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Chapter

Effect of Titanium Oxide Nanofluid over Cattaneo-Christov Model

Hammad Khalil, Tehseen Zahra, Zaffer Elahi and Azeem Shahzad

Abstract

The proposed chapter deals with the study of heat transfer development of titanium oxide nanofluid of platelet shape nanoparticles over a vertical stretching cylinder. The set of nonlinear equations is obtained using suitable transformation on the governing equations that are then solved with numerical scheme BVP4C. The obtained results are interpreted graphically and numerically. The effects of Prandtl, Eckert, and unsteadiness parameters on temperature distribution are depicted. Moreover the skin friction and Nusselt number are also computed.

Keywords: Cattaneo-Christov model, heat transfer, vertical cylinder

1. Introduction

Cattaneo-Christov model is an improved version of Fourier law as Fourier law does not detect the initial temperature disturbance; to overcome this ambiguity, Cattaneo added a thermal relaxation parameter. This parameter covers the ambiguity of Fourier law. The classical Fourier law is obtained while vanishing the relaxation parameter [1]. Cattaneo-Christov heat flux model gives us heat transfer rate in stretching cylinders as well as sheets. Heat transfer is a wonderful natural phenomenon that occurs when two bodies have a thermal difference until both bodies are at thermal equilibrium. The Cattaneo-Christov model is in the form of a heat equation. The thermal convection effect is studied using the Christov heat model in conjunction with the Cattaneo heat model [2]. It has been realized that the development of stretchy surfaces and the flow field that surrounds them speaks to a variety of technological and industrial applications, such as paper making, glass blowing, crystal growth, and aerodynamic plastic sheet extrusion [3]. Heat transfer is a common natural occurrence as long as there is a temperature differential between things or between various regions of the same object, heat transfer will occur. As a result, a lot of effort has gone into predicting the heat transport behavior. In several starting and boundary problems, the uniqueness and structural stability of the solutions for the temperature governing equations using the Cattaneo-Christov heat flow model have been demonstrated. The chapter released uses the Cattaneo-Christov heat flux model to analyze the flow and heat transfer of upper-convective Maxwell fluid across a stretching sheet [4]. Efforts have been undertaken to increase the thermal efficiency of processes during the last many decades. On the one hand, there has been an
attempt to lower the size of the equipment by increasing the thermal exchange surface, such as with fins, and on the other hand, novel fluid exchangers with higher thermal conductivity have been developed. Different NPS types (metallic, nonmetallic, and carbon based) have been synthesized and dispersed in conventional fluids such as water, oil, or ethylene glycol referred to as nanofluids since the advent of nanotechnology and the possibility of synthesizing materials on a nanometric scale [5]. The boundary layer flow and heat transfer caused by stretching flat plates or cylinders are both practical and theoretically interesting in fiber technology and extrusion operations. This method is used to produce polymer sheets and plastic films. The cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyors, the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film in condensation processes, paper production, glass blowing, metal spinning and drawing plastic films, and polymer extrusion are all examples of boundary layers [6]. The aim of this chapter is to manipulate the heat transfer rate of titanium oxide nanofluid with the Cattaneo-Christov heat flux model over a vertical stretching cylinder.

2. Mathematical formulation

In the coordinate plane, assume that the cylinder is taken in the vertical direction along the $z$-axis, and the $r$-axis is normal to the axis of the cylinder. Consider the fluid is moving with surface velocity,

$$U_w = \frac{b z}{1 - \alpha t}$$

In the direction of stretching cylinder under the external magnetic field defined by

$$B(t) = \frac{B_0}{\sqrt{1 - \alpha t}}$$

Further, suppose $u = u(r, z, t)$ and $w = w(r, z, t)$ be the velocity components along the respective axes of the coordinate plane, while $T = T(r, z, t)$ denotes the temperature of the nanofluid, as shown in Figure 1.

![Figure 1.
Representation of the dilemma.](image)
Let the base fluid (water) and platelet-shaped NPS in thermal equilibrium. Under these assumptions, the equation of continuity, momentum equation, and energy equations are obtained, which are as follows:

\[
\frac{\partial (ru)}{\partial r} + \frac{\partial (rw)}{\partial z} = 0 \quad (1)
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \frac{\nu}{C_p} \left( \frac{\partial w}{\partial r} \right) + \kappa \left( \frac{\partial^2 w}{\partial r^2} \right) + \left( \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} \right) \left( \frac{\partial T}{\partial r} \right) + \left( \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) \frac{\partial T}{\partial z} + \frac{\partial^2 T}{\partial t^2} \right) \quad (2)
\]

Subject to the boundary conditions

\[
u = 0, w = U \omega, \frac{\partial T}{\partial r} = 0 \text{ at } r = R,
\]

\[
w \to 0, T \to T_\infty \text{ as } r \to \infty \quad (4)
\]

The thermophysical properties of density ($\rho_{nf}$), dynamic viscosity ($\mu_{nf}$), electric conductivity ($\sigma_{nf}$), diffusivity ($\alpha_{nf}$), and heat capacity ($\rho C_p$) can be defined in Refs. [7, 8], while the ratio of thermal conductivity of nanofluid and base fluid is given by the following equation:

\[
\frac{k_{nf}}{k_f} = \frac{k_i + (m - 1)k_f + (m - 1)(k_i - k_f)\phi}{k_i + (m - 1)k_f - (k_i - k_f)\phi} \quad (5)
\]

where $\phi$ denotes volume-fraction of NPS.

The thermophysical properties of titanium nanofluid with base fluid as water [9] are given in Table 1, while the viscosity coefficients $A1, A2$, and shape factor $m$ values of TiO$_2$ nanofluid [10] are listed in Table 2.

<table>
<thead>
<tr>
<th>Base</th>
<th>Density (kg/m$^3$)</th>
<th>Thermal conductivity (W/m K)</th>
<th>Specific heat (J/kg K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TiO$_2$</td>
<td>3900</td>
<td>8.4</td>
<td>0.8692</td>
</tr>
<tr>
<td>H$_2$O</td>
<td>997.1</td>
<td>0.613</td>
<td>4179</td>
</tr>
</tbody>
</table>

Table 1. Thermophysical properties of base fluid and TiO$_2$ nanoparticles.
Introducing the transformations, as

$$T = T_\infty + (Tw - T_\infty) \theta(\eta), \eta = \left(\frac{c}{\nu(1 - at)}\right)\left(\eta^2 - R^2\right).$$

$$\psi(\eta) = \left(\frac{c\nu}{\nu(1 - at)}\right)2rf(\eta)\theta(\eta)$$

where \(\psi\) is the stream function (describes the flow pattern) and is defined as \(u = \frac{\partial \psi}{\partial \eta}\) and \(w = \frac{\partial \psi}{\partial \eta}\). The governing Eqs. (2)–(5) have been transformed to Eqs. (8)–(10) using similarity variables in Eq. (7), as

$$e_1(1 + 2\eta)f''''(\eta) + 2e_1Cf''(\eta) - e_3M f(\eta) \left[ f(\eta)f'''(\eta) - S\left(f'(\eta) + \frac{\eta}{2}f''(\eta)\right)\right] + \lambda \theta(\eta) = 0.$$  

$$
(1 + 2\eta)\left(e_1Ec f''''(\eta) + \frac{e_2}{Pr}f''(\eta) - S\left(20(\eta) + \frac{11}{2}f''(\eta)\right)\right)

+ \beta \left[ f''(\eta)\theta''(\eta) - f(\eta)f''(\eta)\theta'(\eta) - f(\eta)f''(\eta)\theta(\eta) + f''(\eta)\theta(\eta) \right]

-f''''(\eta)\theta(\eta),$$

where \(\beta\) is the thermal relaxation parameter and is given by,

$$\beta_1 = \frac{c\lambda_1}{(1 - at)}.$$  

Under the boundary condition,

$$f'(0) = 0, f''(0) = 1, \theta'(0) = -\frac{k_f}{k_{surf}} \gamma(1 - 0(0)) \text{ at } \eta = R$$

$$f'(\eta) = 0, \theta(0) \text{ as } \eta \to \infty$$

Now the dimensionless constants, such as \(Ec, Pr, \phi, M,\) and \(S\), and that of \(e_1, e_2,\) and \(e_3\) are used frequently in the above equations, defined in Ref. [11]. For various
values of dimensionless parameters, the value of the local Nusselt number is shown in Table 3. Nusselt number can be defined as,

$$\text{Nu} = \frac{2k_{nf}}{k_f(T_w - T_\infty)} \left[ \frac{\partial T}{\partial r} \right]_{r=R}$$

(11)

The non-dimensionless form of Eq. (11), using Eq. (6), as

$$Re^\frac{\eta}{L} \text{Nu} = - \frac{k_{nf}}{k_f} \theta(0)$$

(12)

3. Method of solution

To find the numerical solution of a nonlinear system (7) and (8), the set of first-order linear equations is obtained by considering the following assumptions. By putting these assumptions in the above equations, we get first-order linear equations, which are then used in MATLAB by using BVP4C scheme to get numerical and graphical results.

$$y_1 = f,$$
$$y'_1 = y_2,$$
$$y'_2 = y_3,$$
$$y'_3 = g_1,$$
$$\theta = y_4,$$
$$y'_4 = g_1,$$
$$y'_5 = g_2,$$

$$y_1(0) = 0, y_2(0) = 1, y_4(0) - 1 = 0 \text{ at } \eta = 0$$
$$y_2(\eta) = 0, y_4(\eta) = 0 \text{ as } \eta \rightarrow \infty$$

<table>
<thead>
<tr>
<th>Physical parameters</th>
<th>Platelet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ee$</td>
<td>$Pr$</td>
</tr>
<tr>
<td>0.0</td>
<td>6.0</td>
</tr>
<tr>
<td>0.5</td>
<td>—</td>
</tr>
<tr>
<td>1.0</td>
<td>—</td>
</tr>
<tr>
<td>1.0</td>
<td>4.0</td>
</tr>
<tr>
<td>—</td>
<td>6.0</td>
</tr>
<tr>
<td>—</td>
<td>8.0</td>
</tr>
</tbody>
</table>

Table 3. Nusselt number of platelet shape nanoparticle.
where

\[ g_1 = \frac{1}{\varepsilon_1(1 + 2\eta C)} \left(\varepsilon_1 M y_2 - 2\varepsilon_1 C y_3 - \lambda y_4 - (y_1 y_3 - y_2^2 - S(y_2 + \frac{\eta}{2} y_3))\right), \]  

(22)

and

\[ g_2 = \frac{1}{Pr} \beta_1 \left(\frac{(S\eta)^2}{4} - S\eta y_1 + y_1^2\right) \left(S(y_4 + \frac{\eta}{2} y_5) - y_4 + y_3 y_4 - \left(\frac{2\varepsilon_2 C y_5}{Pr}\right) + (1 + 2\eta C)\varepsilon_1 E c y_3 - \beta_1 (\varepsilon_1 E c (1 + 2\eta C)(3S y_3^2) + S y_3 y_5' - 2\varepsilon_1 E c C y_4 y_3^2 - 2y_2 y_3 y_5') - \left(S^2 \left(6y_4 + \frac{11}{4} \eta y_5\right) - S \left(5y_2 y_4 - \frac{11}{2} y_2 y_5 + \left(\eta - \frac{1}{2} y_2 y_5 + \frac{\eta}{2} y_4 + y_3 y_5 + y_3 y_4\right)\right)\right)\right). \]

(23)

\( g_1 \) and \( g_2 \) are the obtained first-order linear equations.

### 4. Analysis of results

Obtained numerical results and the effect of various parameters on temperature profile are obtained in both numerical and graphical form and discussed in detail.

#### 4.1 Graphical analysis

Figure 2 describes the influence of Eckert number for platelet shape nanoparticle. By varying the value of Eckert, it can be seen that the temperature is increasing. Physically, it can be seen that the Eckert number enhances the thermal conductivity of the fluid. Figure 3 shows that the increase in the value of the Prandtl number results...
in deceleration in temperature because of the reduction in thermal diffusivity. As the unsteadiness parameter increases, the temperature gradually decreases, as shown in Figure 4. Physically the value of the unsteadiness parameter is grown up, and the thickness of the thermal boundary layer decreases, which results in a decline in the temperature profile.

4.2 Numerical results

The heat transfer rate is calculated for platelet shape nanoparticles, which is given in Table 3. It is inferred that with the rise in Eckert number, Nusselt number increases while the reverse trend is seen for Prandtl number.
5. Conclusion

By numerical computation, the effect of platelet shape nanoparticle on TiO$_2$ nanofluid over a vertical stretching cylinder is seen in this chapter. Influence of different physical parameters, such as Eckerd and Prandtl numbers on temperature profile, is examined both graphically and numerically. The Nusselt number increases for Eckert number Ec, which decreases for Prandtl number Pr. Graphical result shows acceleration in temperature profile, while the reverse trend is found in Figure 2.

Nomenclature

- $u, w$: velocity components along $r, z$ directions (m/s)
- $\alpha_f, \alpha_{nf}$: thermal diffusion of base fluid and nanofluid (m$^2$/s)
- $\rho_f, \rho_{nf}$: density of base fluid and nanofluid (kg/m$^3$)
- $\mu_f, \mu_{nf}$: viscosity of base fluid and nanofluid (kg m/s)
- $\nu_f, \nu_{nf}$: kinematic viscosity of base fluid and nanofluid (m$^2$/s)
- $\sigma_{nf}$: electrical conductivity ($-$)
- $U_w$: surface velocity (m/s)
- $(\rho C_p)_{nf}$: heat capacity of nanofluid ($-$)

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