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Chapter

Searching of Individual Vortices in Experimental Data

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Abstract

The turbulent flows consist of many interacting vortices of all scales, which all together self-organize being responsible for the statistical properties of turbulence. This chapter describes the searching of individual vortices in velocity fields obtained experimentally by Particle Image Velocimetry (PIV) method. The vortex model is directly fitted to the velocity field minimizing the energy of the residual. The zero-th step (which does not a priori use the vortex model) shows the velocity profile of vortices. In the cases dominated by a single vortex, the profile matches the classical solutions, while in turbulent flow field, the profile displays velocity decrease faster than $1/r$. The vortices fitted to measured velocity field past a grid are able to describe around 50% of fluctuation energy by using 15 individual vortices, and by using 100 vortices, the fluctuating field is reconstructed by 75%. The found vortices are studied statistically for different distances and velocities.

Keywords: vortex, turbulence, Particle Image Velocimetry, grid turbulence, individual vortex searching algorithm, vortex model

1. Introduction

Contemporary exploration of turbulent flows focuses on statistical characteristics [1] such as study of distributions [2, 3], Fourier analysis [4], correlations [5, 6], or the Proper Orthogonal Decompositions [7–9]. The success of statistical approach is declared by the large applicability of numerical simulations, which are able to perfectly match the experimental data. Although it is possible to predict the statistical development of turbulent flow, this is still far from understanding the turbulence. The turbulence consists of vortices [10] and other coherent structures [11] whose multi-body interactions are responsible for the life-like behavior—the flow can be infected by turbulence [12], and it dies when it is not fed [13]; turbulence fastens the energy transfer from low-entropy energy source to large-entropy energy (heat) by decreasing its own entropy via self-organization and the rise of coherent structures.

The importance of individual vortices to the turbulent statistics is best shown by the problem of quantum turbulence [14, 15], which consists of quantized vortices [16], which fulfill the Helmholtz circulation theorems [17]—their circulation is constant.
and equal to \( \Gamma_{\text{eq}} = \kappa = 2\pi \hbar/m_4 \approx 9.997 \cdot 10^{-8} \text{m}^2/\text{s} \), \((m_4\) is the mass of single helium 4 atom, it applies \(2 \times m_3\) for helium 3 as it is a fermion); thus the vortices cannot end anywhere in the fluid, only at the fluid domain boundary, or they can form closed loops. The energy cascade can be realized only via vortex interactions, reconnections [16], and the helical Kelvin waves on the quantized vortices leading to phonon emission due to nonlinear interactions [18]. This nature of turbulence made of a tangle of identical vortices instead of different vortices as it is in Richardson cascade leads to polynomial velocity distribution [19] instead of almost-Gaussian distribution observed in classical turbulence [3, 20]. Despite this fact, the large-scale observation of superfluid flows shows the same picture as the classical flows do [21, 22]. The transition between both regimes depends on the length scale [2]. The interacting tangle of quantized vortices builds up the turbulence, whose structure is classical on large scale.

The fluid simulation by using the quantized vortices [23] is able to reconstruct the velocity spectra [18] and overall topology [24]. This method is applied in classical turbulence among others by the group of Ilia Marchevsky [25–27].

The behavior of individual vortices in experiment is studied by many groups; however, it is often limited to the case of some single vortex or vortex system dominating the flow. Among others, let us mention the work of Ben-Gida [28], who detected vortices in a wake past accelerating hydrofoil in stably stratified or mixed water. He used the maxima of \(\lambda_2\) criterion [29]. De Gregorio [30] observed the tip vortex of helicopter rotor blade, and for its detection used the \(\Gamma_2\) criterion [31]. They measured the vortex velocity profile and found that it is similar to the Vatistas model (discussed later here); they studied the development of the tip vortex and observed the interactions of tip vortices of various blades and various turn ages downstream the helicopter jet. Graftieaux et al. [31] developed the the functions \(\Gamma_1\) and \(\Gamma_2\) for the study of swirling flow in a duct. They detected a single vortex in each snapshot and in average field measuring the distribution of the distance of average and instantaneous vortex center. Their scalar function used for vortex detection is effectively similar to smoothed circulation over some neighborhood; therefore it nicely solves the issue of all experimental data: the noise; on the other hand, it introduces a new artificial parameter of the detection: the neighborhood area. Kolář [32] developed probably the most accurate criterion for identifying the three components of velocity gradient tensor—the shear, strain, and rotation. But his method needs a large number of transformations in each point. Maciel et al. [33] noticed that eigen axes of the velocity gradient tensor might do the same job.

In this chapter, the method is based on direct fitting of the instantaneous velocity field by some vortex model with scalar criterion used for the prefit only. In the next section, the available vortex models are introduced, then the velocity profiles on experimental data are shown introducing a new vortex model. Later the prefit function and the fitting procedure are shown, and at the end, some results obtained in the grid turbulence are presented.

1.1 Vortex profiles

A principal disadvantage of any fitting algorithm is the need of a priori knowledge of the functional dependence of the data, in our case, to know the vortex model fitted to the data. There have been a lot of different vortex models developed in the past. The basic idea of a vortex model is the circulation-free potential vortex, whose entire circulation is focused inside an infinitesimal topological singularity—the vortex filament. Everywhere else, the vorticity \(\omega\) is zero.
The vorticity $\omega$ of this potential vortex is a scalar in the discussed simple two-dimensional case:

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\Gamma}{2\pi} \left( \frac{1}{(x^2+y^2)^{3/2}} - \frac{1}{(x^2+y^2)^{1/2}} \right) = \frac{\Gamma}{2\pi} \frac{x^2+y^2}{(x^2+y^2)^{3/2}}$$

Among the infinite velocity of undefined direction in the center, the large velocity gradients smoothen the flow in a way, that there is minimum relative motion at small scales leading to the solid-body rotation with tangential velocity linearly increasing with the distance from the center

$$u_{SVR}^S(r) = \frac{\Gamma}{2\pi} \frac{r}{R}$$

and vorticity $\omega$ being constant everywhere

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\Gamma}{2\pi R^2} \left( \frac{x}{\partial x} - \frac{y}{\partial y} \right) = \frac{\Gamma}{\pi R^2}$$

Simple connection of these two ideas is called Rankine vortex. A new parameter of the vortex is introduced: the vortex core radius $R$ (in solid body rotation vortex (2), $R$ played only the unit role; $r/R$ is dimensionless distance, $\Gamma/2\pi R$ is tangential velocity at dimensionless distance $r/R = 1$). The fluid in this vortex rotates as a solid body inside the sharply bounded vortex core, while it orbits without internal rotation as a potential vortex outside of the circle bounded by $R$

$$u_{SVR}^V(r) = \begin{cases} \frac{\Gamma}{2\pi} \frac{r}{R} & \text{for } r < R \\ \frac{\Gamma}{2\pi} \frac{r}{R} & \text{for } r > R \end{cases}$$

Generally, there are not much sharp changes in the nature; therefore, a smooth solution is introduced by Oseen

$$u_{SVR}^O(r) = \frac{\Gamma}{2\pi} \frac{R}{r} \left( 1 - e^{-\left(\frac{r}{R}\right)^2} \right)$$

This is one of the exact solutions of Navier-Stokes equations containing the temporal evolution as well, and it is called Lamb-Oseen vortex and then the core scales as $R \sim \sqrt{t}$ with time. However, we focus on descriptive analysis of instantaneous two-dimensional velocity fields observed experimentally without the temporal development. There exists more possible exact solutions of Navier-Stokes equations, let us mention at least the Burgers vortex, the Kerr-Dold vortex, or the Amromin vortex [34] with turbulent vortex core and potential envelope.

Mathematical simplification of Oseen vortex is suggested by Kaufmann [35] and later discovered independently by Scully et al. [36]. It uses just the first term of Taylor expansion of the exponential in the Oseen vortex, Eq. (4), as it is shown by Bhagwat and Leishman [37], and it is generalized by Vatistas [38].

$$u_{SVR}^V(r) = \frac{\Gamma}{2\pi} \frac{r}{R} \left[ 1 + \left(\frac{r}{R}\right)^2 \right]^{1/2}$$

which equals to Kaufmann vortex for $n = 1$, and it converges to Rankine vortex for $n \to \infty$. 

3
All the vortex models mentioned up to here display the hyperbolic decrease of tangential velocity with distance, \( u_{\theta} \sim r^{-1} \), see Figure 1. Such a vortex has infinite energy! No matter, which profile is found in its core. Let us integrate the kinetic energy of the orbiting fluid since some distance \( A \) large enough to eliminate the different core descriptions:

\[
E = \int_{A}^{\infty} \frac{1}{2} \left[ \frac{\Gamma}{2\pi R} \right]^2 \frac{u^2(r)}{r} dr = \frac{1}{2} \left( \frac{\Gamma}{2\pi R} \right)^2 2\pi \int_{A}^{\infty} \left( \frac{R}{r} \right)^2 r dr = \pi \left( \frac{\Gamma}{2\pi R} \right)^2 R^2 \left[ \ln \left( \frac{r}{R} \right) \right]_{A}^{\infty} = \infty
\]

(6)

independently on \( A \) or other details near the core. This divergence is often solved by declaring some maximum size \( B \) of the area influenced by the vortex, which is the size of the experimental cell. It could be the size of a laboratory or the circumference of a planet. Anyway, it is an arbitrary parameter the total energy depends on. It signifies that the distant regions have the same weight as the near regions. This is a very uncomfortable property.

A faster decay of tangential velocity can be found in the Taylor vortex [39].

\[
u_{\theta}^{TV}(r) = \frac{\Gamma}{2\pi R} \frac{r}{R} e^{-|r/R|^2},
\]

(7)

which is obtained as the first order of Laguerre polynomials solution for vorticity, whose zero-th order is the already mentioned Oseen vortex. The detailed mathematics can be found in the book [40], specifically, the Section 6.2.1., and it will be not reproduced here. The velocity decays quite fast and thus the energy converges

\[
E = \frac{1}{2} \int_{0}^{\infty} u^2(r) 2\pi r dr = \pi G^2 R^2 \int x^2 e^{-x^2} dx = \frac{\pi}{2} G^2 R^2,
\]

(8)
where \( G = \frac{\Gamma}{2\pi R} \) represents the characteristic vortex core velocity and \( x = \frac{r}{R} \) is the dimensionless distance.

The other hand of faster velocity decay is a skirt of vorticity opposite to that in the core. Let us apply the vorticity operator in cylindrical coordinates

\[
\omega_x(r) = (\nabla \times \mathbf{u})_z = \frac{1}{r} \left( \frac{\partial u_x}{\partial r} - \frac{\partial u_r}{\partial \theta} \right) = \frac{1}{r} r G R e^{-r^2/2R^2} = \frac{G}{R} e^{-r^2/2R^2} \left( 2 - \frac{r^2}{R^2} \right),
\]

which changes the sign at \( r/R = \sqrt{2} \), the opposite vorticity value reaches its maximum at \( r/R = 2 \), and then it decays toward zero. The skirt of opposite vorticity is a property of any profile with tangential velocity decay faster than \( 1/r \), as the profile \( 1/r \) is the limiting case for zero vorticity, see Figure 1.

2. Experimental setup and methods

2.1 Particle Image Velocimetry

The experimental data were obtained by using the standard method of Particle Image Velocimetry (PIV) [41], which is already a standard tool in hydrodynamic research in air or water and even in superfluid helium [42] as well as in high-speed applications [43]. Contrary to the pressure probes, hot wire anemometry, or laser Doppler anemometry, the result of this method is an instantaneous two-dimensional velocity field [44], which opens the exploration of the turbulent flows topology [8, 45, 46]. It is based on the optical observation of small particles [47] carried by the fluid. The particles are illuminated by a double-pulsed laser in order to capture their movement during the time between pulses. There exists even a time-resolved PIV, which uses fast laser and camera, and thus it is able to capture the temporal development and measure, e.g., the temporal spectra [48]. Our system at the University of West Bohemia in Pilsen belongs to the slow ones with repeating frequency 7.4Hz; therefore, only the statistical properties can be studied with quite good spatial resolution 64 × 64 grid points sampled on a 4Mpix (2048 × 2048pix²) camera images.

2.2 Observed velocity profiles

Let us look at the experimental data. To get the velocity profile of a vortex, it is needed to know vortex parameters: position, radius, and circulation or the effective circumferential velocity \( G = \Gamma/2\pi R \) respectively. The listed parameters are results of the fitting procedure; however, the fitting procedure needs to use some vortex model to minimize its energy and, therefore, the result is already a product of the used vortex model. To avoid this back-loop effect, only the prefilt is used. This function is explained later; it uses the spatial distribution of modified Q-invariant of the velocity gradient tensor and does not need any vortex model explicitly. The vortex velocity profile is obtained as an ensemble average of measured velocity profiles across the vortex; the spatial coordinate is normalized by the vortex radius \( R \) and the velocity by the vortex circumferential velocity \( G \). The standard deviation of such ensemble is displayed as a shadow area in Figures 2–6.

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1. The integral \( \int x^3 e^{-x^2} \, dx \) is solved by substituting \( \xi = x^2 \), then \( d\xi = 2x \, dx \) and integral is \( \frac{1}{2} \int \xi e^{-\xi} \, d\xi \); per parts we get \( -\frac{1}{2} \left( \xi e^{-\xi} - \int e^{-\xi} \, d\xi \right) = -\frac{1}{2} e^{-\xi}(\xi + 1) \), i.e. \( -\frac{1}{2} e^{-x^2}(x^2 + 1) \).

2. To obtain the full velocity gradient tensor, Regunath and coworkers had to use 2 laser systems of different colors with slightly shifted planes [44].
Figure 2(a) shows the experimental data measured in a plane perpendicular to suction vortex formed near the inlet to a pump pumping water from reservoir. The flow field is dominated by this single vortex, which has strong divergence component, it slightly moves around the center, and other parameters vary as well.

Figure 2(b) shows that the vortex profile in this case roughly follows the Oseen vortex; however, in one direction (toward the left-hand side in the figure), the velocity decays even slower than the Oseen vortex model predicts. This is caused probably by the reservoir geometry. This data were measured by prof. Uruba.

Figure 3 shows the secondary flow in a corner (bottom and left edge of the figure) of a channel, the main flow is perpendicular to the measured plane. The
displayed vortex forms, when the boundary layers are laminar, this vortex spontaneously brakes the symmetry, and it leads to faster transition to turbulence of the boundary layers at higher velocities. More details about this measurement can be found in our previous publications [49, 50]. In this case, the vortex profile is pushed toward the Taylor profile, which is caused by the presence of solid wall and thus zero velocity at the left and bottom side. In the upper direction, there is observed even an overshoot of the profile caused by the stream supplying the vortex from the central flow.

Figure 4 shows the turbulent flow behind a grid; the distance is 200 mm, i.e., $12.8M$, $M$ is the mesh parameter, Reynolds number is $3.1 \cdot 10^3$. The main flow points from left to right and the convective velocity component is subtracted. More details about this experiment can be found in our previous publication [51]. In this case, the flow field is not dominated by a single vortex; instead, there are more vortices of similar level. A consequence is that the standard deviation is much more massive than in the previous cases. The averaged profile displays velocity decay faster than the potential envelope ($\sim r^{-1/2}$), but not as fast as the Taylor vortex model (7).

A similar velocity profile can be seen in a very different case—the jet flow, see Figure 5, which shows the data measured in a plane perpendicular to the jet axis at distance of one nozzle diameter past the nozzle. The jet-generating device misses the flow straightener; therefore the jet core contains turbulence originating in the fan; more details can be found in our conference contribution [52]. The vortices prefitted within the jet core (depicted by the blue rectangle in Figure 5) display slightly faster velocity decay than the vortices elsewhere, i.e., mainly in the shear layer.

A highly turbulent flow emerges in the steam turbines; the data measured in a model axial air turbine are shown in Figure 6. Here, the strong advection in the axial direction (from left to right in the figure) is subtracted, the rest shows a wide horizontal strip of lower turbulence, which is caused by the rotor jet (fluid passing the interblade channel), this structure overlays a less apparent structure of wakes past rotor wheel, which display as strips of wilder flow in top-bottom direction.
This pattern would be better apparent in an averaged image, but here the instantaneous field is shown. More detailed description of this interesting flow can be found in our article [53]. The vortices in this case display a strong asymmetry—in tangential direction (up-down in the figure), their peak velocity is significantly smaller than the peak velocity in axial direction (left-right). In the axial direction, there is a strong “overshoot” of velocity decay, which is caused by the alternating velocity pattern.

The observation made in very different cases does not support the generally accepted hypothesis of potential envelope around the vortex. This envelope forms, when there is only single vortex dominating the flow.

Figure 5.
(a) Example of instantaneous in-plane velocity field perpendicular to a turbulent jet axis with five vortices in each snapshot, no fitting is used. (b) Profiles of the found vortices. The set is approximately separated into vortices in the jet core and elsewhere according to the blue rectangle in panel (a).

Figure 6.
(a) Example of instantaneous velocity field in the axial × tangential plane inside an axial turbine past the first stage (stator + rotor); the convective velocity in axial direction (from left to right) is subtracted. (b) Profiles of the found vortices. The profiles in axial direction display strong overshoot caused by the pattern of wakes past rotor wheel (such wakes pass the field of view from top to bottom with small left-right drift as the rotor wheel rotates from bottom to top in the field of view perspective).
suction vortex), in the case of turbulent field consisting of multiple vortices, the velocity decay is faster, but not as fast as in the Taylor vortex model (7). Therefore we venture to offer another model of vortex with non-potential envelope

$$u_{\text{VNPE}}(r) = \frac{\Gamma}{2\pi R} \frac{r/R}{\left(1 + \frac{1}{4}(r/R)^2\right)^{\frac{3}{2}}}$$

which decays as $r^{-3}$ at large $r$ and at smaller $r$, it roughly follows the Oseen vortex model, see Figure 1. It is important to note that this vortex is not a solution of Navier-Stokes equations! It is based on the observations only, and there is no any theoretical argument for it.

Similarly as the Taylor vortex, this model displays a skirt of opposite vorticity as well. The vorticity profile is

$$\omega_z(r) = G \frac{2 - \frac{1}{2}(r/R)^2}{\left(1 + \frac{1}{4}(r/R)^2\right)^{\frac{3}{2}}}.$$  

It reaches zero at the distance

$$0 = 2 - \frac{1}{4} \left(\frac{r_0}{R}\right)^2 \Rightarrow r_0 = 2,$$

see Figure 1; since this distance, the vorticity approaches zero from opposite direction as $\sim r^{-4}$ for large $r$. The energy of this vortex model is finite even in unbounded domain:

$$E = \frac{1}{2} \int_{-\infty}^{\infty} \left[ G \frac{r/R}{\left(1 + \frac{1}{4}(r/R)^2\right)^{\frac{3}{2}}} \right]^2 \cdot 2\pi r dr = \frac{4}{3} \pi G^2 R^3.$$  

2.3 Vortex prefit

Any general fitting algorithm falls into some local minimum. This minimum does not need to be the really wanted results, it can be just a small dimple in a wall of huge valley. To avoid this effect, one can (i) modify the fitting algorithm to see larger surroundings of the point, e.g., by using simulated annealing [54, 55], or (ii) just start close to the result. The second possibility solves another small issue—the starting point of the fitting algorithm. In the single particular case solved here, it means to find a peak of appropriate scalar variable, which would signify the presence of vortex.

The prefit is sketched in Figure 7: the starting point is the spatial scalar field of $\sqrt{Q_d \cdot \text{sgn} \, \omega}$, where $\text{sgn} \, \omega = \frac{\omega}{|\omega|}$ is the sign of vorticity. $Q_d$ is the $Q$-invariant with subtracted divergence. Alternatively, any scalar with sparse non-zero values could be used (i.e., not simply the vorticity). Then the separated patches of non-zero signal are detected, see panel (c) of Figure 6. The vortex is built up by using the most energetic patch (label 3 in Figure 7(c)); the vortex position is the center of mass of the patch, the vortex radius $R = \sqrt{n/\pi}$, where $n$ is the number of points of the patch (note that the unit of $R$ is the grid point). The circumferential velocity $G$ is calculated as the average of tangential projection of the measured velocities at eight locations around the vortex in the distance $R$ from its center (crosses in Figure 7(d)).
The just described procedure does not use the vortex model; therefore it is suitable for velocity profile estimation as has been done in the previous section. On the other hand, vortex parameters are only estimated; therefore the fitting is needed to adapt the vortex parameters to the actual velocity field.

### 2.4 Vortex fitting

A single vortex is described by four fitting parameters: the position $x$ and $y$, core radius $R$, and circumferential velocity $G$, which is easier to use than the circulation $\Gamma = 2\pi G$. Of course, this set of parameters describes only the cases, when the vortex tube crosses the measured plane perpendicularly; other angles might produce deformation from the ideal circular shape. But, as John von Neumann said: *With four parameters I can fit an elephant, and with five I can make him wiggle his trunk* [56]. Therefore, it is preferred to avoid using too many fitting parameters; the listed set is considered to be a minimum. This issue will be a true challenge in the case of instantaneous volumetric data in the future.

The used fitting algorithm is called *Amoeba* [55] or *Downhill simplex method* or *Nelder-Mead* by its inventors [57]. The energy of residual velocity field is calculated for each variant serving as a score. Here comes the need of specific vortex model discussed earlier. The algorithm selects single movement from a closed set of movements in the parameter space in order to keep away from areas with high residual energy and converging to some local minimum. The algorithm is in much more detail described in the book [55].

Once the energy residual of a single vortex reaches local minimum, this vortex is subtracted from the velocity field. Then the entire procedure is repeated by using the residual velocity field as the input.

**Figure 8** shows the results of fitting a single instantaneous velocity field by depicted number of vortices. It is clearly visible that the energy decreases as the field is approximated by more and more vortices. It can be seen as a kind of decomposition, although its effectivity is poor in comparison with pure mathematical approaches, e.g., the Proper Orthogonal Decomposition [8, 9]. On the other hand, it describes the fluctuating velocity field by using objects with clear physical interpretation, while the physical interpretation of POD modes is not straightforward [58]. Still, a question remains here: whatever the found vortices are real. To be specific, in **Figure 8**, a large vortex can be seen even in the first set, the core of this...
vortex spans out of the field of view, thus no one knows, if the vortex was still there in the case that areas were measured. For example, a simple advective motion can be explained by a pair of huge vortices up and down the measured area. Of course, that is unphysical. As the number of vortices increases, even smaller and smaller vortices are added converging to a situation, that each single noise vector is described by a single vortex. This limit is unphysical as well, but where is the boundary?

Figure 9 shows the decrease of effectivity of this procedure—as the number of vortices increases, there remains structures less and less similar to a vortex in the instantaneous velocity field. While the first vortex typically covers around 10% of the energy of input fluctuating velocity field (in this case). The convergence of the energy of the rest gets slower, and it becomes to be quite ineffective to describe 75% of the fluctuations by the simple vortices described here. The parameters of found vortices develop as well, see Figure 10, which shows the probability density functions of vortex core radii and circumferential velocities. The vortices found later are typically smaller and have smaller circumferential velocity (the positive and negative values count together in the logarithmic plot of Figure 10(c) and (d)).

Figure 9.
(a) Energy of residual velocity field after subtracting the nth vortex as a function of number of vortices. The area represents the standard deviation of the ensemble of 1,476 snapshots. (b) The energy “saved” by nth vortex, again, the area represents the standard deviation of the ensemble.

Figure 8.
Vortices fitted in a single instantaneous velocity field measured past a grid; the same example field as in previous figures. (a) The input velocity field, (b, c, d, e) the velocity field calculated from theoretical vortex profiles. (f, g, h, ch) The residual field, i.e., input field minus the field of found vortices.
3. Results

The aim of this chapter is mainly to describe the ideas of the developed algorithm. The results and their physical interpretation need more effort in the future. In this section, some ways of result analysis are shown based on the distribution study. As an example case, the grid turbulence is selected, because this is a deeply explored canonical case, see [59] and many more experimental data, e.g., in [4, 60–65].

When exploring the vortices in grid turbulence in dependence on the distance behind the grid or on the Reynolds number, the effectivity of vortex fitting remains almost constant. Figure 11 shows, that the maximum population is around 1 in all cases. There is slow decrease of the number of vortices with low effectivity (i.e., structures with large radius or large velocity causing large theoretical energy, \( E_t \approx R^2 G^2 \), which do not correspond to the energy saved), and there exist cases with saved energy larger than the theoretical one; however, this distribution decreases much faster.

The vortex core radii in Figure 12(a) do not seem to depend on the distance \( x \) past the grid, although it is known that the characteristic turbulent length scales (Kolmogorov and Integral one) typically increase with distance. At the lowest distance, there can be observed a weak wavening of the distribution. The distribution dependence on Reynolds number is weak as well (Figure 12(b)), although a very fine change of vortex core radii scaling at radii larger maximal population. Honestly speaking, the similarity of the distributions is suspicious, and it has to be proven in the future that the shape of the radii distribution is not affected by the measurement spatial resolution (the studied datasets have all the same spatial resolution).

![Figure 10](image-url)
The circumferential velocities $G = \Gamma/2\pi R$ of the vortices move toward smaller values with increasing distance, see Figure 13(a). This effect is clearly caused by the decreasing turbulence intensity [51] as the vortices are searched within the fluctuating velocity field. Figure 13(b) shows that the velocity normalized by the wind tunnel velocity of maximum population increases. At lower velocities, the PDF decrease with increasing $G$ displays two regimes, first it decreases slower, then faster, while at higher velocities, only the fast decay is observable. It has to be mentioned, in the light of observations in Figure 9, that this effect can be caused by the number of fitted vortices, which do not need to be appropriate for the actual datasets. It is quite difficult to distinguish the effects of the method and the physical phenomena.

The distance to nearest other vortex seems to be unaffected by the grid distance and flow velocity, see Figure 14. But the absolute values of the nearest vortex cannot have some physical sense, as this quantity is the first one dependent on the number of searched vortices, thus the vortex density. But the non-changing shape of this distribution suggests that there is nothing like evolution pattern of vortices or vortex lattice.

Figure 11.
Probability density functions (PDFs) of the vortex effectivity, i.e., the ratio of energy saved by the probed vortex and the theoretical energy of the vortex. Left panel (a) shows the data at different distances behind the grid, the mesh-based Reynolds number is $3 \cdot 10^3$; panel (b) shows the data at different Reynolds number, the distance $x/M = 12.8$. The “K” in the legend plays for $10^3$.

Figure 12.
Probability density functions (PDFs) of core radii $R$ found in fields of view in several distances past the grid, panel (a); and at several Reynolds numbers, panel (b). The dotted lines highlight scalings of $R^{-3}$ and $R^{-2}$ the observed data lie in between. It seems that the scaling exponent slightly decreases with $Re$. 
4. Conclusions

The turbulent flows consist of many interacting vortices of all scales, which all together self-organize being responsible for the statistical properties of turbulence. In this contribution, the algorithm for detection of individual vortices via direct fitting of measured velocity field has been presented. It has been shown via the zero-th step of fitting that the velocity profile of vortex in turbulent flow decreases faster than the generally accepted models suggest. This is advantageous, because the energy of vortex with velocity decrease faster than $1/r$ converges. On the other hand, it has a “skirt” of vorticity opposite to the center one. The vortices found in grid turbulence display average radius decreasing with distance and Reynolds number, while the scaling at larger $R$ seems to not depend on those parameters. The effective circumferential velocity $G = \Gamma/2\pi R$ decreases with distance and increases with Reynolds number (faster than expected linear). The algorithm is still under development and mainly the physical interpretation of the results needs more work in the future studying and comparing results of different flow cases.

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Nomenclature

$E$ energy; $E_{in}$ energy of the input velocity field, $E_s$ energy saved by removing fitted vortices, $E_{res}$ energy of the velocity field after removing vortices, $E_t$ theoretical energy of the vortex

$G$ vortex core circumferencial velocity, $G = \frac{\Gamma}{2\pi R}$

$M$ mesh parameter of the grid, i.e., the distance of the rods

$R$ vortex core radius

$Q$ invariant of velocity gradient tensor, in 2D: $Q = \partial_x u \partial_y v - \partial_y u \partial_x v$

$Q_d$ $Q$ invariant without the divergence, in 2D: $Q_d = -\partial_x u \partial_y v - \partial_y u \partial_x v - (\partial_x u)^2 - (\partial_y v)^2$

$u$ instantaneous velocity, $\bar{u}$ vector, $u_\theta$ tangential velocity, $v$ velocity component perpendicular to $u$ in 2D

$\Gamma$ vortex circulation

$\omega$ vorticity $\omega = \nabla \times \bar{u}$

Abbreviations

AV Amromin vortex $u_\theta(r) = G \cdot \frac{r}{r} \left( 1 - \ln \frac{r}{R} \right)$ for $r < R$ and potential vortex outside

KV Kaufmann (often reported as Scully) vortex, $u_\theta(r) = G \cdot \frac{r}{r} \left( 1 - \frac{r^2}{R^2} \right)$

OV Oseen Vortex, $u_\theta(r) = G \cdot \frac{r}{r} \left( 1 - \exp \left( -\frac{r^2}{R^2} \right) \right)$

PDF probability density function

PIV Particle Image Velocimetry

PV potential vortex, $u_\theta(r) = G \cdot \frac{r}{r}$

SBR solid-body rotation, $u_\theta(r) = G \cdot \frac{r}{r}$

TV Taylor vortex, $u_\theta(r) = G \cdot \frac{r}{r} \exp \left( -\frac{r^2}{2R^2} \right)$

VNPE vortex with non-potential envelope, $u_\theta(r) = G \cdot \frac{r}{r} \left( 1 - \frac{r^2}{R^2} \right)$

VV Vatistas vortex system, $u_\theta(r) = G \cdot \frac{r}{r} \left( 1 - \frac{r^2}{R^2} \right)$

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