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Chapter

Flux Reversal Machine Design

Yuting Gao and Yang Liu

Abstract

Flux reversal permanent magnet machines (FRPMMs) have a simple reluctance rotor and a stator with armature windings and permanent magnets (PMs). Due to the high torque density and high efficiency of FRPMMs, they have been widely used in many applications such as electric vehicle, wind power generation, etc. However, the general design method of FRPMMs has not been established in books. Therefore, this chapter will focus on introducing an analytical design method, which allows for fast design of FRPMMs. First of all, the analytical sizing equations are deduced based on a magneto motive force (MMF)-permeance model. After that, the effects of some key performances including average torque, pulsating torque, power factor, and PM demagnetization are analyzed. Moreover, the feasible slot-pole combinations are summarized and the corresponding winding type of each combination is recommended in order to maximize the output torque. Besides, the detailed geometric design of stator and rotor are presented. Finally, a case study is presented to help readers better understand the introduced design methodology.

Keywords: design method, flux reversal permanent magnet machine (FRPMM), sizing equation, finite element analysis (FEA)

1. Introduction

The topology of FRPMM is depicted in Figure 1. As can be seen, it has a slotted rotor without any windings or PMs, and a stator with armature windings and PMs mounted on each stator teeth. First of all, the structural characteristics of FRPMMs and the corresponding performance advantages need to be explained:

1. FRPMMs are excited by PMs instead of the excitation windings, which are different with asynchronous motors and brushed DC motors. So, for FRPMMs, the rotor will not have copper losses, and the efficiency is relatively higher [1, 2].

2. The rotor of FRPMMs has no windings or permanent magnets, thus is suitable for high-speed operation and high-temperature operating conditions [3]. Moreover, the no excitation winding will keep away from the problems of friction noise and electric spark. So, FRPMMs are more reliable and require less maintenance [4, 5]. In addition, the rotor of FRPMMs is light in weight and has a small rotational inertia [6]; hence, the acceleration and deceleration response is faster.
3. The stator windings of FRPMMs are mostly concentrated windings, which are easy to manufacture. Moreover, the electromagnetic isolation of the concentrated windings is better than regular distributed windings, which means that if one winding has faults, the fault is not likely to spread to other windings, and thus the fault tolerance is good [7, 8]. In addition, the concentrated winding has a smaller winding factor, inductance, and a shorter electrical time constant than the distributed windings [9], and thus the dynamic response of concentrated winding is faster.

4. Compared to other stator-PM machines, that is, flux switching PM machines and doubly salient PM machines, FRPMMs have a simpler structure. The PMs of the flux switching PM machines and doubly salient PM machines are inserted into the stator core, which is not convenient for installation. In the flux switching PM machine, putting permanent magnets in the middle of the teeth will reduce the slot area and affect the output torque. In the doubly salient PM machine, placing PMs in the yoke will increase the volume of the motor and reduce the torque density. In the FRPMMs, the PMs are pasted on the inner surface of the stator teeth, thus eliminating the above problems [10].

Finally, the structural characteristics and performance advantages of the flux-reverse motor can be summarized in Table 1.

It can be seen that the FRPMMs have many performance advantages, and these advantages can be utilized in different applications. First of all, the high efficiency, the large torque density, the rapid acceleration, and deceleration response make FRPMMs suitable for various high-speed rotation areas, such as electric vehicles [11–13], electric spindle [14], fans [15, 16], etc. Secondly, the number of rotor pole pairs is usually high, which is also suitable for low-speed areas, meanwhile its torque density is high at the low speeds, making FRPMMs suitable for various low-speed direct-drive occasions [17], for example wind power [18–20], direct drive servo system [21], wave power generation [22], etc. In addition, linear FRPMM has no PMs and copper windings in the secondary, which saves cost and is also very suitable for long rail transit linear motion applications [23, 24].
In most existing literatures, the design of FRPMMs is mainly based on the classical design method [25] with low accuracy or time-consuming finite element algorithm (FEA) [26]. Therefore, in this chapter, the specialized sizing equations for FRPMMs will be deduced and the analytical design method will be introduced, which can be directly employed in the initial design of FRPMMs and allows for fast calculations of machine dimensions.

This chapter is organized as follows. First, the structure and operation principles are introduced in Section 2. Then in Section 3, the magnetic circuit model is built and the sizing equations are analytically derived. After that, in Section 4, the influences of several key parameters (slot-pole combination, airgap radius, electric loading, and equivalent magnetic loading) in the sizing equation on the torque density are analyzed. Also, the effects of the airgap structural parameters on the pulsating torque, power factor, and PM demagnetization performances are investigated. Moreover, in Section 5, the geometric design of stator and rotor are introduced. And in Section 6, the design procedure is illustrated. Besides, to make the analytical design method more readable, a case study is presented and a FRPMM prototype is tested. Finally, conclusions are drawn in Section 7.

### 2. Operation principle of FRPMM

To clearly exhibit the operating principle, a three-phase FRPMM with two pole windings, six stator slots, and eight rotor teeth is cited as an example. The flux distributions at different rotor positions are illustrated in Figure 2. The magnetic flux field is excited only by the PMs, and the difference of each rotor movement is 11.25 mech. degrees (i.e., 1/4 rotor slot pitch). Taking flux linkage of phase A winding as an example, when the rotor position is 0 degree, the flux linkage is 0; when the rotor position is 11.25 mech. degree (90 elec. degree), the flux linkage reaches the positive maximum value; when the rotor position is 22.5 mech. degree (180 elec. degree), the flux linkage is 0; when the rotor position is 33.75 mech. degree (270 elec. degree), the flux linkage reaches the negative maximum value. Therefore, in the duration of one rotor slot pitch (360 elec. degrees), the winding flux linkage reverses the polarity, thus it is called “flux reversal machine.” Then,

### Table 1

<table>
<thead>
<tr>
<th>No.</th>
<th>Structural characteristic</th>
<th>Advantages</th>
</tr>
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<tr>
<td>1</td>
<td>Use rare-earth PMs</td>
<td>1. No excitation loss, high motor efficiency; 2. Rare-earth with high-magnetic energy product increases torque density</td>
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<td>2</td>
<td>No windings or PMs in the rotor</td>
<td>1. Simple rotor structure, suitable for high-speed operation and high temperature conditions; 2. Avoids mechanical friction and electric sparks caused by commutators and brushes, thus improving the reliability; 3. Small rotational inertia, thus fast acceleration and deceleration response</td>
</tr>
<tr>
<td>3</td>
<td>Often use concentrated windings</td>
<td>1. Good fault tolerance and high reliability; 2. Easy processing and manufacturing; 3. Small inductance and electrical time constant</td>
</tr>
<tr>
<td>4</td>
<td>PMs attached to the stator teeth surface</td>
<td>1. Easy to install 2. No reduction in the slot area or increase in the motor volume</td>
</tr>
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</table>

Table 1. Structural characteristics and corresponding advantages of FRPMM.
after obtaining the bipolar flux linkage, as shown in Figure 3, the winding can produce a bipolar back-electromagnetic motive force (EMF). If the armature windings are injected with currents having the same frequency and phase with the back-EMF, a steady torque can be yielded.
3. Sizing equation of FRPMM

3.1 Magnetic circuit model

In order to derive the sizing equation of FRPMMs, the magnetic circuit model should be built at first; then, based on the model, the analytical equations of airgap flux density, back-EMF, and torque will be deduced.

The equivalent magnetic circuit model can be plotted as Figure 4. At No.1 stator tooth, its magnetic field distribution corresponds to the position shown in Figure 2(b), that is, the rotor tooth is closer to the S-pole magnet. The S-pole magnetic generates two paths of magnetic flux, one is pole leakage flux $\Phi_{pl}$, which goes through the adjacent N-pole magnet, the other is main flux $\Phi_{m}$, which goes through the stator tooth, stator yoke, rotor tooth, and rotor yoke, thus can provide winding flux linkage and back-EMF. At No. 2 stator tooth, its magnetic field distribution corresponds to the position shown in Figure 2(c), that is, the rotor axis is at the same distance from the S-pole and N-pole magnets. Thus, at this time, the two magnets can only generate one magnetic flux path, that is, the pole leakage flux $\Phi_{pl}$. At No. 3 stator tooth, its magnetic field distribution corresponds to the position shown in Figure 2(d), that is, the rotor tooth is closer to the N-pole magnet. The N-pole magnetic generates two paths of magnetic flux, one is pole leakage flux $\Phi_{pl}$, which goes through the adjacent S-pole magnet, the other is main flux $\Phi_{m}$, which goes through the stator tooth, stator yoke, rotor tooth, and rotor yoke, thus can provide winding flux linkage and back-EMF. It should be noted that the magnetic flux path of No. 1 stator tooth is just opposite to that of No. 3 stator tooth, so winding flux polarity in these two cases is just opposite to each other.

As mentioned above, Figure 4 provides the magnetic circuit of FRPMMs, which can help analyze the flux distribution of FRPMMs at different rotor positions. However, the magnetic circuit requires the establishment of the whole FRPMM
magnetic path, which is rather complex. Besides, the pole leakage flux, main flux, and the reluctance at each rotor positions should be calculated, which needs high workload. Therefore, a simplified magnetic circuit should be built. Observing Figure 2, it can be seen that a small rotor displacement brings a large rotation in stator flux field. This phenomenon is called as flux modulation effect, i.e. a high-pole slow-speed magnetic field becomes a low-pole high-speed magnetic field through the modulation effect of iron teeth. Therefore, the physical nature of FRPMM is indeed the flux modulation effect. The research of some flux modulation machines are usually based on the PM magnetic motive force (MMF)-airgap permeance model, such as the Vernier machine in [27]. So, this chapter will use this model to analyze FRPMMs.

In PM MMF-airgap permeance model, the no-load airgap flux density \( B(\theta_s, \theta) \) can be written as the product of PM MMF \( F_{PM}(\theta_s) \) and specific airgap permeance \( \Lambda(\theta_s,\theta) \):

\[
B(\theta_s, \theta) = F_{PM}(\theta_s)\Lambda(\theta_s,\theta)
\]

(1)

where the definitions of angles \( \theta \) and \( \theta_s \) are shown in Figure 5. Then, the simplified magnetic circuit model can be given in Figure 6. Once knowing the PM

![Figure 5. Definitions of different angles in FRPMM.](image)

![Figure 6. Simplified equivalent magnetic model of FRPMMs.](image)
MMF and airgap permeance, the no-load airgap flux density can be obtained. Then, the stator flux linkage $\lambda_{ph}(\theta)$ can be deduced using winding function theory:

$$\lambda_{ph}(\theta) = r_g l_{stk} \int_0^{2\pi} B(\theta_s, \theta) N(\theta_s) d\theta_s \tag{2}$$

where $N(\theta_s)$ is the phase winding function. After that, the phase back-EMF $E_{ph}(t)$ and average torque $T_e$ can be calculated as:

$$E_{ph}(t) = \frac{d\lambda_{ph}(\theta)}{dt} \tag{3}$$

$$T_e = \frac{3}{2} E_{ph} I_{ph} \tag{4}$$

where $I_{ph}$ is the peak value of phase current. Therefore, from Eqs. (1–4), it can be found that if the torque equation need to be calculated, the key is to obtain the equation of airgap flux density $B(\theta_s, \theta)$, which is further determined by the PM MMF $F_{PM}(\theta_s)$ and specific airgap permeance $\Lambda(\theta_s, \theta)$. Therefore, in the next parts, the equations of the PM MMF $F_{PM}(\theta_s)$ and specific airgap permeance $\Lambda(\theta_s, \theta)$ will be deduced in detail.

### 3.2 Airgap flux density equation

As aforementioned, to derive the torque equation, the no-load airgap flux density $B(\theta_s, \theta)$ should firstly be known, whose equation can be given as Eq. (1). Then, the next step is to derive the expressions of $F_{PM}(\theta_s)$ and $\Lambda(\theta_s, \theta)$. The PM MMF waveform excited by the magnets is shown in Figure 7, which can be given as:

$$F_{PM}(\theta_s) = \begin{cases} 
F_C; & 0 \leq \theta_s < (1 - SO)\pi/Z_s \\
0; & (1 - SO)\pi/Z_s \leq \theta_s < (1 + SO)\pi/Z_s \\
F_C; & (1 + SO)\pi/Z_s \leq \theta_s < 2\pi/Z_s \\
-F_C; & 2\pi/Z_s \leq \theta_s < (3 - SO)\pi/Z_s \\
0; & (3 - SO)\pi/Z_s \leq \theta_s < (3 + SO)\pi/Z_s \\
-F_C; & (3 + SO)\pi/Z_s \leq \theta_s < 4\pi/Z_s 
\end{cases} \tag{5}$$

![Figure 7. Magnet MMF waveform.](image-url)
where $F_C$ is:

$$F_C = \frac{B_i h_m}{\mu_0 \mu_r}$$  \hspace{1cm} (6)

Then, it can be written in Fourier series as follows:

$$F_{PM}(\theta_s) = \sum_{i=1,3,5}^\infty F_i \sin \left( \frac{iZ_s \theta_s}{2} \right)$$  \hspace{1cm} (7)

where the magnitude $F_i$ is

$$F_i = \frac{41 B_i h_m}{\pi i \mu_0 \mu_r} \left[ 1 + (-1)^{i+1} \sin \left( \frac{i\pi}{2} \right) \right]$$  \hspace{1cm} (8)

Then, the next step is to derive the specific airgap permeance $\Lambda(\theta_s, \theta)$ in Eq. (1). Since the stator slotting effect has already been considered in Eqs. (5–8), the specific airgap permeance $\Lambda(\theta_s, \theta)$ can be replaced by the airgap permeance with smoothed stator and slotted rotor $\Lambda_r(\theta_s, \theta)$. The model of smoothed stator and slotted rotor is shown in Figure 8. Then, the $\Lambda_r(\theta_s, \theta)$ can be expressed by:

$$\Lambda_r(\theta_s, \theta) \approx \Lambda_{0r} + \Lambda_{1r} \cos [Z_r(\theta_s - \theta)]$$  \hspace{1cm} (9)

The coefficients of the airgap permeance function $\Lambda_{0r}$ and $\Lambda_{1r}$ in Eq. (9) can be obtained using the conformal mapping method [28, 29]:

$$\Lambda_{0r} = \frac{\mu_0}{g'} \left( 1 - 1.6 \beta \frac{b_o}{t} \right)$$  \hspace{1cm} (10)

$$g' = g + h_m/\mu_r$$  \hspace{1cm} (11)

$$\Lambda_{1r} = \frac{\mu_0}{g' \pi} \left[ 0.5 + \frac{(b_o/t)^2}{0.78125 - 2(b_o/t)^2} \right] \sin \left( 1.6\pi \frac{b_o}{t} \right)$$  \hspace{1cm} (12)

$$\beta = 0.5 - \frac{1}{2 \sqrt{1 + \left( \frac{b_o \pi}{2} \right)^2}}$$  \hspace{1cm} (13)

![Figure 8](image-url)  
Schematic of single-side salient structure on rotor.
where $b_o$ is the rotor slot opening width and $t$ is the rotor slot pitch, as shown in Figure 8. Combining Eq. (1), Eqs. (5–13), the no-load airgap flux density $B(\theta_s, \theta)$ can be finally calculated as:

$$ B(\theta_s, \theta) = \sum_{i=1,3}^{\infty} B_i \sin \left( \frac{iZ_{s}}{2} \pm Z_r \theta \right) $$

(14)

where the magnitude $B_i$ is

$$ B_i = \frac{1}{2} F_i \Lambda_{1r}, \ i = 1,3,5 \ldots $$

(15)

### 3.3 Slot-pole combinations

As can be seen in Eq. (14), the number of pole pairs in the air gap flux density is $iZ_{s}/2 \pm Z_r$, $i = 1,3,5 \ldots$ Then, in order to make the flux density induce EMF in the armature windings, the pole pair number of the armature windings $P$ should be equal to $iZ_{s}/2 \pm Z_r$, $i = 1,3,5 \ldots$ Besides, for three phase symmetry, the winding pole pair number must also meet the following requirement:

$$ \frac{Z_{s}}{\text{GCD}(Z_{s}, P)} = 3k, \ k = 1,2,3 \ldots $$

(16)

All in all, the slot-pole combination of three-phase FRPMs is ruled by the following equation:

$$ P = \min \left\{ P = \frac{iZ_{s}}{2} \pm Z_r; \ \frac{Z_{s}}{\text{GCD}(Z_{s}, P)} = 3k \right\} $$

(17)

where min means to select the minimum number of these qualified harmonic orders so as to obtain a maximal pole ratio of FRPMs. Therefore, the feasible slot-pole combinations can be summarized as Table 2. Non-overlapping windings (i.e., concentrated windings) are usually used in FRPMs because of the higher fault tolerance and easier manufacture than regular overlapping windings. However, some FRPMs are suggested to employ overlapping windings in order to have a larger winding factor and thus a higher torque density. Therefore, both winding factors, that is, $k_{wn}$ (using non-overlapping winding) and $k_{wr}$ (using overlapping winding) are calculated for each FRPM so as to see the difference of using different winding types.

### 3.4 Torque equation

Once the stator winding pole pair is selected, the stator flux linkage can be deduced using winding function theory, just as mentioned in Eq. (2). The winding function $N(\theta)$ in Eq. (2) can be written as:

$$ N(\theta) = \sum_{i=1,3,5}^{\infty} N_i \cos (iP\theta) $$

(18)

$$ N_i = \frac{2}{i\pi} k_{wn} $$

(19)
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**PS:** Non-overlapping winding is recommended.  
**Other:** Overlapping winding is recommended.

\(k_{sw}\) and \(k_{swr}\) are fundamental winding factors calculated based on non-overlapping winding type and recommended winding types, respectively.

Table 2. 
Slot-pole combinations of three-phase FRPMM.
where $N_i$ is the $i$th harmonics of the winding function and $k_{w1}$ is the winding factor of the $i$th harmonics. As can be seen in Eq. (17), the pole pair number is $iZ_s/2 \pm Z_r (i = 1, 3, 5 \ldots)$. So, the sum or difference of any two pole pair harmonics $P_{i1}$ and $P_{i2}$ is a multiple of stator slot number, that is,

$$
\begin{align*}
P_{i1} &= iZ_s/2 \pm Z_r \\
P_{i2} &= iZ_s/2 \pm Z_r
\end{align*}
$$

Therefore, all the flux density harmonics are tooth harmonics of each other, that is, they have the same absolute values of winding factors, and their absolute winding factor equals the fundamental winding factor $k_{w1}$:

$$
|k_{wP_{i1}}| = |k_{wP_{i2}}| = k_{w1}
$$

Then, combining Eq. (2), Eq. (3), Eqs. (18–21), the back-EMF can be finally obtained as:

$$
E_{ph} = 2\omega_m r_g l_{stk} N_s k_{w1} \sum_{i=1}^{\infty} \text{sgn} * \frac{B_i}{(\frac{iZ_s}{2} \pm Z_r)/P}
$$

where

$$
\text{sgn} = \begin{cases} 
1, \text{winding factor of } (iZ_s/2 \pm Z_r)^{th} \text{ harmonic equals } k_{w1} \\
-1, \text{winding factor of } (iZ_s/2 \pm Z_r)^{th} \text{ harmonic equals } -k_{w1}
\end{cases}
$$

Since the reluctance torque of FRPMM is negligible, the electromagnetic torque under $i_d = 0$ control can be expressed as Eq. (4). Then, combining Eq. (4) and Eq. (22), the average torque $T_e$ is able to be calculated as:

$$
T_e = 3I_{ph} r_g l_{stk} N_s k_{w1} \sum_{i=1}^{\infty} \text{sgn} * \frac{B_i}{(\frac{iZ_s}{2} \pm Z_r)/P}
$$

So far, the general torque equation has been obtained as Eq. (24), but in this equation, some parameters such as $B_i$, $I_{ph}$ cannot be determined in the initial design stage of FRPMMs, so it is desirable that Eq. (24) can be transformed to a combination of several basic parameters, such as electric loading, magnetic loading, which can be easily determined in the initial design stage.

As known for electrical machines, the electric loading $A_e$ can be written as:

$$
A_e = \frac{6N_s I_{ph}}{2\sqrt{2\pi} r_g}
$$

Then, the equivalent magnetic loading of three-phase FRPMM $B_m$ is defined as:

$$
B_m = \sum_{i=1}^{\infty} \text{sgn} * \frac{B_i}{(\frac{iZ_s}{2} \pm Z_r)/P}
$$

So, the torque expression in Eq. (24) can be rewritten as:

$$
T_e = \sqrt{2\pi} r_g l_{stk} k_{w1} A_e B_m
$$
Thus, the rotor volume $V_r$, which equals $\pi l_{stk} r_g^2$, can be obtained:

$$V_r = \frac{T_e}{\sqrt{2}k_w Z_r A_e B_m}$$  \hspace{1cm} (28)$$

and then the airgap radius $r_g$ and the stack length $l_{stk}$ can be derived as:

$$r_g = \sqrt{\frac{V_r}{\pi k_{tr}}}$$  \hspace{1cm} (29)$$

$$l_{stk} = \sqrt[3]{\frac{V_r k_{tr}^2}{\pi}}$$  \hspace{1cm} (30)$$

where $k_{tr}$ is the aspect ratio, equals to the ratio of $r_g$ to $l_{stk}$. It can be found in Eq. (27) that the key parameters affecting the torque density are the airgap radius $r_g$, stack length $l_{stk}$, winding factor $k_w$, rotor slot number $Z_r$, electric loading $A_e$, and equivalent magnetic loading $B_m$, among which the stack length $l_{stk}$ can be determined by the volume requirement, and winding factor $k_w$ is approximate to 1. So, the remaining parameters $r_g$, $Z_r$, $A_e$, $B_m$ should be determined at the initial stage of the design process. Thus, the influences of the above key parameters on important performances, such as average torque, pulsating torque, power factor, PM demagnetization performance, will be investigated in the following parts.

4. Influence of design parameters on key performances

4.1 Average torque performances

4.1.1 Influence of slot-pole combinations on average torque

As aforementioned, the rotor slot number $Z_r$ is one of key parameters that should be determined in the first design stage. How to determine the rotor slot number is a question. In this part, the influence of $Z_r$ on the torque performance will be investigated, giving instruction on how to select $Z_r$. The parameters of the FRPMM models are listed in Table 3. These parameters are kept the same for the FRPMMs in order to have a reasonable comparison of their torque performance. That is to say, the airgap radius $r_g$, stack length $l_{stk}$, and electric loading $A_e$ are the same.

Figure 9 shows the influence of rotor slot number on the output torque when non-overlapping windings and recommended windings are used respectively. For Figure 9(a), when non-overlapping windings are adopted, the average torque is mainly related to the product of winding factor and rotor slot number, that is,

<table>
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<th>Parameter</th>
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<th>Parameter</th>
<th>Value</th>
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<td>Rated speed</td>
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<td>Magnet remanence</td>
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</table>

Table 3. Parameters of the three-phase FRPMM models.
Since the machine volume and PM usage are kept the same, the equivalent magnet loading $B_m$ is mainly determined by the pole ratio (PR). So, the variation trend of torque is similar to that of $k_w^* Z_r^* \text{PR}$. It can be seen that the torque achieves the maximum value when the rotor slot number is 8, 14, and 21 for 6 stator slots, 12 stator slots, and 18 stator slots, respectively. When the recommend windings are used, which means that the winding factor are maximized, the main factor that affects the torque is the $Z_r^* \text{PR}$. As shown in Table 2, the variation of PR is irregular, hence the variation of torque with rotor slot number is irregular. As can be seen, for recommended winding types, the torque achieves the maximal value when the rotor slot number is 8, 10 and 17 for 6 stator slots, 12 stator slots, and 18 stator slots, respectively.

### 4.1.2 Influence of airgap radius on average torque

As shown in Eq. (27), the airgap radius $r_g$ is also very important for the output torque. Figure 10 investigates the effect of optimal rotor slot number $Z_r$ at different $r_g$. For 6, 12, 18 stator slots, their rotor slot numbers are selected as 8, 14, and 21.
respectively. Moreover, non-overlapping windings are used in these models because non-overlapping winding is simple and has the same end winding length. It can be seen that when the airgap radius is small, the optimal rotor slot number is small. This is because when the airgap radius is small, the leakage flux between adjacent rotor teeth occupies a large percent, so the optimal rotor slot number should be small to reduce the leakage flux as much as possible. When the airgap radius gets larger and larger, the leakage flux decreases gradually. Hence, the optimal rotor slot number increases.

Then, keeping the stator outer diameter as a constant, that is, 170 mm, the effects of airgap radius of average torque are analyzed in Figure 11. It indicates that when the airgap radius increases, the output torque goes up. This is because the torque is proportional to the square of airgap radius. The larger the airgap radius, the higher the torque. However, the torque is not only influenced by the airgap radius, but also the electric loading $A_e$. With the increase of airgap radius, the inner diameter of the stator increases, and thus the slot area decreases, leading to the decrease of winding turns per slot and the electric loading. Therefore, as the airgap radius keeps increasing, the output torque decreases afterwards.

4.1.3 Influence of magnetic loading and equivalent electric loading on average torque

In addition to the rotor slot number $Z_r$, airgap radius $r_g$, the rest of key parameters affecting the torque in Eq. (27) are the electric loading $A_e$ and the equivalent magnetic loading $B_m$. Figure 12 analyzes the influence of $A_e$ and $B_m$ on the average torque at different stator slot number. For these models, the airgap radius is fixed as 55 mm and their rotor slot number is chosen as their corresponding optimal value. Also, non-overlapping windings are adopted. As can be seen, the output torque increases with the electric loading. This reason is very simple, that is, a larger current, a higher torque. But for the equivalent magnetic loading, the variation trend of torque does not monotonically increase with the equivalent magnetic loading. This is due to the saturation effect of the iron core. Moreover, it can be seen that the knee point of the equivalent magnet loading increases with the stator slot number. Since the winding pole pair of the 18-stator-slot FRPMM is 6, which is larger than 1-winding-pole-pair of the 6-stator-slot and 4-winding-pole-pair of the
12-stator-slot FRPMM, the stator iron of the 18-stator-slot FRPMM is less likely to saturate than the others.

4.2 Pulsating torque performances

4.2.1 Influence of slot-pole combinations on pulsating torque

Apart from the torque density, pulsating torque is also very important because a large pulsating torque will increase the vibration and noise of machines. Figure 13 shows the cogging torque and ripple torque waveforms of 13-, 14-, 16-, 17-, and 19-rotor-slot FRPMMs. The stator slot number of these models is all chosen as 12. For the rated torque, we can see in Figure 13(b) that the 14-rotor-slot FRPMM yields the largest among the five models. As for the pulsating torque, we can see that the cogging torque and ripple torque of 16-rotor-slot FRPMM are the largest, and that of 19-rotor-slot FRPMM is the least. This phenomenon is related to the least common multiple of stator slot number and rotor slot number. The larger least common multiple, the lower pulsating torque. The least common multiples of the 13-, 14-, 16-, 17-, and 19-rotor-slot FRPMMs are 156, 84, 48, 204, and 228, respectively. Therefore, the 19-rotor-slot FRPMM exhibit the lowest cogging torque and ripple torque. However, attentions should be paid to use odd rotor number because it will cause other problems such as eccentricity stress.

Figure 14 compares the radial stress of the five FRPMM models. It can be seen that for the even rotor slot number FRPMMs, that is, 14 and 16 rotor slots, the stress harmonics only have even orders, which will not lead to eccentricity. However, for the odd rotor slot number

12-stator-slot FRPMM, the stator iron of the 18-stator-slot FRPMM is less likely to saturate than the others.
Figure 13. Effect of slot-pole combination on pulsating torque performances: (a) cogging torque waveforms (%); (b) rated torque waveforms.

Figure 14. Radial stress analysis of the FRPMs: (a) 13-rotor-slot; (b) 14-rotor-slot; (c) 16-rotor-slot; (d) 17-rotor-slot; (e) 19-rotor-slot.
FRPMs, that is, 13, 17, and 19 rotor slots, there are many odd stress harmonics. Since the first-order harmonic is dominant for the eccentricity, the 13-rotor-slot FRPMM has a large eccentricity stress. Therefore, 13-rotor-slot is not recommended. The first-order stress harmonic for 17 and 19 rotor slots are very small, so their eccentricity can be neglected.

4.2.2 Influence of PM thickness and split ratio on pulsating torque

The influences of split ratio and PM thickness on cogging torque and ripple torque of FRPMMs are also analyzed in Figure 15. This figure is plotted based on the 14-rotor-slot, which is chosen because it has the largest torque density and a relatively low pulsating torque, as shown in Figure 13. It can be found in Figure 15(a) that the cogging torque increases with the PM thickness and the split ratio. When the PM thickness increases, the airgap flux density increases, and thus the interaction between the PMs and slot-teeth becomes greater, which leads to a higher cogging torque. As the split ratio increases, the airgap radius increases, hence the cogging torque increases with the split ratio [30]. As for the ripple torque, the ripple torque has the maximum value when the split ratio is around 0.66. This is because the ripple torque is not only related to the slot structure but also influenced by the electric loading. As aforementioned, the pulsating torque resulting from the slot structure is increased with the split ratio. However, as the split ratio increases, the slot area is reduced and the electric loading gets smaller and smaller, so the ripple torque resulting from the electric loading becomes lower. Considering these two impacts, the ripple torque has a maximal value when the split ratio changes.

4.2.3 Influence of slot opening ratios on pulsating torque

As we know, the airgap structure is significant for the pulsating torque because the pulsating torque results from the interaction between the two sides of the airgap, that is, stator and rotor. Therefore, this chapter also analyzes the influences of stator slot opening ratio and rotor slot opening ratio on cogging torque and ripple torque. Here, the stator/rotor slot opening ratio is defined as the ratio of stator/rotor slot opening width to the stator/rotor slot pitch. Figure 16 shows the variation of cogging torque and ripple torque with the two slot opening ratios. It can be seen that the cogging torque increases with the stator slot opening ratio. The reason is that a larger stator slot opening ratio reduces the PM width and the smoothness of PM MMF, thus the changing of the PM MMF along the tangential direction increases the cogging torque. Additionally, the ripple torque decreases with the stator slot opening ratio, because the slot opening ratio reduces the electric loading.
the cogging torque. As for the rotor slot opening ratio, which simultaneously influences all the harmonic contents of the airgap permeance, it has great and nonlinear impact on the pulsating torque. Since the pulsating torque results from the interaction of multi permeance harmonics, the variation of pulsating torque changes nonlinearly with the rotor slot opening ratio. It can be seen in Figure 16 that the optimal cogging torque and ripple torque can be achieved when the stator slot opening ratio and rotor slot opening ratio are around 0.25 and 0.7, respectively.

4.3 Power factor performances

4.3.1 Influence of stator inner diameter and PM thickness on power factor

Since the power factor of FRPMs is usually low, which is around 0.4–0.7, meanwhile a low power factor will increase the converter capacity and cost, the influences of key parameters on the power factor should be also analyzed to achieve a relatively high power factor. The power factor can be given as:

$$PF = \frac{1}{\sqrt{1 + \left(\frac{L_s I_s}{\psi_m}\right)^2}}$$  \hspace{1cm} (31)

where $I_s$ is the winding current, $L_s$ is the synchronous inductance (because the saliency ratio is approximate to 1, $L_d \approx L_q$), and $\psi_m$ is the PM flux linkage. Then, the effect of stator inner diameter on power factor is shown in Figure 17. Here, the stator outer diameter is kept as 124 mm, and the airgap length is fixed as 0.5 mm. It can be found that with the increase of stator inner diameter, the power factor increases continuously. The reason is that with the increase of stator inner diameter, the slot area decreases, so the winding turns per phase decreases, thus leading to the reduction of the synchronous inductance $L_s$. The lower $L_s$, the higher power factor, as shown in Eq. (31). Apart from the stator inner diameter, another important parameter affecting the power factor is the PM thickness $h_m$. Figure 18 investigates the variation of power factor with respect to the PM thickness. It indicates that the power factor initially increases with the PM thickness but then decreases. The reason is explained as follows. As the PM thickness increases, the PM flux linkage $\psi_m$ becomes larger, so the power factor increases. However, the synchronous inductance $L_s$ also increases with the PM thickness, which leads to the reduction of power factor afterwards. Therefore, there is an optimal PM thickness for a maximum achievable power factor.
4.3.2 Influence of slot opening ratio on power factor

Another important parameter that influences the airgap structure is the slot opening ratio. Hence, Figures 19 and 20 analyzes the effect of stator slot opening ratio and rotor slot opening ratio on pulsating torque performances, respectively. It can be seen in Figure 19 that the maximum power factor can be obtained when the stator slot opening ratio is approximately to 0.3. The explanation is as follows. When the stator slot opening ratio is too small, the slot leakage flux between the stator tips is large, thus the main flux is reduced, and the back-EMF is lowered, resulting in smaller back-EMF. And when the stator slot opening ratio is too large, the PM width will be narrower. Although the slot leakage flux is reduced, the main flux is not high due to the narrower PMs, thus the back-EMF is lowered. Therefore, the stator slot opening ratio cannot be too small or too large, that is, there is an optimal value for the stator slot opening ratio.

Then, the influences of rotor slot opening ratio on power factor can be seen in Figure 20. It indicates that when the rotor slot opening ratio is around 0.7, the
power factor reaches the maximal value. This is because the power factor is mainly influenced by the back-EMF. When the rotor slot opening ratio increases, the effective airgap length becomes smaller, thus the main flux is increased and the back-EMF is improved. As a result, the power factor is increased. When the rotor slot opening ratio keeps increasing, the flux modulation effect of the rotor teeth becomes weaker and weaker, thus the smaller modulated flux, and the lower back-EMF. Therefore, there is also an optimal value for rotor slot opening ratio when a high power factor is demanded.

4.4 PM demagnetization performances

For PM machines, PM demagnetization performances are very important because it is highly related to the safe operation and machine reliability. Therefore, the PM demagnetization performances of FRPMMs should be analyzed in this
chapter. Since the magnetic properties of PM materials are sensitive to temperature, and the temperature coefficient of NdFeB magnet is as high as $-0.126\%/K$. When the current of FRPMMs is large, the winding heating can easily affect the PMs attached to the stator teeth surface, causing the decrease of PM magnetic performances. On the other hand, when the winding current is large, the demagnetizing effect of the armature field is enhanced, and thus the PMs have the possibility to be demagnetized. Therefore, it is of great importance to investigate the PM demagnetization performances of FRPMMs at different conditions.

Figure 21 shows the demagnetization curve of the magnets. The upper half is a straight line, and lower half under the knee point $B_{\text{knee}}$ is a curved line. When the FRPMM works on the straight line (such as point $P_1$), the return line coincides with the demagnetization curve, and the magnetic performance of the magnets will not be lost. However, when the armature equivalent MMF $H_a'$ is too large at load condition, or the knee point is too high, the working point $B_{\text{knee}}$ is moved to $P_2$. At this time, the recovery line does not coincide with the original demagnetization line, thus the intersection of the $B$-axis changes from $B_r$ to $B_{r1}$, causing the irreversible demagnetization. Then, the PM properties and machine performances will no longer return to the original. So, the PM flux density should be examined in order to check the risk of irreversible demagnetization. As we know, the PM flux density is determined by the design parameters such as electric loading $A_e$, PM thickness $h_m$, rotor slot opening ratio, etc. So, in this chapter, the effects of electric loading $A_e$, PM thickness $h_m$, rotor slot opening ratio $b_o/t$ on PM demagnetization performances of FRPMMs will be studied. For instance, the PM material is selected as N38SH, and knee point of the PM flux density at 100°C is 0.35 T.

Figure 22 shows the PM flux density of a 12-stator-slot/14-rotor-slot FRPMM when the electric loading $A_e$ is 1600A/cm, the PM thickness $h_m$ is 3 mm, rotor slot opening ratio $b_o/t$ is 0.65. It can be seen that the PM flux density distribution varies with the rotor position. When the rotor position is 140°, the PM does not demagnetize, while at 0° and 340°, the PM will demagnetize. Hence, in the following analysis, the PM flux density at the most severe moment of demagnetization is selected.

Figure 23 studies the magnetic flux density distribution in the PMs under different electric loadings. It can be found that the larger electric loading $A_e$, the smaller minimum flux density. This is because the larger electric loading, the higher armature MMF $H_a'$, and the more left operating point $P_2$, so the lower flux density
in the magnets. When the electric loading $A_e$ is 1400A/cm, the PM irreversible demagnetization just occurs. In addition, it can be seen that the entire magnetic flux density map is skewed to the right. This is because the N-pole magnet is intercepted in this analysis, and there is an S-pole magnet next to the N-pole magnet. There is PM pole leakage flux between the S-pole magnet (negative axis) and the N-pole magnet (positive axis), so the magnetic flux density around the 0 position is lower, and away from the 0 position, the magnetic flux density gradually rises.

**Figure 22.**
PM demagnetization at different rotor positions: (a) rotor position $= 0^\circ$; (b) rotor position $= 140^\circ$; (c) rotor position $= 340^\circ$.

**Figure 23.**
Influence of $A_e$ on PM demagnetization.

**Figure 24** analyzes the effect of PM thickness $h_m$ on the PM demagnetization performances. At this time, the electric loading is chosen as 800 A/cm, and the rotor slot opening ratio is selected as 0.65. It can be seen in **Figure 24** that when the PM thickness $h_m$ is less than 2.5 mm, the irreversible demagnetization will happen, while when the PM thickness $h_m$ is larger than 2.5 mm, the irreversible demagnetization will not. In this model, the airgap length is 0.5 mm. Therefore, in the design stage, the PM thickness should be better to set as five times or more the airgap length. Considering the back-EMF, it is claimed in [3] that when the PM thickness is about three times the airgap length, the back-EMF will reach the maximum. But considering both back-EMF and PM demagnetization risk, it is safer to set the PM thickness as about five times airgap length.

**Figure 25** shows the influences of rotor slot opening ratio $b_o/t$ on the flux density distribution inside the PMs. At this time, the electric loading is chosen as 800 A/cm,
and the PM thickness is selected as five times the airgap length, that is, 3 mm. The larger rotor slot opening ratio, the narrower rotor teeth, thus the more saturated rotor teeth, and the smaller magnetic reluctance. As shown in Figure 20, when the magnetic gets smaller, the more left operating point $P_2$, and thus the lower PM flux density. It can be seen in Figure 25 that when the rotor slot opening ratio $b_o/t$ is 0.9, the irreversible PM demagnetization just occurs. In Ref. [28], it is claimed that the maximum back-EMF can be achieved when the rotor slot opening ratio $b_o/t$ is around 0.6. So, during the design process, the optimal rotor slot opening ratio can be directly applied without consideration of the PM demagnetization risk.

5. Geometric design of stator and rotor

5.1 Stator design

The geometrical parameters of stator and rotor are shown in Figure 26. The no-load flux of each winding pole could be calculated as:
where $\lambda_w$ is the winding pitch. If full-pitch winding is adopted, the winding pitch is able to be written as:

$$\lambda_w = \frac{2\pi r_g}{2P}$$

Then, the no-load flux of each winding pole $\phi_m$ in Eq. (32) could change to:

$$\phi_m = \frac{2r_gl_{stk}B_m}{P}$$

Defining the average flux density at the stator yoke as $B_y$, the stator yoke thickness $h_y$ can therefore be deduced as:

$$h_y = \frac{\phi_m}{2B_y l_{stk}} = \frac{r_g B_m}{PB_y l_{stk}}$$

Similarly, defining the average flux density at the middle of stator tooth as $B_t$, the stator tooth width is able to be worked out:

$$w_t = \frac{\phi_m}{3SPPk_{stk}l_{stk}B_t} = \frac{4r_g B_m}{Z_s k_{stk}B_t}$$

Moreover, in order to simultaneously maintain a relatively large torque density as well as reduce the risk of PM demagnetization, the PM thickness is recommended to be:

$$h_m = 4g \sim 6g$$

Figure 26.
Geometry of stator and rotor.
Then, next step is to calculate the stator outer radius \( r_o \). Firstly, the total slot area of all the stator slots \( A_{\text{slot}} \) can be written based on the winding electric loading \( A_e \) and the current density \( J_e \):

\[
A_{\text{slot}} = 2\pi (r_g + h_m) A_e / J_e S_{\text{fg}}
\]  

(39)

where \( S_{\text{fg}} \) is the slot fill factor. Meanwhile, the total slot area of all the stator slots \( A_{\text{slot}} \) can be also derived out using the structural parameters:

\[
A_{\text{slot}} = \pi (r_g + h_m + h_1 + h_y)^2 - \pi (r_g + h_m + h_1)^2 - Z_s w_t h_j
\]  

(40)

Combining the Eqs. (39) and (40), the slot depth \( h_s \) can be determined. Then, the stator outer radius \( r_o \) can be given as:

\[
r_o = r_g + h_m + h_1 + h_y + h_y
\]  

(41)

5.2 Rotor design

Defining the average flux density of each rotor yoke and middle of rotor tooth as \( B_{ry} \) and \( B_{rt} \), respectively, the rotor yoke thickness \( h_{ry} \) and rotor tooth width \( w_{rt} \) are able to be achieved using the similar derivation procedure as Eq. (35) and Eq. (36). Finally, the \( h_{ry} \) and \( w_{rt} \) are given as:

\[
h_{ry} = r_g B_m / Z_r B_{ry} k_{stk}
\]  

(42)

\[
w_{rt} = 4r_g B_m / Z_r k_{stk} B_{rt}
\]  

(43)

Then, the rotor slot depth \( h_{rs} \) is determined as:

\[
h_{rs} = r_s + h_{ry}
\]  

(44)

6. Design methodology and evaluations

6.1 Design procedure

Based on the analytical equations and the investigations of key performances in the former parts, a quick and accurate analytical design of a FRPMM can be realized by following these procedures (as depicted in Figure 27):

1. Based on the performance investigations in Figures 9–24, the initial design values, including combination of stator slot and rotor slot number, electric loading, equivalent magnetic loading, airgap length, materials of active parts, etc. can be firstly selected.

2. Then, assuming an appropriate aspect ratio \( k_{lr} \), the airgap radius \( r_g \) and the stack length \( l_{stk} \) can be worked out using Eqs. (29) and (30).

3. Based on Eqs. (32–44), the detailed geometric parameters of the stator core and the rotor core are able to be obtained. Therefore, the stator outer diameter \( r_o \) and machine total length \( l_o \) can be finally determined.

4. After that, check if the stator outer diameter and the machine total length satisfy the required design specifications. If so, proceed to the FEA
verifications of machine performances. If not, reset the initial values such as combination of stator slot and rotor slot number, electric loading, equivalent magnetic loading, etc.

Figure 27.
Design flow of FRPMM.
5. Conducting FEA simulations, the electromagnetic performances such as back-EMF, average torque, pulsating torque, power factor, efficiency, etc. can be obtained. Check if all the performances satisfy the design specifications. If not, adjust the design parameters in the former steps and iterate the design flow until every output meets the requirement.

6. Finally, it is the result output.

6.2 Case study

In order to show the effectiveness of the introduced analytical method, a FRPMM is designed based on the method. Table 4 shows the specifications of the FRPMM, which mainly includes the rated torque, machine volume, cooling method, rated power, and speed. According to the rated torque, a design margin of 5% is suggested so as to make sure the torque output. Therefore, the requirement of the torque is 8.4 Nm for this design. Then, the combination of stator slots and rotor slots is determined in the first place. This combination is selected due to its high torque density and low pulsating torque, as shown in Figure 13. Then, since the cooling method is natural cooling, the electric loading and the equivalent magnetic loading are chosen as 300A/cm and 0.2 T, respectively. After that, based on the output torque value 8.4 Nm and Eq. (29), the airgap radius is determined as 38.5 mm. Furthermore, assuming the yoke flux density of stator core and rotor core as 1.0 T, and the teeth flux density of stator core and rotor core as 1.2 T, the detailed

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Table 4.
Design specifications of a three-phase FRPMM.

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<td>Slot depth</td>
<td>13.5 mm</td>
<td>Yoke flux density</td>
<td>1.0 T</td>
</tr>
<tr>
<td>Winding pole pair</td>
<td>1</td>
<td>Teeth flux density</td>
<td>1.1 T</td>
</tr>
<tr>
<td>Magnet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PM thickness</td>
<td>3 mm</td>
<td>Magnet width</td>
<td>7.8 mm</td>
</tr>
<tr>
<td>Rotor</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outer diameter</td>
<td>77.8 mm</td>
<td>Slot depth</td>
<td>10.4 mm</td>
</tr>
<tr>
<td>Teeth flux density</td>
<td>1.2 T</td>
<td>Yoke thickness</td>
<td>12.5 mm</td>
</tr>
<tr>
<td>Inner diameter</td>
<td>32 mm</td>
<td>Yoke flux density</td>
<td>1.0 T</td>
</tr>
<tr>
<td>Teeth width</td>
<td>4 mm</td>
<td>Slot number</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 5.
Design parameters of the FRPMM using the design method.
Table 6.
Results comparison of the design method and 2D FEA.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Analytical design method</th>
<th>FEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>PM flux linkage</td>
<td>1.43 Wb</td>
<td>1.35 Wb</td>
</tr>
<tr>
<td>Back-EMF</td>
<td>44.8 V</td>
<td>42.4 V</td>
</tr>
<tr>
<td>Torque</td>
<td>8.4 Nm</td>
<td>7.97 Nm</td>
</tr>
</tbody>
</table>

Figure 28.
12-slot/17-pole FRPMM prototype: (a) stator; (b) rotor.

Figure 29.
Test bed of the FRPMM prototype.

Figure 30.
Back-EMF waveforms at rated speed 300 rpm: (a) waveform; (b) FFT analysis.
geometric parameters can all be determined. At last, the stator outer diameter is worked out as 124 mm, which is less than the requirement 130 mm. So far, this design is effective. Table 5 summarizes the design parameters of the FRPMM.

Finally, in order to verify the accuracy of the proposed analytical design method, the FEA model is built, and the simulated performances are compared to the analytical designed values. It can be seen in Table 6 that the FEA simulated results match well with the analytical method. More importantly, the simulated performance output satisfies the design specifications. Therefore, this analytical design is successful.

6.3 Experimental study

To verify the calculated results by the analytical method and FEA, the FRPMM prototype has been built. Its major parameters are listed in Table 4. The structure and test bed of the prototype are shown in Figures 28 and 29, respectively.

![Figure 31. Output torque vs. phase current.](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>FEA</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average torque at rated current</td>
<td>7.97 Nm</td>
<td>7.24 Nm</td>
</tr>
<tr>
<td>Torque per weight</td>
<td>0.66 Nm/kg</td>
<td>0.60 Nm/kg</td>
</tr>
<tr>
<td>Phase back-EMF magnitude at 300 rpm</td>
<td>42.4 V</td>
<td>41.2 V</td>
</tr>
<tr>
<td>THD of the phase back-EMF at 300 rpm</td>
<td>1.26%</td>
<td>2.63%</td>
</tr>
<tr>
<td>Total losses</td>
<td>99.5 W</td>
<td>116.7 W</td>
</tr>
<tr>
<td>Efficiency</td>
<td>60.3%</td>
<td>57.3%</td>
</tr>
<tr>
<td>Power factor</td>
<td>0.756</td>
<td>0.746</td>
</tr>
</tbody>
</table>

Table 7.
Result comparison of FEA and experiment of the FRPMM prototype.
Figure 30 compares the phase back-EMF waveform and spectrum at 300 rpm. It can be seen that the back-EMF waveforms are very sinusoidal. This is because the total harmonic distortion (THD) of FEA and experiments are only 1.26% and 2.63%, respectively. The sinusoidal back-EMF is inherent without any special design techniques such as skewing or pole shaping. Then, Figure 31 shows the FEA simulated and experimental results of average torque at different winding current values. In addition, the analytical design value is also plotted as the blue triangle. It indicates that the simulated, analytical and experimental results have reached good agreements. Finally, Table 7 compares the electromagnetic performances by FEA and experiments. Thus, the feasibility of the analytical design method can be seen.

7. Conclusions

The design of FRPMMs is usually based on time-stepping FEA, which are accurate but time-consuming. To save the design time meanwhile maintain the accuracy, this chapter proposes an analytical design method of FRPMMs. First, the sizing equation is derived, and then the dimensional parameters of stator and rotor are calculated. Finally, based on the above equations, an analytical design procedure is established. Moreover, in order to help to choose the initial design parameters in the sizing equation, including number of stator slots and rotor slots, airgap radius, electrical loading, and equivalent magnetic loading, their effects on the average torque, cogging torque, torque ripple, and power factor are investigated, providing reliable guidance for designers. At last, in order to make the introduced design methodology easier to understand, a FRPMM is designed and tested.

Acknowledgements

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Nomenclature

- \( B_r \) : remanent flux density
- \( \mu_r \) : relative permeability of magnets
- \( g' \) : effective airgap length considering PM thickness
- \( h_m \) : PM height along the magnetization direction
- \( SO \) : stator slot opening ratio (=slot opening width/slot pitch)
- \( g \) : airgap length
- \( r_g \) : airgap radius
- \( N_s \) : number of series turns per phase
- \( P \) : number of stator winding pole pairs
- \( l_{stk} \) : active stack length
- \( Z_r \) : number of rotor teeth
- \( \omega_m \) : mechanical angular speed of rotor
- \( Z_s \) : number of stator teeth
- \( SPP \) : slot per pole per phase
- \( \theta \) : angular position of rotor axis with respect to the axis of phase \( a \)
- \( \theta_s \) : particular position in the stator reference frame measured from the axis of phase \( a \)
- \( PR \) : pole ratio (=rotor pole number/winding pole pair)
References


[16] Dmitrievskii V, Prakht V, Mikhaliutsyn A. A new single-phase flux reversal motor with the cores made of


