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Chapter

A Review Note on Laplace Transform and Its Applications in Dynamical Systems

Shivram Sharma, Praveen Kumar Sharma and Jitendra Kaushik

Abstract

Laplace Transform is one of the essential transform techniques. It has many applications in engineering and science. The Laplace transform techniques can be used to solve various partial differential equations and ordinary differential equations that cannot be resolved using conventional techniques. The Laplace transform approach is practically the essential functional method for engineers. The Laplace transform and variations like the fuzzy Laplace transform are advantageous because they directly solve issues such as initial value problems, fuzzy initial value problems, and nonhomogeneous differential equations without first resolving the corresponding homogeneous equation. This chapter uses the Laplace transform and its variations to dynamical systems.

Keywords: Laplace transform, Inverse Laplace transform, Fuzzy Laplace transform properties, applications, initial value problem, electrical circuits

1. Introduction

A problem can be solved easily in another domain using an integral transform, and the solution is then transformed using an inverse transform back to the original domain. The Laplace transformation is one such transformation that Pierre-Simon Laplace found in 1785. A popular integral transform in mathematics with several uses in science and engineering is the Laplace Transform. A transformation from the time domain, where the inputs and outputs are time functions, to the frequency domain, where the inputs and outputs are functions of complex angular frequency, is what the Laplace Transform entails. The Laplace transform is frequently used to convert a differential equation system into an algebraic equation system and to multiply a convolution [1–5]. It entails decomposing a system of differential equations into a set of linear equations that must be solved and then applying the inverse Laplace transform to return the answer to the time domain. In many circumstances, the result is divided into “patterns,” for which the inverse transform is known. The inverse Laplace transform is then used to return the solution to the time domain.

Definition 1.1. Suppose that \( f \) is real or complex-valued function of time \( t > 0 \), and \( p \) is a real or complex parameter, then the Laplace transform of \( f(t) \) is defined as...
\[ \mathcal{L}[f(t)] = \int_0^\infty e^{-pt} f(t) \, dt = F(p) \tag{1} \]

(Provided the integral defined in (1) exists)

**Example 1.1.** If \( f(t) = 1 \) for \( t \geq 0 \), then The Laplace Transform of this function \( f(t) = 1 \) can be obtained by using (1) as:

\[ \mathcal{L}[1] = \int_0^\infty e^{-pt} \cdot 1 \, dt = \int_0^\infty e^{-pt} \, dt = \lim_{t \to \infty} \frac{e^{-pt}}{-p} \bigg|_0^\infty = \frac{1}{p} \]

Thus, we have Laplace transform of \( f(t) = 1 \), which is given by \( \mathcal{L}[1] = \frac{1}{p} \).

Taking this formula’s, Inverse Laplace Transform \( (\mathcal{L}^{-1}) \), we get the inverse Laplace Transform of \( \frac{1}{p} \), which is given by \( \mathcal{L}^{-1} \left( \frac{1}{p} \right) = 1 \).

Similarly, following the above procedure, we can find the Laplace transform and inverse Laplace transforms of other valuable functions.

Some essential formulae of LT and ILT are given below in tabular form **Table 1**, [5], which are very useful for finding out the results of problems/systems by the Laplace transform method.

<table>
<thead>
<tr>
<th>S. No</th>
<th>Laplace transform</th>
<th>Inverse Laplace transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>( \frac{1}{p} )</td>
</tr>
<tr>
<td>2</td>
<td>( e^t )</td>
<td>( \frac{1}{p} )</td>
</tr>
<tr>
<td>3</td>
<td>( t^n, n = 1, 2, 3 )</td>
<td>( \frac{n!}{p^n} )</td>
</tr>
<tr>
<td>4</td>
<td>( t^n, n &gt; -1 )</td>
<td>( \frac{(n+1)!}{p^{n+1}} )</td>
</tr>
<tr>
<td>5</td>
<td>( \sin(at) )</td>
<td>( \frac{a}{p^2 + a^2} )</td>
</tr>
<tr>
<td>6</td>
<td>( \cos(at) )</td>
<td>( \frac{p}{p^2 + a^2} )</td>
</tr>
<tr>
<td>7</td>
<td>( \sinh(at) )</td>
<td>( \frac{a}{p^2 - a^2} )</td>
</tr>
<tr>
<td>8</td>
<td>( \cosh(at) )</td>
<td>( \frac{p}{p^2 - a^2} )</td>
</tr>
<tr>
<td>9</td>
<td>( f(t) )</td>
<td>( \frac{1}{p} F(\zeta) )</td>
</tr>
<tr>
<td>10</td>
<td>( e^t f(t) )</td>
<td>( F(p - c) )</td>
</tr>
<tr>
<td>11</td>
<td>( t^n f(t), n = 1, 2, 3 \ldots )</td>
<td>( (-1)^n p^n F^n(\zeta) )</td>
</tr>
<tr>
<td>12</td>
<td>( f'(t) )</td>
<td>( \int_0^p F(u) , du )</td>
</tr>
<tr>
<td>13</td>
<td>( \int_0^p f(u) , du )</td>
<td>( \frac{F(\zeta)}{p} )</td>
</tr>
<tr>
<td>14</td>
<td>( \int_0^p f(t - \Gamma g(T)) , d\Gamma )</td>
<td>( F(p) G(p) )</td>
</tr>
<tr>
<td>15</td>
<td>( f(t + T) = f(t) )</td>
<td>( \frac{\int_0^p f(u) , du}{p} )</td>
</tr>
<tr>
<td>16</td>
<td>( f'(t) )</td>
<td>( p F(p - f(0)) )</td>
</tr>
<tr>
<td>17</td>
<td>( f''(t) )</td>
<td>( p^2 F(p) - pf(0) - f'(0) )</td>
</tr>
</tbody>
</table>

**Table 1.**

Formulate of LT and ILT.
2. A sufficient condition for the Existence of Laplace transforms any function

The existence of LT of any function \( f(t) \) depends on the piecewise continuity of the function on the interval \([0, \infty)\) and the exponential order of the function.

If a function is piecewise continuous on the interval \([0, \infty)\) and has exponential order \( \alpha \) (for \( t \geq 0 \), for some \( p > \alpha \)), then the LT of function \( f(t) \) exists.

The above conditions are only sufficient conditions but not necessary conditions for the existence of the Laplace Transform of any function. A function may have Laplace transform even if it violates the above conditions/existence conditions.

For example: The function \( f(t) = t^{-\frac{1}{2}} \) is not continuous at \( t = 0 \), even though its Laplace transform exists.

Moreover, when the analysis depends on a mathematical model of differential equations due to the uncertainty of future aspects, we require mathematical tools to prescience other risks. We can categorize uncertainties into two categories. 1. Possible sets or fuzzy sets handle uncertainty. 2. Unpredictable uncertainty, which probability models shall deal with.

The publication of a seminal paper by Zadeh [6], a computer scientist from the University of California, was a crucial turning point in developing the modern concept of uncertainty. In his seminal paper, the USA was the first to introduce the concept of fuzzy set theory as a new way to represent vagueness in our daily lives. Zadeh defined “fuzzy sets” in his theory as sets with ill-defined bounds. This idea is employed and found to be superior for solving issues across many fields.

Recently, the analysis of dynamical systems has obtained more attention due to mathematical models of fuzzy applications through Laplace transforms. Hence, pre-eminence and optimal solution are required through fuzzy concepts of H- derivatives and SGH derivatives (1) gH-derivative, g-derivative and gr-derivative. Few researchers such as Najariyan, & Farahi [7], Najariyan, & Farahi [8], Najariyan, & Zhao [9], Mazandarani & Pariz [10] and Najariyan & Zhao [11] attempt the work of optimal solution through the dynamic systems of the fuzzy approach.

Allahviranloo and Ahmadi [12] recently proposed a fuzzy Laplace transformation for first-order fuzzy differentiation equations. They keep focused on the fuzzy Laplace transform concept yet to explain the fuzzy valued conditions. Then, Salahshour and Allahviranloo [13] focused on first- and second-order derivative Laplace transform for linear, continuous, uniform, and convergence problems. They considered FIVP, H-differentiation, and second-order derivatives. The large-size fuzzy-valued function can be executed by fuzzy Laplace transformation.

Using fuzzy differential equations (FDEs) to model dynamic systems with uncertainty makes sense. First-order linear fuzzy differential equations are among the most fundamental FDEs in several applications. Chang and Zadeh initially presented the fuzzy derivative concept in 1972 [14]. Kandel and Byatt [15, 16] analyzed fuzzy dynamical issues using the idea of FDEs. The fuzzy Laplace transform method solves FDEs and their fuzzy beginning and boundary value problems. Fuzzy Laplace transforms reduce an FDE to an algebraic problem, which facilitates its solution.

Here in this chapter, we aim to study the applications of Laplace transform in dynamical systems, so we present our study under four subsections 2.1, 2.2, 2.3, and 2.4. At the end of this chapter, we give a brief conclusion about the proposed research.
3. Main results/discussion/application of Laplace transform in a dynamic system

In this chapter, our main aim is to apply Laplace transform in a dynamic system. A dynamical system [17] is one in which something evolves over time or in which a function describes the time dependence of a point in the surrounding space. For instance, mathematical representations of the pendulum of a clock, the flow of water through a pipe, the number of fish in a lake each spring, population growth, and so forth.

Both a continuous timeline and discrete time increments can be used to depict a dynamic system.

Discrete-time dynamical system [17]

\[ x_t = F(x_{t-1}, t) \]  

This type of model is called a difference equation, a recurrence equation, or an iterative map (if the right-hand side is not dependent on).

Continuous-time dynamical system [17]

\[ \frac{dx}{dt} = F(x, t) \]

This type of model is called a differential equation.

The system's state variable at time \( t \) in both scenarios is \( x_t \) or \( x \), which may take a scalar or vector value. The rule by which the system changes its state over time is determined by a function called \( F \).

Differential equations often model dynamical systems.

So, here we discuss an application of Laplace transform and its variant (e.g., fuzzy Laplace transform) to solve differential equations/electrical circuits/mechanical systems/fuzzy differential equations:

3.1 Solution of differential equations (including IVP and system of simultaneous differential equations) by using Laplace transform technique

Example 2.1.1. Consider an IVP

\[ \frac{dy}{dt} + y = \sin t; y(0) = 1 \]

Taking the Laplace transform of both sides of the above equation, we have

\[ \mathcal{L}\left[\frac{dy}{dt}\right] + \mathcal{L}[y] = \mathcal{L}[\sin t] \]

\[ p\mathcal{L}[y(t)] - y(0) + \mathcal{L}[y(t)] = \frac{1}{p^2 + 1} \]

\[ (p + 1)\mathcal{L}[y(t)] = 1 + \frac{1}{p^2 + 1} \]

\[ \mathcal{L}[y(t)] = \frac{1}{p + 1} + \frac{1}{(p^2 + 1)(p + 1)} \]
Taking the Inverse Laplace transform of both sides of the above equation, we have

\[
y(t) = \mathcal{L}^{-1}\left[\frac{1}{p+1}\right] + \mathcal{L}^{-1}\left[\frac{1}{(p^2+1)(p+1)}\right]
\]

\[
y(t) = e^{-t} + \frac{1}{2} \mathcal{L}^{-1}\left[\frac{1}{(p+1)} - \frac{p}{p^2+1} + \frac{1}{p^2+1}\right]
\]

\[
y(t) = e^{-t} + \frac{1}{2} \left[ e^{-t} - \cos t + \sin t \right]
\]

\[
y(t) = \frac{3}{2} e^{-t} + \frac{1}{2} \left[ \sin t - \cos t \right]
\]

Which is the required solution for given IVP.

Remark 2.1.1. Mathematical modeling of the system is a must for analyzing/predicting the nature and behavior of any physical system. There are many physical systems wherein we get IVP, for example, the growth and decay population model of Malthus, which is represented by \( \frac{dy}{dt} = ky \) where \( y(t) \) is the population at any time \( t \) and the constant of proportionality \( k \) is the growth constant and is used for finding the bacteria in a culture, life of radioactive substance and Newton's law of cooling. A solution to these problems can be obtained by following the procedure in example 2.1.1 using Laplace transform techniques.

Example 2.1.2. Consider a system of simultaneous equations

\[
\begin{align*}
\frac{dx}{dt} + y &= \sin t \\
\frac{dy}{dt} + x &= 1 + \cos t
\end{align*}
\]

with \( x(0) = 0 \) & \( y(0) = 0 \) (4)

Using Laplace transform, we have

\[
\mathcal{L}\left[\frac{dx}{dt} + y\right] = \mathcal{L}[\sin t], \mathcal{L}\left[\frac{dy}{dt} + x\right] = \mathcal{L}[t + \cos t]
\]

\[
p \mathcal{L}[x(t)] - x(0) + \mathcal{L}[y(t)] = \frac{1}{p^2+1} \mathcal{L}[y(t)] - y(0) + \mathcal{L}[x(t)] = \frac{1}{p} + \frac{p}{p^2+1}
\]

\[
p \mathcal{L}[x(t)] + \mathcal{L}[y(t)] = \frac{1}{p^2+1} \mathcal{L}[y(t)] + \mathcal{L}[x(t)] = \frac{1}{p} + \frac{p}{p^2+1}
\]

On solving the above pair of equations for \( \mathcal{L}[\{et\}] \), we have

\[
\mathcal{L}[y(t)] = \frac{2}{p - 1} - \frac{2}{(p - 1)(p^2 + 1)}
\]

Taking the Inverse Laplace Transform of both sides of the above equation, we have

\[
y(t) = \mathcal{L}^{-1}\left[\frac{2}{p - 1}\right] - \mathcal{L}^{-1}\left[\frac{2}{(p - 1)(p^2 + 1)}\right]
\]

\[
y(t) = 2e^t - \mathcal{L}^{-1}\left[\frac{1}{p - 1} - \frac{p}{p^2 + 1} - \frac{1}{p^2 + 1}\right]
\]
\[ y(t) = e^t - \cos t - \sin t \] (5)

Putting the above value of \( y(t) \) in the second equation of equation no. (4)

\[
x(t) = 1 + \cos t - \frac{d}{dt}[e^t - \cos t - \sin t]
\]

\[ x(t) = 1 + \cos t - [e^t + \sin t - \cos t] \]

\[ x(t) = 1 - e^t - \sin t \] (6)

Equations 2nd and 3rd together give the solution of the given system of simultaneous differential equations.

**Remark 2.1.2.** In many physical problems/situations like particle movement along any curve at time \( t \), two tanks in mixing problems, and two circuits in electrical networks, etc., we get a system of ordinary differential equations. The solution to all these problems can also be obtained by following the procedure in example 2.1.2 using Laplace transform techniques.

### 3.2 Solution/response/current in electrical circuits by using Laplace transform technique

This section finds the electrical circuits’ response/solution/current \( i(t) \) using Laplace transform techniques. Many authors have done work in this field, but their results were different and proved using a different methodology. Some basic rules and principles of the area are required to prove results under this section. Although there are many advanced books where we can get these basic rules/definitions/principles, we refer to [5] for this purpose.

**Definition 2.2.1 [5]. Kirchhoff’s Laws**

I. **Voltage Law:** the algebraic sum of the voltage drops around any closed circuit equals the circuit’s resultant electromotive force (EMF).

II. **Current Law:** at a junction or node, current coming is current going.

**Example 2.2.1. (Response/Solution/Current \( i(t) \) in the L-R Series Circuit)**

A simple R-L series circuit is shown in **Figure 1**, where resistance (R) and Inductance (L) are in series with voltage source \( E(t) \). Let \( i(t) \) be the current flowing in the circuit at any time \( t \).

![R-L series circuit](image_url)
Then, the model of the L-R series circuit is given by:

\[
R \frac{di}{dt} + \frac{L}{i} = E(t) \quad \text{(By voltage law)}
\]

(7)

It is assumed that the current is initially zero, i.e., \(i = 0\) at \(t = 0\).

Taking the Laplace transform of both sides of the above equation, we get

\[
\mathcal{L} \left[ \frac{di}{dt} + \frac{R}{L} i \right] = \mathcal{L} \left[ \frac{E}{L} \right]
\]

\[
\mathcal{L} \left[ \frac{di}{dt} \right] + \frac{R}{L} \mathcal{L}[i] = \frac{E}{L} \mathcal{L}[1]
\]

(8)

Taking the inverse Laplace transform of both sides of the above equation, we get

\[
i(t) = \frac{E}{L} \mathcal{L}^{-1} \left[ \frac{1}{p} \right]
\]

\[
i(t) = \frac{E}{R} \mathcal{L}^{-1} \left[ \frac{1}{p} - \frac{1}{p + \frac{E}{R}} \right]
\]

\[
i(t) = \frac{E}{R} \left[ 1 - e^{-\frac{E}{R}} \right]
\]

(9)

Which is the required solution of the given L-R series circuit.

**Example 2.2.2. (Response/Solution/Current \(i(t)\) in the RLC circuit)**

In Figure 2, an RLC circuit is shown. An RLC circuit is obtained from an RL circuit by adding a capacitor. We have modeled the RL circuit in example 2.2.1, which is given by

\[
R \ i + L \frac{di}{dt} = E(t)
\]

Figure 2.

*RLC circuit.*
We obtain the RLC circuit simply by adding the voltage drop $Q/C$ across the capacitor.

Here, current $i(t) = \frac{dQ}{dt}$ (or) $Q(t) = \int i(t) dt$

Assuming a sinusoidal EMF as in the figure. Thus, the model of the RLC circuit is given by:

$$L \frac{d^2i}{dt^2} + iR + \frac{1}{C} \int i(t) dt = E(t) \quad (10)$$

Differentiating 1st concerning $t$, we have

$$L \frac{d^2i}{dt^2} + R \frac{di}{dt} + \frac{1}{C} i = E'(t) = E \quad (11)$$

The solution of 2nd will give the current $I$ in the RLC circuit

Taking Laplace transform of both sides of 2nd, we have

$$\frac{L}{s^2} [i(t)] + \frac{R}{s} [i(t)] + \frac{1}{sC} [i(t)] = \frac{E}{s} \quad (11)$$

(As current and capacitor charge are 0 when $t = 0$, so $i(0) = 0$, and $i'(0) = 0$)

This implies that

$$L[i(t)] = \frac{1}{p\left(\frac{R}{p} + \frac{\gamma}{p} + 1\right)} \quad (12)$$

(where $\gamma = RC, \delta = LC$

Taking Inverse Laplace Transform of both sides of the above equation we have

$$i(t) = \frac{\gamma + \delta p}{1 + \gamma p + \delta p^2} \quad (12)$$
This implies that

$$i(t) = EC - EC(\gamma) \left( \frac{2\delta}{\sqrt{4\delta - \gamma^2}} \right) e^{-\frac{\gamma}{2\delta} t} \sin \left( \frac{\sqrt{4\delta - \gamma^2}}{2\delta} \right) t$$

$$- EC(\delta)e^{-\frac{\gamma}{2\delta} t} \left[ \cos \left( \frac{\sqrt{4\delta - \gamma^2}}{2\delta} \right) t - \left( \frac{\gamma}{\sqrt{4\delta - \gamma^2}} \right) \sin \left( \frac{\sqrt{4\delta - \gamma^2}}{2\delta} \right) t \right]$$

(13)

(where, $\gamma = RC, \delta = LC$)

Which is the required solution of the given RLC circuit.

3.3 Solution of mechanical systems by using Laplace transform technique

Some basic rules and principles of the field are required to prove our main result in this section. We refer to [5] for this purpose.

**Example 2.3.1. (Torsional Pendulum Experiment)**

In Figure 3, a Torsional Pendulum is demonstrated.

It consists of a disk or rod suspended at the end of the wire.

When the end of the wire is twisted at an angle $\Theta$

The restoring torque $\tau$ arises

Then by Hooke’s Law: $\tau = -k\Theta$ ... (1) (where $k$ is called the torsional constant).

If the wire is twisted and released, the oscillating system is called a torsional pendulum.

By Newton’s Second Law: $\tau = I\alpha$ ... (2) (where $= \frac{d^2\Theta}{dt^2}$)

By 1st and 2nd and we have

$$-k\Theta = I \frac{d^2\Theta}{dt^2}$$

Which can be rewritten as

$$\frac{d^2\Theta}{dt^2} + k \frac{\Theta}{I} = 0$$

(Or)

$$\frac{d^2\Theta}{dt^2} + \omega^2 \Theta = 0$$

(14)

This is called the equation of a simple harmonic oscillator, whose angular frequency is $\omega = \sqrt{\frac{k}{I}}$ and period is $T = 2\pi \sqrt{\frac{I}{k}}$

![A Torsional Pendulum.](A_Torsional_Pendulum.png)
Now taking Laplace transform of both sides of the above equation, we have

\[ \mathcal{L}\left[\frac{d^2 \phi}{dt^2}\right] + \omega^2 \mathcal{L}[\phi] = 0 \]

\[ [p^2 \mathcal{L}[\phi(t)] - p\phi(0) - \phi'(0)] + \omega^2 \mathcal{L}[\phi(t)] = 0 \]

[Let, \( \phi(0) = A \) and \( \phi'(0) = B \)]

\[ (p^2 + \omega^2) \mathcal{L}[\phi(t)] = pA + B \]

\[ \mathcal{L}[\phi(t)] = \frac{pA + B}{p^2 + \omega^2} \] (15)

Now taking the Inverse Laplace Transform of both sides of Eq. (16)

\[ \phi(t) = \mathcal{L}^{-1}\left[\frac{pA + B}{p^2 + \omega^2}\right] \]

\[ \phi(t) = A \mathcal{L}^{-1}\left[\frac{p}{p^2 + \omega^2}\right] + B \mathcal{L}^{-1}\left[\frac{1}{p^2 + \omega^2}\right] \]

\[ \phi(t) = A \cos \omega t + B \sin \omega t \] (16)

Which is the required solution of the given Mechanical System.

**Remark 2.3.1.** The balance wheel in a clock or wristwatch is an example of a torsional pendulum. All simple harmonic oscillators satisfy a differential equation 3rd of the above example. Hence, the general solution (displacement) of all bodies moving with simple harmonic motion can be obtained by following the procedure given in the above example using Laplace transform technique.

### 3.4 Solution of fuzzy differential equations (FDEs) by using fuzzy Laplace transform

The chapter by Sharma et al. [17] titled “Applications of fuzzy set and fixed point theory in Dynamical systems” and published in the open-access book “Qualitative and Computational Aspects of Dynamical Systems” with the ISBN: 978-1-80356-567-5 contains a detailed discussion of this section (one can refer to this chapter and can go through the definition 2.1, theorem2.2, formulae 2.3, and remark 2.5 of this chapter to understand the concept of this sub-section 2.4).

**Example 2.4.1.** Consider the initial value problem

\[ \begin{align*}
    y'(t) &= y(t), \quad 0 \leq t \leq T \\
    y(0) &= (y(0), \alpha), \quad (y(0), \alpha)
\end{align*} \]

by using the fuzzy Laplace transform method, we have

\[ L[y'(t)] = L[y(t)], \quad \text{and} \quad L[y'(t)] = \int_0^t y'(t) \odot e^{-sp} \, dt \]

in (i)-differentiable then by using Case (1), we have

\[ L[y'(t)] = (s \cdot L[y(t)]) \odot y(0) \]

Therefore, \( L[y(t)] = s \cdot L[y(t)] \odot y(0) \)
\[ l[y(t, \alpha)] = s l[y(t, \alpha)] - y(0, \alpha) \]
\[ l\left[y(t, \alpha)\right] = s l[y(t, \alpha)] - y(0, \alpha) \ldots \] (17)

Hence, the solution of system (18) is:
\[ l[y(t, \alpha)] = -y(0, \alpha) \left(\frac{s}{s^2 - 1}\right) + y(0, \alpha) \left(-\frac{1}{s^2 - 1}\right) \]
\[ l\left[y(t, \alpha)\right] = -y(0, \alpha) \left(\frac{s}{s^2 - 1}\right) + y(0, \alpha) \left(-\frac{1}{s^2 - 1}\right) \]

Thus
\[ y(t, \alpha) = -y(0, \alpha) l^{-1} \left[\left(\frac{s}{s^2 - 1}\right) + y(0, \alpha) t^{-1} \left(-\frac{1}{s^2 - 1}\right)\right] \]
\[ y(t, \alpha) = -y(0, \alpha) l^{-1} \left[\left(\frac{s}{s^2 - 1}\right) + y(0, \alpha) t^{-1} \left(-\frac{1}{s^2 - 1}\right)\right] \]

Finally, we have:
\[ y(t, \alpha) = e^{-t} \left(\frac{y(0, \alpha) - y(0, \alpha)}{2}\right) - e^{t} \left(\frac{y(0, \alpha) + y(0, \alpha)}{2}\right) \]
\[ y(t, \alpha) = e^{-t} \left(\frac{-y(0, \alpha) + y(0, \alpha)}{2}\right) - e^{t} \left(\frac{y(0, \alpha) + y(0, \alpha)}{2}\right) \]

If \( y'(t) \) in (ii)-is differentiable, then by using Case II, we have
\[ L[y'(t)] = -(y(0)) \oplus (-s L[y(t)]) \]. Therefore, \( L[y(t)] = (-y(0)) \oplus (-s L[y(t)]) \)
\[ l[y(t, \alpha)] = s l[y(t, \alpha)] - y(0, \alpha) \]
\[ l\left[y(t, \alpha)\right] = s l[y(t, \alpha)] - y(0, \alpha) \ldots \] (18)

Hence, the solution of system (2) is:
\[ l[y(t, \alpha)] = -y(0, \alpha) \left(\frac{1}{1+s}\right) \]
\[ l\left[y(t, \alpha)\right] = -y(0, \alpha) \left(\frac{1}{1+s}\right) \]

Thus
\[ y(t, \alpha) = -y(0, \alpha) l^{-1} \left(\frac{1}{1+s}\right) \]
Finally, we have:
\[ y(t, \alpha) = -y(0, \alpha)l^{-1}\left(\frac{1}{1+s}\right) \]

4. Conclusion

This chapter discussed the Laplace transform and fuzzy Laplace transform in dynamic systems. Using the Laplace transform and the fuzzy Laplace transform methods, we provided solutions to first- and second-order ordinary differential equations and first- and second-order fuzzy ordinary differential equations. We demonstrated how the Laplace transform method could determine the solution (current flow) of first- and second-order electrical circuits and a mechanical system's solution (vibration frequency).

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References


