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Chapter

Thermo-Rheological Effect on Weak Nonlinear Rayleigh-Benard Convection under Rotation Speed Modulation

S.H. Manjula and Palle Kiran

Abstract

The effects of rotation speed modulation and temperature-dependent viscosity on Rayleigh-Benard convection were investigated using a non-autonomous Ginzburg-Landau equation. The rotating temperature-dependent viscous fluid layer has been considered. The momentum equation with the Coriolis term has been used to describe finite-amplitude convective flow. The system is considered to be rotating about its vertical axis with a non-uniform rotation speed. In particular, we assume that the rotation speed is varying sinusoidally with time. Nusselt number is obtained in terms of the system parameters and graphically evaluated their effects. The effect of the modulated system diminishes the heat transfer more than the un-modulated system. Further, thermo-rheological parameter $V_T$ is found to destabilize the system.

Keywords: rotation speed modulation, Coriolis force, thermo-rheological parameter, weak nonlinear theory

1. Introduction

For the past few decades, researchers are trying to explore various possibilities of controlling convective phenomena in a horizontal fluid layer by imposing external physical constraints. Some of them are temperature modulation, gravity modulation, magnetic field modulation and rotation speed modulation, etc. Motivated by the experiments of Donnelly [1], about the effect of rotation speed modulation on the onset of instability in fluid flow between two concentric cylinders, Venezian [2] was the first to perform a linear stability analysis of Rayleigh-Benard convection under temperature modulation by considering free-free surfaces. Later on, Rosenblat and Tanaka [3] studied the same problem of temperature modulation for rigid-rigid boundaries, using the Galerkin method. Bhadauria and Kiran [4] investigated thermal modulation on an-isotropic porous media. Their investigation reveals that an-isotropy of the medium and modulation has been used to regulate heat transfer in the system. It was concluded that an-isotropy play a role in instability of the porous media.
The study of rotation and thermal modulation on onset convection in a rotating fluid layer was investigated by Rauscher and Kelly [5]. Their results conclude that, for small values of Pr an oscillatory mode exists. However, they found that deviation of instability occurs at Pr = 1. Liu and Ecke [6] analyzed heat transport of turbulent Benard convection under rotational effect. Malashetty and Swamy [7], investigated the thermal instability of a heated fluid layer subjected to both boundary temperature modulation and rotation. Kloosterziel and Carnevale [8] investigated the thermal modulation effect on the stability analysis of the modulated system. They have analytically determined critical points on the marginal stability boundary above which an increase of either viscosity or diffusivity is destabilizing. They also show that, if the fluid has zero viscosity the system is always unstable, in contradiction to Chandrasekhar's conclusion. Bhadauria [9] studied the effect of rotation on thermal instability in a fluid-saturated porous medium under temperature modulation. Further, Bhadauria [10] studied the double-diffusive convection in a rotating porous layer with temperature modulation of the boundaries. Also, Bhadauria [11] studied the magneto fluid convection in a rotating porous layer under modulated temperature on the boundaries. Malashetty and Swamy [12] studied the effect of thermal modulation on the onset of instability in a rotating fluid layer. Bhadauria [13] investigated the rotational influence on Darcy convection and found that both rotation and permeability suppress the onset of thermal instability. Bhadauria et al. [14] investigated the nonlinear thermal instability in a rotating viscous fluid layer under temperature/ gravity modulation. They have concluded that both modulations may use alternatively to regulate heat transfer in the system.

The idea of rotation speed modulation was the originating idea of thermal as well as gravity modulation, but research work in this field is scarce. There are many studies that reported only linear thermal instability than nonlinear thermal instability. The effect of temperature modulation and the effect of rotation speed modulation on thermal instability has been investigated in detail both theoretically and experimentally by Niemela and Donnelly [15], Kumar et al. [16], Meyer et al. [17], Walsh and Donnelly [18]. In general, the effect of thermal modulation is supposed to stabilize conduction state. Also, it breaks the reflection symmetry about the mid-plane and forms hexagons, rather than cylinders. Then it constitutes the convection plan form immediately above the threshold. However, the above problem may be avoided if the rotation speed is modulated. Amongst the literature, the study due to Bhattacharjee [19] is of great importance, in which he studied the effect of rotation speed modulation on Rayleigh-Benard convection in an ordinary fluid layer. He found that the effect of modulation is to stabilize for most of the configurations. Bhadauria and Suthar [20] investigated the effect of the rotation speed modulation on the onset of free convection in a rotating porous layer placed farther away from the axis of rotation. Further, Om et al. [20] investigated the effect of rotation speed modulation on the onset of free Darcy convection. It is found that by applying modulation of proper frequency to the rotation speed, it is possible to delay or advance the onset of convection.

It is known that viscosity [4] plays a significant role in the study of fluid flows. It is mainly affected by temperature fluctuation due to its nature. From the above literature, the fluid viscosity is considered to be constant. However, in certain situations, fluid viscosity is not constant. This may vary with temperature, pressure, and distance. In most of the applications related to thermal transportation problems the temperature distribution is not uniform, i.e., the fluid viscosity may change noticeably if large temperature differences exist. That is why one needs to consider temperature-dependent viscosity in the energy equation.
The top and bottom structures of the fluid layer are different when the fluid viscosity varies with temperature. Kafoussius and Williams [21], investigated the effect of variable viscosity on the free convective boundary layer flow along with a vertical isothermal plate. It is concluded that when the viscosity of a working fluid is sensitive to the variation of temperature, care must be taken to include this effect, otherwise, considerable error can result in the heat transfer processes. Kafoussius et al. [22] examined the effect of temperature-dependent viscosity on the mixed convection laminar boundary layer flow along with a vertical isothermal plate. Molla et al. [23], studied the natural convection flow from an isothermal circular cylinder with temperature-dependent viscosity. Pal and Mondal [24] examined the effect of temperature-dependent viscosity and thermal radiation on MHD-forced convection over a non-isothermal wedge. Ching and Cheng [25], investigated temperature viscosity effects on natural convection for boundary layer flow. Nadeem and Akbar [25, 26], studied the effects of temperature-dependent viscosity on the peristaltic flow of a Jeffrey-six constant fluid in a uniform vertical tube. However, most of these studies are done with a steady temperature gradient across the fluid layer. Nield [27], investigated the effect of temperature-dependent viscosity on the onset of convection in a saturated porous medium. There is a stabilizing effect when the Rayleigh number is a function of mean viscosity.

Bhaduria and Kiran [28] investigated weak nonlinear analysis of magneto-convection [29] under magnetic field modulation. Using Landau mode, the effect of rotational speed modulation on heat transport in a fluid layer with temperature-dependent viscosity and internal heat source has been studied by Bhaduria and Kiran [29]. Centrifugally driven convection in a nanofluid saturated rotating porous medium with modulation has been investigated by Kiran and Narasimhulu [30]. The effect of gravity modulation on different flow models has been presented in [31-37]. These studies present results of gravity modulation on thermal instability in fluid or porous layer. Also, the effect of thermal modulation on convective instability has been investigated in [38–56]. The effect of thermal modulation on chaotic convection [38], throughflow effects in the presence of thermal modulation [39, 41, 42, 49, 56], thermal modulation with internal heating [40], thermal modulation on magneto-convection [43, 45, 48, 54, 55] have been investigated. The effect of rotation on nonlinear fluid convection under temperature modulation has been reported by Kiran and Bhaduria [57].

In most of the previous studies, only linear theory has been considered. This linear theory is not supportive to quantify heat/mass transfer. Also, other studies in the literature provide gravity modulation and thermal modulation on convective flows than rotation modulation. To the best of the authors’ knowledge, no nonlinear study is available in the literature in which the effect of rotation speed modulation has been considered in a temperature-dependent viscous fluid layer. This situation motivated us to study a rotational speed modulation on temperature-dependent viscous fluid layer.

2. Governing equations

An infinitely and horizontally extended viscous-incompressible fluid layer has been considered. It is confined between two parallel planes which are at \( z = 0 \), lower plane and \( z = d \), upper plane. The lower surface is heated and the upper surface is
cooled to maintain an adverse temperature gradient across the fluid layer. We assume that the system is rotating with variable rotation speed $\Omega(t) = (0, 0, \Omega(t))$, about the $z$-axis as shown in Figure 1. The effect of rotation speed is restricted to the Coriolis term; thus, we neglect the centrifugal force term. The effect of density variation is given by the Boussinesq approximation. With these assumptions the basic governing equations are:

$$\nabla \cdot \vec{q} = 0$$

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} + 2(\vec{\Omega}(t) \times \vec{q}) = -\frac{1}{\rho_0} \nabla p - \frac{\rho}{\rho_0} \vec{g} + \frac{\mu(T)}{\rho_0} \nabla^2 \vec{q}$$

$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = \kappa_T \nabla^2 T$$

$$\rho = \rho_0 (1 - \alpha_T (T - T_0))$$

$$\mu(T) = \frac{\mu_0}{1 + \epsilon^2 \delta_0 (T - T_0)}$$

Eq. 5 has been considered by many authors [4, 25–27, 29]. The rotation speed is assumed to vary sinusoidally with respect to time, as

$$\vec{\Omega}(t) = \Omega_0 (1 + \epsilon \delta \cos (\omega t)) \hat{k}$$

where $\vec{q}$ is velocity, $\nu$ is a kinematic viscosity, $p$ is reduced pressure, $\kappa_T$ is the thermal diffusivity, $\alpha_T$ is thermal expansion coefficient, $\rho$ is the density, $\rho_0$ and $T_0$ are the reference density and temperature, $\vec{g} = (0,0,-g)$ is gravitational acceleration, $\delta$, $\omega$ are the small amplitude and frequency of modulation, $\epsilon^2$ is a quantity that indicates the smallness in order of magnitude of modulation and $t$ is the time. The gravity is being considered in the downward direction.

The constants and variables used in the above Eqs. (1)–(5) have their usual meanings and are given in the nomenclature. The thermo-rheological relationship in Eq. (5) is guided by Nield [27] and Bhadauria and Kiran [29]. The considered thermal boundary conditions are:

![Figure 1](https://example.com/figure1.png)

_Figure 1._ Physical configuration of the problem.
We consider the following steady thermal boundary conditions are:

\[
\begin{align*}
T &= T_0 + \nabla T \quad \text{at } z = 0 \\
T &= T_0 \quad \text{at } z = d
\end{align*}
\] (7)

The conduction state is assumed to be quiescent i.e. \( q_b = (0,0,0) \). The other quantities of the basic state are:

\[
\rho = \rho_b(z), \quad p = p_0(z) \quad \text{and} \quad T = T_b(z)
\] (8)

Substituting the Eq. (8) into Eqs. (1)-(5), we get the basic state equations, respectively for pressure and temperature, as

\[
\frac{\partial p_b}{\partial z} = -\rho_b g
\] (9)

\[
\frac{d^2 T_b}{dz^2} = 0
\] (10)

\[
\rho_b = \rho_0(1 - \alpha T_b - T_0)
\] (11)

where “b” refers the basic state. The Eq. (10) is solved for \( T_b(z) \) subject to the boundary condition Eq. (7), we get

\[
T_b = T_0 + \Delta T \left(1 - \frac{z}{d}\right)
\] (12)

The perturbations on the basic state are superposed in the form

\[
\tilde{q} = q_b + \tilde{q}, \quad T = T_b + T', \quad \rho = \rho_b + \rho' \quad \text{and} \quad p = p_b + p'
\] (13)

where the perturbations are of finite-amplitude. Substituting Eq. (13) i in Eqs. (1)-(5) and using the basic state results Eqs. (9)-(12), we obtain

\[
\nabla \cdot \tilde{q}' = 0
\] (14)

\[
\frac{\partial \tilde{q}'}{\partial t} + \left(\tilde{q}' \cdot \nabla\right)\tilde{q}' + 2 \left(\nabla(T) \times \tilde{q}'\right) = -\frac{1}{\rho_0} \nabla p + \alpha \left(\frac{T'}{T_b}\right) + \frac{\mu(T)}{\rho_0} \nabla^2 \tilde{q}'
\] (15)

\[
\frac{\partial T'}{\partial t} + \omega \frac{dT_b}{dz} + \left(\tilde{q}' \cdot \nabla\right)T' = \kappa \nabla^2 T'
\] (16)

We consider, two-dimensional convection in our study, introducing, stream function \( \psi \) as \( u = \frac{\partial \psi}{\partial z} \) and \( \omega = -\frac{\partial \psi}{\partial x} \). Non-dimensionalizing the physical variables \((x,y,z) = (x^*, y^*, z^*)\) \( t = \frac{\alpha}{\nu} t^*, \quad \tilde{q} = \frac{\nu}{\nu} \tilde{q}^*, \quad \psi = \kappa_T \psi^*, \quad T' = \Delta T T^* \) and \( \omega = \frac{\nu}{\nu} \omega^* \) by eliminating the pressure term and finally dropping the asterisk, we obtain:

\[
\frac{1}{Pr} \frac{\partial}{\partial t} \nabla^2 \psi - \frac{1}{Pr} \frac{\partial (\psi, \nabla^2 \psi)}{\partial (x, z)} = -RaT \frac{\partial T}{\partial \xi} + R \frac{\partial \tilde{p}}{\partial \xi} \tilde{V}^2 \psi + \sqrt{T \alpha \left(1 + \nu^2 \cos(\omega t)\right)} \frac{\partial \tilde{V}}{\partial \xi} + \frac{\partial \tilde{p}(T) \frac{\partial}{\partial \xi}}{\partial \xi} \nabla^2 \psi
\] (17)
\[- \frac{dT}{dz} \frac{\partial \psi}{\partial x} - \nabla^2 T = - \frac{\partial T}{\partial t} \frac{\partial (\psi, T)}{\partial (x, z)} \quad (18)\]

\[\frac{1}{Pr} \frac{\partial V}{\partial x} - \nu(T) \nabla^2 V = - \sqrt{T_A} (1 + \varepsilon^2 \delta \cos(\omega t)) \frac{\partial \psi}{\partial z} + \frac{1}{Pr} \frac{\partial (\psi, V)}{\partial (x, z)} \quad (19)\]

The non-dimensional parameters which are obtained in the above equations are:

\[Pr = \frac{\nu}{\kappa T} \] is the Prandtl number,

\[RaT = \frac{\eta \Delta T^0}{\kappa v} \] is thermal Rayleigh number,

\[Ta = \frac{4 \Omega^4 0 d^4 \nu^2}{\nu} \] is the Taylor number,

\[\nu = \frac{\mu}{\rho_0} \] is the kinematic viscosity. We assume small variations of time and re-scaling it as \( t = \varepsilon^2 \tau \) to study the stationary convection of the system. It is to be noted that in our study we are not considering overstable solutions of the system.

To solve Eqs. (17)–(19) we considering stress-free and isothermal boundary conditions as given bellow

\[\psi = \frac{\partial^2 \psi}{\partial z^2} = \frac{\partial V}{\partial z} = T = 0 \quad \text{at} \quad T = 0 \quad \text{and} \quad T = z \quad (20)\]

The above boundary conditions are free-free and stress-free isothermal, they are used by [28, 29, 32, 45, 54, 55].

3. Heat transport for stationary instability

We introduce the following asymptotic expansions (also used in [31–36]) in the above Eqs. (17)–(19):

\[(RaT, \psi, T, V) = R_{oc} + \varepsilon^2 R_2 + \varepsilon^2 R_3 + \ldots, \varepsilon \psi_1 + \varepsilon^2 \psi_2 + \ldots, \varepsilon T_1 + \varepsilon^2 T_2 + \ldots \varepsilon V_1 + \varepsilon^2 V_2 + \ldots \quad (21)\]

Where \( R_{oc} \) is the critical value of the Rayleigh number at which the onset of convection takes place in the absence of modulation. Now we solve the system for different orders of \( \varepsilon \). For reference see the studies of [28, 29, 32, 45, 51].

At the lowest order, we get the following system:

\[
\begin{bmatrix}
- \nabla^4 & R_{oc} \frac{\partial}{\partial x} & - \sqrt{T_A} \frac{\partial}{\partial z} \\
\frac{\partial}{\partial x} & - \nabla^2 & 0 \\
\sqrt{T_A} \frac{\partial}{\partial z} & 0 & - \nabla^2 
\end{bmatrix}
\begin{bmatrix}
\psi_1 \\
T_1 \\
V_1 
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
0 
\end{bmatrix} \quad (22)
\]

The solution of the lowest order system subject to the boundary conditions Eq. (20), is

\[\psi_1 = A(\tau) \sin (k_c x) \sin (\pi z)\]

\[T_1 = - \frac{k_c}{\beta^2} A(\tau) \cos (k_c x) \sin (\pi z)\]

\[V_1 = - \frac{\pi \sqrt{T_A}}{\beta^2} A(\tau) \sin (k_c x) \cos (\pi z) \quad (23)\]
where $\beta^2 = \pi^2 + k_c^2$ is the total wavenumber. The critical value of the Rayleigh number for the onset of stationary convection is calculated numerically and the expression is given by

$$R_{oc} = \frac{\beta^6 + \pi^2 Ta}{k_c^2} \quad (24)$$

For the system without rotation i.e. $Ta = 0$, we get $R_{oc} = \frac{\beta^6}{k_c^2}$ and its critical wavenumber is given as $k_c = \frac{\sqrt{2}}{\beta}$, which is the classical results obtained by Chandrasekhar [58]. At the second order, we have the following terms on RHS (similar to the system Eq. (22)).

$$R_{21} = 0 \quad (25)$$

$$R_{22} = \frac{\partial(y_1, T_1)}{\partial(x, z)} \quad (26)$$

$$R_{23} = \frac{\partial(y_1, V_1)}{\partial(x, z)} \quad (27)$$

The second order solutions subjected to the boundary conditions Eq. (20), is obtained as follows

$$\psi_2 = 0 \quad (28)$$

$$T_2 = -\frac{k_c^2}{8\pi^2} A(\tau)^2 \sin (2\pi z) \quad (29)$$

$$V_2 = \frac{\pi^2 \sqrt{Ta}}{8k_c \Pr^2} A(\tau)^2 \sin (2k_c x) \quad (30)$$

The horizontally averaged Nusselt number $Nu(\tau)$, for the stationary mode of convection is given by:

$$Nu(\tau) = 1 + \left( \frac{k_c}{BC} \left( \frac{dT_2}{dz} \right) dx \right) \bigg|_{z=0} \quad (31)$$

Substituting the expression of $T_2, T_b$ in the Eq. (31) and simplifying we get:

$$Nu(\tau) = 1 + \frac{k_c^2}{4\beta^2} A(\tau)^2 \quad (32)$$

At third order system the expressions in RHS (Eq. (33)) will have many terms and it is difficult to simplify and find expressions for $(\psi_3, T_3, V_3)$. 

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The expression given in the Eq. (33) may be obtained at third order system as:

$$ R_{31} = \frac{1}{Pr} \frac{\partial}{\partial t} \nabla^2 \psi_1 - R_2 \frac{\partial T_1}{\partial x} \sqrt{T_a} (1 + \delta \cos(\omega t)) \frac{\partial V_1}{\partial z} + V_T T_b V^4 \psi_1 $$  \hspace{1cm} (34)

$$ R_{32} = -\frac{\partial T_1}{\partial t} + \frac{\partial \psi_1}{\partial x} \frac{\partial T_2}{\partial z} $$ \hspace{1cm} (35)

$$ R_{33} = -\frac{1}{Pr} \frac{\partial V_1}{\partial t} - \sqrt{T_a} (V_T T_b + \delta \cos(\omega t)) \frac{\partial \psi_1}{\partial z} + \frac{1}{Pr} \frac{\partial \psi_1}{\partial z} \frac{\partial V_2}{\partial x} $$ \hspace{1cm} (36)

Substituting the first and second order solutions into Eqs. (34)–(36), we can easily simplify the expressions $R_{31}, R_{32}$ and $R_{33}$.

Now, by applying the solvability condition for the existence of third order solution, we get the Ginzburg-Landau equation for stationary mode \cite{28-36, 38-45, 52-56} of convection, with time-periodic coefficients, in the form:

$$ Q_1 \frac{\partial A(\tau)}{\partial \tau} - Q_2 A(\tau) + Q_3 A(\tau)^3 = 0 $$ \hspace{1cm} (37)

Where $Q_1 = \frac{\beta^2}{Pr} + \frac{R_0 k^4}{\rho^4} \frac{\beta^2 T_a}{Pr^2}$, $Q_2 = \frac{R_0 k^4}{\rho^4} - \frac{2 \beta T_a}{4Pr} \delta \cos(\omega t) - H_1$, and $Q_3 = \frac{R_0 k^4}{\rho^4} + \frac{\beta^2 T_a}{4Pr} + \frac{\delta}{Pr}$.

The equations given by Eq. (37) is known as the Bernoulli equation. This equation is solved numerically using NDSolve of Mathematica 8. A suitable initial condition $A(0) = a_0$ maybe chosen to solve the amplitude equation. In our calculations we take $R_2 = R_0$ to analyze instability near to critical Rayleigh number.

### 4. Analytical solution for un-modulated case

In the case of un-modulated system $\delta = 0$, we obtain the following analytical expression for $A_u(\tau)$

$$ A_u(\tau) = \frac{1}{\sqrt{A_1 + C_1 e^{\frac{2\pi \tau}{\eta}}}} $$ \hspace{1cm} (38)

where $C_1$ is an integrating constant to be calculated for a given initial condition. The horizontal Nusselt number is obtained from the Eq. (32) substituting $A_u(\tau)$ in place of $A(\tau)$.  


5. Results and discussion

In this problem, we address a weakly nonlinear Rayleigh-Benard convection with a variable viscous liquid under rotational speed modulation. Here, we have presented the results of weakly nonlinear stability analysis to investigate the effect of rotational speed modulation and thermo-rheological parameters on heat transport. The modulation of the Rayleigh-Benard system has been assumed to be of order $O(\varepsilon^2)$, which shows, that we consider the only small amplitude of rotation speed modulation (see the article of [29–36, 38–45] for different modulations at the third order of Landau equation). This assumption will help us in obtaining the amplitude equation in a simple manner, and much easier than the Lorenz model. The present work is important to study nonlinear convection and quantify heat transport. External regulation of convection is important in the study of thermal instability in a fluid layer, therefore, in this paper, we have considered rotational speed modulation for either enhancing or inhibiting convective heat transport as is required by a real-life application.

The results of rotational speed modulation on heat transport have been depicted in the Figures 2–10. Here, we have drawn the figures Nu versus slow time. We observe the nature of the graphs where Nu is a function of time and vary with respect to different parameters. To see the results corresponding to the parameters $Pr, V_T, Tu, \delta,$ and $\omega$, we have drawn figures with variations in slow time. The mentioned parameters describe the convective flow of heat transport. The first two parameters are related to the fluid layer and the next three parameters concern the external mechanism of controlling convection. The fluid layer is not considered to be highly viscous, therefore, only moderate values of $Pr$ are taken for calculations. Because of small amplitude modulation, the values of $\delta$ are considered around 0.5. Further, modulation considers low frequency, due to maximum heat transport. The effect of frequencies of modulation influences the onset of convection as well as on heat transport. The thermo-rheological parameter $V_T$, has taken to be small values.

![Image](image.png)

**Figure 2.** Nu versus $\tau$ for different values of $Pr$. 

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Before writing the discussion of the results, we mention some features of the following aspects of the problem.

1. The importance of nonlinear stability analysis to study heat transfer.
2. The relation of the problem to the real application (Temperature, viscosity).
3. The selection of all the dimensional parameters utilized in computations.

In Figures 2–8, we have plotted the Nusselt number \( Nu(\tau) \) with slow time \( \tau \) for the case of rotational speed modulation. From the figures, we find that for small-time \( \tau \),
the value of $Nu(\tau)$ does not alter and remains almost constant, then it increases on increasing $\tau$, and finally becomes oscillatory on further increasing $\tau$. It is clear from the figure that $Nu(\tau)$ starts with one showing the conduction state, initially.

In Figure 2, we see that $Nu(\tau)$ increases upon increasing $Pr$ for fixed values of other parameters (see the studies of [46–48]). This may happen due to the dominating role of thermal diffusivity $\kappa_T$ over kinematic viscosity $\nu$. As Prandtl number $Pr$ increases, then for no change in kinematic viscosity, probably there is a large decrement in thermal diffusivity, and this makes the sudden increase in the temperature gradient. So, convection takes place early, and there is an enhancement in heat transfer. Thus, the effect of an increment in Prandtl number $Pr$ is to advance the convection. There are few studies, which relate the effect of $Pr$ given by [28, 29, 32, 43,
45, 48, 52]. A similar effect may be observed for $V_T$ given in Figure 3. The reader may notify that the heat transport is more in the presence of variable viscosity. We have

$$\text{Nu}_{Pr=0.5} < \text{Nu}_{Pr=0.6} < \text{Nu}_{Pr=0.7}$$

$$\text{Nu}_{V_T=0.1} < \text{Nu}_{V_T=0.2} < \text{Nu}_{V_T=0.3}$$

From the Figure 4, we depict the effect of the Taylor number on $\text{Nu}(\tau)$ for fixed values of other parameters. An increment in $Ta$ increases the value of critical Rayleigh number $R_{0c}$, and it delays the onset of convection, hence, heat transport decreases.

Figure 4 shows that as $Ta$ increases, the amplitude of modulation enhances. From the Eq. (37) we observe that the rotation is multiple of amplitude modulation. This means that the amplitude is dependent on rotation. Generally, if there is no rotation ($Ta = 0$), it is meaningless to talk about rotation speed modulation. Further, for no rotation $Ta = 0$, the effect of frequency diminishes, so the effect of frequency of modulation can be seen when rotation is not there. In our study consider $Ta$ non-zero otherwise modulation effect disappears.

$$\text{Nu}_{Ta=30} < \text{Nu}_{Ta=25} < \text{Nu}_{Ta=20}$$

For reference, the reader may observe the studies of [29, 41, 50, 51]. In Figure 5, we depict the effect of amplitude of modulation for moderate values of $Ta$ and for the fixed values of other parameters. On increasing the value of $\delta$, the value of $Nu(\tau)$ increases, hence advancing the convection so the heat transport. This means that an increasing amplitude of modulation increases heat transfer.

In the case of un-modulated $\delta = 0$ system shows no influence on heat transport for larger values of time $\tau$. Similar results can be obtained analytically for an unmodulated system given by Eq. (38).

$$\text{Nu}_{\delta=0.0} < \text{Nu}_{\delta=0.5} < \text{Nu}_{\delta=1.0} < \text{Nu}_{\delta=1.5}$$

Figure 7. $\text{Nu}$ verses $\tau$ with or without $Ta$. 

[Image 135x639 to 476x855]
Figure 6, shows that the effect of frequency on \( \nu \), for small values of \( \omega \), heat transports is more. An increment in the value of \( \omega \) decreases the magnitude of heat transfer and shortens the wavelength. Upon frequency increasing from 2 to 100, \( \nu(\tau) \) decreases. Further value of \( \omega \), the effect of modulation disappears altogether. Hence, the effect of \( \omega \) is to stabilize the system. It is clear from the studies of [29–36, 38–43] that frequency of modulation can reduce heat transfer. We have the following inequality...
The results corresponding to $\delta$ and $\omega$ one can see the recent results obtained by the studies of [49–52]. **Figure 7** shows the heat transport is more when there is no rotation $Ta = 0$ (which means no modulation) than in the presence of rotation and modulation. Hence, rotation strongly stabilizes the system. The results of rotational effects on convection may compare with the results obtained in [29, 44, 57].

**Figure 9.**
Isotherms (a) $\tau=0.0$ (b) $\tau=0.1$ (c) $\tau=0.4$ (d) $\tau=0.7$ (e) $\tau=1.0$ (f) $\tau=1.5$.

$$\text{Nu}_{\omega=70} < \text{Nu}_{\omega=30} < \text{Nu}_{\omega=10} < \text{Nu}_{\omega=2}$$

The results corresponding to $\delta$ and $\omega$ one can see the recent results obtained by the studies of [49–52]. **Figure 7** shows the heat transport is more when there is no rotation $Ta = 0$ (which means no modulation) than in the presence of rotation and modulation. Hence, rotation strongly stabilizes the system. The results of rotational effects on convection may compare with the results obtained in [29, 44, 57].
We have drawn streamlines and isotherms in Figures 8–10. The fixed values of \( \tau = 0.0, 0.1, 0.4, 0.7, 1.0 \) and 1.5 for \( Pr = 0.5, Ta = 20.0, \delta = 0.5 \) and \( \omega = 2.0 \) has been consider.

We found that initially, the magnitude of streamlines is small (Figure 8a, b), and isotherms are straight (Figure 9a, b) showing the conduction state. However, as time increases, the magnitude of streamlines increases, and the isotherms lose their evenness. This shows that convection is taking place in the system. Convection becomes faster by further increasing the value of time \( \tau \). However, steady-state achieves beyond \( \tau = 1.0 \) as there is no change in the Figures 8d–f and 9d–f. Figure 10 shows that as \( V_T \) increases the isotherms lose their evenness which shows heat transfer increasing as \( V_T \). This nature of streamlines and isotherms is quite natural for convective flow models. Similar results can be observed from the studies of [31–36].

6. Conclusions

Heat transfer results are obtained in terms of Nusselt number, which is derived from Ginzburg-Landau equation and the effect of various parameters depicted

\[
\begin{align*}
\text{Nu}_{Ta \neq 0} & < \text{Nu}_{Ta = 0, \omega \neq 0}
\end{align*}
\]

Figure 10. Isotherms (a) \( V_T = 0.2 \) (b) \( V_T = 0.6 \) (c) \( V_T = 1.0 \) (d) \( V_T = 1.4 \).
graphically. The Prandtl number $Pr$ and variable viscosity $V_T$ is to increase heat transfer. The modulation loses its effect at sufficiently large values of frequency of modulation. Overall, the effect of rotation speed modulation is highly significant and can be used to reduce heat transfer. As the time $\tau$ passes the magnitude of streamlines increases, and isotherms lose their evenness, showing that convection is taking place. At $\tau \approx 1.0$ the system achieves an equilibrium state. Further, It was observed that amplitude and frequency of modulation have no effect on un-modulated system but, in the case of the modulated system shows sinusoidal behavior.

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Author details

S.H. Manjula¹ and Palle Kiran²*

1 Department of Mathematics (S & H), Vignan’s Foundation for Science, Technology and Research (VFSTR), Guntur, Andhra Pradesh, India

2 Department of Mathematics, Chaitanya Bharathi Institute of Technology, Hyderabad, Telangana, India

*Address all correspondence to: pallekiran_maths@cbit.ac.in

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