Chapter

Energy Harvesting Prediction from Piezoelectric Materials with a Dynamic System Model

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Abstract

Piezoelectric vibration energy harvesting has been investigated for different applications due to the amount of wasted vibration from dynamic systems. In the case of piezoelectric materials, this energy lost to the environment can be recovered through the vibration of energy harvesting devices, which convert mechanical vibration into useful electrical energy. In this context, this chapter aims to present the mechanical/electrical coupling on a simple dynamic system model in which a linear piezoelectric material model is incorporated. For this purpose, a mechanical/electrical element of a piezoelectric disk is developed and integrated into a lumped mass, viscous damping, and spring assembling, similar to a quarter car suspension system. Equations of motion for this dynamic system in the time domain can be solved using the finite element method. The recovered electric power and energy density for PZT (Lead Zirconate Titanate) from the wasted vibration can be predicted considering that the road roughness is introduced as an input mode.

Keywords: harvesting energy, wasted vibration, dynamic system, linear piezoelectric model, PZT

1. Introduction

Energy use is widely discussed nowadays, as energy conversion and management. In this way, new sources of energy are required to be investigated. Thus, one of the energy sources that can be used is from vibration systems, which can be subject to different excitations. This wasted energy to the environment can be recovered through vibration energy harvesting devices, which convert mechanical vibration into useful electrical energy in a way that low power devices may utilize.

Piezoelectric materials are known to have the electro/mechanical coupling effect. This property has a large range of applications in engineering. Currently, they are extensively used as sensors and actuators in vibration control systems. As a sensor, it can monitor the vibrations when bonded to a flexible structure. As an actuator, it can control the vibration level by introducing a restored force or by adding damping to the system.

In the context of recovered energy from mechanical vibrations based on the conversion of piezoelectric harvesting devices and its application on powering electronic devices, this subject has received the attention from various researchers [1–3]. An example of this type of recovered energy is the suspension system vibration for use of the vehicle itself, such as an energetic source for an active and semi-active suspension.

On a typical road, vehicles suffer accelerations due to its roughness, which excite undesired vibration. Some recently conducted reviews mentioned the potential of recovering a few hundred watts for a passenger car driven in experimental tests as well as some mathematical models [4, 5]. One of the ways to convert the mechanical energy from the vehicle suspension to electric energy is through piezoelectric materials [6]. Therefore, the objective of this chapter is to present the coupling between a piezoelectric element and a dynamic system in the context of predicting the recovered electric power and energy density for piezoelectric materials, especially the PZT (Lead Zirconate Titanate).

2. Piezoelectric material modelling

Mathematical models for predicting the harvesting energy in piezoelectric materials submitted to axial loads consider its geometric properties, diameter D_p and thickness h_p, and its mechanical and electrical properties, Young's modulus is c_{zz}^E, the piezoelectric constant is e_{zz} and the dielectric constant is ϵ_{zz}^S, as shown in **Figure 1**. Points 1 and 2 represent the two faces of the piezoelectric disk, and w and V are the mechanical displacement and electric potential, respectively, at these two points.

The electromechanical coupling effect of the piezoelectric material can be described using a set of basic equations as given in the IEEE Standard on Piezoelectricity [7]:

$$\begin{aligned} \sigma_z &= c_{zz}^E \frac{\partial w}{\partial z} - e_{zz} \frac{\partial V}{\partial z} \\ D_z &= e_{zz} \frac{\partial w}{\partial z} + \epsilon_{zz}^S \frac{\partial V}{\partial z} \end{aligned} \tag{1}$$

Where σ_z is the normal stress, D_z is the electric flux density, both in direction z. As seen in the above equations, the electro/mechanical coupling occurs due to the piezoelectric constant e_{zz}.

The mechanical strain energy U_m and the electric energy U_e of the piezoelectric material are written as [7]:

$$\begin{aligned} U_m &= \frac{1}{2}\int_V \sigma_z \frac{\partial w}{\partial z} dV = \frac{1}{2}\int_V \left(c_{zz}^E \frac{\partial w}{\partial z} - e_{zz} \frac{\partial V}{\partial z} \right) \frac{\partial w}{\partial z} dV \\ U_e &= \frac{1}{2}\int_V D_z \frac{\partial V}{\partial z} dV = \frac{1}{2}\int_V \left(e_{zz} \frac{\partial w}{\partial z} + \epsilon_z^S \frac{\partial V}{\partial z} \right) \frac{\partial V}{\partial z} dV \end{aligned} \tag{2}$$

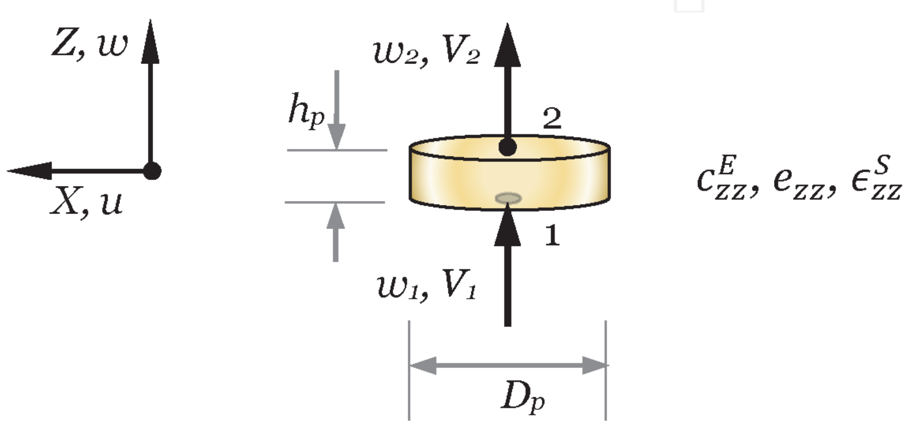

Figure 1.
The piezoelectric material model.

The electrical power can be calculated as the partial derivative of the electric energy, presented by Eq. (2), in respect to time:

$$P_e = \frac{\partial U_e}{\partial t} = \frac{1}{2}\int_V \frac{\partial}{\partial t}\left(D_z \frac{\partial V}{\partial z}\right) dV = \frac{1}{2}\int_V \frac{\partial}{\partial t}\left(e_{zz}\frac{\partial w}{\partial z} + \epsilon_z^S \frac{\partial V}{\partial z}\right)\frac{\partial V}{\partial z} dV \qquad (3)$$

3. Dynamic system modelling

This chapter aims to predict the wasted energy from vibration systems that can be further transformed into electrical energy. A typical vibration system can be described as mass, spring and damper elements, and that can represent a suspension system assembly.

The suspension system is an assembly of suspension arms or linkages, springs and shock absorbers that connect the wheels to the vehicle's chassis in order to isolate passengers from vibrations due to bumps and roughness of the road. Furthermore, it must maintain the contact of the wheels with the road to ensure drivability. Thus, the suspension system is the mechanical system where the stability and handling of the vehicle, besides energy harvesting, must be equilibrated.

Mathematical models were initially developed for vertical vehicle performance, and the one-dimensional quarter car model is the simplest from the frequently used suspension system [8]. It is composed of the sprung mass m_s, which represents ¼ of the vehicle's body and the unsprung mass m_u, which represents the wheel assembling mass. Both are considered rigid bodies. Its displacements are w_s and w_u, respectively and both are vertically aligned. There are some studies that include a third degree in the system to describe road roughness excitation w_r. In this case, only the bounce input mode, or the vertical displacement can be implemented. Other elements of the suspension system are included, such as tire stiffness k_t, suspension stiffness k_s, and viscous damping c_s. All vertical displacements are a function of the independent variable t that represents the time. **Figure 2** illustrates this 1D quarter car model, which could represent both the front and rear of the vehicle.

The expressions of kinetical energy from the masses, strain energy and dissipation function from the shock absorber, and the virtual work from the road roughness excitation are written as:

$$\begin{aligned} T &= \frac{1}{2}m_s\dot{w}_s^2 + \frac{1}{2}m_u\dot{w}_u^2 \\ U &= \frac{1}{2}k_s(w_s - w_u)^2 \\ R &= \frac{1}{2}c_s\left(\dot{w}_s^2 - \dot{w}_u^2\right) \\ \delta W &= F_r(t)\delta w_u = k_t w_r(t)\delta w_u \end{aligned} \qquad (4)$$

4. The suspension system and piezoelectric disk coupling

The conversion of the mechanical energy from the vehicle suspension to electric energy through piezoelectric materials can be predicted by piezoelectric disk coupling illustrated in **Figure 1** and the suspension system illustrated in **Figure 2**. Since the conversion of mechanical energy to electrical energy in this system is produced by compressive efforts, the piezoelectric disk is located between the shock absorber

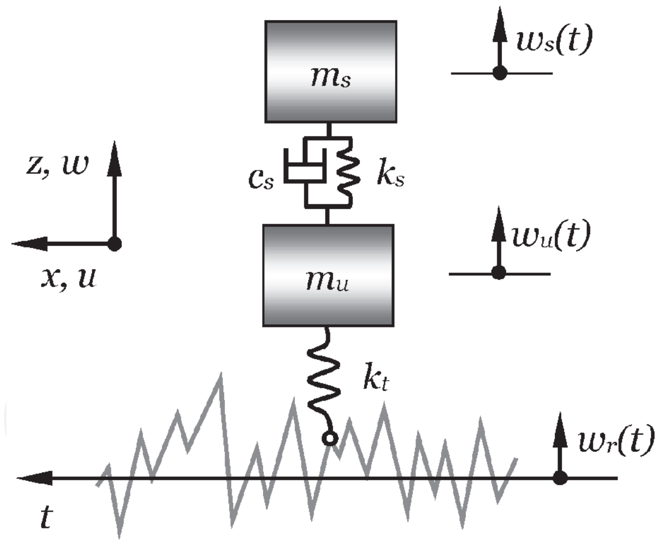

Figure 2.
1D quarter car model.

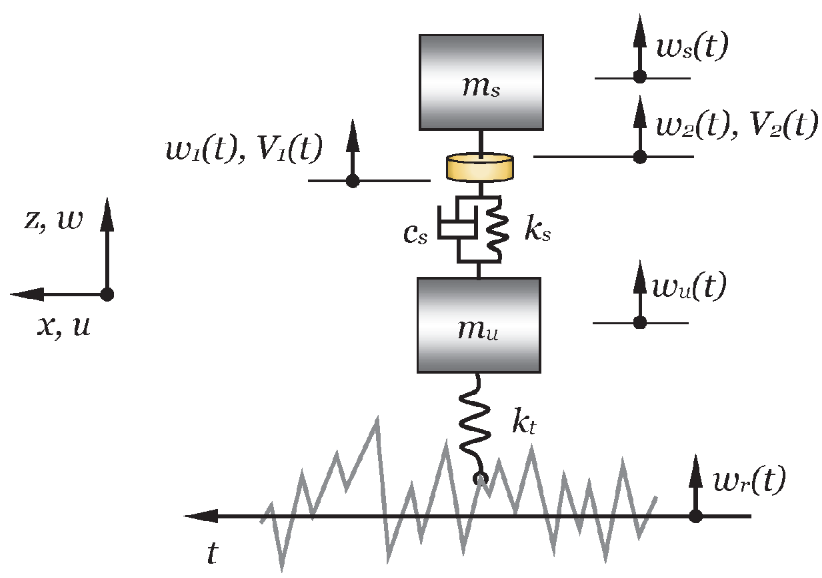

Figure 3.
The suspension system and piezoelectric coupling model.

system composed by stiffness k_s and viscous damping c_s on the bottom side, along with the sprung mass m_s on the upper side, as illustrated in **Figure 3**. As stated previously, vertical displacements w and now the electric potential V, are all a function of the independent variable t.

5. The vertical displacement and electric potential approach

Understanding physical problems can be accomplished when the numerical simulation of equations within the variables that describe them are represented. In the case of the electric energy prediction from the vehicle suspension system, the variables are vertical displacement w and electric potential V of different points, or nodes, and the domain within the model is considered valid.

Some numerical methods are often used to understand physical problems, and among them, the most widely used in engineering is the finite element method. Within this method, the variables in the equations that describe the physical problem must be approximated to the polynomial functions of these variables. For the application of this method in solving problems, all elements that compose it must be represented by matrices, called elementary matrices, which are the result of the adopted polynomial functions. For further details on this method, the following references [9, 10] are recommended.

If the linear approximation function of vertical displacement w and electric potential V over thickness h_p in direction z of the piezoelectric disk are considered, stiffness elementary matrices of the piezoelectric finite element can be obtained by applying the Lagrange equations [11] over the energy expressions presented by Eq. (2).

$$
\begin{aligned}
[K_m] &= \frac{c_{zz}^E S_p}{h_p}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k_m \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \\
[K_{m\_el}] &= -\frac{e_{zz} S_p}{2h_p}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = -k_{m\_el} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \\
[K_{el}] &= \frac{\epsilon_z^S S_p}{h_p}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k_{el} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}
\end{aligned} \tag{5}
$$

Where $[K_m]$, $[K_{m\_el}]$ and $[K_{el}]$ are mechanical elementary stiffness, electromechanical coupling elementary stiffness and electric elementary stiffness matrices, respectively, and S_p is the cross-sectional area of the piezoelectric disk.

Moreover, if the same linear approximation function of the vertical displacement w over stiffness k_s and viscous damping c_s of the shock absorber in direction z are considered, the suspension system's differential equation of motion, illustrated in **Figure 2**, can be obtained by applying the Lagrange equations [11] over the energy expressions presented by Eq. (4).

$$[M]\{\ddot{w}\} + [C]\{\dot{w}\} + [K]\{w\} = \{F(t)\} \tag{6}$$

Where the elementary matrices and the vectors are:

$$
\begin{aligned}
[M] &= \begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix} \\
[K] &= \begin{bmatrix} k_s & -k_s \\ -k_s & k_s \end{bmatrix} \\
[C] &= \begin{bmatrix} c_s & 0 \\ 0 & c_s \end{bmatrix} \\
\{w\} &= \begin{Bmatrix} w_s(t) \\ w_u(t) \end{Bmatrix} \\
\{\dot{w}\} &= \begin{Bmatrix} \dot{w}_s(t) \\ \dot{w}_u(t) \end{Bmatrix} \\
\{\ddot{w}\} &= \begin{Bmatrix} \ddot{w}_s(t) \\ \ddot{w}_u(t) \end{Bmatrix} \\
\{F(t)\} &= \begin{Bmatrix} 0 \\ k_t w_r(t) \end{Bmatrix}
\end{aligned} \tag{7}
$$

Using the technique of assembling the elementary matrices of the finite element method, the suspension system's differential equation of motion and piezoelectric disk coupled model, as illustrated in **Figure 3**, is as shown in Eq. (8):

$$[M_m]\{\ddot{w}\} + [C_m]\{\dot{w}\} + [K_m]\{w\} + [K_{m\_el}]\{V\} = \{F(t)\} \tag{8}$$

$$[K_{m\_el}]^t\{w\} + [K_{el}]\{V\} = \{0\} \tag{9}$$

Eq. (9) can be manipulated and substituted for Eq. (8). Thus, the final equation of motion as a function of only mechanical variables is:

$$[M_m]\{\ddot{w}\} + [C_m]\{\dot{w}\} + \left[[K_m] - [K_{m\_el}][K_{el}]^{-1}[K_{m\_el}]^t\right]\{w\} = \{F(t)\} \tag{10}$$

Eq. (10) can be solved in the time domain with an integration method, such as the Newmark Method [12]. The mechanical force is due to road roughness as well as the tire characteristics of the wheel as shown in Eq. (7). These data are used to apply the condition in each time step in solving Eq. (10), in which all variables $w_s = w_2$, w_1, and w_u and their time derivatives are obtained.

The response in the time domain in respect to vertical displacements is obtained by using Eq. (10) and subsequently, the electric potential is obtained as:

$$\{V(t)\} = -[K_{el}]^{-1}[K_{mel}]^t\{w(t)\} \tag{11}$$

The electrical energy can be calculated as presented by Eq. (2). The development of this equation follows:

$$U_e = \frac{1}{2}\int_0^{h_p}\int_{S_p}\left[e_{zz}\left(\frac{\partial w}{\partial z}\right)\ \frac{\partial V}{\partial z}\ + \epsilon_z^S\ \left(\frac{\partial V}{\partial z}\right)^2\right]dS\ dz \tag{12}$$

Thus, the expression of the electrical energy due to the piezoelectric disk on this suspension system is as:

$$U_e = \frac{1}{2}\frac{e_{zz}S_p}{h_p}\ (w_2 - w_1)\ (V_2 - V_1) + \frac{\epsilon_z^S S_p}{h_p}\ (V_2 - V_1)^2 \tag{13}$$

In addition, the electrical power can be calculated as presented by Eq. (3) and its development is as follows:

$$P_e = \frac{1}{2}\int_0^{h_p}\int_{S_p}\left[e_{zz}\ \left(\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right)\ \frac{\partial V}{\partial z} + \frac{\partial w}{\partial z}\ \frac{\partial}{\partial t}\left(\frac{\partial V}{\partial z}\right)\right) + \epsilon_z^S\ \frac{\partial}{\partial t}\left(\frac{\partial V}{\partial z}\right)^2\right]dS\ dz \tag{14}$$

Thus, the expression of the electrical power due to the piezoelectric disk on this suspension system is as:

$$\begin{aligned} P_e = &\frac{1}{2}\frac{e_{zz}S_p}{h_p}\ \left[(\dot{w}_2 - \dot{w}_1)\ (V_2 - V_1) + (w_2 - w_1)\ \left(\dot{V}_2 - \dot{V}_1\right)\right] \\ &+ \frac{\epsilon_z^S S_p}{h_p}\ (V_2 - V_1)\ \left(\dot{V}_2 - \dot{V}_1\right) \end{aligned} \tag{15}$$

Where $\dot{w}_1$, $\dot{w}_2$, $\dot{V}_1$ and $\dot{V}_2$ are time derivatives of the displacements and electric potential of nodes 1 and 2 of the piezoelectric disk.

6. Application example

For a simpler demonstration of the potential for harvesting energy in a suspension system by means of piezoelectric material, a MATLAB® code to obtain the results was developed and presented below.

The dimensions of the piezoelectric disk are diameter $D_p = 0.065\,m$ and thickness $h_p = 0.025\,m$. The vehicle data and the properties of piezoelectric material PZT-5H are shown in **Tables 1** and **2**, as indicated in references [13, 14], respectively.

Property description	Value
Body and wheel mass	
¼ Vehicle sprung mass (m_u)	362.5 kg
Unsprung mass (m_s)	39 kg
Tire stiffness (k_t)	200 kN/m
Shock absorber suspension	
Suspension stiffness (k_s)	30 kN/m
Suspension damping (c_s)	4 kN s/m

Table 1.
Vehicle data.

	PZT-5H
Piezoelectric constant – e_{zz} [C/m^2]	23.30
Dielectric constant – ϵ_{zz}^S [F/m]	1.30 x 10^{-8}
Density – ρ [kg/m^3]	7500
Young's modulus – c_{zz}^E [GPa]	23.0

Table 2.
Properties of piezoelectric material.

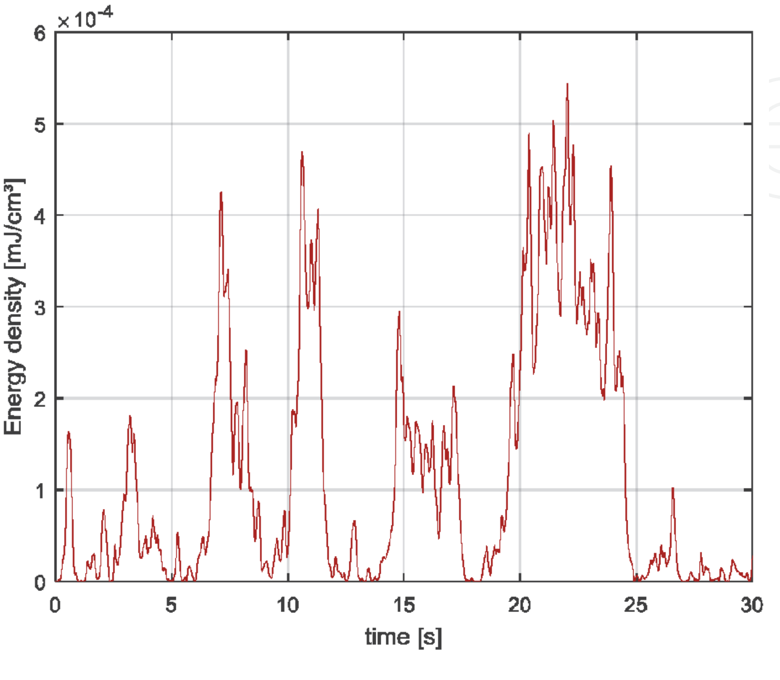

Figure 4.
Electric energy density response for PZT-5H.

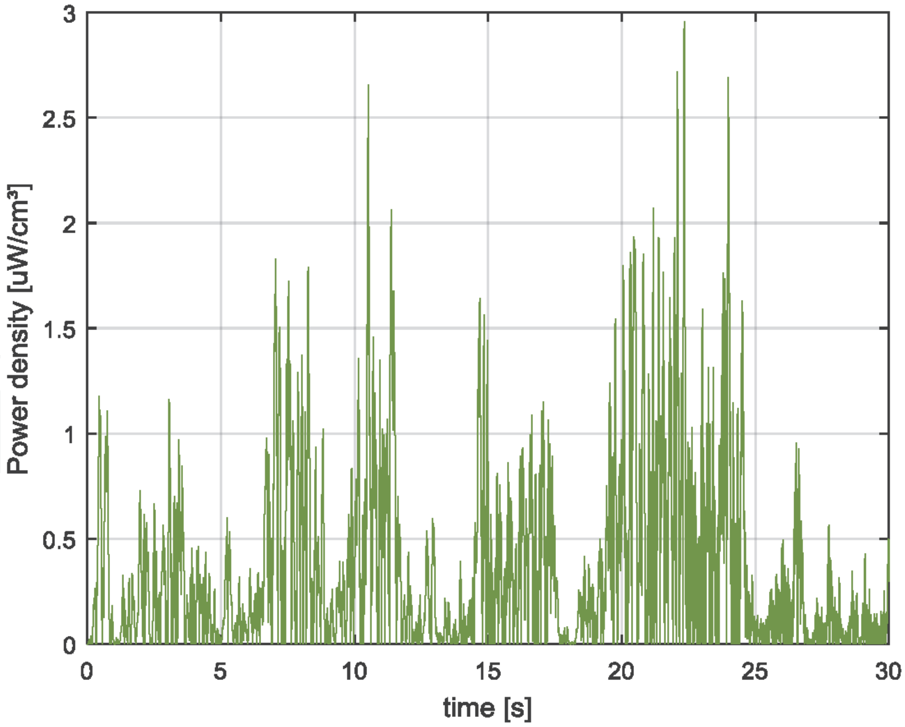

Figure 5.
Electric power density response for PZT-5H.

The road excitation is as depicted in [6]. The excitation is the bounce input mode vertical position of the road $w_r(t)$ with class D road (poor) and a driving speed for a vehicle of 20 m/s.

In **Figure 4**, the instantaneous electric energy density for piezoelectric material PZT-5H calculated by Eq. (13), with a simulation time of 30 s and a fixed step time of 0.00001 s.

Figure 5 exhibits the overall instantaneous electric power density response for piezoelectric material PZT-5H calculated by Eq. (15).

7. Conclusions

This chapter proposes a coupled suspension system and piezoelectric model to predict the potential of harvested electric power in vehicle suspension systems. The performance of piezoelectric material PZT-5H was investigated, in respect to harvesting energy based on energy density and electric power density.

The approach presented in this chapter is a way to simulate the electric power generated in vehicle suspension systems established by piezoelectric harvesting. Nonetheless, these results would need to be compared with experimental results to demonstrate the validity of the proposed model.

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