & * & -\gamma^2\mathbf{I}_l & \mathbf{0} & \mathbf{0} \\ * & * & * & -\gamma^2\mathbf{I}_l & \mathbf{0} \\ * & * & * & * & -\mathbf{I}_m \end{bmatrix} \tag{188}$$

where

$$\mathbf{\Phi}_{22} = \mathbf{R}\mathbf{A} - \mathbf{Z}\mathbf{C} + \mathbf{A}^T\mathbf{R} - \mathbf{C}^T\mathbf{Z}^T, \quad \mathbf{\Phi}_{33} = \mathbf{V} - \mathbf{W}\mathbf{D}_f + \mathbf{V}^T - \mathbf{D}_f^T\mathbf{W}^T \tag{189}$$

The estimators matrix parameters are then given as

$$\mathbf{M} = \mathbf{S}^{-1}\mathbf{V}, \quad \mathbf{N} = \mathbf{S}^{-1}\mathbf{W}, \quad \mathbf{J} = \mathbf{R}^{-1}\mathbf{Z} \tag{190}$$

Proof. Supposing that $\mathbf{q}(t) = \mathbf{0}$ then (149) is reduced as follows

$$\begin{bmatrix} \mathbf{\Phi}_{22} & \mathbf{R}(\mathbf{B}_f - \mathbf{J}\mathbf{D}_f) - (\mathbf{S}\mathbf{N}\mathbf{C})^T & \mathbf{0} & \mathbf{0} & (\mathbf{D}_u\mathbf{K})^T \\ * & \mathbf{\Phi}_{33} & -\mathbf{S}\mathbf{M} & \mathbf{S} & (\mathbf{D}_u\mathbf{L})^T \\ * & * & -\gamma^2\mathbf{I}_l & \mathbf{0} & \mathbf{0} \\ * & * & * & -\gamma^2\mathbf{I}_l & \mathbf{0} \\ * & * & * & * & -\mathbf{I}_m \end{bmatrix} < 0 \tag{191}$$

where

$$\mathbf{\Phi}_{22} = \mathbf{R}(\mathbf{A} - \mathbf{J}\mathbf{C}) + (\mathbf{A} - \mathbf{J}\mathbf{C})^T\mathbf{R}, \quad \mathbf{\Phi}_{33} = \mathbf{S}(\mathbf{M} - \mathbf{N}\mathbf{D}_f) + (\mathbf{M} - \mathbf{N}\mathbf{D}_f)^T\mathbf{S} \tag{192}$$

Thus, with notation

$$\mathbf{S}\mathbf{M} = \mathbf{V}, \qquad \mathbf{S}\mathbf{N} = \mathbf{W}, \qquad \mathbf{R}\mathbf{J} = \mathbf{Z} \tag{193}$$

(191), (192) implies (188), (189). This concludes the proof. ■

It is obvious that $\mathbf{F}_e = \mathbf{A} - \mathbf{J}\mathbf{C}$, as well as $\mathbf{M}$ have to be stable matrices.

6.4 Illustrative example

To demonstrate algorithm properties it was assumed that the system is given by (1), (2) where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -9 & -5 \end{bmatrix}, \quad \mathbf{B}_u = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 1 & 5 \end{bmatrix}, \quad \mathbf{B}_f = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \quad \mathbf{D}_u = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{D}_f = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

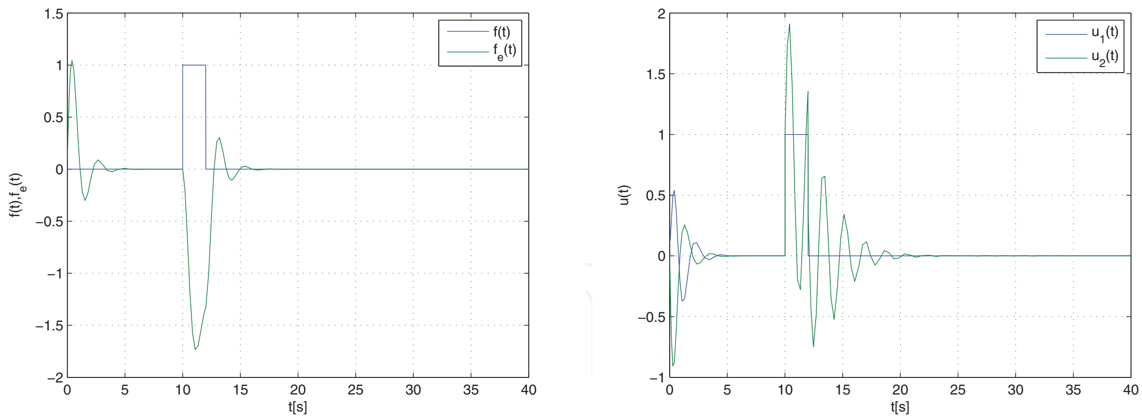

Fig. 8. The first actuator fault as well as its estimation and system input variables

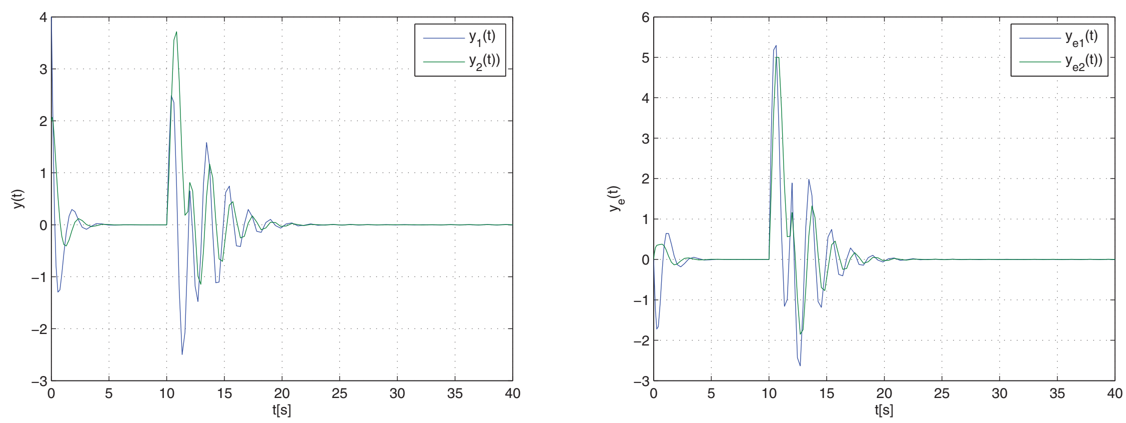

Fig. 9. System output and its estimation

Solving (175), (176) with respect to the LMI matrix variables γ, $\mathbf{X}$, and $\mathbf{Y}$ using Self-Dual-Minimization (SeDuMi) package for Matlab, the feedback gain matrix design problem was feasible with the result

$$\mathbf{X} = \begin{bmatrix} 1.7454 & -0.8739 & 0.0393 \\ -0.8739 & 1.3075 & -0.5109 \\ 0.0393 & -0.5109 & 2.0436 \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} 0.9591 & 1.2907 & -0.1049 \\ -0.1950 & -0.5166 & -0.4480 \end{bmatrix}, \quad \gamma = 1.8509$$

$$\mathbf{K} = \begin{bmatrix} 1.2524 & 1.7652 & 0.0436 \\ -0.0488 & -0.2624 & -0.3428 \end{bmatrix}$$

In the next step the solution to (186) – (188) using design parameters $\gamma = 1.8509$ was also feasible giving the LMI variables

$$\mathbf{V} = -1.3690, \quad \mathbf{S} = 1.1307, \quad \mathbf{W} = \begin{bmatrix} 0.9831 & 0.7989 \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} 1.7475 & 0.0013 & 0.0128 \\ 0.0013 & 1.4330 & 0.0709 \\ 0.0128 & 0.0709 & 0.6918 \end{bmatrix}, \quad \mathbf{Z} = \begin{bmatrix} -0.0320 & 1.0384 \\ 0.1972 & 0.1420 \\ -2.0509 & -1.1577 \end{bmatrix}$$

which gives

$$\mathbf{J} = \begin{bmatrix} 0.0035 & 0.6066 \\ 0.2857 & 0.1828 \\ -2.9938 & -1.7033 \end{bmatrix}, \quad \mathbf{N} = \begin{bmatrix} 0.8694 & 0.7066 \end{bmatrix}, \quad \mathbf{M} = -1.2108$$

Since $\mathbf{M} < 0$ it is evident that the fault estimator is stable and verifying the rest subsystem stability it can see that

$$\mathbf{A}_c = \mathbf{A} - \mathbf{B}_u \mathbf{K} = \begin{bmatrix} -1.1062 & 0.0282 & 0.9847 \\ -2.4561 & -3.2659 & 1.2555 \\ -6.0087 & -9.4430 & -3.3297 \end{bmatrix}, \quad \varrho(\mathbf{A}_c) = \{-0.7110 \;\; -3.4954 \pm \mathrm{i}\, 4.3387\}$$

$$\mathbf{A}_{qe} = \mathbf{A} - \mathbf{J}\mathbf{C} = \begin{bmatrix} -0.6101 & 0.3864 & -0.0035 \\ -0.4684 & -0.7541 & 0.7143 \\ -0.3029 & -1.3092 & -2.0062 \end{bmatrix}, \quad \varrho(\mathbf{A}_{qe}) = \{-1.0000 \;\; -1.1852 \pm \mathrm{i}\, 0.7328\}$$

where $\varrho(\cdot)$ is eigenvalue spectrum of a real square matrix. It is evident that the designed observer-based control structure results the stable system.
The example is shown of the closed-loop system response in the autonomous mode where Fig. 8 represents the first actuator fault as well as its estimation, and the system input variables, respectively, and Fig. 9 is concerned with the system outputs and its estimation, respectively.

7. Concluding remarks

This chapter provides an introduction to the aspects of reconfigurable control design method with emphasis on the stability conditions and related system properties. Presented viewpoint has been that non-expansive system properties formulated in the H_∞ design conditions underpins the nature of dynamic and feedback properties. Sufficient conditions of asymptotic stability of systems have thus been central to this approach. Obtained closed-loop eigenvalues express the internal dynamics of the system and they are directly related to aspects of system performance as well as affected by the different types of faults. On the other hand, control structures alternation achieved under virtual sensors, or by design or re-design of an actuator fault estimation can be done robust with respect of unaccepted faults. The role and significance of another reconfiguration principles may be found e.g. in the literature (Blanke et al.,2003), (Krokavec and Filasova,2007), (Noura et al.,2009), and references therein.

8. Acknowledgment

The main results presented in this chapter was derived from works supported by VEGA, Grant Agency of Ministry of Education and Academy of Science of Slovak Republic, under Grant No. 1/0256/11. This support is very gratefully acknowledged.

9. References

[Blanke et al.,2003] Blanke, M.; Kinnaert, M.; Lunze, J. & Staroswiecki, M. (2003). *Diagnosis and Fault-tolerant Control.* Springer, ISBN 3-540-01056-4, Berlin.

[Boyd et al.,1994] Boyd, D.; El Ghaoui, L.; Peron, E. & and Balakrishnan, V. (1994). *Linear Matrix Inequalities in System and Control Theory.* SIAM Society for Industrial and Applied Mathematics, ISBN, 0-89871-334-X, Philadelphia

[Chen and Patton,1999] Chen, J. & Patton, R.J. (1999). *Robust Model-Based Fault Diagnosis for Dynamic Systems*. Kluwer Academic Publishers, ISBN 0-7923-8411-3, Norwell.

[Chen et al.,1999] Chen, J.; Patton, R.J. & Chen, Z. (1999). Active fault-tolerant flight control systems design using the linear matrix inequality method. *Transactions of the Institute of Measurement and Control*, ol. 21, No. 2, (1999), pp. 77-84, ISSN 0142-3312.

[Chiang et al.,2001] Chiang, L.H.; Russell, E.L. & Braatz, R.D. (2001). *Fault Detection and Diagnosis in Industrial Systems*. Springer, ISBN 1-85233-327-8, London.

[Ding,2008] Ding, S.X. (2008). *Model-based Fault Diagnosis Techniques: Design Schemes, Alghorithms, and Tools.* Springer, ISBN 978-3-540-76304-8, Berlin.

[Dong et al.,2009] Dong, Q.; Zhong, M. & Ding, S.X. (2009). On active fault tolerant control for a class of time-delay systems. *Preprints of 7th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes, SAFEPROCESS 2009*, pp. 882-886, Barcelona, Spain, June 30, - July 3, 2009.

[Ducard,2009] Ducard, G.J.J. (2009). *Fault-tolerant Flight Control and Guidance Systems. Practical Methods for Small Unmanned Aerial Vehicles*. Springer, ISBN 978-1-84882-560-4, London.

[Filasova and Krokavec,2009] Filasová, A. & Krokavec, D. (2009). LMI-supported design of residual generators based on unknown-input estimator scheme. *Preprints ot the 6th IFAC Symposium on Robust Control Design ROCOND '09*, pp. 313-319, Haifa, Israel, June 16-18, 2009,

[Gahinet et al.,1995] P. Gahinet, P.; Nemirovski, A.; Laub, A.J. & Chilali, M. (1995). *LMI Control Toolbox User's Guide*, The MathWorks, Natick.

[Herrmann et al.,2007] Herrmann, G.; Turner, M.C. & Postlethwaite, I. (2007). Linear matrix inequalities in control. *Mathematical Methods for Robust and Nonlinear Control*, Turner, M.C. and Bates, D.G. (Eds.), pp. 123-142, Springer, ISBN 978-1-84800-024-7, Berlin.

[Jiang,2005] Jiang, J. (2005). Fault-tolerant Control Systems. An Introductory Overview. *Acta Automatica Sinica*, Vol. 31, No. 1, (2005), pp. 161-174, ISSN 0254-4156.

[Krokavec and Filasova,2007] Krokavec, D. & Filasová, A. (2007). *Dynamic Systems Diagnosis*. Elfa, ISBN 978-80-8086-060-8, Košice (in Slovak).

[Krokavec and Filasova,2008] Krokavec, D. & Filasová, A. (2008) Diagnostics and reconfiguration of control systems. *Advances in Electrical and Electronic Engineering*, Vol. 7, No. 1-2, (2008), pp. 15-20, ISSN 1336-1376.

[Krokavec and Filasova,2008] Krokavec D. & A. Filasová, A. (2008). Performance of reconfiguration structures based on the constrained control. *Proceedigs of the 17th IFAC World Congress 2008*, pp. 1243-1248, Seoul, Korea, July 06-11, 2008.

[Krokavec and Filasova,2009] Krokavec, D. & Filasová, A. (2009). Control reconfiguration based on the constrained LQ control algorithms. *Preprints of 7th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes SAFEPROCESS 2009*, pp. 686-691, Barcelona, Spain, June 30, - July 3, 2009.

[Liao et al.,2002] Liao, F.; Wang, J.L. & Yang, G.H. (2002). Reliable robust flight tracking control. An LMI approach. *IEEE Transactions on Control Systems Technology*, Vol. 10, No. 1, (2002), pp. 76-89, ISSN 1063-6536.

[Nesterov and Nemirovsky,1994] Nesterov, Y.; & Nemirovsky, A. (1994). *Interior Point Polynomial Methods in Convex Programming. Theory and Applications*, SIAM, ISBN 0-89871-319-6, Philadelphia.

[Nobrega et al.,2000] Nobrega, E.G.; Abdalla, M.O. & Grigoriadis, K.M. (2000). LMI-based filter design for fault detection and isolation. *Proceedings of the 39th IEEE Conference Decision and Control 2000*, Vol. 5, pp. 4329-4334, Sydney, Australia, December 12-15, 2000.

[Noura et al.,2009] Noura, H.; Theilliol, D.; Ponsart, J.C. & Chamseddine, A. (2009). *Fault-tolerant Control Systems. Design and Practical Applications*. Springer, ISBN 978-1-84882-652-6, Berlin.

[Patton,1997] Patton. R.J. (1997). Fault-tolerant control. The 1997 situation. *Proceedings of the 3rd IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes SAFEPROCESSŠ97*, Vol. 2, pp. 1033–1054, Hull, England, August 26-28, 1997.

[Peaucelle et al.,2002] Peaucelle, D.; Henrion, D.; Labit, Y. & Taitz, K. (2002). *User's Guide for Sedumi Interface 1.04*. LAAS-CNRS, Toulouse.

[Simani et al.,2003] Simani, S.; Fantuzzi, C. & Patton, R.J. (2003). *Model-based Fault Diagnosis in Dynamic Systems Using Identification Techniques*. Springer, ISBN 1-85233-685-4, London.

[Skelton et al.,1998] Skelton, R.E., Iwasaki, T. & Grigoriadis, K. (1998) A Unified Algebraic Approach to Linear Control Design. Taylor & Francis, ISBN 0-7484-0592-5, London.

[Staroswiecki,2005] Staroswiecki, M. (2005). Fault tolerant control. The pseudo-inverse method revisited. *Proceedings of the 16th IFAC World Congress 2005*, Prag, Czech Repulic, July 4-8, 2005.

[Theilliol et al.2008] Theilliol, D.; Join, C. & Zhang, Y.M. (2008). Actuator fault tolerant control design based on a reconfigurable reference input. *International Journal of Applied Mathematics and Computer Science*, Vol.18, No.4, (2008), pp. 553-560, ISSN 1641-876X.

[Zhang and Jiang,2008] Zhang, Y.M. & Jiang, J. (2008). Bibliographical review on reconfigurable fault-tolerant control systems. *Annual Reviews in Control*, Vol. 32, (2008), pp. 229Ű-252, ISSN 1367-5788.

[Zhou and Ren,2001] Zhou, K.M. & Ren, Z. (2001). A new controller architecture for high performance, robust and fault tolerant control. *IEEE Transactions on Automatic Control*, Vol. 40, No. 10, (2001), pp. 1613-1618, ISSN 0018-9286.

www.intechopen.com

Robust Control, Theory and Applications
Edited by Prof. Andrzej Bartoszewicz

ISBN 978-953-307-229-6
Hard cover, 678 pages
Publisher InTech
Published online 11, April, 2011
Published in print edition April, 2011

The main objective of this monograph is to present a broad range of well worked out, recent theoretical and application studies in the field of robust control system analysis and design. The contributions presented here include but are not limited to robust PID, H-infinity, sliding mode, fault tolerant, fuzzy and QFT based control systems. They advance the current progress in the field, and motivate and encourage new ideas and solutions in the robust control area.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:

Anna Filasová and Dušan Krokavec (2011). Design Principles of Active Robust Fault Tolerant Control Systems, Robust Control, Theory and Applications, Prof. Andrzej Bartoszewicz (Ed.), ISBN: 978-953-307-229-6, InTech, Available from: http://www.intechopen.com/books/robust-control-theory-and-applications/design-principles-of-active-robust-fault-tolerant-control-systems

InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821

© 2011 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the Creative Commons Attribution-NonCommercial-ShareAlike-3.0 License, which permits use, distribution and reproduction for non-commercial purposes, provided the original is properly cited and derivative works building on this content are distributed under the same license.

IntechOpen

IntechOpen