Chapter

An Innovative Maintenance Scheduling Framework for Preventive, Predictive Maintenance Using Ant Colony Optimization

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Abstract

The fourth industrial revelation has brought exponential technologies in the area of digitization, internet of things (IOT), artificial intelligence (AI), and optimization which has helped mining companies to increase the availability and utilization of equipment's. As mining companies implement predictive maintenance technologies, to improve overall equipment availability, there is more value to be unearthed if predictive maintenance is optimized with the production schedules. Ant colony optimization (ACO) is a metaheuristic that is inspired from the behavior of real ants to solve combinatorial optimization problems. This chapter describes an innovative maintenance scheduling framework in the context of optimizing schedules for preventive maintenance and predictive maintenance, with multiple constraints for optimized dynamic schedule to reduce the maintenance time, and production losses.

Keywords: ant colony optimization, predictive maintenance, preventive maintenance, mining, schedule, optimizing mining equipment schedule

1. Introduction

The high and volatile commodity prices are caused by unanticipated changes demand and supply [1]. These volatile prices put cost pressure on mining organization to optimize operations. The availability and utilization of mining equipment's, is the major contributor for an organization to manage costs and supply disruptions.

Traditionally, maintenance activity for mining equipment, relies on a series of time based or equipment running hours based checks for scheduling maintenance activities. The fourth industrial revolution provide organizations with a balanced approach to reduce costs with safety. As new technologies get deployed, the operations and maintenance landscape is continually being digitized by mechanization, automation, industrial internet of things (IIoT) and IT-OT (information technology – operational technology) integration. These technologies provide visibility to real time operations data. Analysing the data with artificial intelligence (AI) adds the ability to predict and respond to operational disruptions, for example – predict the next failure date of the asset and provide perspective guidance for maintenance. Further, optimization adds the capability to synchronizing the scheduled, predicted maintenance activity with production schedule to minimize the maintenance costs and production losses.

Ant colony optimization (ACO) [2, 3] is a metaheuristic for solving hard combinatorial optimization problems. This was proposed by Dorigo et al., inspired from the behaviour of real ants, which use pheromones as a communication medium to find the shortest path to food from the colony. Analogous to the biological example, ACO is based on indirect communication within a colony of simple agents, called (artificial) ants, mediated by (artificial) pheromone trails. In ACO algorithms there are several generations of artificial ants which search for good solutions. In each generation, each ants find a solution by going step by step through many probabilistic decision till a solution is found. Ants that found good solutions put some amount of pheromone on the edges of path to mark their path. This will help attract the next generation of ants to find solutions near the good space. Generally pheromone values of ants are guided by the specific heuristic that is used for evaluating decisions.

In this chapter, we will focus on the framework for optimizing the preventive maintenance, predictive maintenance and production schedule. The first section will cover the maintenance strategies and framework. The second section will give a brief overview of the ACO. The third section will cover the maintenance solution framework for mining equipment's. In the last section we will conclude our recommendations for mine equipment maintenance scheduling.

2. Maintenance strategies and framework

The systematic, optimally sequenced activities and framework, through which mining companies can sustainably manage their equipment, its performance, risks and operating expenses over their entire life cycle, for the purpose of achieving its organizational objectives and plan, can be defined as an enterprise asset management. Mining equipment maintenance requires series of checks by the equipment operator, for better diligence apart from unscheduled fixes. The frequency is dependent on the combination of equipment performance and the running hours in a specified time interval.

Maintenance costs can be upto 30% of direct costs [4] and more in terms of operations disruptions. To control maintenance costs, mining organization have centred their efforts on areas such as optimizing scheduled maintenance operations, deferring non-essential maintenance, reducing maintenance manpower, controlling inventories of spare parts even more adequately, and using contract maintenance support [5]. Below all the maintenance types, which are performed by the maintenance team at varied frequency depending on the nature of inputs from various sources are described

2.1 Regular maintenance—Type I

The first daily checks and safety practices are mandated by the OEM and occur the start of every shift or during operator changes. These proactive checks involve the inspection of all major system parameters such as engine temperature, tyre pressures, oil levels and control surfaces and are performed at the mine site itself. These checks can be part of the total productive maintenance (TPM) strategy and incorporates activities and actions performed by equipment operators with the intention of to ensure failure-free operations, fewer breakdowns and efficiency [6].

2.2 Preventive maintenance—Type II

Routine planned maintenance is about avoiding, reducing or eliminating the consequences of failures. The frequency of performance-based maintenance is done primarily on performance (running hours, KM run) of the equipment, time based (weekly, monthly, quarterly, yearly, and multiple of years) or as recommended by OEM and updated time to time.

The determination of inspection intervals is based on the reliability level to prevent potential catastrophic failures that augments extensive maintenance and increased downtime costs. The insights are considered to develop a preventive maintenance plan that is based on the deterioration of equipment key components and renewed life after repair [7]. The mechanical parts deterioration factors depend on multiple factors which are wear and tear, corrosion, operational fatigue and weather, and operator skills. This deterioration is a continuous process which is time and usage dependent [8].

2.3 Breakdown or corrective maintenance—Type III

The unscheduled breakdown has a major impact in both – production and expedited maintenance costs. Furthermore, the ability to fix it right first time is fundamental to ensure equipment is back to operation quickly. The core objective for repairing the equipment is in less time, with higher accuracy and support maintenance technicians to diagnose the failure properly is key for improved utilization. The breakdown and corrective maintenance require timely availability of spares and management of spares just in time is one of the main challenges of delay in maintaining of the equipment.

Condition based maintenance (CBM) provides the insights of the equipment component, either real time or at a specific frequency to analyse the condition for effective decision making for dynamic corrective maintenance schedule for restoration. CBM insights can be utilized either for preventive or corrective maintenance depending on the organizational maintenance strategy [9].

2.4 Overhaul and shutdown maintenance—Type IV

This type of maintenance is mostly capital in nature and performed at relatively large intervals. It may be done for several reasons due to severe break- down, accident or to overhaul the entire equipment to enhance the useful life of the equipment. This overhaul or shutdown maintenance is managed as a project with procurement of spare parts and maintenance activities are scheduled and synchronized as per the timeline agreed upon. Overhaul is complete check and review of the mining equipment and depends on the performance of the equipment or between the mid to end of useful life to enhance the useful life of the equipment. This type of maintenance is performed mostly on the large equipment for example, dragline and shovels. This maintenance can run up-to multiple months and are managed as a project with a sequence of activities and schedules, to minimize equipment downtime.

2.5 Predictive maintenance—Type V

This type of maintenance is performed based on accurate prediction when an equipment or any of its components are going to fail. If the prediction attribute is derived the maintenance can be executed just before such failure is predicted to occur.

There are systems like vehicle health monitoring system (VHMS) which provide, frequency-based sample data about the equipment performance for example, running hours, speed, rpm, load, engine temperature, payload so on and so forth. Similarly, weather related data is collected through weather application programming interface (API), which helps to understand the impact of ambient temperature, precipitation, humidity etc. on the equipment performance. The main idea behind the IIoT is to connect computers, devices, sensors, and industrial equipment and applications within an organization and to continually collect data, such as system errors and machine telemetry, from all of these with the aim of analysing and acting on this data in order to optimize operational efficiencies. Predictive maintenance is more effective than performing preventive maintenance at frequent intervals, which could also be costlier because unnecessary maintenance may be applied on equipment.

The above types of maintenance strategies help organization to develop a comprehensive maintenance framework to maximize value and realise benefits. Specifically for mining equipment, the maintenance planning can be done at

- Workshop—typically for moving equipment like dump trucks, haulers, dozers, graders, and wheel loaders

- Onsite at mine—typically for large slowly/non-movable, equipment like dragline, large shovels

2.6 Strategies and framework

Overall, maintenance affects, all aspects of business efficacy, safety, environmental impact, energy efficiency, product quality, customer service, plant availability and cost. Many times scheduled maintenance activity is seldom integrated with the production [10] which leads to unplanned production losses due to planning of scheduled and preventive maintenance activities.

Therefore, the selection of the right maintenance framework plays a significant role in preserving the functions of the equipment and supporting mining organization value drivers:

- Improve revenue—increased asset availability and greater reliability in line with production schedule i.e. grow more revenue from the same asset base

- Reduce operating costs and expenses—more timely and precise interventions, increase asset life, less downtime, high utilization

In order to achieve these benefits, efficient combination of preventive (Type II) and predictive maintenance (Type V) with production schedule is required to reduce the overall maintenance execution time and maximize production.

3. Understanding ant colony optimization

Ant colony optimization (ACO) was inspired by the observation of the behaviour of real ants. Real ants, which use pheromones as a communication medium to find the shortest path to food from the colony [11]. As in the case of real ants, the problem is to find the food, in the case of artificial ants, it is to find a good solution to a given optimization problem.

One ant (either a real or an artificial one) can find a solution to its problem, but only cooperation among many individual ants through stigmergy enables them to find good solutions [12]. In real ants stigmergic communication happens via the pheromone that ants deposit on the ground. Artificial ants live in a virtual world, hence they only modify numeric values (called for analogy artificial pheromones) associated with problem states they visit while building solutions to the optimization problem. Real ants simply walk, choosing a direction based on local pheromone concentrations and a stochastic decision policy. Artificial ants also create solutions step by step, moving through available problem states and making stochastic decisions at each step.

The ACO metaheuristics has an initialization step and then a loop over three basic components. In one iteration of the loop, there are steps to construct the solution by all ants, improve (optional) the solutions with local search also and then an update of the pheromones.

Algorithm for ant colony optimization metaheuristic

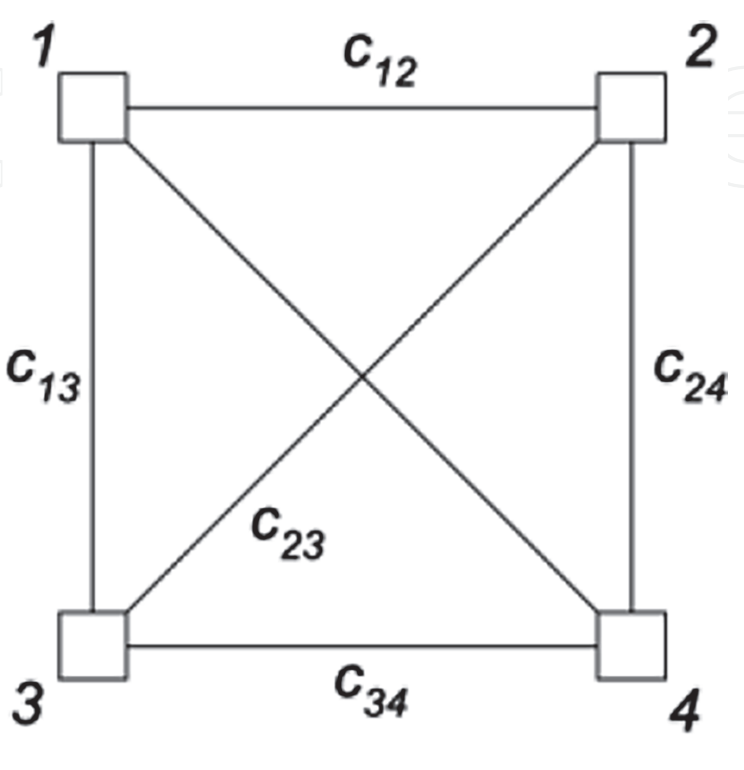
```
Set initial values and initialize pheromone trails
while
    termination conditions not met do
        Construct Ant Solutions
        Apply Local Search {optional}
        Update Pheromones
end while
```

In the next section ACO is explained using travelling salesman problem example.

3.1 Example: the traveling salesman problem

Travelling salesman problem (TSP) can be easily applied to the Ant colony optimization. In this problem, there are a set of locations (cities) where the travelling salesman has to visit. The key constraints are to visit each location and visit only once. The distance between cities (locations) are given and the objective is to find the shortest distance between them.

In the example below, there are four cities, c1, c2, c3 and c4. The lengths of the edges between vertices is proportional to the distance between cities i.e. c13 is the distance between city 1 and city 3.



The pheromone is associated with the edges of the graph. Each ant starts from a randomly selected city and then at each construction step it moves along the edges of the graph. An ant chooses probabilistically the edge to follow among the available ones (those that lead to yet unvisited vertices).

3.1.1 Sample equation for implementation

$$p(cij|s^p) = \frac{\lambda^{\alpha}{}_{ij} \cdot \eta(c_{ij})^{\beta}}{\sum_{c_{ij} \in N(s^p)} \lambda^{\alpha}{}_{ij} \cdot \eta(c_{ij})^{\beta} c}, \forall c_{ij} \in N(s^p). \tag{1}$$

where pheromone value associated with the component cij is λij. Function that assigns at each construction step a heuristic value to each feasible solution component $c_{ij} \in N(s^p)$ is η(·) which is commonly called heuristic information. Positive parameters, whose values determine the relative importance of pheromone versus heuristic information are α and β.

The solution is constructed once the ant has visited all the vertices of the graph. When all the ants have constructed the solutions by visiting the vertices of the graph, pheromone levels on the edges are updated positively for good solutions and reduced for the bad solutions. The update function, typically does two things one is to increase the pheromone values for set of good solutions and second is to reduce the pheromone value by implementing an evaporation function. This helps to avoid rapid convergence of the algorithm and helps in the exploration of new areas.

3.1.2 Sample equation for pheromone update

$$\lambda_{ij} \leftarrow (1-\rho)\tau_{ij} + \rho \sum_{s \in S_{\text{upd}} | c_{ij} \in s} F(s) \tag{2}$$

where set of solution that are used for update are S_{upd}, the parameter that is called for evaporation rate is $\rho \in (0,1]$, and $F : S \to R+0$ a function such that $f(s) < f(s') \Rightarrow F(s) \geq F(s'), \forall s \neq s' \in S$. F(·) is commonly called the fitness function.

Ant colony optimization has been shown to perform quite well on the TSP [13].

3.2 Other applications of ACO: Scheduling problems

ACO has been used for many applications including scheduling problem, vehicle routing problem, assignment problem, set problem, device sizing problem in nanoelectronics physical design, antennas optimization and synthesis, image processing. In this chapter our focus is on the scheduling problems.

In scheduling problems, jobs have to be processed on one or many machines such that some objective function is optimized. For these problems the following is true (a) the processing time of jobs is known beforehand and (b) processes of jobs cannot be interrupted. Typically the construction graph for scheduling problems is represented by the set of jobs (for single-machine problems). Some of the key research papers published for scheduling problems are:

- Group-shop scheduling problem (GSP) [14]
- Sequential ordering problem (SOP) [15]
- Job-shop scheduling problem (JSP) [16]

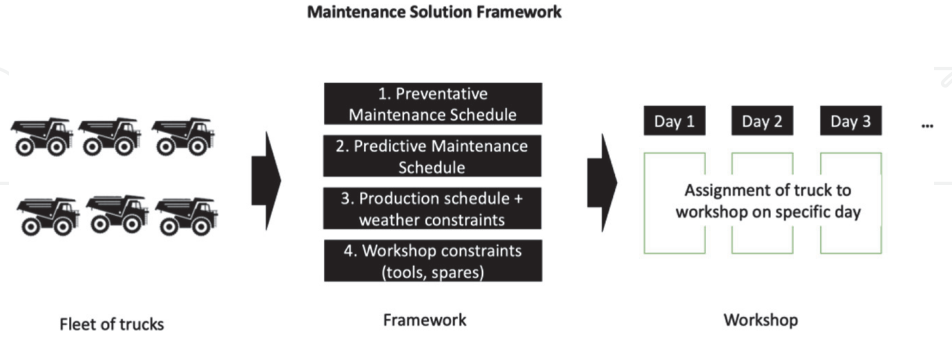
- Multistage flowshop scheduling problem (MFSP) with sequence dependent setup/changeover times [17]
- Permutation flow shop problem (PFSP) [18]
- Open-shop scheduling problem (OSP) [19, 20]
- Single-machine total tardiness problem with sequence dependent setup times (SMTTPDST) [21]
- Single machine total tardiness problem (SMTTP) [22]
- Resource-constrained project scheduling problem (RCPSP) [23]
- Single machine total weighted tardiness problem (SMTWTP) [24–26]

Out of the above scheduling applications, the SMTWTP has the best application for our maintenance schedule. In the subsequent sections, SMTWTP is described in greater detail for developing the optimal schedule for maintenance of mining equipment's.

4. Maintenance solution framework

The optimized maintenance scheduling framework is recommended to be built using ant colony optimization model using structured content found in a typical maintenance ecosystem.

The first step is to predict the failure based on survival analysis, with certain confidence level before its occurrence. The next step is to combine the predictive maintenance and preventative maintenance schedule using optimization model. This optimization model determines which equipment should be assigned to which day in the maintenance workshop bay for minimizing waiting time, maximizing production and ultimately increase the availability.



In the solution framework, two types of maintenance activities are considered: first preventive maintenance which is based on the recommendation of OEM. The second maintenance is predictive maintenance which is based on probabilistic failure. The next section will cover the derivation of predictive maintenance schedule.

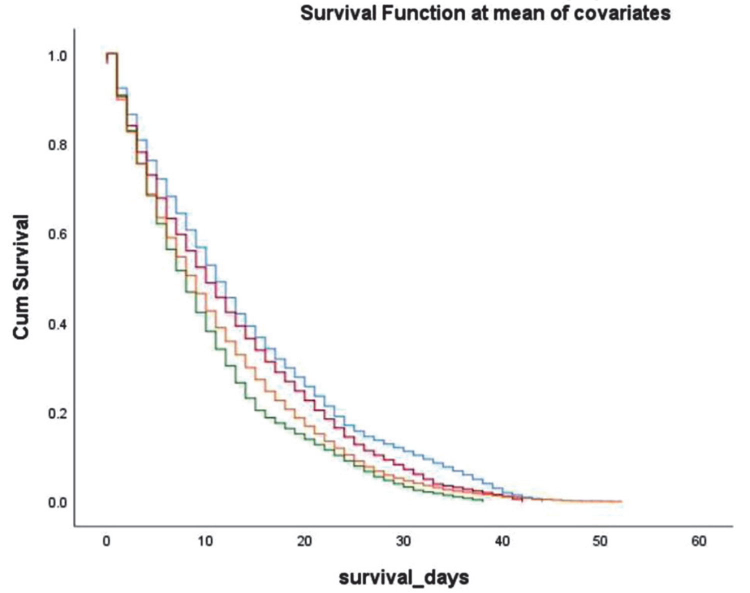
4.1 Predictive maintenance schedule

The predictive maintenance schedule is derived from Cox regression model which gives survival probability distribution function. The Cox regression model is shown in below

$$H(t) = H_0(t)\exp(\beta 1Y1 + \beta 2Y2 + \cdots + \beta nYn \quad (3)$$

Where the expected hazard is $H(t)$ at time t, the baseline hazard is $H_0(t)$ and it represents the hazard when all of the independent variables) $Y1, Y2, \ldots Yn$ when they are equal to zero. Based on the collected data the model estimates $\beta 1, \beta 2, \ldots \beta n$.

The expected hazard function increases as the days progress. This function is converted to survival days and remaining useful life (RUL). RUL is defined as the duration left for the occurrence of breakdown based on the probability threshold of failure i.e. how many days when the cumulative probability falls below 60%. The RUL has been extensively used in calculating the reliability-based research in the mine system to derive the occurrence of the failure, so the appropriate action can be taken proactively



This survival data (predictive failure day) is used in combination with the preventive maintenance, production schedule and other constraints to optimizing the maintenance schedule.

4.2 Optimization of maintenance schedule

The optimization of maintenance schedule requires to determine the optimal maintenance day for each truck in a time horizon so that the maintenance time, production loss is minimised while meeting the preventative maintenance schedule requirements and minimizing the probability of failure (predictive maintenance – survival days). In the above sections we discussed on the application of ACO in the context of SMTWTP.

In order to formulate the problem using SMTWTP, it is assumed that there is one workshop and it has one bay for carrying out maintenance activities (single machine). One truck is represented as one job and a fleet has n trucks which need to be scheduled for maintenance. Based on historical analysis, the time for each job i.e. time for carrying out maintenance activities is available (processing time p_j). The due date (d_j) of processing is provided by the preventive maintenance schedule for each job (truck). The completion time of job j is defined as C_j. The earliness of job is defined as E_j, if the job is completed early and the tardiness of job is defined as T_j, if the job is completed late. The probability of failure (f_j) at C_j is provided by the predictive maintenance schedule (RUL). The cost functions (w) for earliness and lateness take the probability of failure (f_j) into consideration.

The objective is to find the truck fleet maintenance scheduling sequence that minimizes the function as below.

$$1|d_j| \sum \underset{-j}{w} E_j + \bar{w}_j T_j \tag{4}$$

where

d_j – due date of job (preventative maintenance scheduled day for truck)

$\underset{-j}{w}$ – unit cost of earliness i.e. cost of maintenance too early based

E_j = max{0,d_j–C_j}; earliness of job

T_j = max{0,C_j–d_j}; tardiness of job

$\bar{w}_j$ – unit cost of tardiness i.e. cost of lost production if failure before maintenance (f_j)

N = {1,…,n}; n trucks (jobs) have to be sequentially processed (1 job = 1 truck maintenance activities) at workshop (1 bay)

This function can be modelled to minimizes the total weighted earliness tardiness (z) as below

$$z = \min \sum_{j \in N} \underset{-j}{w} E_j + \bar{w}_j T_j \tag{5}$$

Mainly there are three key requirements of the ACO algorithm.

- A construction graph – The construction graph consist of C components for the n jobs that need to be assigned at the optimal positions. In the graph each points is connected by L arcs.
- Problem constraints – The main constraint is that all the jobs have to be scheduled and scheduled only once.
- Update pheromone trails – This refer to the attractiveness of scheduling or assigning the job j to position i.

Applying the ACO algorithm, in the initialization step, a colony A of m ants is generated, where each ant corresponds to a random feasible solution. The next is the iterative step where the acquired knowledge (pheromone level) is fetched and job assignment attractiveness is calculated. Next the ant generations are merged and only the best ants are retained for the optimal solution. Lastly pheromone evaporation and deposit are updated and the process continues till the maximum number of Generations are reached.

ACO pseudo code
Input parameters

- N, is a set of n trucks (jobs that need to be processed in workshop)
- C, the number of colonies
- n, the number of ants in the colony (i.e. size)

Output solution

- A (near) global optimum S* of cost z_{a^*}

Steps

1. Setup
    1. Initialize the generation counter g=0
    2. Create an initial colony A of size n
    3. Set the best solution S* to the ant a ∈ A with the least weighted earliness tardiness
    4. Initialize the pheromone level $\rho(0)$ using a subset of the colony A, and set $\rho(1) = \rho(0)$
2. Loop Step
    1. Set g=g+1
    2. Build colony of ants while taking into account for knowledge acquired ρ (g) and attractiveness η (g), and apply a dynamic visibility function
    3. Merge colonies of generations g and (g−1), and retain the best n ants
    4. Update optimal solution S*
    5. Update the pheromone level ρ(g+1) by accounting for the evaporation and deposit
3. Stopping Criterion
    1. If g<C, then go-to Step 2.

At the end of step 2.3 to further enhance quality of the solution i.e. the retained n ants and speed up the convergence towards near optimal solution, a local search criteria can be applied. Hybrid approaches with local search criterial include beam search [20], scatter search, tabu search [27], threshold accepting [28], and neighbourhood search [29]. These search criteria help to efficiently guide the ants movements towards global optima. In the paper by M'Hallah and Alhajraf [30] ant colony systems for the single-machine total weighted earliness tardiness scheduling problem, they provide empirical evidence of using variable neighbourhood search (VNS) to improve the overall quality of the retained ants and converge towards a near global optimum.

By applying ACO to SMTWTP, the total cost for early or late maintenance is reduced by optimally assigning the truck to the workshop for maintenance activities based on the preventive maintenance schedule and predicted maintenance (RUL).

5. Conclusion

In this chapter, the importance of optimally planning maintenance activities for mining organization was discussed. A solution framework for optimizing the preventive maintenance, predictive maintenance and production schedule was proposed using ant colony optimization. Many mining organization can benefit by using this solution framework to reduce the overall maintenance costs and production losses.

As mining companies adopt and implement Industry 4.0, this maintenance solution framework has the potential to evolve beyond maintenance schedules to allocation of ore to customer demand, planning truck routing, and even to mine planning. ACO as part of the wider Swarm intelligence algorithms presents the capacity to achieve Industry 4.0 vision, where individual machines cooperate through self-organization, that is, without any form of central control to achieve the organization KPIs.

Conflict of interest

The views expressed in this chapter are my own and are not representative of my employer.

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References

[1] Commodity Price Volatility. Available from: https://treasury.gov.au/sites/default/files/2019-03/01_Commodity_price_volatility.pdf [Accessed: 27 December 2021]

[2] Dorigo M, Di Caro G. The ant colony optimization meta-heuristic. In: Corne D, Dorigo M, Glover F, editors. New Ideas in Optimization. London, UK: McGraw Hill; 1999. pp. 11-32

[3] Dorigo M, Di Caro G, Gambardella LM. Ant algorithms for discrete optimization. Artificial Life. 1999;**5**(2):137-172

[4] Henderson K, Pahlenkemper G, Kraska O. Integrated asset management—An investment in sustainability. Procedia Engineering. 2014;**83**:448-454. DOI: 10.1016/j.proeng.2014.09.077

[5] Unger EJ. An Examination of the Relationship Between Usage and Operating and Support Costs for Air Force Aircraft. Santa Monica, CA: RAND Corp.; 2007

[6] Brodny J, Tutak M. Application of elements of TPM strategy for operation analysis of mining machine. In: IOP Conference Series: Earth and Environmental Science journal. Vol. 95. IOP Publishing; 2017. p. 42019. Available from: https://iopscience.iop.org/article/10.1088/1755-1315/95/4/042019

[7] Angeles E, Kumral M. Optimal inspection and preventive maintenance scheduling of mining equipment. Journal of Failure Analysis and Prevention. 2020:1-9. DOI: 10.1007/s11668-020-00949-z. Available from: https://www.researchgate.net/publication/343413414_Optimal_Inspection_and_Preventive_Maintenance_Scheduling_of_Mining_Equipment.

[8] Changyou L, Haiyang L, Song G, Yimin Z, Zhenyuan L. Gradual reliability sensitivity analysis of mechanical part considering preventive maintenance. Advances in Mechanical Engineering. 2014;**6**:829850

[9] Marseguerra M, Zio E, Podofillini L. Condition-based maintenance optimization by means of genetic algorithms and Monte Carlo simulation. Reliability Engineering & System Safety. 2002;**77**(2):151-165

[10] Liao W, Pan E, Xi L. Preventive maintenance scheduling for repairable system with deterioration. Journal of Intelligent Manufacturing. 2010;**21**(6): 875-884

[11] Deneubourg J-L, Aron S, Goss S, Pasteels J-M. The self- organizing exploratory pattern of the argentine ant. Journal of Insect Behavior. 1990;**3**:159

[12] Dorigo M, Socha K. An introduction to ant colony optimization. IRIDIA—Technical Report Series. 2006

[13] Stützle T, Dorigo M. ACO algorithms for the traveling salesman problem. In: Miettinen K, Mäkelä MM, Neittaanmäki P, Périaux J, editors. Evolutionary Algorithms in Engineering and Computer Science. New York: Wiley; 1999. p. 163

[14] Blum C. ACO applied to group shop scheduling: A case study on intensification and diversification [permanent dead link]. In: Proceedings of ANTS 2002. Vol. 2463 of Lecture Notes in Computer Science. 2002. pp. 14-27

[15] Gambardella LM, Dorigo M. An ant colony system hybridized with a new local search for the sequential ordering problem. INFORMS Journal on Computing. 2000;**12**(3):237-255

[16] Martens D, De Backer M, Haesen R, Vanthienen J, Snoeck M, Baesens B. Classification with Ant Colony Optimization. IEEE Transactions on Evolutionary Computation. 2007;**11**(5): 651-665

[17] Donati AV, Darley V, Ramachandran B. An ant-bidding algorithm for multistage flowshop scheduling problem: Optimization and phase transitions. In: Advances in Metaheuristics for Hard Optimization. Springer; 2008. pp. 111-138. Available from: https://www.springerprofessional.de/en/an-ant-bidding-algorithm-for-multistage-flowshop-scheduling-prob/2812100

[18] Stützle T. An ant approach to the flow shop problem. Technical Report AIDA-97-07. 1997

[19] Pfahring B. Multi-agent search for open scheduling: Adapting the ant-Q formalism. Technical Report TR-96-09. 1996

[20] Blem C. Beam-ACO, hybridizing ant colony optimization with beam search. An application to open shop scheduling. Technical Report TR/IRIDIA/2003-17. 2003

[21] Gagné C, Price WL, Gravel M. Comparing an ACO algorithm with other heuristics for the single machine scheduling problem with sequence-dependent setup times. Journal of the Operational Research Society. 2002;**53**: 895-906

[22] Bauer A, Bullnheimer B, Hartl RF, Strauss C. Minimizing total tardiness on a single machine using ant colony optimization. Central European Journal for Operations Research and Economics. 2000;**8**(2):125-141

[23] Merkle D, Middendorf M, Schmeck H. Ant colony optimization for resource-constrained project scheduling. In: Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2000). 2000. pp. 893-900. DOI: 10.5555/2933718.2933886

[24] den Besten M. Ants for the single machine total weighted tardiness problem [master's thesis]. University of Amsterdam; 2000. Available from: https://www.academia.edu/9009128/An_Ant_Colony_Optimization_Application_to_the_Single_Machine_Total_Weighted_Tardiness_Problem

[25] den Bseten M, Stützle T, Dorigo M. Ant colony optimization for the total weighted tardiness problem. In: Proceedings of PPSN-VI, Sixth International Conference on Parallel Problem Solving from Nature. Vol. 1917 of Lecture Notes in Computer Science. 2000. pp. 611-620. DOI: 10.5555/645825.669098

[26] Merkle D, Middendorf M. An ant algorithm with a new pheromone evaluation rule for total tardiness problems. In: Real World Applications of Evolutionary Computing. Vol. 1803 of Lecture Notes in Computer Science. 2000. pp. 287-296

[27] Huang KL, Liao CJ. Ant colony optimization combined with taboo search for the job shop scheduling problem. Computers & Operations Research. 2008;**35**:1030-1046

[28] Marimuthu S, Ponnambalam SG, Jawahar N. Threshold accepting and ant-colony optimization algorithms for scheduling m-machine flow shops with lot streaming. Journal of Materials Processing Technology. 2009;**209**: 1026-1041

[29] Behnamian J, Fatemi Ghomi SMT, Zandieh M. Development of a hybrid meta heuristic to minimise earliness and tardiness in a hybrid flow shop with sequence-dependent setup times. International Journal of Production Research. 2010;**48**(5):1415-1438

[30] M'Hallah R, Alhajraf A. Ant colony systems for the single-machine total weighted earliness tardiness scheduling problem. Journal of Scheduling. 2015;**19**. DOI: 10.1007/s10951-015-0429-x. Available from: https://www.researchgate.net/publication/275220942_Ant_colony_systems_for_the_single-machine_total_weighted_earliness_tardiness_scheduling_problem