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Redundant Actuation
of Parallel Manipulators

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1. Introduction

High stiffness, low inertia, large accelerations, and high precision are desirable properties attributed to parallel kinematics machines (PKM). However, relatively small workspace and the abundance of singularities within the workspace partly annihilate the aforementioned advantages. Redundant actuation and novel redundant kinematics are means to tackle these shortcomings. Redundant parallel kinematics machines are ideal candidates for use in high-precision applications, such as robot-assisted surgery. Their advantageous features promise to deliver the needed accuracy, stiffness, dexterity and reliability. Redundant actuation admits to eliminate singularities, increase the usable workspace, augment the dexterity, and partially control the internal forces. Actuator redundancy is also a means to improve fault tolerance, as redundant actuators can compensate the failure of other actuators. Redundant actuation increases the payload and acceleration, can yield an optimal load distribution among the actuators, or can reduce the power consumption of the individual drives. Actuator redundancy can also improve the force transmission properties and the manipulator stiffness. It can be purposefully exploited for secondary tasks, such as the generation of internal prestress and the generation of a desired compliance of the PKM. The first can be used to avoid backlash, whereas the second admits to homogenize the stiffness properties within the workspace. Kinematically redundant PKM, i.e. systems that possess a higher mobility then required for the task, allow to circumvent singularities as well as obstacles, and to increase the dexterity.

The control of redundantly actuated PKM poses additional challenges, rooted in the resolution of the redundancy within the control schemes. Whereas, model-based control techniques can be directly applied to the control of non-redundantly actuated PKM, redundancy, however, brings up two specific problems, one is the computationally efficient resolution of the actuation redundancy, and the other is the occurrence of unintentional antagonistic actuation due to model uncertainties.

This chapter is devoted to the modeling and control of redundantly actuated PKM. The aim of the chapter is to summarize concepts for dynamic modeling of redundantly actuated PKM, with emphasize on the inverse dynamics and control, and to clarify the terminology used in the context of redundant actuation. Based on a mathematical model, PKM are regarded as non-linear control systems.

The chapter is organized as follows. A short literature review in section 2 is meant to familiarize the reader with current developments and research directions. In order to point out the potential of redundantly actuated PKM, a motivating example is given in section 3.

The PKM motion equations are recalled in section 4 as basis for the subsequent considerations. The associated non-linear control problem is formulated in section 5, and used for the definition of actuation and redundancy in section 6. Section 7 is devoted to the resolution of actuator redundancy. For the important case of simply-redundant actuation a closed form solution to the inverse dynamics problem is given, and actuator redundancy is exploited for secondary tasks. The applicability of standard model based control schemes to redundantly actuated PKM is studied in section 8. The effect of geometric uncertainties is analyzed and shown to lead to interference effects that are peculiar to redundantly actuated PKM. An amended version of the augmented PD and computed torque control schemes is proposed that eliminates these effects. The chapter closes with a conclusion and hints to open problems in section 9.

2. Literature review

Compared to serial manipulators PKM exhibit a much richer phenomenology, and give rise to more types of redundancy. A brief overview of redundancy in PKM can be found in (Merlet, 1996). Redundantly actuated PKM were analyzed with regard to their kinematic and dynamic properties, and in view of singularities in (Alba et al., 2007; Dasgupta & Mruthyunjaya, 1998; Firmani & Podhorodeski, 2004; Gardner et al., 1989; Kim et al., 2001; Kock & Schumacher, 1998; Kurtz & Hayward, 1992; Liao et al., 2004; Mohamed & Gosselin, 1999; Müller, 2005; O'Brien & Wen, 1999; Valasek, 2002; Zhang et al., 2007).

It was shown by a number of authors that redundant actuation is a means to eliminate singularities and so enlarges the usable workspace. Redundant actuation can be achieved in different ways, and there are two directions: the actuation of passive joints, and the inclusion of additional kinematic chains without increasing the PKM DOF. Most authors propose using additional chains, such as planar 3RRR (Alba et al., 2007; Buttolo & Hannaford, 2005; Kock & Schumacher, 1998), planar 4RRR (Valasek, 2002), spherical wrists (Kurtz & Hayward, 1992) and shoulder (Yi et al., 1994), Stewart platforms with one (O'Brien & Wen, 1999) or two (Valasek, 2002) additional struts, or the Eclipse (Kim et al., 2001). The improvement of kinematic manipulability or dexterity via redundant actuation has been investigated in (O'Brien & Wen, 1999). The optimal design of a robotic wrist aiming to maximize manipulability was addressed in (Kurtz & Hayward, 1992). Actuation redundancy was successfully applied to maximize and to homogenize the force output of a haptic force display (Buttolo & Hannaford, 2005). Other redundantly actuated PKM were developed for use as robot hands (Lee et al., 1998). Temporarily redundant actuation was proposed as a way to cope with singularities (Ganovski et al., 2004). The basic idea was to equip the PKM with more drives than needed, and to activate the 'excess drives' whenever the main drives are unable to properly control the machine. Systems with variable topology are also temporarily redundantly. The inverse dynamics of such systems was addressed in (Nahon & Angeles, 1989).

Kinematically redundant PKM possess multiple inverse kinematics solutions that can be applied for various purposes, such as maximizing dexterity or stiffness, avoiding singularities or obstacles, and minimizing drive power or the overall joint motions. A singularity avoiding inverse kinematics algorithm was proposed in (Alba et al., 2007) for a kinematically redundant planar 3RRR positioning PKM, where the concept of feasibility maps for serial manipulators, was adopted for the identification of working modes of PKM, i.e. singularity-free regions in joint space. Kinematic redundancy was further used in

(Mohamed & Gosselin, 1999) as a means to reshape the manipulator's platform. Kinematic calibration and model identification is much more involved as shown in (Jeong et al., 2004) and (Abdellatif et al., 2007).

The force capability of redundantly actuated PKM were investigated in (Nokleby et al., 2005). A peculiarity of redundantly actuated PKM is the ability to generate internal prestress, via antagonistic control of the redundant drives, without generating end-effector forces. This feature was employed in (Chakarov, 2004; Kock & Schumacher, 1998; Kock & Schumacher, 2000; Müller, 2006; Yi et al., 1994) for the generation of a desired (tangential) EE-stiffness. Prestress was further used for the avoidance of joint backlash, which is critical in the presence of joint clearing and DC motor hysteresis (Müller, 2005; Valasek, 2002).

Dynamic modelling is crucial for control of redundantly actuated PKM. Modelling and control were addressed in (Cheng et al, 2003; Liu et al., 2003; Müller, 2005; Nakamura & Ghodoussi, 1989). In (Cheng et al, 2003) model based motion control of redundantly actuated PKM was considered, and it was proposed to adopt the established computed torque and augmented PD control schemes (Murray, et al., 1993). In (Garrido, 2004) these control schemes were extended by allowing for measurement uncertainties, and in (Gourdeau et al., 1999) a computed torque control scheme without velocity measurement was proposed. An important issue for the inverse dynamics of redundantly actuated PKM is a goal-oriented resolution of the redundancy. The resolution is achieved using a weighed pseudoinverse, which is however computationally expensive to evaluate. For simply redundantly actuated PKM a closed form solution for the pseudoinverse was presented in (Müller, 2005).

Another type of redundancy is related to the placement of sensors. Sensor redundancy was shown in (Yiu & Li, 2003) to be beneficial for the solution of the forward kinematics problem.

3. A motivating example

For demonstration purpose consider the R$\underline{P}$/2R$\underline{P}$R PKM in figure 1. This PKM has the DOF 2, and is controlled by actuation of the three prismatic joints. The PKM could be uniquely positioned using two of the prismatic joints only. Therefore, the R$\underline{P}$/2R$\underline{P}$R is redundantly full-actuated.

Manipulability/Dexterity/Singularities: An attractive feature of redundantly actuated PKM is the fact that singularities are eliminated that would occur in the non-redundant counterpart. A PKM is in a singularity if the EE-motion can not be determined by the actuated joints, reflected by a drop of its manipulability. Assumed, however, that an additional actuated kinematic chain is suitably attached to the moving platform, it shall be possible to overcome the indeterminacy.

The kinematic capability of robotic manipulators can be quantified by manipulability or dexterity measures, introduced in (Murray, et al., 1993) and (Yoshikawa, 1985), that characterize the velocity and force transformation. The EE-twist is determined by a set of independent joint velocities via $\mathbf{V} = \mathbf{J}_E \dot{\mathbf{q}}_2$, and away from singularities, $\dot{\mathbf{q}}_2 = \mathbf{J}_E^{-1} \mathbf{V}$. For redundant PKM there are more velocities of actuated joints than independent ones. But, there exists a Matrix $\mathbf{A}$, such that the actuator velocities are $\dot{\mathbf{q}}_a = \mathbf{A} \dot{\mathbf{q}}_2 = \mathbf{A} \mathbf{J}_E^{-1} \mathbf{V}$ (section 4). Hence, the manipulability measures

$$\mu' := \det(\mathbf{J}_E^{-T}\mathbf{A}^T\mathbf{A}\mathbf{J}_E^{-1}), \ \mu'' := 1/\mathrm{cond}(\mathbf{J}_E^{-T}\mathbf{A}^T\mathbf{A}\mathbf{J}_E^{-1})$$

characterize the velocity transmission from EE to actuators. Figure 2 shows the distribution of the two measures in the main part of the workspace of the RP/2RPR PKM. For comparison, the manipulability measures are also shown for a non-redundant RP/RPR PKM. The RP/2RPR PKM arises from the non-redundant PKM by addition of another RPR limb. Obviously the manipulability of the RP/2RPR PKM is much higher and more homogeneously distributed in the workspace. In particular, the singularities of the RP/RPR PKM, at the bottom of the workspace, are removed (singularities are marked by vanishing manipulability measure).

Actuator loads: Beside eliminating singularities, additional redundant actuators allow to distribute the required work load among the drives. In this way, the individual drive loads can be reduced. The resolution and optimal distribution of control forces among the drives is achieved by a strategic inverse dynamics, as derived in section 7. In the RP/2RPR example, the third strut compensates a large part of EE-loads, that cause high control forces in the RP/RPR PKM. Clearly, redundant actuation increases the dynamical capability of the PKM.

Stiffness/Compliance: Under working conditions, the accuracy of PKM is strongly related to its structural stiffness, and a realistic analysis must take into account the link flexibility. Using the same argument as for the load distribution among the drives, it is clear that the overall EE-stiffness increases with the addition of redundant struts. Clearly, the stiffness apparent at the EE depends on the PKM's pose.

Fault tolerance: It is clear from the manipulability analysis of the RP/2RPR PKM that the system is manipulable even if one of the actuators fails. For example, if the third actuator fails, then the PKM is still maneuverable as a RP/RPR manipulator, apart from singular postures.
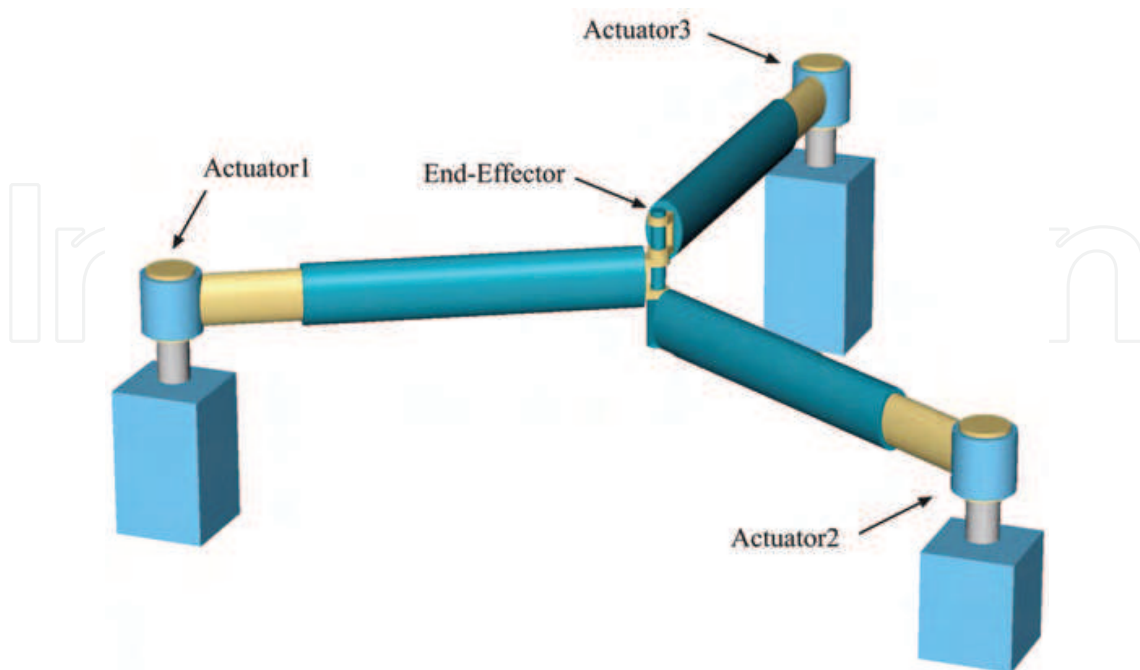

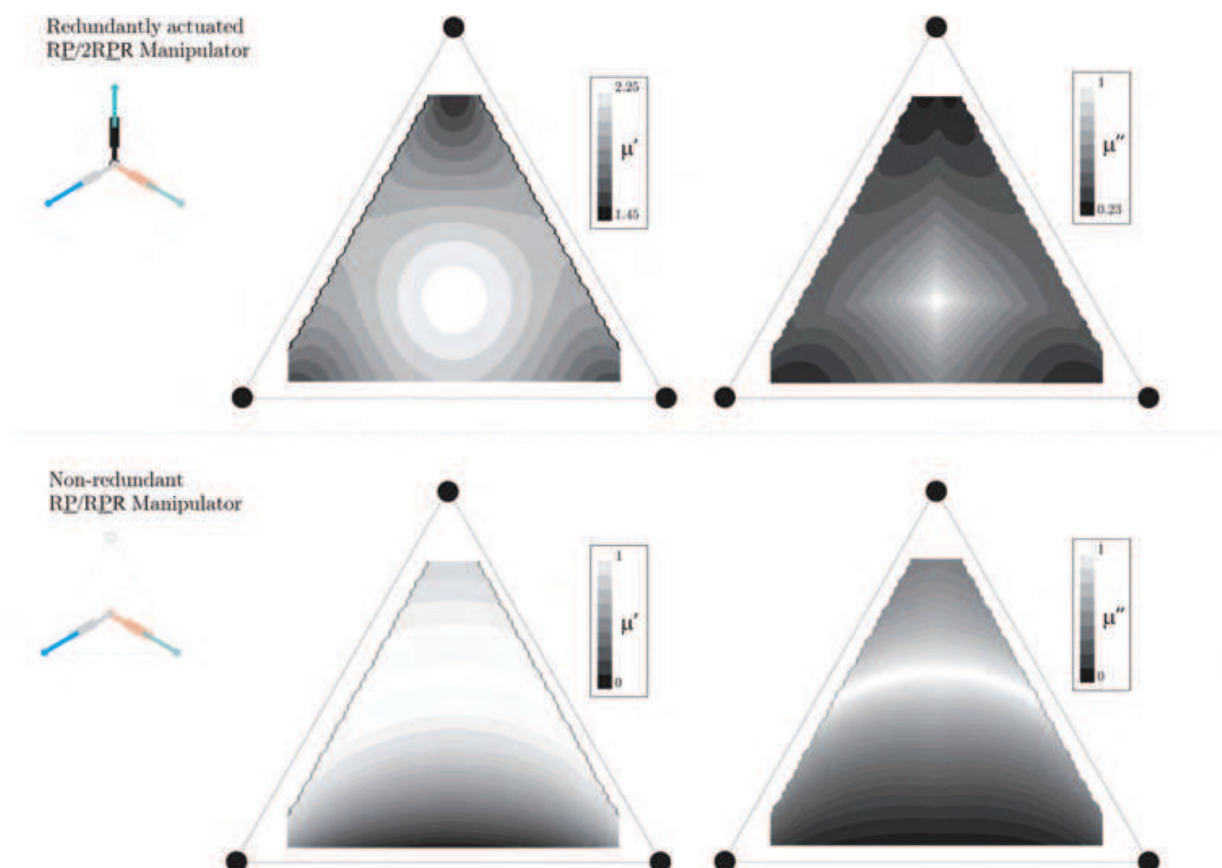Figure 1. Redundantly full-actuated planar RP/2RPR manipulator.

Figure 2. Manipulability distribution for the redundantly actuated R$\underline{P}$/2R$\underline{P}$R and the non-redundant R$\underline{P}$/R$\underline{P}$R manipulator.

4. Dynamic modeling

A PKM is a controlled, holonomically constrained multibody system (MBS), where the constraints embody the geometric closure conditions of kinematic loops. In applications where the manipulator interacts with its environment, the PKM is subject to additional possibly non-holonomic constraints. The latter will not be taken into account here.

The Lagrangian motion equations of second kind for a PKM can be derived with the standard methods for MBS with kinematic loops (Maisser, 1997; Müller, 2006; Papastavridis, 2002) as it was pursued in (Cheng et al, 2003; Liu et al., 2003; Müller, 2005; Nakamura & Ghodoussi, 1989). This proceeds by transforming the MBS with kinematic loops into an MBS with tree topology, subject to the closure constraint that enforce the loop closure. In each fundamental loop of the topological graph one joint (the cut-joint) is removed, and corresponding cutjoint constraints (closure conditions) are imposed to the resulting MBS with tree topology (Müller, 2006). Figure 3 shows the topological graph of the R$\underline{P}$/2R$\underline{P}$R PKM in figure 1. Two fundamental loops can be identified according to the indicated cut-joints. Each loop gives rise to two closure constraints.

Denote with $\mathbf{q} \in \mathbb{V}^n$ the vector of joint variables q^a, $a = 1, \ldots, n$ (higher DOF joints are split into one DOF joints) of the tree MBS, where $\mathbb{V}^n := \mathbb{T}^{nR} \times \mathbb{R}^{nP}$ if the PKM comprises nR revolute and nP prismatic/screw joints. $\mathbf{q} \in \mathbb{V}^n$ is called the configuration of the PKM. A

configuration is admissible only if it fulfils the *r* geometric loop closure conditions. Now, the fundamental loops give rise to a set of *r* geometric constraint $0 = \mathbf{h}(\mathbf{q})$, $\mathbf{h}(\mathbf{q}) \in \mathbb{R}^r$. In case of the R$\underline{P}$/2R$\underline{P}$R PKM in figure 3 this is a system of 4 constraints for the $n = 6$ joint variables of the tree system. Time differentiation yields the kinematic constraints

$$\mathbf{0} = \mathbf{J}(\mathbf{q})\,\dot{\mathbf{q}}, \; \mathbf{J}(\mathbf{q}) \in \mathbb{R}^{r,n} \tag{1}$$

The geometric constraints define the configuration space of the PKM

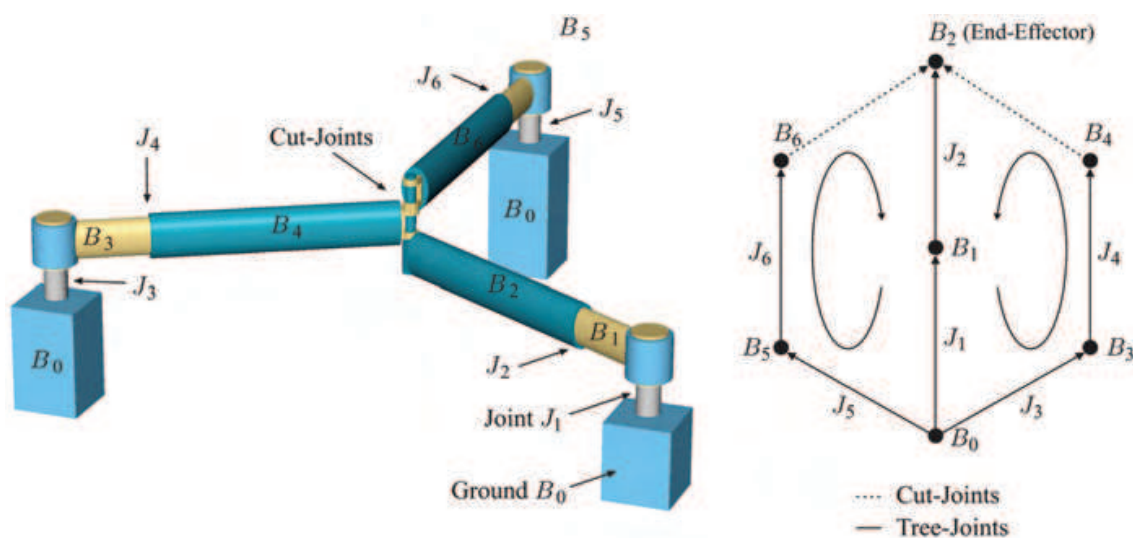$$V := \{\mathbf{q} \in \mathbb{V}^n | \mathbf{h}(\mathbf{q}) = \mathbf{0}\}. \tag{2}$$

Figure 3. Topological graph, and spanning tree of the R$\underline{P}$/2R$\underline{P}$R manipulator.

The configuration space is the set of all admissible configurations of the PKM. V is an analytic variety and only locally a smooth manifold. The manifolds are separated by the singular points of V, where the rank of **J** changes. The latter are called c-space singularities. Their determination is vital for a reliable operation of the PKM. If the r constraints are locally independent, the local DOF of the PKM is $\delta := n - r$.

The admissible configuration **q** is locally determined by $\delta := n - r$ independent generalized coordinates. Denoting the vector of dependent and independent variables respectively with $\mathbf{q}_1$ and $\mathbf{q}_2$, the kinematic constraints are

$$\mathbf{J}_1\dot{\mathbf{q}}_1 + \mathbf{J}_2\dot{\mathbf{q}}_2 = \mathbf{0}, \tag{3}$$

where $\mathbf{J} = (\mathbf{J}_1, \mathbf{J}_2)$, with $\mathbf{J}_1(q) \in \mathbb{R}^{r,r}$, $\mathbf{J}_2(q) \in \mathbb{R}^{r,\delta}$. The independent coordinates can be chosen so that J_2 is full rank, and the generalized velocities are

$$\dot{\mathbf{q}} = \mathbf{F}\dot{\mathbf{q}}_2, \; \text{where} \;\; \mathbf{F} := \begin{pmatrix} -\mathbf{J}_1^{-1}\mathbf{J}_2 \\ \mathbf{I}_\delta \end{pmatrix}. \tag{4}$$

F is an orthogonal complement of **J**, i.e. **JF** ≡ 0. The accelerations follow with $\ddot{\mathbf{q}} = \mathbf{F}\ddot{\mathbf{q}}_2 + \dot{\mathbf{F}}\dot{\mathbf{q}}_2$. The constituent feature of any PKM is that a moving platform, carrying an end-effector (EE), is connected to the base by several (possibly identical) kinematic chains (limbs, struts, legs) containing actuated joints. The EE is represented by an EE-frame. The configuration of the EE-frame w.r.t. a inertial (world) frame is represented by $\mathbf{C} \in SE(3)$. The **EE-map** $f_E : \mathbb{V}^n \rightarrow SE(3)$, gives the EE-configuration $\mathbf{C} = f_E(\mathbf{q})$ in the configuration **q**. The workspace of the PKM is

$$W := \{f_E(\mathbf{q}) \mid \mathbf{q} \in V\} \subset SE(3).$$ (5)

The **EE-Jacobian** $\mathbf{J}_E(\mathbf{q}) : T_q\mathbb{V}^n \rightarrow se(3)$ yields the EE-twist $\mathbf{V} = \mathbf{J}_E(\mathbf{q})\dot{\mathbf{q}}$, in terms of the state of the PKM. If $\boldsymbol{\tau} \in se^*(3)$ is an EE-wrench, then $\mathbf{Q} = \mathbf{J}_E^T\boldsymbol{\tau}$ is the corresponding vector of generalized forces.

Now, the dynamics of a force-controlled holonomic constrained MBS with kinematical tree structure is governed by the Lagrangian motion equations

$$\mathbf{G}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{Q}(\mathbf{q}, \dot{\mathbf{q}}, t) + \mathbf{J}_E^T(\mathbf{q})\boldsymbol{\tau} + \mathbf{J}^T(\mathbf{q})\boldsymbol{\lambda} = \mathbf{u},$$ (6)

where **G** is the generalized mass matrix, $\mathbf{C}\dot{\mathbf{q}}$ represents generalized Coriolis and centrifugal forces, **Q** represents all remaining, including generalized potential forces, and **u** are the generalized control forces. The Lagrange multipliers $\boldsymbol{\lambda}$ can be identified with the constraint reactions in cut-joints.

For a PKM some of the possible control forces in u are identically zero, and only m control forces corresponding to active joints are present. Denote with $\mathbf{c} \equiv (c_1, \ldots, c_m)$ the vector of generalized control forces in the actuated joints. Let **A** be the relevant part of **F** so that $\mathbf{F}^T\mathbf{u} = \mathbf{A}^T\mathbf{c}$. Projecting the Lagrangian equations (6) onto the configuration space *V*, with the help of the orthogonal complement **F** and the relation (4), yields the Voronets equations (Maisser, 1997; Papastavridis, 2002)

$$\mathbf{G}(\mathbf{q})\ddot{\mathbf{q}}_2 + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_2 + \mathbf{Q}(\mathbf{q}, \dot{\mathbf{q}}, t) + \mathbf{J}_E^T(\mathbf{q})\boldsymbol{\tau} = \mathbf{A}^T\mathbf{c},$$ (7)

where

$$\mathbf{G} := \mathbf{F}^T\mathbf{G}\mathbf{F}, \quad \mathbf{C} := \mathbf{F}^T(\mathbf{C}\mathbf{F} + \mathbf{G}\dot{\mathbf{F}}), \quad \mathbf{Q} := \mathbf{F}^T\mathbf{Q}, \quad \mathbf{J}_E := \mathbf{J}_E\mathbf{F}.$$

Clearly, only those c that are not in the kernel of $\mathbf{A}^T$ are effective control forces. The system (7) together with the kinematic constraints in (1) yield *n* differential equations in $\mathbf{q} \in \mathbb{V}^n$, that completely determine the MBS dynamics.

Note that the motion equations are formulated in terms of minimal coordinates $\mathbf{q} \in \mathbb{V}^n$, for the purpose of deriving an unconstrained control system. One has to be cautious, however, since the removal of cut-joints can lead to dependent closure constraints that have no physical meaning. These artifacts are merely due to the parameterization. Geometrically, *V*

is only a section of the 'complete' configuration space, including cut-joint variables. This issue is important for model based control, as pointed out in (Liu et al., 2003), since the actual controller is built upon the model (7), i.e. using a certain V. In fact, it may be necessary to switch between PKM models with different cut-joints.

5. The associated non-linear control systems

A PKM is a force-controlled holonomically constrained dynamical system, whose dynamics is governed by (7). The control purpose is to manipulate the EE, which embodies the system's primary output. A PKM can be regarded as a second order control-affine control system on the configuration space V, which can be transformed to the first order control system on the $2n$-dimensional state space TV

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \sum_{i=1}^{m} \mathbf{g}_i(\mathbf{x}) c^i \tag{8}$$

$$\mathbf{C} = f_{\mathrm{E}}(\mathbf{x})$$

with state vector $\mathbf{x} := (\mathbf{q}_2, \dot{\mathbf{q}}_2)$. Therein

$$\mathbf{f} := \begin{pmatrix} \dot{\mathbf{q}}_2 \\ -\mathbf{G}^{-1}(\mathbf{C}\dot{\mathbf{q}}_2 + \mathbf{Q} + \mathbf{J}_{\mathrm{E}}^T \tau) \end{pmatrix}, \tag{9}$$

is the drift vector field, and the columns g_i, $i = 1, \ldots, m \le n$ of

$$\mathbf{g} := \begin{pmatrix} \mathbf{0} \\ \mathbf{G}^{-1}\mathbf{A}^T \end{pmatrix} \tag{10}$$

define the control vector fields, via which the control forces affect the system.
From a control point of view, one is interested in the controllability and observability of the PKM. That is, one is concerned with whether the PKM can be steered between two given configurations (Nijmeijer & van der Schaft).

6. Actuation and redundancy

The terms actuation and redundancy are differently used in the literature. In order clarify this notion a stringent definition is given based on the above control system. The following definitions refer to a regular configuration $\mathbf{q}$, i.e. a configuration for which the orthogonal complement $\mathbf{F}$ and its submatrix $\mathbf{A}$ has full rank in a neighborhood of $\mathbf{q}$ in V. The dependence on $\mathbf{q}$ is omitted, and δ denotes the DOF.

Definition 1. The rank of the input vector field is called the **degree of actuation** (DOA)

$$\alpha := \mathrm{rank}(\mathbf{g}) = \mathrm{rank}(\mathbf{A}).$$

If $\alpha < \delta$ the PKM is **underactuated** and if $\alpha = \delta_{loc}$ the PKM is **full-actuated**. The **degree of redundancy** of the actuation is $\rho_\alpha := m - \alpha$. The PKM is called **redundantly actuated** if $\rho_\alpha > 0$ and **nonredundantly actuated** if $\rho_\alpha = 0$.

Actuation refers to the effect that control forces have on the state change of a system. The above definition is in accordance with this notion, though it refers to the ability to influence the PKM's acceleration. This is so because a PKM (as considered here) is a holonomically constrained system, so that prescribing the acceleration also determines the velocity and configuration, with known initial conditions. Actuation is a pointwise property, and the DOA changes in singular configurations. The effect of the actuation on the motion is described by the controllability of the system. This is a local property, i.e. considering the effects over a small time (Nijmeijer & van der Schaft). Redundantly actuated PKM are occasionally termed 'overactuated'. Notwithstanding that redundantly actuated PKM can be underactuated, a full-actuated PKM is completely actuated, and an improvement is impossible. Therefore, the term 'overactuation' makes no sense.

7. Resolution of actuation redundancy

7.1 Inverse dynamics of redundantly full-actuated PKM

The first step in navigating PKM consists in task/motion planning and a subsequent solution of the inverse kinematics, i.e. the determination of required actuator motions. The inverse dynamics problem is to determine the actuator forces required for this motion to take place. The DOA of a full-actuated PKM equals its DOF ($\delta = \alpha$). The number $m = \delta + \rho$ of active drives of a redundantly full-actuated PKM exceeds its DOF by ρ. Without loss of generality, the joint variables can be arranged as $\mathbf{q} \equiv (\mathbf{q}_p, \mathbf{q}_a)$, with $\mathbf{q}_a \equiv (\dots, \mathbf{q}_2)$. Accordingly, the generalized control vector has the form $\mathbf{u} = (\mathbf{0}, \mathbf{c})$, with $\mathbf{c} \equiv (c_1, \dots, c_m)$. The orthogonal complement takes on the form

$$\mathbf{F} = \begin{pmatrix} -\mathbf{J}_1^{-1}\mathbf{J}_2 \\ \mathbf{I}_\delta \end{pmatrix} \equiv \begin{pmatrix} \mathbf{P} \\ \mathbf{A} \end{pmatrix}, \tag{11}$$

$$\mathbf{P}(\mathbf{q}) \in \mathbb{R}^{n-m,\delta}, \quad \mathbf{A}(\mathbf{q}) = \begin{pmatrix} \mathbf{A}_1(\mathbf{q}) \\ \mathbf{I}_\delta \end{pmatrix} \in \mathbb{R}^{m,\delta}, \quad \mathbf{A}_1(\mathbf{q}) \in \mathbb{R}^{\rho,\delta},$$

where $\mathbf{P}$ contains the first $n - m$ and $\mathbf{A}_1$ the remaining rows of $-\mathbf{J}_1^{-1}\mathbf{J}_2$. $\mathbf{A}$ is full rank δ. The kernel of $\mathbf{A}^{\mathrm{T}}$ is ρ-dimensional, so that (7) can not be uniquely solved for the controls $\mathbf{c}$. As consequence, 1) the load distribution among the drives is not unique, and 2) one can generate control forces in the null-space of $\mathbf{A}^{\mathrm{T}}$ that have no effect on the motion, so-called prestress.

Let $\mathbf{c}^0 \in \mathbb{R}^m$ be a desired prestress vector, then a solution for the controls $\mathbf{c}$ such that $(\mathbf{c} - \mathbf{c}^0)^{\mathrm{T}}\mathbf{M}(\mathbf{c} - \mathbf{c}^0) \to \min$ is

$$\mathbf{c} = \left(\mathbf{A}^T\right)_{\mathbf{M}}^+ \left(\overline{\mathbf{G}}(\mathbf{q})\ddot{\mathbf{q}}_2 + \overline{\mathbf{C}}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_2 + \overline{\mathbf{Q}} + \overline{\mathbf{J}}^T(\mathbf{q})\boldsymbol{\tau}\right) + \mathbf{N}_{\mathbf{A}^T}\mathbf{c}^0 \tag{12}$$

where $(\mathbf{A}^T)^+_M := \mathbf{M}^{-1}\mathbf{A}(\mathbf{A}^T\mathbf{M}^{-1}\mathbf{A})^{-1}$ is the weighed right pseudoinverse, and $\mathbf{N}_{\mathbf{A}^T} := (\mathbf{I}_m - (\mathbf{A}^T)^+\mathbf{A}^T)$ is a projector to the null-space of $\mathbf{A}^T$. Complete knowledge of the system and the EE-load λ is assumed. $\mathbf{M}$ is a positive definite weighting matrix for the drive forces.

For the important case of simply redundant actuation ($\rho = 1$) a close form solution, with $\mathbf{M} = \mathbf{I}$, was derived in (Müller, 2005). In this case $\mathbf{A}_1$ is a row vector, and

$$(\mathbf{A}^T)^+ = \begin{pmatrix} \mathbf{A}_1 \\ \mathbf{I}_\delta \end{pmatrix} \left(\mathbf{I}_\delta - \frac{1}{1+\|\mathbf{A}_1\|^2} \mathbf{A}_1^T \mathbf{A}_1 \right)$$
$$\mathbf{N}_{\mathbf{A}^T} = \begin{pmatrix} \mathbf{I}_\rho \\ -\mathbf{A}_1^T \end{pmatrix}.$$

(13)

Note that no matrix inversion is necessary, which is numerically advantageous.

On the basis of a preceding path planing and inverse kinematics solution of redundantly actuated PKM, the inverse dynamics is not unique and can take into account various goals. Actuation redundancy can be used to reduce the load of individual drives by strategically distributing the required control forces. On the other hand, the null-space components of the control forces can be employed for 'secondary' tasks.

7.2 Optimal distribution of control forces
An immediate application of the redundancy is a purposeful allocation of the control forces (Kock & Schumacher, 1998). This is achieved via the weighing matrix $\mathbf{M}$. Without prestress, i.e. with $\mathbf{c}^0 = 0$, the inverse dynamics solution (12) is such that $\mathbf{c}^T\mathbf{M}\mathbf{c} \to \min$. Usually $\mathbf{M}$ is a diagonal matrix, and its entries scale the control forces according to their drive performances. The lower the force capability of a drive the higher its weight. E.g., one can think of a lowpowered redundant drive, used to balance and reduce otherwise high force peaks in the main drives.

Note, that these force considerations are essentially static, and do not take into account the PKM dynamics. For highly dynamic applications, the driving power distribution may significantly differ from the force distribution.

7.3 Backlash avoiding control
In (Müller, 2005; Valasek, 2002; Valasek, 2002) it was proposed to use internal prestress $\mathbf{c}^0$ to avoid actuator backlash, which refers to situations, where the sign of the control forces changes. One practical motivation for this is to eliminate the negative effects of joint clearings, and another is rooted in the observation of DC motor hysteresis. Also, for tendon driven PKM actuator signs must remain constant.

The main idea is to include the generation of internal prestress in the control scheme of the PKM. The condition for backlash free control is that the magnitude of each particular control force c_a remains above a certain level $c_a^{\min}$ and that its sign remains constant during the considered task with a duration T. Denote with $s_a \in \{-1, 1\}$ the required sign of c_a, then the condition

$$s_a c_a(t) \geq c_a^{\min}, \; t \in [0, T]$$

(14)

must be satisfied with $c_a^{\min} > 0$.

In (Müller, 2005) a method for backlash avoiding control of simply rendundantly actuated PKM was presented. In this case $\mathbf{A}^T$ has a one-dimensional null-space that can be parameterized by a prestress parameter $\sigma(t)$, so that

$$\mathbf{c} = \begin{pmatrix} \mathbf{A}_1 \\ \mathbf{I}_\delta \end{pmatrix} \left(\mathbf{I}_\delta - \frac{1}{1+\|\mathbf{A}_1\|^2} \mathbf{A}_1^T \mathbf{A}_1 \right) \boldsymbol{\varphi} + \sigma \begin{pmatrix} \mathbf{I}_\rho \\ -\mathbf{A}_1^T \end{pmatrix} \tag{15}$$

with $\varphi := \overline{G}(\mathbf{q}) \, \ddot{\mathbf{q}}_2 + \overline{C}(\mathbf{q}, \dot{\mathbf{q}}) \, \dot{\mathbf{q}}_2 + \overline{Q}$.

Given a prescribed trajectory $\mathbf{q}^d(t)$, the control problem at time instant t_i consists in determining the prestress parameter $\sigma(t_i)$ such that (14) holds and an objective functional $L(\mathbf{q}^d(t_i), \sigma(t_i))$ is minimized. The latter can be the weighed sum of squared control forces or the overall driving power. In summary the one-dimensional optimization problem

$$\left\{ \begin{array}{c} L \to \min, \ t \in [0, T] \\ s_a c_a \geq c_a^{\min} \\ s_a c_a \leq c_a^{\max} \end{array} \right\} \tag{16}$$

with $\mathbf{c}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \sigma)$ in (15), has to be solved at any time step. This can either be solved independently at each time instant, or the σ can be approximated as a function of time, which results in a smoother behavior.
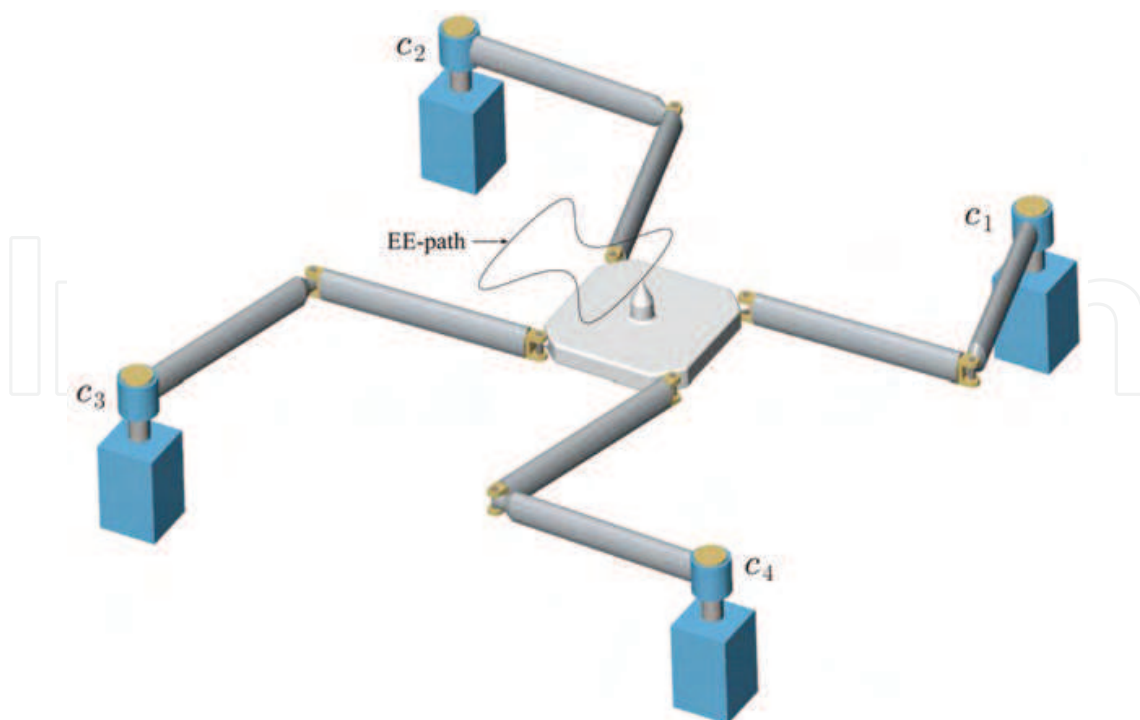

Figure 4. Planar 4$\underline{R}$RR manipulator.

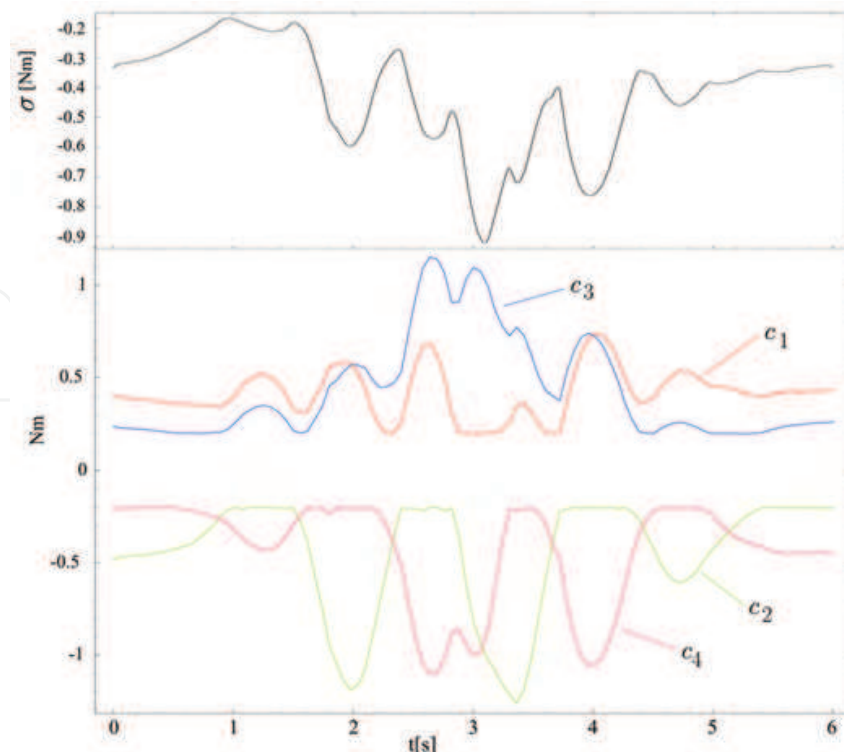Figure 5. Prestress parameter and control torques for the EE motion in figure 4 with sign vector s = (−1, 1,−1, 1).

For illustration purpose, we recall an example from (Müller, 2005), where the redundantly full-actuated planer PKM in figure 4 is navigated along the shown EE-path with fixed EE orientation. Figure 5 shows the drive torques, where a minimum drive torque of 0.2 Nm was required for prestress, with sign vector (s_a) = (−1, 1,−1, 1).

7.4 Stiffness control

Stiffness or impedance control has long been proposed and developed for serial manipulators (Asada & Slotine, 1986). These concepts can straightforwardly be adopted for non-redundantly actuated PKM, thanks to the identical structure of the motion equations. Essentially, stiffness control (more precisely the control of the tangential stiffness since a PKM is a highly non-linear dynamical system) aims to mimic the force-deflection properties of an elastic medium, so that an applied EE-wrench causes an 'elastic' evasive deflection. This is achieved by generating control forces as reactions to joint motions caused by EE-motions. It is thus the result of a control cycle, which operates in discrete time steps. The actual behavior is therefore only 'elastic' for sufficiently slow effects, due to the latency in the force response to due to a perturbation.

Now, redundantly actuated PKM possesses the potential for another approach that does not suffer from the control latency. Redundant actuation allows for the generation of prestress, using controls in the null-space of $\mathbf{A}^T$. Since $\mathbf{A}^T$ and thus $\mathbf{N}_{\mathbf{A}^T}$ are configuration dependent, part of the null-space component of a given control vector c becomes effective when the configuration is perturbed. Hence, there is an immediate! response to EE-deflection. In order to exploit this effect, the control forces in the null-space of $\mathbf{A}^T$ must be such that the change of $\mathbf{A}^T$ due to a EE-perturbation yields a desired EE-wrench. This was attempted in (Müller,

2006; Yi et al., 1989). It turned out that, for the considered PKM, a large number of redundant actuators is required for stable control of all stiffness components. This requirement is eased if only some stiffness components are to be controlled. Moreover, so far, stiffness control was not taken into account in the PKM design, and other novel PKM structures may need lower actuator redundancy to control EE stiffness via prestress.

8. PKM control

Upon the motion equations (7), established non-linear control methods can be applied to PKM, and shall exhibit the known stability properties. Model based motion control of redundantly actuated PKM was addressed in (Cheng et al, 2003). It was proposed to adopt the established computed torque and augmented PD control schemes, where perfect knowledge of the PKM model as well as perfect measurement of position, velocity and acceleration was presumed. The assumption of perfect measurements was abandoned in (Garrido, 2004), and a standard PD control in conjunction with a velocity estimator was proposed. In (Gourdeau et al., 1999) a computed torque control scheme without velocity measurement was proposed. Common to the control methods proposed so far, is the assumption of a perfect model. Robust control of redundantly actuated PKM has not yet been attempted.

In the following we briefly recall the standard model bases control schemes and their application to PKM control, and point out problems specific to redundantly actuated PKM that arise in the presence of model uncertainties. For notational simplicity the weighting matrix $\mathbf{M} = \mathbf{I}$ is assumed.

8.1 Model-based control schemes

Two model-based control schemes frequently used for the control of robotic manipulators: the augmented PD and the computed torque control (Asada & Slotine, 1986; Murray, et al., 1993). The unaltered augmented PD control attains the form

$$\mathbf{c} = \left(\mathbf{A}^{T}\right)^{+} \left[\overline{\mathbf{G}}\ddot{\mathbf{q}}_2^{\mathrm{d}} + \overline{\mathbf{C}}\dot{\mathbf{q}}_2^{\mathrm{d}} + \overline{\mathbf{Q}} \right.$$
$$\left. -\mathbf{K}_{\mathrm{D}}\dot{\mathbf{e}} - \mathbf{K}_{\mathrm{P}}\mathbf{e}\right] + \mathbf{N}_{\mathbf{A}^T}\mathbf{c}^{0}, \tag{17}$$

with the desired nominal path $\mathbf{q}^{\mathrm{d}}(t)$, and the tracking error $\mathbf{e}(t) := \mathbf{q}(t) - \mathbf{q}^{\mathrm{d}}(t)$ (Cheng et al., 2003). The computed torque control law adopted for PKM is

$$\mathbf{c} = \left(\mathbf{A}^{T}\right)^{+} \left[\overline{\mathbf{G}}\mathbf{v} + \overline{\mathbf{C}}\dot{\mathbf{q}}_2 + \overline{\mathbf{Q}}\right] + \mathbf{N}_{\mathbf{A}^T}\mathbf{c}^{0}, \tag{18}$$

setting $\mathbf{v} := \ddot{\mathbf{q}}_2^{d} - \mathbf{K}_{\mathrm{D}}\dot{\mathbf{e}} - \mathbf{K}_{\mathrm{P}}\mathbf{e}$. Perfect matching of model and plant presumed, both control laws applied to (7) result in exponential trajectory tracking for sufficiently large gains $\mathbf{K}_{\mathrm{D}}$ and $\mathbf{K}_{\mathrm{P}}$, provided $\overline{\mathbf{G}}$ is regular. The latter assumption fails in configuration space singularities.

8.2 Model uncertainties

The aforementioned control laws yields exponential stability for the nominal system only. Any real-life manipulator will differ from the nominal model used in the control scheme due to inevitable model uncertainties. There is a plethora of methods for the estimation of kinematic and other model parameters of serial manipulators, such as inertia, stiffness, and friction. Adaptations of these algorithms to PKM were proposed in (Valasek, 2002). Friction identification in particular was attempted in (Abdellatif et al., 2007).

A parameter estimation, whatsoever, will not achieve perfect matching of model and plant. To tackle this uncertainties, a number of robust control schemes have been proposed for serial manipulators. Robust control is still a field of active research, and we will not attempt to develop a such for redundant PKM here. The interested reader is referred to (Asada & Slotine, 1986) for the fundamentals. It is nevertheless instructive to investigate the effect of model uncertainties. In contrast to non-redundant manipulators, where model uncertainties cause incorrect positioning, geometric uncertainties of redundantly actuated PKM may cause (possibly high) actuator loads that have no effect on the motion.

Deviations from the nominal geometry alter the geometric constraints and thus the configuration space V. That is, a configuration $\mathbf{q} \in \mathbb{V}^n$ that complies with the nominal constraints will not do so for the uncertain system. If the variations are small, the PKM configuration will still be expressible in terms of the independent coordinates $\mathbf{q}_2$. The constraint Jacobian in (1) changes according to

$$\underline{\mathbf{J}} \equiv (\underline{\mathbf{J}}_1, \underline{\mathbf{J}}_2) := \mathbf{J} + \Delta\mathbf{J}, \quad \Delta\mathbf{J} := (\Delta\mathbf{J}_1, \Delta\mathbf{J}_2). \tag{19}$$

Underlines indicate perturbed objects. For $\Delta\mathbf{f}$ small compared to $\mathbf{f}$, and neglecting second order terms of $\Delta\mathbf{J}$, yields

$$\underline{\mathbf{J}}_1^{-1}\underline{\mathbf{J}}_2 = \left(\mathbf{I} - \mathbf{J}_1^{-1}\Delta\mathbf{J}_1\right)\mathbf{J}_1^{-1}\mathbf{J}_2 + \mathbf{J}_1^{-1}\Delta\mathbf{J}_2, \tag{20}$$

The perturbed orthogonal complement is then

$$\underline{\mathbf{F}} := \mathbf{F} + \Delta\mathbf{F}, \quad \Delta\mathbf{F} = \begin{pmatrix} \mathbf{J}_1^{-1}\Delta\mathbf{J}_1\mathbf{J}_1^{-1}\mathbf{J}_2 - \mathbf{J}_1^{-1}\Delta\mathbf{J}_2 \\ \mathbf{0} \end{pmatrix}. \tag{21}$$

The splitting (11) according to active and passive joints yields

$$\underline{\mathbf{F}} \equiv \begin{pmatrix} \mathbf{P} \\ \underline{\mathbf{A}} \end{pmatrix} := \begin{pmatrix} \mathbf{P} + \Delta\mathbf{P} \\ \mathbf{A} + \Delta\mathbf{A} \end{pmatrix}, \quad \Delta\mathbf{A} := \begin{pmatrix} \Delta\mathbf{A}_1 \\ \mathbf{0} \end{pmatrix} \tag{22}$$

where $\Delta\mathbf{A}$ comprises the last $\rho = m - \delta$ rows of $\mathbf{J}_1^{-1}\,\Delta\mathbf{J}_1\,\mathbf{J}_1^{-1}\,\mathbf{J}_2 - \mathbf{J}_1^{-1}\,\Delta\mathbf{J}_2$. Thus, the pseudoinverse of $\underline{\mathbf{A}}^T$ is

$$\begin{aligned} (\underline{\mathbf{A}}^T)^+ &= (\mathbf{A}^T)^+ + \mathbf{B}, \\ \mathbf{B} &:= \mathbf{N}_{\mathbf{A}^T}\Delta\mathbf{A}^T\left(\mathbf{A}^T\mathbf{A}\right)^{-1} - (\mathbf{A}^T)^+\Delta\mathbf{A}^T(\mathbf{A}^T)^+. \end{aligned} \tag{23}$$

The projector to the null-space of $\underline{\mathbf{A}}^\mathrm{T}$ is

$$\mathbf{N}_{\underline{\mathbf{A}}^T} = \mathbf{N}_{\mathbf{A}^T} + \Delta\mathbf{N}_{\mathbf{A}^T}$$
$$\Delta\mathbf{N}_{\mathbf{A}^T} := -\left(\mathbf{A}^T\right)^+ \Delta\mathbf{A}^T\mathbf{N}_{\mathbf{A}^T} - \mathbf{N}_{\mathbf{A}^T}\Delta\mathbf{A}\mathbf{A}^+. \tag{24}$$

The null-space difference makes part of the control forces ineffective causing unintentional prestress, and part of the control forces applied to the uncertain system are annihilated by $\Delta\mathbf{N}_{\mathbf{A}^T}$.

The objects in the motion equations change accordingly,

$$\underline{\overline{\mathbf{G}}} := \overline{\mathbf{G}} + \Delta\overline{\mathbf{G}}, \quad \underline{\overline{\mathbf{C}}} := \overline{\mathbf{C}} + \Delta\overline{\mathbf{C}}, \quad \underline{\overline{\mathbf{Q}}} := \overline{\mathbf{Q}} + \Delta\overline{\mathbf{Q}}, \tag{25}$$

that give rise to the motion equations of the uncertain PKM

$$\underline{\overline{\mathbf{G}}}\ddot{\mathbf{q}}_2 + \underline{\overline{\mathbf{C}}}\dot{\mathbf{q}}_2 + \underline{\overline{\mathbf{Q}}} = \underline{\mathbf{A}}^T\mathbf{c}. \tag{26}$$

Application of the augmented PD controller (17) to (26), results in the error dynamics governed by

$$\overline{\mathbf{G}}\ddot{\mathbf{e}} + \overline{\mathbf{C}}\dot{\mathbf{e}} + (\mathbf{I}+\mathbf{S})\mathbf{K}_\mathrm{D}\dot{\mathbf{e}} + (\mathbf{I}+\mathbf{S})\mathbf{K}_\mathrm{P}\mathbf{e}$$
$$+ \Delta\overline{\mathbf{G}}\ddot{\mathbf{q}}_2 + \Delta\overline{\mathbf{C}}\dot{\mathbf{q}}_2 + \Delta\overline{\mathbf{Q}}$$
$$- \mathbf{S}\left(\overline{\mathbf{G}}\ddot{\mathbf{q}}_2^\mathrm{d} + \overline{\mathbf{C}}\dot{\mathbf{q}}_2 + \overline{\mathbf{Q}}\right) - \Delta\mathbf{A}^T\mathbf{N}_{\mathbf{A}^T}\mathbf{c}^0 = \mathbf{0}, \tag{27}$$

with

$$\mathbf{S} := \Delta\mathbf{A}^T\left(\mathbf{A}^T\right)^+. \tag{28}$$

It is obvious that, with the perturbation $\mathbf{S}$ of the gain matrices, model uncertainties do not only affect the dynamics of the controlled system but also interfere with the PD feedback. This is a peculiarity of the redundant actuation. The extent of the effect depends on the degree of non-linearity of the geometric constraints. With, usually large gains, the parasitic control forces due to $\mathbf{SK}_\mathrm{D}\dot{\mathbf{e}}$ and $\mathbf{SK}_\mathrm{P}\mathbf{e}$ may be large too. Moreover, the critical point to observe here is that these parasitic forces can never be equilibrated by adjusting the gains. Also observe that in view of $\Delta\mathbf{A}^\mathrm{T}\mathbf{N}_{\mathbf{A}^T}\mathbf{c}^0$, the control forces, deduced from the nominal model, are partially annihilated, whereas some null-space components (according to a secondary task, e.g. prestress generation) become effective and interfere with the motion control. The latter is due to the mismatch of the null-space of $\mathbf{A}^\mathrm{T}$ and $\underline{\mathbf{A}}^\mathrm{T}$ that, with (24), can be inferred from $\underline{\mathbf{A}}^T\mathbf{N}_{\underline{\mathbf{A}}^T} \neq \mathbf{0}$.

8.3 Amended control schemes

Parasitic control forces can be avoided by restricting the linear feedback to the subspace of independent coordinates $\mathbf{q}_2$, which are a subset of $\mathbf{q}_a$, since $\mathbf{q}_a \equiv (. . . , \mathbf{q}_2)$. This gives rise to the following adapted augmented PD control law for redundantly actuated PKM

$$
\begin{aligned}
\mathbf{c} = \left(\mathbf{A}^T\right)^+ &\left(\overline{\mathbf{G}}\left(\mathbf{q}\right) \ddot{\mathbf{q}}_2^{\mathrm{d}} + \overline{\mathbf{C}}\left(\mathbf{q}, \dot{\mathbf{q}}\right) \dot{\mathbf{q}}_2^{\mathrm{d}} + \overline{\mathbf{Q}}\right) \\
&+ \mathbf{N}_{\mathbf{A}^T}\mathbf{c}^0 - \begin{pmatrix} \mathbf{0} \\ \mathbf{I}_\delta \end{pmatrix} \left(\mathbf{K}_D\dot{\mathbf{e}} + \mathbf{K}_P\mathbf{e}\right).
\end{aligned}
\tag{29}
$$

The adapted computed torque control law is

$$
\begin{aligned}
\mathbf{c} = \left(\mathbf{A}^T\right)^+ &\left(\overline{\mathbf{G}}\left(\mathbf{q}\right) \ddot{\mathbf{q}}_2^{\mathrm{d}} + \overline{\mathbf{C}}\left(\mathbf{q}, \dot{\mathbf{q}}\right) \dot{\mathbf{q}}_2 + \overline{\mathbf{Q}}\right) \\
&+ \mathbf{N}_{\mathbf{A}^T}\mathbf{c}^0 - \begin{pmatrix} \mathbf{0} \\ \mathbf{G} \end{pmatrix} \left(\mathbf{K}_D\dot{\mathbf{e}} + \mathbf{K}_P\mathbf{e}\right).
\end{aligned}
\tag{30}
$$

It is vital that both control schemes work stable for the nominal system. To see this, consider the error dynamics of the closed loop control law (29) that is governed by $\overline{\mathbf{G}}\left(\mathbf{q}\right) \left(\ddot{\mathbf{e}} + \mathbf{K}_D\dot{\mathbf{e}} + \mathbf{K}_P\mathbf{e}\right) = 0$, and the error dynamics for (30) governed by $\overline{\mathbf{G}}\left(\mathbf{q}\right) \ddot{\mathbf{e}} + \overline{\mathbf{G}}\left(\mathbf{q}, \dot{\mathbf{q}}\right) \dot{\mathbf{e}} + \mathbf{K}_D\dot{\mathbf{e}} + \mathbf{K}_P\mathbf{e} = 0$. Thereupon, with the classical stability results (Murray, et al., 1993) it can be shown that the control laws (29) and (30) applied to the nominal system (7) are exponentially stable.

Having concluded stability for the nominal system, it remains to show the claimed elimination of parasitic control forces. This is obvious from the closed loop dynamics

$$
\begin{aligned}
\overline{\mathbf{G}}\ddot{\mathbf{e}} &+ \overline{\mathbf{C}}\dot{\mathbf{e}} + \mathbf{K}_D\dot{\mathbf{e}} + \mathbf{K}_P\mathbf{e} \\
&+ \Delta\overline{\mathbf{G}}\ddot{\mathbf{q}}_2 + \Delta\overline{\mathbf{C}}\dot{\mathbf{q}}_2 + \Delta\overline{\mathbf{Q}} \\
&- \mathbf{S}\left(\overline{\mathbf{G}}\ddot{\mathbf{q}}_2^{\mathrm{d}} + \overline{\mathbf{C}}\dot{\mathbf{q}}_2^{\mathrm{d}} + \overline{\mathbf{Q}}\right) - \Delta\mathbf{A}^T\mathbf{N}_{\mathbf{A}^T}\mathbf{c}^0 = \mathbf{0},
\end{aligned}
\tag{31}
$$

when (29) is applied to the uncertain system (26), and from

$$
\begin{aligned}
\overline{\mathbf{G}}(\ddot{\mathbf{e}} &+ \mathbf{K}_D\dot{\mathbf{e}} + \mathbf{K}_P\mathbf{e}) \\
&+ \Delta\overline{\mathbf{G}}\ddot{\mathbf{q}}_2 + \Delta\overline{\mathbf{C}}\dot{\mathbf{q}}_2 + \Delta\overline{\mathbf{Q}} \\
&- \mathbf{S}\left(\overline{\mathbf{G}}\ddot{\mathbf{q}}_2^{d} + \overline{\mathbf{C}}\dot{\mathbf{q}}_2^{d} + \overline{\mathbf{Q}}\right) - \Delta\mathbf{A}^T\mathbf{N}_{\mathbf{A}^T}\mathbf{c}^0 = \mathbf{0},
\end{aligned}
\tag{32}
$$

when the computed torque controller (30) is applied.

Now the control forces act freely upon the uncertain system, in contrast to (29) and (30). Therewith the uncertainties affect the dynamics of the controlled PKM, but not the way the controls act upon the system. The second and third lines in (31) and (32) embody the uncertain dynamics that is not balanced by the controller.

The proposed adapted control schemes shall motivate the development of tailored model-based robust control concepts for redundantly actuated PKM.

8.4 Example

For illustration purpose the effect of geometric uncertainties of the planar RP̲/2RP̲R PKM in figure 6 is analyzed. This is a fully-parallel but not symmetric PKM. There is no moving platform, and the EE is mounted on one of the limbs. The EE is connected to the base by one RP̲ and two RP̲R chains. The PKM is obtained from a non-redundant RP̲/RP̲R by adding one RP̲R chain.
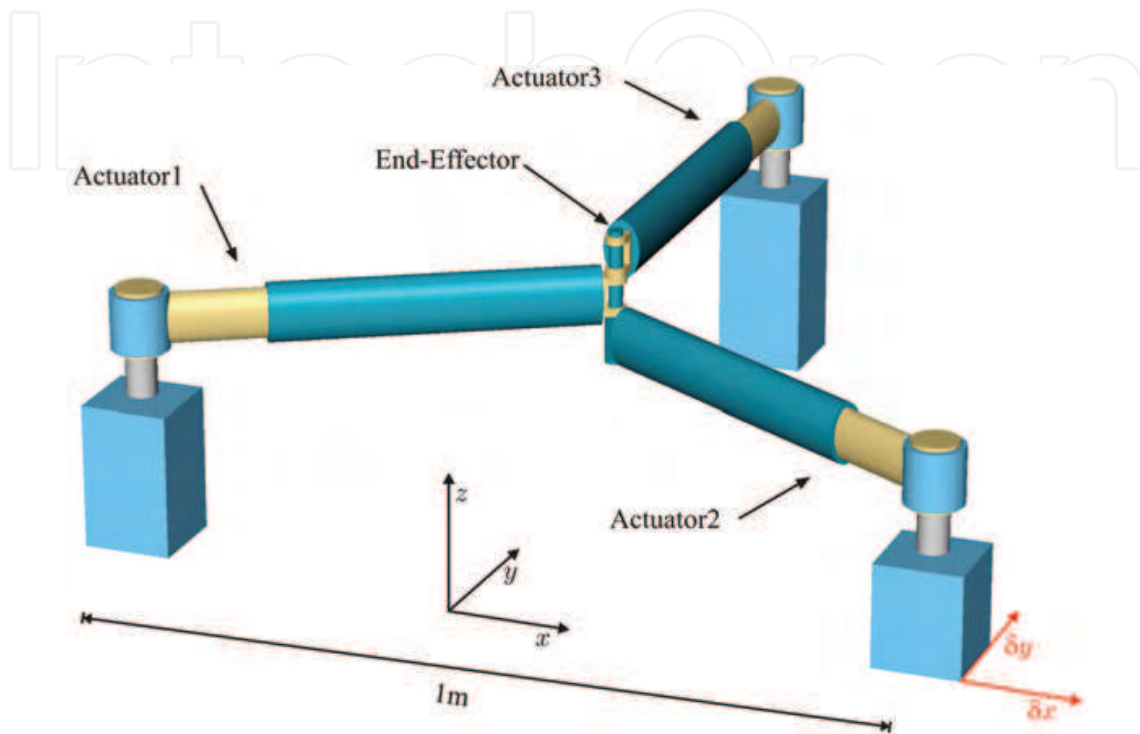

Figure 6. Planar 2RP̲R/RP̲ PKM with DOF 2.

The drive units are mounted on the base at the corners of an equilateral triangle. A disturbance frequently encountered in setting up a PKM is the misplacement of joints. Now assume that one of the drive units is displaced on the ground plane as indicated in figure 6. This leads to a perturbed plant with input matrix $\underline{\mathbf{A}}^T$. The control forces are deduced from the nominal model with $\mathbf{A}^T$. Consequently, the inverse dynamics solution (12) applied to the perturbed system (26) can not perfectly reproduce the desired control forces, due to $\underline{\mathbf{A}}^T (\mathbf{A}^T)^+ \neq \mathbf{I}$. This leads to desired forces in the null-space of $\mathbf{A}^T$ becoming effective, due to $\underline{\mathbf{A}}^T \mathbf{N}_{\mathbf{A}^T} \neq 0$. For a quantitative analysis the drive unite has been displaced by 5% of the triangle side length, as shown in figure 7. The perfect model and the perturbed plant are evaluated along the indicated EE path. For this PKM the null-space projector and thus $\underline{\mathbf{A}}^T \mathbf{N}_{\mathbf{A}^T}$ is two dimensional vector (being zero for perfect matching). Figure 7 shows the two components evaluated for the EE positions on the indicated path. It turns out that the matrix $\mathbf{S} = \Delta\mathbf{A}^T (\mathbf{A}^T)^+$ leads to uncontrollable counter action of the drives. For the RP̲/2RP̲R PKM this is a 2 × 2 matrix, which is identically zero for a perfect match of plant and model. The norm of $\mathbf{S}$ is shown in figure 7. It is clear that even for this simple PKM the effect of geometric uncertainties can not be neglected.
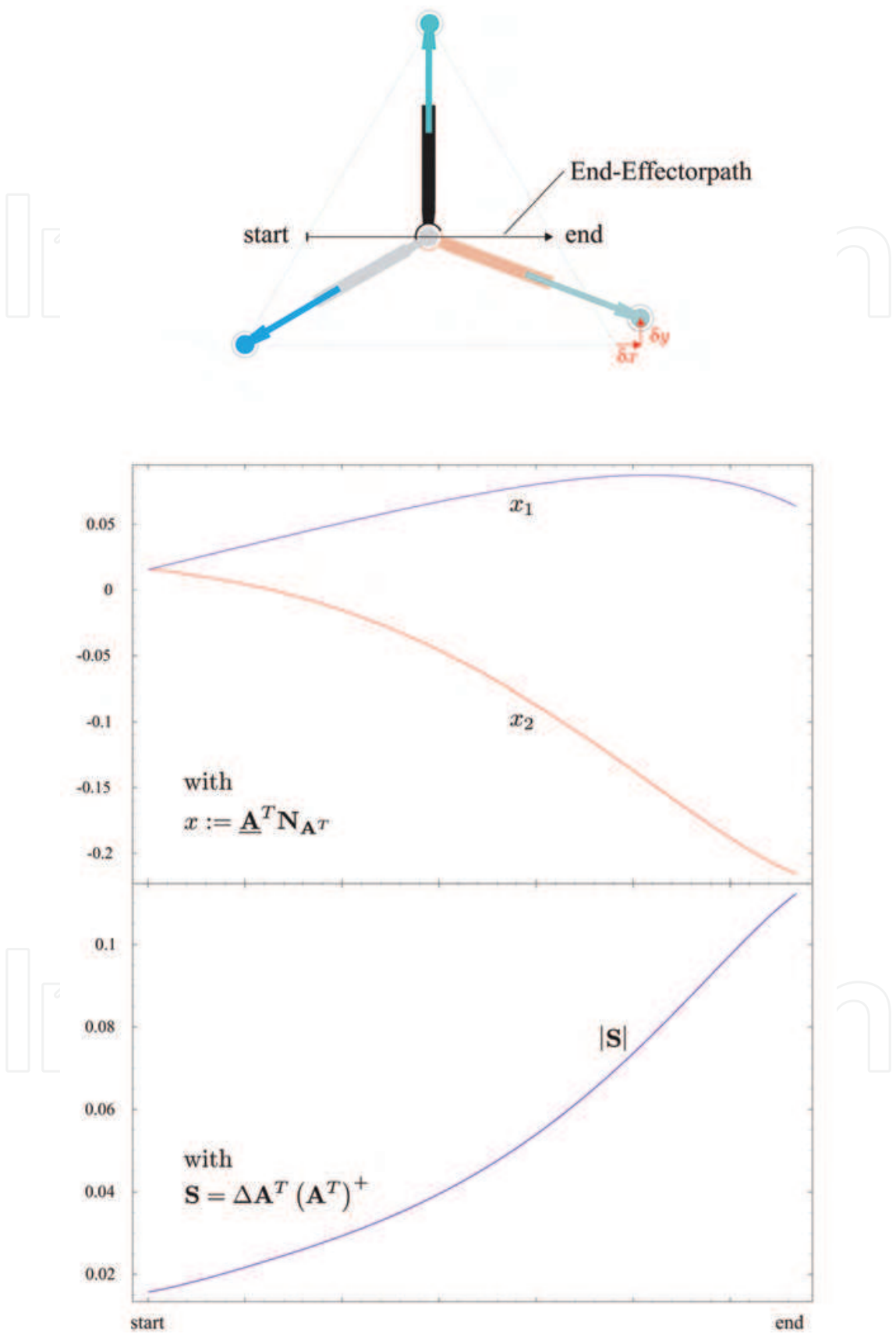
Figure 7. Effect of displacement of a drive unit of the PKM in figure 6.

9. Conclusions and open problems

In this chapter the dynamics modeling of redundantly actuated PKM is reviewed, and the redundancy resolution is addressed. The resolution takes into account different secondary tasks, such as backlash avoidance and stiffness control.

It was aimed to point out the potential of redundant actuation, but also the challenges that need to be addressed. In this contribution the effect of kinematic parameter uncertainties on the control of redundantly actuated PKM is analyzed. It is shown how geometric uncertainties effect the control system. The application of standard model-based control schemes to redundantly actuated PKM is shown not only to change the control system, but also to change the way in which control forces act upon the system. A consequence thereof is that the perturbation forces, due geometric uncertainties, can not be compensated by the actuation. To overcome these effects, an amended augmented PD and computed torque control scheme for redundantly actuated PKM is introduced. This is a first step that at least ensures the applicability of the control schemes. It shall be clear that robust control of uncertain redundantly actuated PKM is a critical issue for redundant PKM.

10. References

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Parallel Manipulators, towards New Applications

Edited by Huapeng Wu

In recent years, parallel kinematics mechanisms have attracted a lot of attention from the academic and industrial communities due to potential applications not only as robot manipulators but also as machine tools. Generally, the criteria used to compare the performance of traditional serial robots and parallel robots are the workspace, the ratio between the payload and the robot mass, accuracy, and dynamic behaviour. In addition to the reduced coupling effect between joints, parallel robots bring the benefits of much higher payload-robot mass ratios, superior accuracy and greater stiffness; qualities which lead to better dynamic performance. The main drawback with parallel robots is the relatively small workspace. A great deal of research on parallel robots has been carried out worldwide, and a large number of parallel mechanism systems have been built for various applications, such as remote handling, machine tools, medical robots, simulators, micro-robots, and humanoid robots. This book opens a window to exceptional research and development work on parallel mechanisms contributed by authors from around the world. Through this window the reader can get a good view of current parallel robot research and applications.

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