28

Fuzzy Pattern Modelling of Data Inherent Structures Based on Aggregation of Data with heterogeneous Fuzziness

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1. Introduction

Nowadays we are drowning in data but starving knowledge. In order to arise knowledgeable from the flood of data it has to be analysed. The goal of such an analysis is the creation of a model or the classification to a known phenomenon, e.g. modelling of the traffic flow in cities, medical or machine diagnosis (Herbst, 2008; Hempel, 2008a; Weihs, 2005).

Basically there are two main philosophies to deduce a model, namely theoretical and experimental modelling. In experimental modelling it is assumed that measurement data (objects) reflect a phenomenon by data–inherent structures. Unfortunately every observation is afflicted with inaccuracies such that the data might depict interesting phenomena characteristics just vaguely.

The knowledge about occurring imprecision is additional and valuable information. With the help of the fuzzy set theory it can be taken into account as a supplementary model feature (Zadeh, 1965). By understanding the whole modelling problem as a fuzzy classification task, where specific fuzzy sets referred to as fuzzy pattern classes form a model equivalent, the fuzzy pattern modelling method represents such a capable approach. Among several sophisticated solutions for such a task, which in general apply nonparametric fuzzy sets or a composition of different fuzzy sets (Bezdek, 2005) the main philosophy behind this work is the exclusive usage of a single specific parametrical fuzzy set to model data–inherent structures as well as the data itself.

Its key feature — a closed and uniform framework — provides the ability to incorporate occurring imprecision into the so called fuzzy pattern class model. The same framework allows an automatic deduction of fuzzy pattern class models based on a set of learning data with heterogeneous fuzziness.

The mission of the chapter is in the first place the introduction of the afore mentioned data–driven fuzzy modelling method to an audience applying experimental modelling (e.g. scientists, engineers, medical scientists or machine diagnosis specialists etc.). Another objective is to make a novel contribution to the field of experimental modelling. Consequently it is the concern of this work to provide a more general view onto the method.

In order to give an easy understandable survey about fuzzy pattern modelling the chapter will be organized in four main sections:

- Definition of the fuzzy pattern class model.

The second section will establish the fundamental terminology, the mathematics and the most general case of the multivariate fuzzy pattern class concept. Besides this definition part the application of fuzzy pattern classes as well as the representation of data with fuzzy pattern classes is introduced.

- Data–driven design of fuzzy pattern classes.

The automated design of fuzzy pattern models will be presented in section three step by step.

- Properties of fuzzy pattern class models.

In order to complete the survey, section four will provide information about advantages, disadvantages and limitations of fuzzy pattern models.

- State of the art research.

The last section will introduce ways to overcome the afore elaborated limitations of simple fuzzy pattern class models by sketching the state of the art research about networks of fuzzy pattern classes.

2. Definition of the Fuzzy Pattern Class Model

The multivariate fuzzy pattern class (FPC) is determined by set of basis functions. However, due to the here pursued type of modelling a basis function defines also a one–dimensional fuzzy pattern class model. Consequently a preliminary study of the one–dimensional class definition provides an easy access to derive the multivariate case.
In its most general form a one–dimensional fuzzy pattern class A is defined by the following unimodal side–specific parametrical prototype over a class specific reference system U, the so called class space.

$$\mu^A(u) = \begin{cases} \dfrac{a}{1+\left(\dfrac{1}{b_l}-1\right)\left|\dfrac{u}{c_l}\right|^{d_l}} & u < 0 \\ \dfrac{a}{1+\left(\dfrac{1}{b_r}-1\right)\left|\dfrac{u}{c_r}\right|^{d_r}} & u \geq 0 \end{cases} \tag{1}$$

With its graph illustrated by figure 1:

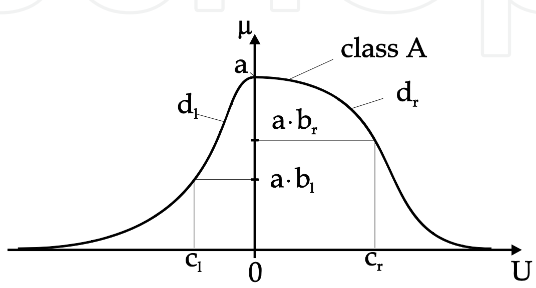


Fig. 1. One–dimensional class membership function with parameters

According to figure 1 it becomes obvious that the unidimensional function concept is based upon a set of seven parameters a and $\vec{p} = (b_l, b_r, c_l, c_r, d_l, d_r)$. The further specification of these parameters results from the fact that the parameter a characterises an entire fuzzy pattern class (FPC), whereas the parameters combined in $\vec{p}$ are related to a specific dimension of the class space (Bocklisch, 1987). Beyond their mere mathematical functionality all parameters possess the following semantic meaning:

- The parameter a represents the maximum membership value of the fuzzy pattern class $\mu^A(u)$. Regarding a structure of classes a expresses the weight of a specific class. Considering a dynamic classification process it embodies the topicality or authenticity of the information represented by that class (Hempel, 2005; Paessler, 1998).
- In the normalised case $a = 1$, the parameters $b_l, b_r \in [0,1]$ assign the left– and right–sided membership values at the class borders $u = -c_l$ and $u = c_r$.
- c_l, c_r mark the support of a class in a crisp sense. Both parameters characterise the left– and right–sided expansions of a fuzzy pattern class.
- The continuous descent of the membership function is specified by the parameters d_l, d_r. From a graphical point of view d_l, d_r determine the shape of the membership function, or in other words, the fuzziness of a class. From a modelling perspective this means that the d-parameters allow the incorporation of imprecision into a class model.
  The smoothest class shape is obtained for $d_{lr} = 2$, whereas the crisp case results for $d_{lr} \to \infty$. However for calculation purposes $d_{lr} = 20$ has proven to be a sufficient value to represent the crisp case.

Based on the introduced one–dimensional fuzzy pattern class model the multivariate fuzzy pattern class A derives from the intersection of such basis functions using the N–fold compensatory Hamacher intersection operator (2), where n denotes the index of the basis functions and N the total number of dimensions (Scheunert, 2001).

$$\cap_{Ham}\, \mu^A = \left(\frac{1}{N}\sum_{n=1}^{N}\frac{1}{\mu_n^A}\right)^{-1} \tag{2}$$

Regarding the main philosophy behind this paper, the key feature of this intersection is the conservation of the parametrical class concept for the multidimensional case, see (3).

$$\mu(\vec{u}) = a \cdot \left[\begin{array}{l} 1 + \dfrac{1}{2N}\displaystyle\sum_{n=1}^{N}\big(1 - sgn(u)\big)\left(\frac{1}{b_{ln}} - 1\right)\left(\frac{|u_n|}{c_{ln}}\right)^{d_{ln}} \\ + \dfrac{1}{2N}\displaystyle\sum_{n=1}^{N}(sgn(u) + 1)\left(\frac{1}{b_{rn}} - 1\right)\left(\frac{|u_n|}{c_{rn}}\right)^{d_{rn}} \end{array}\right]^{-1} \tag{3}$$

The definition of the fuzzy pattern class model is completed with the augmentation of the class describing set of parameters by a class space position $\vec{u}_0$ in the original feature space and a class space orientation $\vec{\varphi}$. $\vec{u}_0$ is also denoted as class representative since it determines the spot of the highest class membership.

Figure 2 depicts the influence of the additional parameters for a two–dimensional three class structure. The different location of each class results from the representatives

$\vec{u}_{c10} = (0.25\,, 0.85)^T$, $\vec{u}_{c20} = (0.5\,, 0.5)^T$ and $\vec{u}_{c30} = (0.75\,, 0.25)^T$, whereas an additional class orientation φ of 50° has been applied to the middle class.

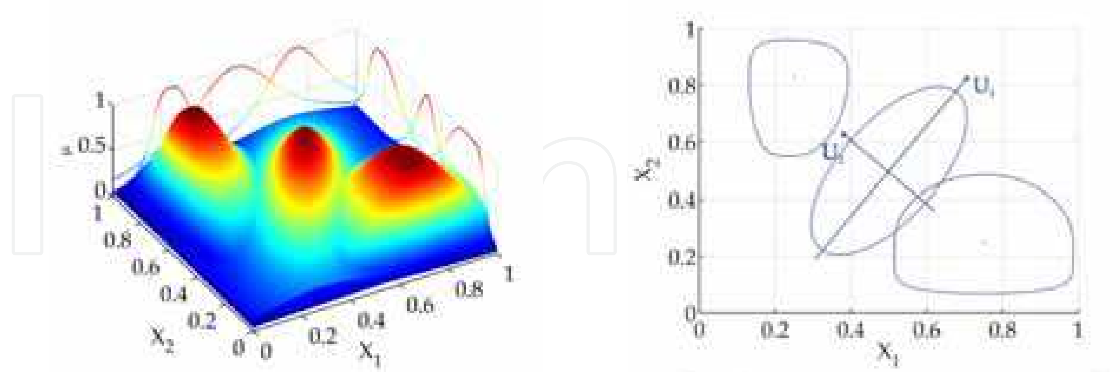


Fig. 2. complete representation left, border–curve representation right

As it can be seen there are two different ways of depicting fuzzy pattern class models a so called complete representation and a border–curve presentation. The complete representation is the most meaningful. It unifies the influence of all class parameters by mapping an N–dimensional class into an $N+1$ dimensional space, in particular the effects of the shape defining parameters d can be observerd. As a consequence a complete representation requires a lot of computational costs. On the contrary the border–curves of a class are defined by the geometrical locus with same class memberships. It follows naturally that the border–curve presentation based on equipotential membership lines can be computed with less effort while capturing a class's location, orientation and extension. The border–curve representation equates a mapping of an N–dimensional class into an N dimensional space. Figure 2 exhibits this border–curve mapping for the class membership $\mu(u) = 0.5$. Additionally it is pointing out the difference between the class space U and the feature space X. Due to the fact that U is a class specific reference system each point or object given in the feature space X has to be referred to U to be meaningful. The relation between both is given by transformation (4) where u_0 corresponds to the centre of a class space and the matrix T realises the class space rotation.

$$u = T(x - u_0)\ |u \in U, x \in X \tag{4}$$

2.1 Data Representation with the help of Fuzzy Pattern Classes

One outstanding feature of the here discussed method is the treatment of measured objects as fuzzy pattern entities. This means each object of a data set is considered to be a so called atomic fuzzy pattern class. The fuzzy perception of objects is justified by the fact that every observation (measurement) inheres an inaccuracy a so called elementary fuzziness (e.g. imprecision of a sensor), see figure 3.

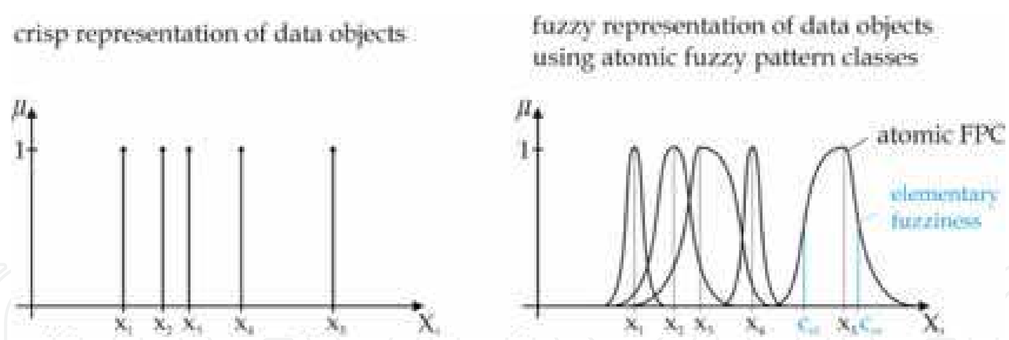


Fig. 3. Set of fuzzy objects with heterogeneous elementary fuzziness

The denotation atomic results from the fact that objects are the smallest informational unit available; they are the atoms of a data set. In order to signify that a fuzzy pattern class represents an object it is defined upon a specific set of parameters. Due to the fact that it is modelling the smallest informational entity the atomic class carries the weight $a = 1$ and its border memberships are set to $b_{lr} = 0.5$. Furthermore it is unlikely for objects to exhibit "internal" distributions therefore the class shape is assigned to $d_{lr} = 2$ leading to the atomic FPC description (5).

$$\mu^{Obj}(\vec{x}) = \left[\begin{array}{l} 1 + \frac{1}{2N} \sum_{n=1}^{N} \left(1 - sgn(x_n)\right) \left(\frac{|x_n - x_{0n}|}{c_{eln}} \right)^2 \\ + \frac{1}{2N} \sum_{n=1}^{N} \left(sgn(x_n) + 1\right) \left(\frac{|u_n - x_{0n}|}{c_{ern}} \right)^2 \end{array} \right]^{-1} \tag{5}$$

The only parameter that has not been specified yet is the expansion of a fuzzy object referred to as elementary fuzziness. The elementary fuzziness expresses the measuring accuracy of a sensor or the trust behind the position of the object in the feature space (if it is for example given verbally by an expert).

If there no such information available the elementary fuzziness is set symmetrically according to the given sensor inaccuracy (e.g. two percent of the measurement scale). If on the contrary there is access to such information the elementary fuzziness can take an asymmetrical or heterogeneous shape which consequently has to be imbued to the modelling process. Typical sources for elementary fuzziness are data sheets of a sensor, sensor characteristics as well as statements of experts. Figure 4 sketches the emergence of heterogeneous elementary fuzziness with the help of a nonlinear sensor characteristic (another example is diode characteristics).

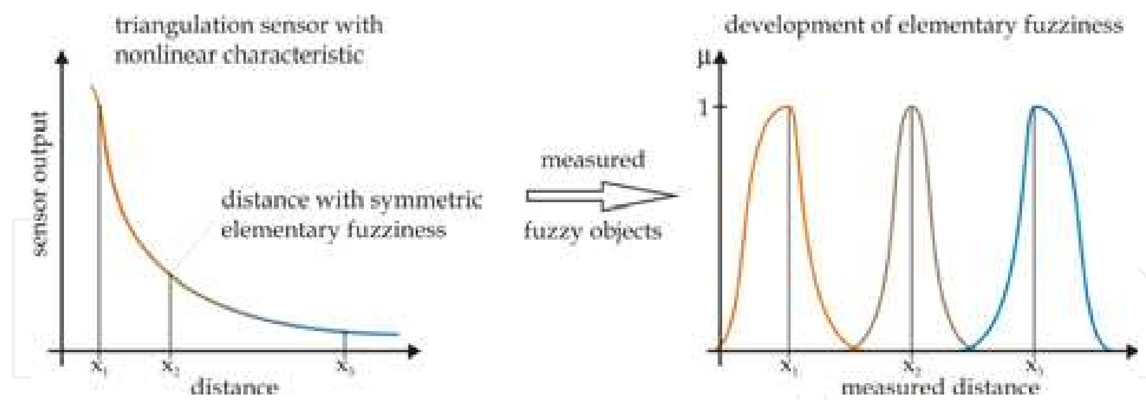


Fig. 4. Set of fuzzy objects with heterogeneous elementary fuzziness

Assuming a feature space spanned by two of such sensors it is likely to obtain the following exemplary set of fuzzy objects, see figure 5.
Especially when considering the border–curve ($\mu(u) = 0.5$) presentation, it can be imagined that the elementary object fuzziness might affect an experimental deduced model. The latter example motivates the reason to imbue elementary fuzziness into the modelling process. Hence the here introduced concept of fuzzy data will serve as foundation for the aggregation process.

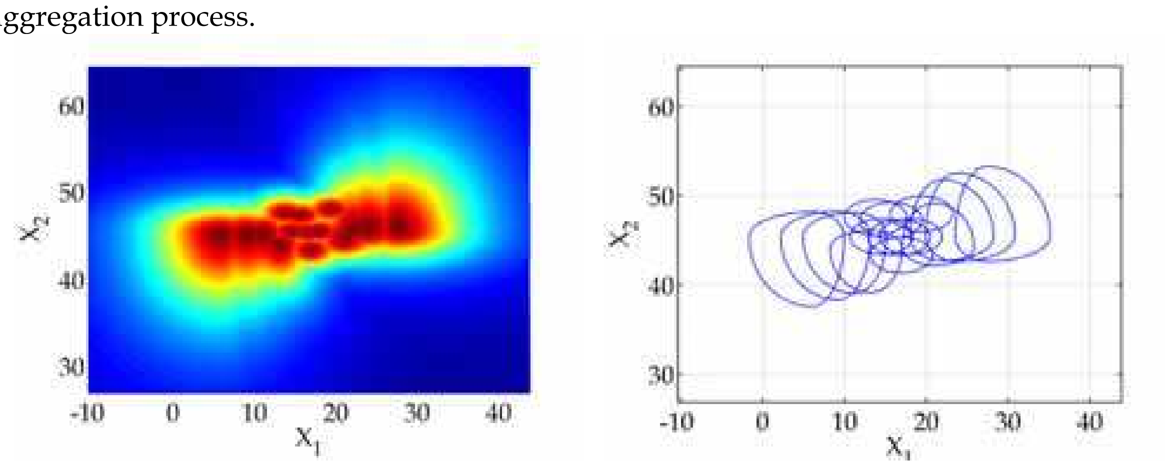


Fig. 5. Exemplary set of fuzzy objects with heterogeneous elementary fuzziness

2.2 Fuzzy Patter Classification

As introductorily alluded the entire modelling task is understood as a fuzzy classification task. In the framework of classification a more comprehensive model is characterised by a set of meaningful classes. For this purpose all model relevant fuzzy pattern classes are grouped together in a so called fuzzy pattern classifier. In operating mode the fuzzy pattern classifier then assigns unknown objects to this class structure. The results of the classification process are stored into a so called vector of sympathy $\vec{s} = (s_1, s_2, \dots, s_K)^T$. Each component of $\vec{s}$ denotes the membership of a classified object to the corresponding class, where K is the total number of classes.
The gradual membership of an object to a given class is calculated using (1).

$$s_k = \mu^k(\vec{x}) \;\text{ for }\; k = 1,2,\dots,K \tag{6}$$

According to the last section the objects to be classified are considered to be fuzzy entities. However the classification of fuzzy pattern objects goes beyond the scope of this work such that each object to be classified is denoted just by a vector of features $\vec{x} = (x_1, x_2, \dots, x_N)^T$ where N represents the number of feature dimensions. All full review about the classification of fuzzy objects can be found in (Hempel, 2005)
Figure 6 illustrates the process of classification with the help of a one–dimensional three class structure and the alongside listed classification results.

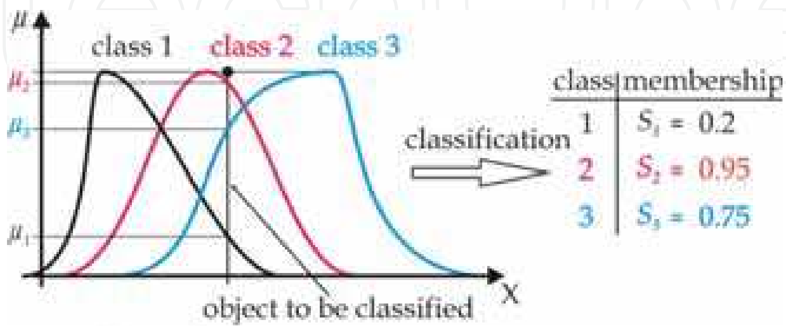


Fig. 6. One–dimensional classification process

The vector of sympathy describes an assignment of the object to the class structure with respect to its location in the feature space. Since the classifier comprises three fuzzy pattern classes it assigns three values of membership.

3. Aggregation of Fuzzy Pattern Classes from Data with Heterogeneous Fuzziness

As mentioned earlier the concept of classification embodies three subtasks: discovery of (data–inherent) structures, modelling of these structures and finally the usage or application of these models to classify unknown data.
At this point it is assumed that the first task, discovery of a structure within a set of data, has been resolved by a preliminary conducted cluster analysis or any other structure discovering algorithm (Bacher, 1996; Jain 1988). This section revolves around the question how to model an *already structured* set of data applying the afore introduced definition of fuzzy pattern classes as a modelling framework. Hence it is addressing the second task.
Basically fuzzy pattern class models can be obtained via two different ways (Bocklisch, 1987). First via definition by expertise, where an expert interprets the data at hand and determines all class parameters based upon task and domain specific knowledge. This approach is not pursued here.
The here featured second approach is a data–driven method, strongly advocating the goal to model known data–inherent structures. Based upon the class labelled set of "learning" data all class parameters are assigned automatically by a two step aggregation procedure, according to figure 7 (Hempel, 2005; Päßler, 1998).

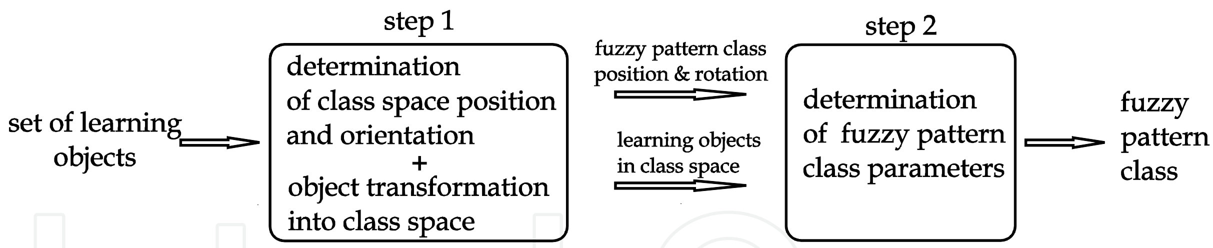


Fig. 7. Signal flow and general progression of the object aggregation to fuzzy pattern classes

As figure 7 indicates the class construction starts with the determination of the class location and rotation in the first step and is completed with assignment of the class parameters in the second step. The main features during this procedure are:

- consideration of the data fuzziness throughout the entire process,
- mapping of data distribution onto the class shape and
- data sequence independency.

Besides its specialisation on fuzzy data the class aggregation can also be applied to usual (crisp) data. In order to perform a congeneric aggregation the crisp learning dataset is just extended to a set of fuzzy objects, using the introduced function concept section 2.2.
Since the multivariate fuzzy pattern class model derives from its basis functions the task of deducing the class parameters can be performed in a dimension wise manner. Hence, for the sake of clarity all computations will be only shown for an arbitrary dimension, all further dimensions follow analogously.

3.1 First Step of Aggregation: Determination of the Class Space

As mentioned before the aggregation is based on a class labelled set of data being split up according to its class labels. Each subset is treated separately but in the same manner by the subsequent algorithm, resulting in different fuzzy pattern classes. Because of its uniformity the class construction will be shown with an exemplary subset x given in the original feature space, where N represents the number of object dimensions and M total number of objects.

$$x = \begin{pmatrix} x_{11} & \cdots & x_{1M} \\ \vdots & \ddots & \vdots \\ x_{N1} & \cdots & x_{NM} \end{pmatrix} \tag{7}$$

Origin of the class space

The origin $\vec{u}_0 = (u_{01}, \dots, u_{0i}, \dots, u_{0N})^T$ of the class space is referred to as class representative it marks the location of the highest class membership and is the reference point for all other parameters. It is calculated dimension–wise as the mean over all class supporting objects. In order to allow for heterogeneous elementary fuzziness each object representative is adjusted based on its specific elementary fuzziness, (see figure 8).

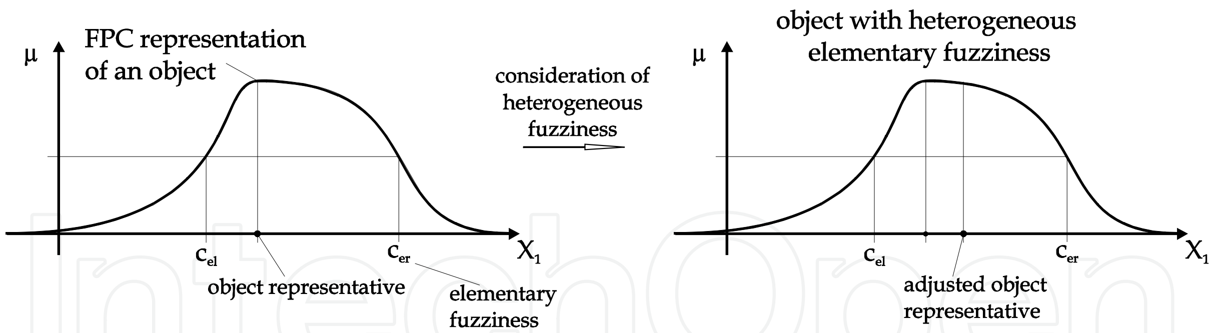


Fig. 8. Consideration of object specific elementary fuzziness via object representative adjustment

The above illustrated adjustment is realised by a weighted mean over the object representatives, the left–sided and right–sided elementary fuzziness in every dimension, see equation (8).

$$\tilde{x}_{i,j} = \frac{1}{1+2g}\left(x_{i,j} + g\left(2x_{i,j} - c_{eli,j} + c_{eri,j}\right)\right) \tag{8}$$

yielding:

$$\tilde{x} = \begin{pmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1M} \\ \vdots & \ddots & \vdots \\ \tilde{x}_{N1} & \cdots & \tilde{x}_{NM} \end{pmatrix} \tag{9}$$

The weight of the object borders c_{el}, c_{er} is be specified by the parameter $g\epsilon[0,1]$, where $g = 0$ blanks out the influence of the elementary fuzziness and $g = 1$ values object representative and the elementary fuzziness equally. For the fully automatic class construction g is set according to the membership of c_{el}, c_{er} ($g = 0.5$).

Figure 9 demonstrates the effect of the adjustment procedure for a two–dimensional object.

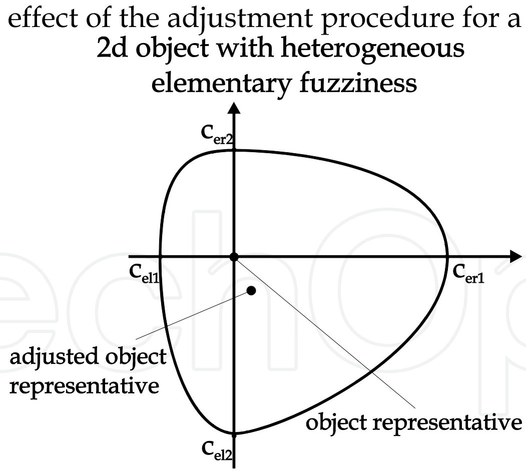


Fig. 9. Fuzzy induced representative adjustment for two–dimensional object

Rotation of the class space

In order to realise an optimal FPC–model of the data structure the class specific reference system U is rotated into the neutral axis of x. Thus the alignment of a class space U^N is defined by a set of $N-1$ rotation angles and stored in the parameter vector $\vec{\varphi} = (\varphi_1, \varphi_2, \ldots, \varphi_{N-1})^T$.

Dealing with fuzzy objects it has to be born in mind that the elementary fuzziness might affect the class orientation. Similar to the above considerations the elementary fuzziness is taken into account by the adjusted object representative (9).
The entire rotation can be denoted in matrix representation:

$$T = T_{N-1} \cdot T_{N-2} \cdot \ldots \cdot T_i \cdot \ldots \cdot T_1 \;\; where \;\; T_i \in \mathbb{R}^{N \times N}$$
$$T_i = \left(t_{i_1 i_2}\right) \left| t_{i_1 i_2} = \begin{cases} \cos\varphi_i & for\ i_1 = i_2 = 1\ or\ i \\ \sin\varphi_i & for\ i_1 = 1, i_2 = i \\ -\sin\varphi_i & for\ i_1 = i, i_2 = 1 \\ 1 & for\ 1 \neq i_1 = i_2 \neq i \\ 0 & else \end{cases} \right. \tag{10}$$

3.2 Second Aggregation Step: Determination of the Class Parameters

After the determination of the class space U all class parameters can be deduced based on the position and fuzziness of the class supporting objects. Since the origin of the class space is the reference point for all class parameters it is also necessary to refer the class supporting objects to their class space.

Transformation of the objects into the class space

Such a reference is generated by an affine object transformation into the class space, see equation 11.

$$u = T(x - u_0) \;\; where \;\; u = \begin{pmatrix} u_{11} & \cdots & u_{1M} \\ \vdots & \ddots & \vdots \\ u_{N1} & \cdots & u_{NM} \end{pmatrix} \tag{11}$$

Due to the fact that the objects themselves are fuzzy pattern entities a mere transformation of the object representatives according to (11) is insufficient, as their elementary fuzziness has to be transformed into the class space as well. Figure 10 depicts this concern for the two dimensional case.

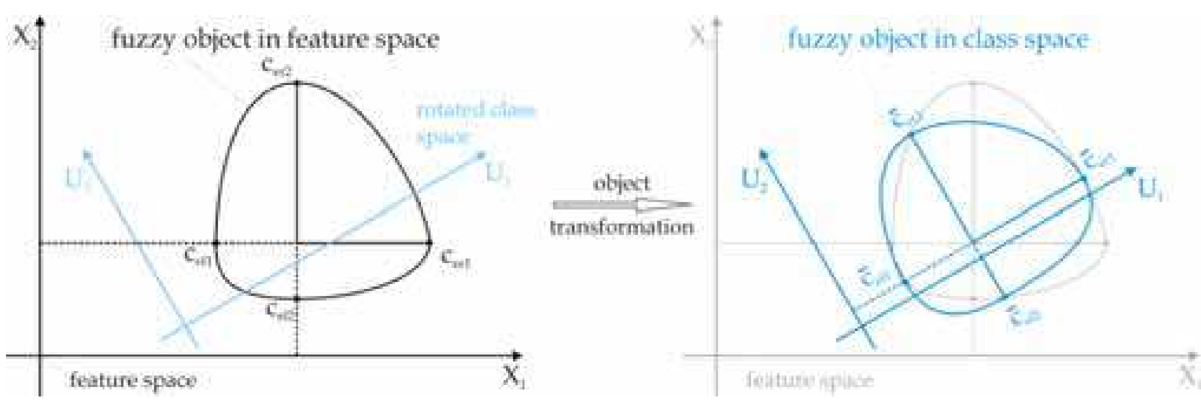


Fig. 10. Transformation of fuzzy pattern objects

Several methods to transform a fuzzy object have been reviewed considering interpretability, errors and computational costs (Hempel, 2005) The most promising method takes the heterogeneous elementary fuzziness into account using the back–transformed unity vectors of the class axis and their intersection points with the border curve ($\mu(u) = 0.5$) of the fuzzy object.

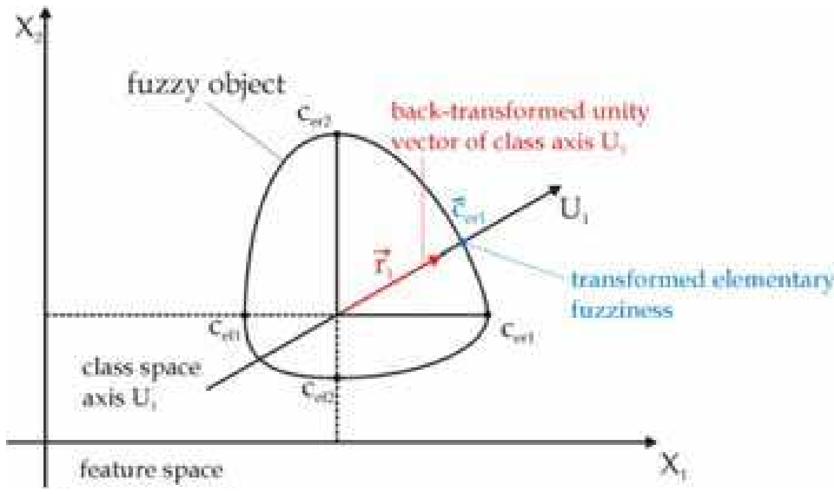


Fig. 11. Principle of object transformation

As figure 11 exemplifies, the values of the elementary fuzziness for a specific class axis (U_1) results from the dilation factor of its transformed unity vector $\vec{r}_1$. Due to the side specific modelling and the heterogeneous elementary fuzziness the significant ce–parameter are selected by quadrant discrimination during calculation.
After their transformation the set of objects is split up into two ordered sets of left–sided and right–sided objects O_{li}, O_{ri}.

$$O_{ri} = \{u\} \ \left| u_{0i} \leq u_{i,1} \leq u_{i,2} \leq \cdots \leq u_{i,R} \right.$$
$$O_{li} = \{u\} \ \left| u_{0i} \geq u_{i,1} \geq u_{i,2} \geq \cdots \geq u_{i,L} \right. \tag{12}$$

Specification of the class borders
In each class space dimension the extensions c_l, c_r of a class are determined by the outermost objects including the elementary fuzziness, (14)

$$c_{ri} = \max_{s=1 \ldots R} \left(u_{i,s} + c_{eri,s} \right)$$
$$c_{li} = \min_{s=1 \ldots L} \left(u_{i,s} - c_{eli,s} \right) \tag{13}$$

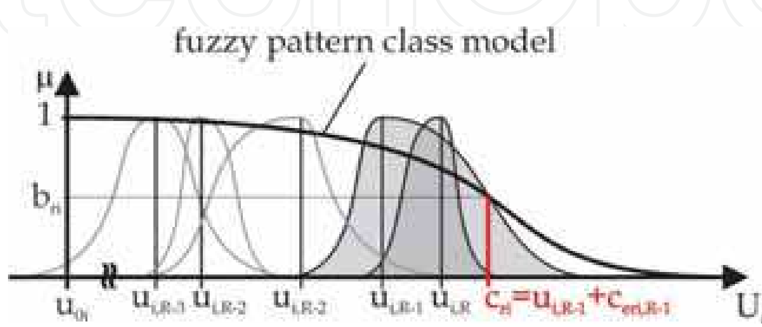


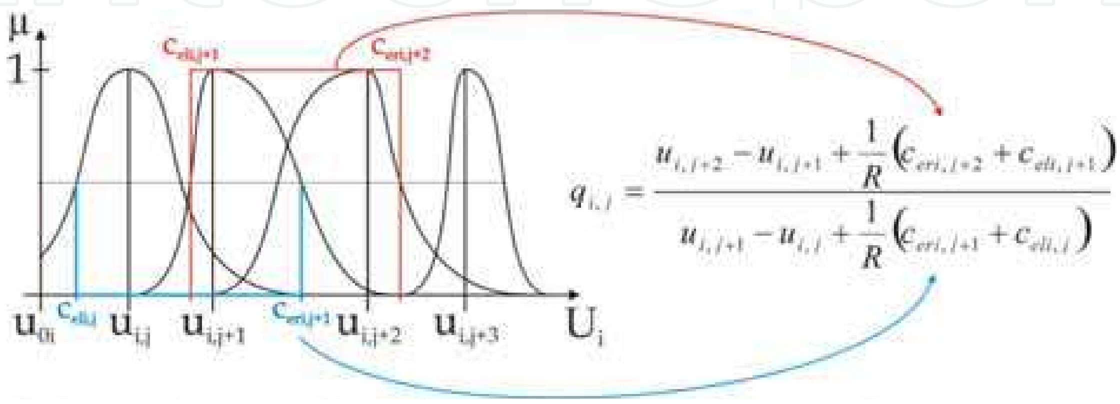
Fig. 12. Specification of the class borders in the i^{th} dimension

Since the class parameters d_{lr}, b_{lr} are obtained analogously for each class space dimension as well as for the left– and right–handed function branch the following considerations are straitened to the right–sided function branch.

Determination of the class shape

The shape of a fuzzy pattern class (d_r) is assigned based on the agglomeration properties of the class supporting objects. The more the data resembles an agglomeration according to a geometric series the smoother the class shape. The rate of resemblance is determined by the mean distance alteration between two adjacent objects $q_{i,j}$.

Figure 13 depicts the calculation of this rate $q_{i,j}$ for an arbitrary class dimension.



$$q_{i,j} = \frac{u_{i,j+2} - u_{i,j+1} + \frac{1}{R}\left(c_{eri,j+2} + c_{eli,j+1}\right)}{u_{i,j+1} - u_{i,j} + \frac{1}{R}\left(c_{eri,j+1} + c_{eli,j}\right)}$$

Fig. 13. Distance between adjacent objects including their elementary fuzziness

Based on the mean change of distance between two adjacent objects the class shape d_{ir} is determined in such a manner that the smoothest class shape $d_{lr} = 2$ is realised when the objects are cumulating in the centre of the class conform to a geometric series. On the contrary the crisp class shape $d_{lr} = 20$ is obtained when the objects are at least equally distributed over the class space.

The effects of the object specific elementary fuzziness $c_{eri,j}$, $c_{eli,j}$ is balanced out against the number of class supporting objects R. Correspondingly, the shape of classes being supported by a small number of objects is mainly characterised by the elementary fuzziness, whereas classes with a high number of class supporting objects experience a reduced influence of the elementary fuzziness onto their shape.

Assignment of the border membership

The value for border membership b_r is derived under the conservation of the object cardinality. This means that the sum of the area under all objects is required to be equal to the area under the fuzzy pattern class function, see figure (14).

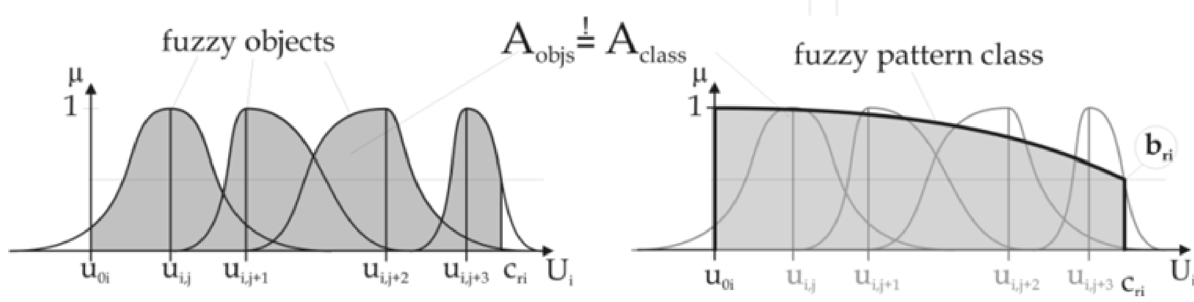


Fig. 14. Claim for the conservation of the object cardinality

The border memberships b_r are estimated over the unity interval $b_r \in [0,1]$ by (14) taking into account the prior results for c_r and d_r.

$$A_{class} = \int_0^{c_{ri}} \frac{du}{1 + \left(\frac{1}{b_{ri}} - 1\right)\left(\frac{u - u_{0i}}{c_{ri}}\right)^{d_{ri}}} = A_{objs} \;\; |b_{ri} \epsilon [0,1] \tag{14}$$

Assignment of the class weight

The weight of a class within a structure of fuzzy pattern classes depends on the total number of its class supporting objects M, $(a = f(M))$. Under the assumption of an evolutionary growing class weight a results from (15).

$$a(M) = a_{max}\left(1 - \left(\frac{a_{max} - 1}{a_{max}}\right)^M\right) \tag{15}$$

The assumption of an evolutionary growing according to (Peschel, 1986) is motivated by the fact that the emergence of a class from elementary observations is a structural transition from a quantitative growing to a qualitative one (classes are superordinate entities compared to objects).

4. Properties of Fuzzy Pattern Class models

The introduced fuzzy modelling concept is round off by having its major properties, advantages and drawbacks discussed subsequently.

The most characterising features of fuzzy pattern class models namely versatility, uniformity, treatment of fuzzy data and a closed modelling framework emanate from the unimodal, side–specific and parametric class membership function.

In its most general case fuzzy pattern classes offer multivariate fuzzy models with various asymmetric shapes, ranging from peak– over bell– to crisp forms (see figure 15). Together with the introduced data–driven design fuzzy patter classes allow to map class internal object distributions as well as correlative relations.

Besides all multi–dimensionality and flexibility the parameters of fuzzy pattern classes remain semantically motivated ensuring its the interpretability and transparency.

Furthermore the parametric class concept provides a good trade off between data compression, computational cost and generality. Especially for high dimensional models a sufficient level of data compression is reached since each fuzzy pattern class is defined upon a set of eight parameters per dimension. By connecting the basis function of every dimension exclusively on parameter level the chosen conjunction operator saves computational costs.

Both advantages are traded off for generality in so far as fuzzy pattern classes are convex models, specifying a convex area of the feature space. Consequently FPCs are most suited when it comes to model convex data–inherent structures.

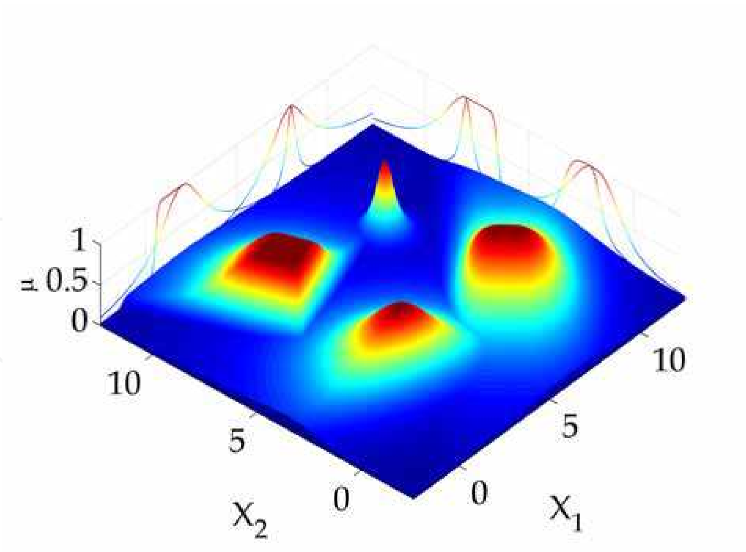


Fig. 15. Side–specific shape variety of fuzzy pattern classes

However when it comes to model nonconvex data–inherent structures fuzzy pattern class models are afflicted with errors. Figure 16 illustrates such an error by having an enclosed central object accumulation aggregated to a fuzzy pattern class.

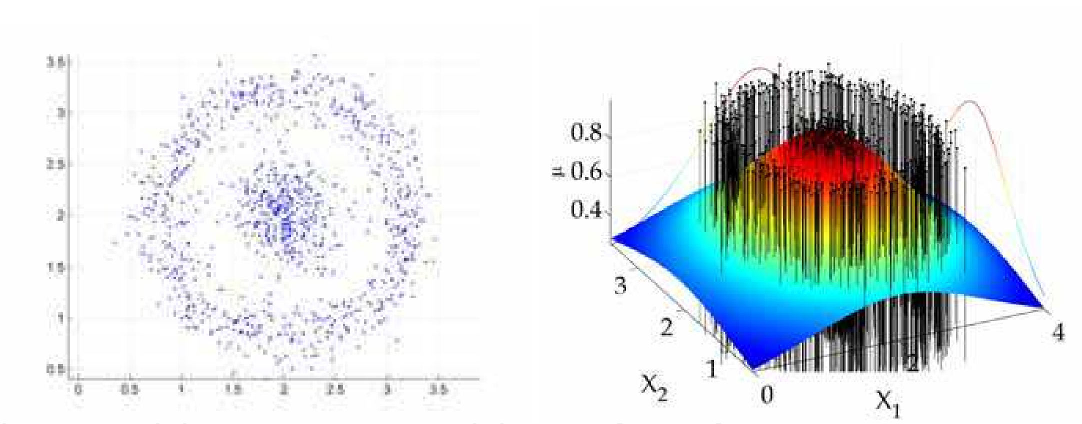


Fig. 16. right: data–inherent structure; left: associated fuzzy pattern class

The region between the ring shaped object arrangement and the central object accumulation does not belong to the given data structure. Nonetheless it will be associated with high grades of memberships by the corresponding fuzzy pattern model.

5. Networks of Fuzzy Pattern Classifiers

In order to circumvent this major drawback two possibilities have been thought of, leading to the state of the art fuzzy pattern research. The first way is a cluster based approach rendering a nonconvex describable by segmenting the data into convex subsets. The second access to dissolve the convexity drawback arises from the adoption of fuzzy pattern anti classes (FPAC) (negating fuzzy pattern models).

Both approaches lead to hereafter introduced networks of fuzzy pattern classifiers or so called fuzzy pattern classifier network. A Fuzzy Pattern Classifier Network (FPCN) consists of interconnected Fuzzy Pattern Classifier (FPC) nodes, representing its functional core. It is a modelling approach combining fuzzy set theory and network theory, as powerful and flexible tools of modelling.

5.1 Cluster Based Fuzzy Pattern Classifier Networks

In the cluster based approach the layout of the network structure and the configuration of classifier nodes, will be addressed by a hierarchical clustering and selection strategy. Aside from the fact that a mere clustering would work on every data–inherent structure it might create considerably large structures of fuzzy pattern classes at the expenses of model clarity. In order to maintain model clarity different layers of detail corresponding to a certain structural resolution have been introduced.

In its current implementation the cluster based fuzzy pattern network evolves from coarse to fine structures (Hempel, 2008a; Hempel, 2008b). Starting with the entire set of data a cluster analysis is conducted. Since each cluster method uses its specific strategy to discover structures some phenomena typical structures remain indiscoverable by a certain method (Jain, 1978). That is why at least an ensemble of sufficiently diverse cluster algorithms is applied (Strehl, 2005). Based on the clustering results (class labels) the data set is split into the most stable cluster configuration. All subset are modelled as fuzzy pattern class (see section 3) in an associated classifier node and subsequently treated separately but in the same manner, producing the next level of detail. As an overall result the cluster based approach leads to a network oriented hierarchical fuzzy pattern model.

An exemplary cluster based FPCN with the supporting set of data is given by figure 17.

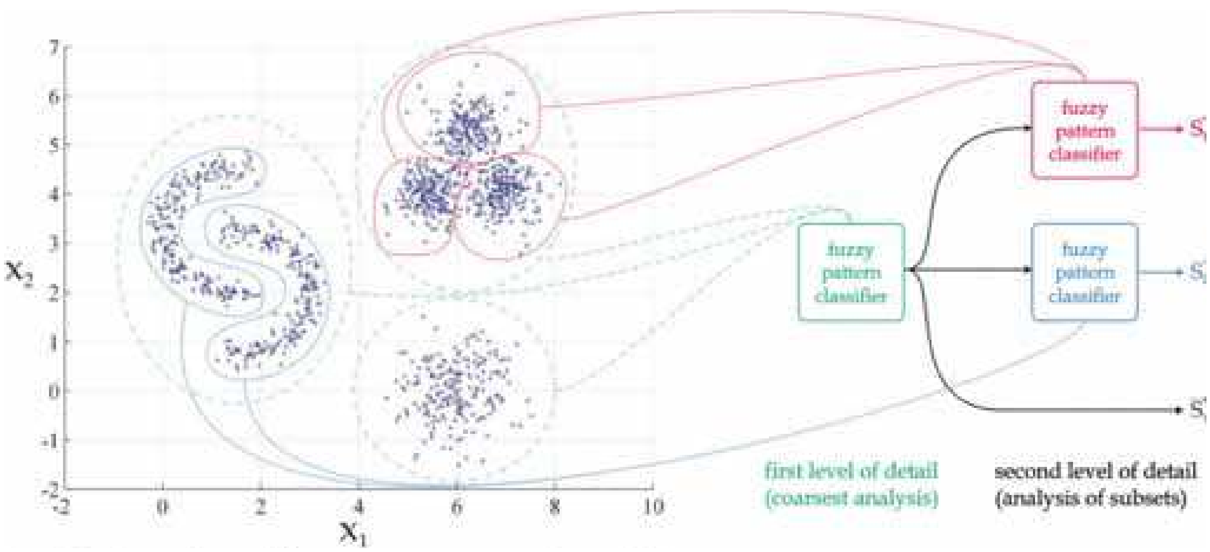


Fig. 17. cluster based fuzzy pattern network model

In the first level of detail (entire data set) the cluster analysis returned the highlighted (dashed lines) three class structure stored in the first fuzzy pattern classifier node. In the second level analysis each subset is treated separately generating a three class structure for the upper left subset summarised by the upper classifier node and a two class structure captured in the lower classifier node. Finally, each classifier is connected with its preceding node, eventually creating the tree–like FPCN. In detail the classifier nodes are connected with respect to the components of the sympathy vector that is originating from the fuzzy pattern classes of preceding node. This clear connection facilitates the information propagation throughout the network. The node activation is based on the highest component of the sympathy vector.

5.2 Fuzzy Pattern Classifier Networks with Anti–Classes

Instead of applying current clustering methods the fuzzy pattern anti–class (FPAC) strategy exploits the inverse of a data structure to create a fuzzy model. The principal idea behind this approach consists in the negation of a class assertion over its unsupported class space, see figure 18.

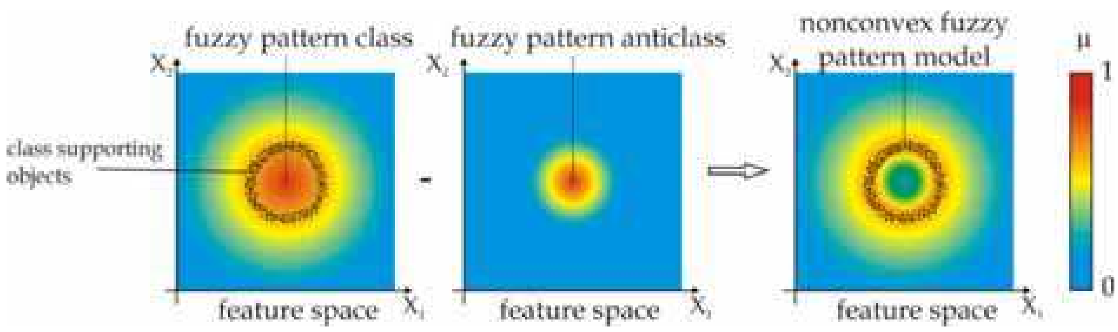


Fig. 18. Anti–class approach for a ring shaped data structure

Like it is depicted, the negation works on semantical level and from this point of view FPACs can be seen as a further specification of a preceding fuzzy pattern class. The repeated specification of fuzzy pattern classes and anti–classes can be interpreted as a network of such classes.

In order to conserve the modelling framework, the automated model generation and the model properties (such as flexibility, interpretability, computational efficiency, etc.) the negating anti–classes are defined upon the same membership function concept as the fuzzy pattern classes. Due to this definition it is also valid that FPACs, like usual fuzzy pattern classes, can be supported by objects, or better so called anti–objects, and that the before elaborated aggregation procedure can be applied on these anti–objects.

The crucial point of this approach lies in the determination of a set of anti–objects forming an inverse data–inherent structure. Unfortunately these anti–objects are unavailable prior to the design such that, they have to be generated and distributed over the class space. Concretely this generation and distribution process is driven by the policy that anti–objects will exclusively accumulate in the unsupported class space within the borders of an object modelling FPC.

Figure 19 illustrates the results of the above outlined anti–object generation with the help of the example given in figure 16. Similar to figure 16 original objects are highlighted in blue, whereas the constructed anti–objects are displayed in orange.

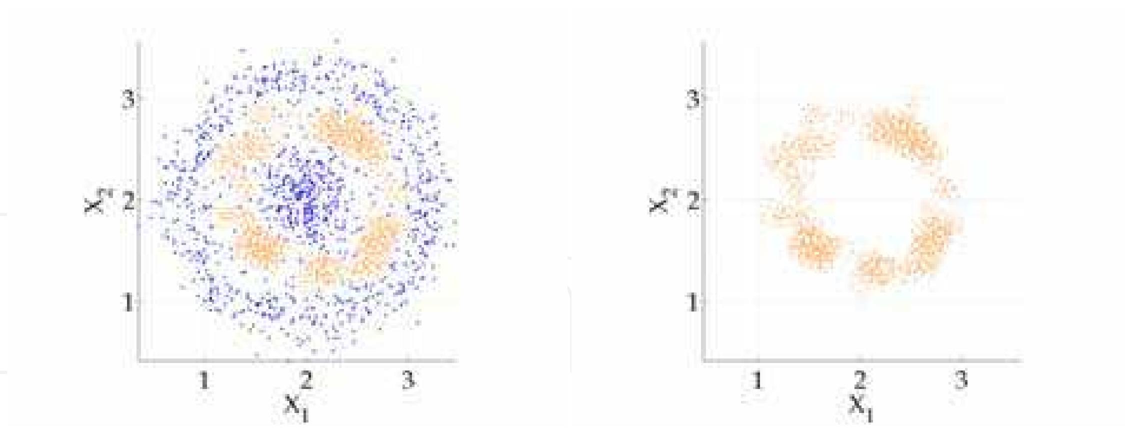


Fig. 19. Right: objects and generated anti–objects; left: anti–objects only

Due to distribution policy the anti–objects accumulate in the area between the ring and the central object agglomerations (see right side of figure 19) forming itself a ring–like anti or inverse data–inherent structure (see left side of figure 19). Because this ring–like anti–object structure is again a nonconvex structure the modelling fuzzy pattern anti–class will be inadequate and it might appear that the whole problem was just shifted to the anti–object structure. This is not the case since the entire procedure can be repeated on the anti–class yielding a convex set of anti–anti-objects and hence an appropriate anti–anti–class (see figure 18).
In sum the fuzzy pattern anti–class approach results in a sequence of three fuzzy pattern classifiers. The first one is a model over all given (original) objects it is further specified by the second classifier (anti–node) comprising all anti–objects being itself specified by the last classifier (anti–node). Figure 20 summarises the resulting fuzzy pattern model with the help of its memberships (right) and its network presentation (left).

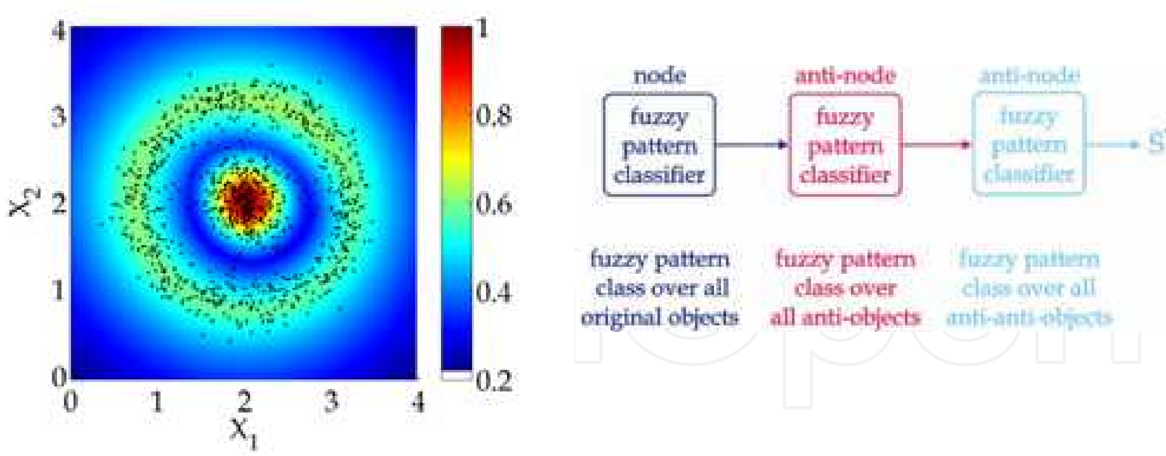


Fig. 20. Right: complete fuzzy pattern model; left: associated fuzzy pattern classifier network

The fuzzy pattern network works similar to its design. An unknown object is evaluated by the first classifier node ensuring a general membership to the data structure. Thereafter it is processed by the first anti–node, excluding a membership if it is situated in the centre. In the last step (third classifier node) this anti–class membership is negated for central located objects, ensuring that the region between centre and outer ring maintains a low membership.

6. Conclusion

This chapter dedicates itself towards the establishment of a closed fuzzy modelling framework for data–inherent structures. By and large the entire modelling process is conceived as fuzzy classification task, where superordinate fuzzy classes agglomerate structures of related data.
The closeness of this modelling framework is ensured by a *side–specific, parametric, basis function motivated, multivariate* membership function concept holding for data as well as for classes. Due to its central role the class membership function has been explicitly defined, its adoptions for objects of data have been motivated and its application has been sketched.
The main concern this chapter lies in the presentation of a data–driven algorithm to agglomerate fuzzy data to fuzzy class models without leaving the modelling framework. The innovation regarding this agglomeration is the treatment of data that exhibits heterogeneous elementary fuzziness (asymmetric measurement insecurities) and the consideration of these heterogeneous elementary fuzziness throughout the whole agglomeration process.
The resulting fuzzy pattern class model embraces advantageous properties like multi–dimensionality, shape diversity, semantic interpretability, transparency, unimodality and computational efficiency. The major drawback of the fuzzy class model arises from its convexity.
The patronage of fuzzy pattern classes for convex shaped data sets can be resolved with a network oriented design paradigm. In detail two state–of–the–art design approaches for networks of fuzzy pattern classifiers have been sketched. Their data–driven design and their combination are of particular interests for further research.
Another aspect of the here pursued type of structure modelling is that it works in the original feature space without a transformations applied for fuzzy support vector classifiers (Schölkopf, 2001; Li, 2008).

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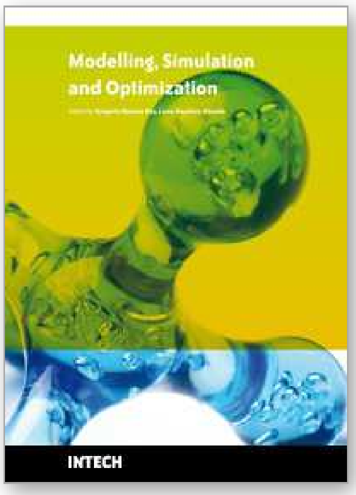

Modelling Simulation and Optimization

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Computer-Aided Design and system analysis aim to find mathematical models that allow emulating the behaviour of components and facilities. The high competitiveness in industry, the little time available for product development and the high cost in terms of time and money of producing the initial prototypes means that the computer-aided design and analysis of products are taking on major importance. On the other hand, in most areas of engineering the components of a system are interconnected and belong to different domains of physics (mechanics, electrics, hydraulics, thermal...). When developing a complete multidisciplinary system, it needs to integrate a design procedure to ensure that it will be successfully achieved. Engineering systems require an analysis of their dynamic behaviour (evolution over time or path of their different variables). The purpose of modelling and simulating dynamic systems is to generate a set of algebraic and differential equations or a mathematical model. In order to perform rapid product optimisation iterations, the models must be formulated and evaluated in the most efficient way. Automated environments contribute to this. One of the pioneers of simulation technology in medicine defines simulation as a technique, not a technology, that replaces real experiences with guided experiences reproducing important aspects of the real world in a fully interactive fashion [iii]. In the following chapters the reader will be introduced to the world of simulation in topics of current interest such as medicine, military purposes and their use in industry for diverse applications that range from the use of networks to combining thermal, chemical or electrical aspects, among others. We hope that after reading the different sections of this book we will have succeeded in bringing across what the scientific community is doing in the field of simulation and that it will be to your interest and liking. Lastly, we would like to thank all the authors for their excellent contributions in the different areas of simulation.