Chapter

Trajectory Tracking Using Adaptive Fractional PID Control of Biped Robots with Time-Delay Feedback

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Abstract

This paper presents the application of fractional order time-delay adaptive neural networks to the trajectory tracking for chaos synchronization between Fractional Order delayed plant, reference and fractional order time-delay adaptive neural networks. For this purpose, we obtained two control laws and laws of adaptive weights online, obtained using the fractional order Lyapunov-Krasovskii stability analysis methodology. The main methodologies, on which the approach is based, are fractional order PID the fractional order Lyapunov-Krasovskii functions methodology, although the results we obtain are applied to a wide class of non-linear systems, we will apply it in this chapter to a bipedal robot. The structure of the biped robot is designed with two degrees of freedom per leg, corresponding to the knee and hip joints. Since torso and ankle are not considered, it is obtained a 4-DOF system, and each leg, we try to force this biped robot to track a reference signal given by undamped Duffing equation. To verify the analytical results, an example of dynamical network is simulated, and two theorems are proposed to ensure the tracking of the nonlinear system. The tracking error is globally asymptotically stabilized by two control laws derived based on a Lyapunov-Krasovskii functional.

Keywords: biped robot, fractional time-delay adaptive neural networks, fractional order PID control, fractional Lyapunov-Krasovskii functions, trajectory tracking

1. Introduction

Fractional calculus is a generalization of differential and integral calculus which involves generalized functions. The first to work this new branch of mathematics was Leibniz. Due to the growing interest in the applications of fractional calculation, in this work we obtain conditions that guarantee the tracking of trajectories of nonlinear systems generated by differential equations of fractional order which we will call plants (This term is widely used in engineering), which in our case will be a mechanical arm, a helicopter, a plane or limbs of a humanoid, all of fractional order.

The problem of tracking control of trajectories is very important, since the control function allows the non-linear system to carry out a previously assigned

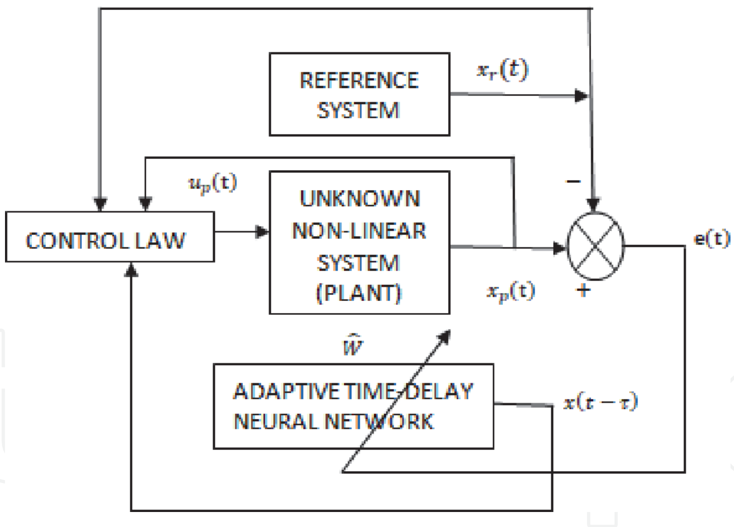

Figure 1.
Adaptive recurrent control diagram.

task, work or trajectory, for example, a mechanical arm and its objective is to cut a piece with a previously generated form, or the coupling of two aircraft in space.

In this chapter we use adaptive recurrent neural networks with time delay, since its use allows us to work with systems whose mathematical model is unknown and with the presence of uncertainties, this is a well-known problem of robust control.

We include mathematical models with time delay, since the processing and transmission of information is important in this type of systems, which depending on the delay, these systems can generate undesirable oscillatory or chaotic dynamics, and cause instability in the mathematical model that describes the trajectory tracking error.

The chapter is organized as follows: first, the general mathematical model of non-linear systems is proposed, as a second part, the Neural Network is proposed that will adapt to the non-linear system and the reference signal that both must follow, as a third part obtains the dynamics of the tracking error between the non-linear system and the reference, after obtaining conditions in the laws of adaptation of weights in the Neural Network and obtaining the control law that guarantees that the tracking error converges to zero, so that the non-linear system will follow the indicated reference signal, which is what was wanted to be demonstrated. Finally simulations are presented, which illustrate the theoretical results previously demonstrated. The proposed new control scheme is applied via simulations to control of a 4-DOF Biped Robot [1].

We use the scheme of **Figure 1** to indicate the procedure used in the obtaining of the laws of adaptation of weights and the laws of control that guarantee that the tracking error between the non-linear system, the neural network and the reference signal converges to zero.

2. Time-delay adaptive neural network and the reference

2.1 List of variables

W^* is the matrix weights.
$e = x_p - x_r$, error between the plant and the reference.
$\hat{W}$ is part of the approach, given by W^*.
Ωu_1, Ωu_2, u_p, u_n are the controls
$PI^{\lambda}D^{\alpha} = K_p e(t) + K_i a D_t^{-\lambda} e(t) + K_d a D_t^{\alpha} e(t)$ control law
τ is time delay

$tr\left\{aD_t^\alpha \tilde{W}^T \tilde{W}\right\} = -e^T \tilde{W}\sigma(x(t-\tau))$, learning law from the neural network weights
$\int_{t-\tau}^{t} \left[\emptyset_\sigma^T(s)\hat{W}^T \hat{W}\emptyset_\sigma(s)\right] ds$, Lyapunov-Krasovskii Function
$D(q(t)\ddot{q}(t)) + C(q(t),\dot{q}(t))\dot{q}(t) + G(q(t)) = B\tau(t)$, dynamics of the bipedal robot
$D(q(t))$ is the inertia matrix
$C(q(t),\dot{q}(t))$ is the matrix of Coriolis and centripetal forces
$G(q(t))$ represents a matrix of gravitational effects
B defines the input matrix

There are several ways to define the fractional calculation, in this research we will use the well-known derivative of Caputo, which has the following notation:

$$aD_t^\alpha f(t) = \frac{1}{\Gamma(\alpha - n)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \tag{1}$$

For $(n-1<\alpha<n)$.

The nonlinear system, Eq. (2), which is forced to follow a reference signal:

$$aD_t^\alpha x_p = f_p(\mathrm{t}, x_p(\mathrm{t}) + x_p(\mathrm{t}-\tau)), \mathrm{t} \in [0,\mathrm{T}], 0<\alpha \le 1, \tag{2}$$

$$x_p(\mathrm{t}) = \mathrm{g}(\mathrm{t})$$

$$x_p, f_p \in \mathbb{R}^n, u \in \mathbb{R}^m, g_p \in \mathbb{R}^{nxn}.$$

The differential equation will be modeled by the neural network [2]:

$$aD_t^\alpha x_p = \mathrm{A}(x) + \mathrm{W}^*\Gamma_\mathrm{z}(x(t-\tau) + \Omega_\mathrm{u}.$$

The tracking error between these two systems:

$$w_{per} = x - x_p \tag{3}$$

We use the next hypotheses.

$$aD_t^\alpha w_{per} = -k w_{per} \tag{4}$$

In this research we will use $k=1$, so that, Eq. (5), $aD_t^\alpha w_{per} = aD_t^\alpha x - aD_t^\alpha x_p$, so:

$$aD_t^\alpha x_p = aD_t^\alpha x + w_{per}$$

The nonlinear system is [3]:

$$aD_t^\alpha x_p = aD_t^\alpha x + w_{per} = \mathrm{A}(x) + \mathrm{W}^*\Gamma_\mathrm{z}[x(t-\tau)] + w_{per} + \Omega_\mathrm{u} \tag{5}$$

where the W^* is the matrix weights.

3. Tracking error problem

In this part, we will analyze the trajectory tracking problem generated by

$$aD_t^\alpha x_r = f_r(x_r, u_r), w_r, x_r \in \mathbb{R}^n \tag{6}$$

are the state space vector, input vector and f_r, is a nonlinear vectorial function.

To achieve our goal of trajectory tracking, we propose the error between the plant and the reference as: $e = x_p - x_r = (x_p - x_n) + (x_n - x_r) = (x_p - x) + (x - x_r)$.

Let $e_p = x_p - x$, and $e_n = x - x_r$, be the trajectory tracking error and $e = e_p + e_r$

$$e_n = x - x_r \tag{7}$$

The time derivative of the error is:

$$aD_t^\alpha e_n = aD_t^\alpha x - aD_t^\alpha x_r = \mathrm{A}(x) + \mathrm{W}^*\Gamma_\mathrm{z}[x(t-\tau)] + w_{per} + \Omega_\mathrm{u} - f_r(x_r, u_r) \tag{8}$$

Eq. (8), can be rewritten as follows, adding and subtracting, the next terms $\hat{W}\Gamma_\mathrm{z}(x_r x(t-\tau))$, $\alpha_r(\mathrm{t}, \hat{W})$, Ae and $w_{per} = x - x_p$, then,

$$\begin{aligned} aD_t^\alpha e = {} & \mathrm{A}(x) + \mathrm{W}^*\Gamma_\mathrm{z}(x(t-\tau)) + x - x_p + \Omega_\mathrm{u} - f_r(x_r, u_r) + \hat{W}\Gamma_\mathrm{z}(x_r(t-\tau)) \\ & -\hat{W}\Gamma_\mathrm{z}(x_r(t-\tau)) + \Omega\alpha_r(\mathrm{t}, \hat{W}) - \Omega\alpha_r(\mathrm{t}, \hat{W}) + Ae - Ae \\ aD_t^\alpha e = {} & \mathrm{A}e + \mathrm{W}^*\Gamma_\mathrm{z}(x(t-\tau)) + \Omega_\mathrm{u} - f_r(x_r, u_r) + \hat{W}\Gamma_\mathrm{z}(x_r(t-\tau)) \\ & +\Omega\alpha_r(\mathrm{t}, \hat{W}) - \Omega\alpha_r(\mathrm{t}, \hat{W}) - e - x_r - Ae + x + A(x) - \hat{W}\Gamma_\mathrm{z}(x_r(t-\tau)) \end{aligned} \tag{9}$$

The unknown plant will follow the fractional order reference signal, if:

$$Ax_r + \hat{W}\Gamma_\mathrm{z}(x_r(t-\tau)) + x_r - x_p + \Omega\alpha_r(\mathrm{t}, \hat{W}) = f_r(x_r, u_r),$$

where

$$\Omega\alpha_r(\mathrm{t}, \hat{W}) = f_r(x_r, u_r) - Ax_r - \hat{W}\Gamma_\mathrm{z}(x_r(t-\tau)) - x_r + x_p \tag{10}$$

$$\begin{aligned} aD_t^\alpha e = {} & \mathrm{A}e + \mathrm{W}^*\Gamma_\mathrm{z}(x(t-\tau)) - \hat{W}\Gamma_\mathrm{z}(x_r(t-\tau)) - \mathrm{A}e + (A+I)(x - x_r) \\ & +\Omega(u - \alpha_r(\mathrm{t}, \hat{W})) \end{aligned} \tag{11}$$

Now, $\hat{W}$ is part of the approach, given by W^*. Eq. (11) can be expressed as Eq. (12), adding and subtracting the term $\hat{W}\Gamma_\mathrm{z}(x(t-\tau))$ and if $\Gamma_\mathrm{z}(x(t-\tau)) = \Gamma(z(x(t-\tau)) - z(x_r(t-\tau)))$

$$\begin{aligned} aD_t^\alpha e = {} & \mathrm{A}e + (\mathrm{W}^* - \hat{W})\Gamma_\mathrm{z}(x(t-\tau)) + \hat{W}\Gamma(z(x(t-\tau)) - z(x_r(t-\tau))) \\ & +(A+I)(x - x_r) - \mathrm{A}e + \Omega(u - \alpha_r(\mathrm{t}, \hat{W})) \end{aligned} \tag{12}$$

If

$$\tilde{W} = \mathrm{W}^* - \hat{W} \text{ and } \tilde{u} = u - \alpha_r(\mathrm{t}, \hat{W}) \tag{13}$$

And by replacing Eq. (13) in Eq. (12), we have:

$$\begin{aligned} aD_t^\alpha e = {} & \mathrm{A}e + \tilde{W}\Gamma_\mathrm{z}(x(t-\tau)) + \hat{W}\Gamma(z(x(t-\tau)) - z(x_r(t-\tau))) \\ & +(A+I)(x - x_r) - \mathrm{A}e + \Omega\tilde{u} \end{aligned}$$

$$\begin{aligned} aD_t^\alpha e = {} & \mathrm{A}e + \tilde{W}\Gamma_\mathrm{z}(x(t-\tau)) \\ & + \hat{W}\Gamma\big(z(x(t-\tau)) - z(x_p(t-\tau)) + z(x_p(t-\tau)) - z(x_r(t-\tau))\big) \\ & + (A+I)(x - x_p + x_p - x_r) - \mathrm{A}e + \Omega\tilde{u} \end{aligned} \tag{14}$$

And:

$$\tilde{u} = u_1 + u_2 \tag{15}$$

So, the result for Ωu_1 is

$$\Omega u_1 = -\hat{W}\Gamma\left(z(x(t-\tau)) - z\left(x_p(t-\tau)\right)\right) - (A+I)\left(x - x_p\right) \tag{16}$$

and Eq. (14), is simplified:

$$\begin{aligned} aD_t^{\alpha} e = \mathrm{A}e + \tilde{W}\Gamma_z(x(t-\tau)) + \hat{W}\Gamma\left(z\left(x_p(t-\tau)\right) - z(x_r(t-\tau))\right) \\ +(A+I)\left(x_p - x_r\right) - \mathrm{A}e + \Omega\tilde{u} \end{aligned}$$

Taking into account that $e = x_p - x_r$, shortening notation a little bit by setting $\sigma = \Gamma_z$, and defining $\varnothing_\sigma(t-\tau) = \sigma\left(x_p(t-\tau)\right) - \sigma(x_r(t-\tau))$, the equation for aD_t^{α}e is

$$aD_t^{\alpha} e = (A+I)e + \tilde{W}\sigma(x(t-\tau)) + \hat{W}\varnothing_\sigma(t-\tau) + \Omega u_2 \tag{17}$$

Now, the problem is to find the control law Ωu_2, which it stabilizes to the system Eq. (20). The control law, we will obtain using the fractional order Lyapunov-Krasovskii methodology.

4. Study of trajectory tracking error

Our mathematical model of the dynamics in the tracking error is described in (17). In this equation we can see that an equilibrium state of this system is $\left(e, \hat{W}\right) = 0$.

Without loss of generality we can assume that the matrix A is given $A = -\lambda I$, $\lambda > 0$, where I is the identity matrix of order nxn.

For the study of the stability of the tracking error we propose the following PID control law [4], widely used in science and engineering.

We will determine conditions in the parameters that guarantee that the tracking error converges to zero, and we will also use the following control law [5].

$$\Omega u_2 = K_p e + K_i aD_t^{-\alpha} e + K_v aD_t^{\alpha} e - \gamma\left(\frac{1}{2} + \frac{1}{2}\left\|\hat{W}\right\|^2 L_\phi^2\right)e \tag{18}$$

We also include the following control law, $PI^{\lambda}D^{\alpha}$ [6]:

$$u(t) = K_p e(t) + K_i aD_t^{-\lambda} e(t) + K_d aD_t^{\alpha} e(t)$$

Substituting Eq. (18) in Eq. (17):

$$\begin{aligned} aD_t^{\alpha} e = (A+I)e + \tilde{W}\sigma(x(t-\tau)) + \hat{W}\varnothing_\sigma(t-\tau) \\ + K_p e + K_i aD_t^{-\alpha} e + K_v aD_t^{\alpha} e - \gamma\left(\frac{1}{2} + \frac{1}{2}\left\|\hat{W}\right\|^2 L_\phi^2\right)e, \end{aligned}$$

then

$$\begin{aligned} (1-K_v) aD_t^{\alpha} e = (A+I)e + \tilde{W}\sigma(x(t-\tau)) + \hat{W}\varnothing_\sigma(t-\tau) \\ + K_p e + K_i aD_t^{-\alpha} e - \gamma\left(\frac{1}{2} + \frac{1}{2}\left\|\hat{W}\right\|^2 L_\phi^2\right)e. \end{aligned}$$

If $a = (1 - K_v)$, then

$$aD_t^\alpha e = \frac{1}{a}(A+I)e + \frac{1}{a}\tilde{W}\sigma(x(t-\tau)) + \frac{1}{a}\hat{W}\O_\sigma(t-\tau) + \frac{1}{a}K_p e + \frac{1}{a}K_i aD_t^{-\alpha}e - \frac{\gamma}{a}\left(\frac{1}{2} + \frac{1}{2}\|\hat{W}\|^2 L_\phi^2\right)e \tag{19}$$

$$aD_t^\alpha e = \frac{-1}{a}(\lambda - 1 + K_p)e + \frac{1}{a}\tilde{W}\sigma(x(t-\tau)) + \frac{1}{a}\hat{W}\O_\sigma(t-\tau) + \frac{1}{a}K_i aD_t^{-\alpha}e - \frac{\gamma}{a}\left(\frac{1}{2} + \frac{1}{2}\|\hat{W}\|^2 L_\phi^2\right)e \tag{20}$$

And if $w = \frac{1}{a}K_i aD_t^{-\alpha}e$, then $aD_t^\alpha w = \frac{1}{a}K_i e(t)$, [7], then Eq. (20) we rewrite as:

$$aD_t^\alpha e_n = \frac{-1}{a}(\lambda - 1 + K_p)e + \frac{1}{a}\tilde{W}\sigma(x(t-\tau)) + \frac{1}{a}\hat{W}\O_\sigma(t-\tau) + w - \frac{\gamma}{a}\left(\frac{1}{2} + \frac{1}{2}\|\hat{W}\|^2 L_\phi^2\right)e \tag{21}$$

We will show, the new state $(e_n, w)^T$ is asymptotically stable, and the equilibrium point is $(e_n e, w)^T = (0,0)^T$, when $\tilde{W}\sigma(x_r(t-\tau)) = 0$, as an external disturbance.
Let V be, the next candidate Lyapunov function as [8, 9]:

$$V = \frac{1}{2}\left(e_n{}^T, w^T\right)(e_n, w)^T + \frac{1}{2a}tr\left\{\tilde{W}^T\tilde{W}\right\} + \frac{1}{a}\int_{t-\tau}^{t}\left[\O_\sigma^T(s)\hat{W}^T\hat{W}\O_\sigma(s)\right]ds \tag{22}$$

The fractional order time derivative of (22) along the trajectories of Eq. (21) is:

$$aD_t^\alpha V = e^T aD_t^\alpha e + w^T aD_t^\alpha w + \frac{1}{a}tr\left\{aD_t^\alpha \tilde{W}^T\tilde{W}\right\} + \frac{1}{a}\left[\O_\sigma^T(t)\hat{W}^T\hat{W}\O_\sigma(t) - \O_\sigma^T(t-\tau)\hat{W}^T\hat{W}\O_\sigma(t-\tau)\right] \tag{23}$$

$$aD_t^\alpha V = e^T\left(\frac{-1}{a}(\lambda - 1 + K_p)e + \frac{1}{a}\tilde{W}\sigma(x(t-\tau)) + \frac{1}{a}\hat{W}\O_\sigma(t-\tau) + w - \frac{\gamma}{a}\left(\frac{1}{2} + \frac{1}{2}\|\hat{W}\|^2 L_\phi^2\right)e\right) + \frac{1}{a}\tilde{W}^T K_i e + \frac{1}{a}tr\left\{aD_t^\alpha \tilde{W}^T\tilde{W}\right\} + \frac{1}{a}\left[\O_\sigma^T(t)\hat{W}^T\hat{W}\O_\sigma(t) - \O_\sigma^T(t-\tau)\hat{W}^T\hat{W}\O_\sigma(t-\tau)\right] \tag{24}$$

In this part, we select the next learning law from the neural network weights as in [10, 11]:

$$tr\left\{aD_t^\alpha \tilde{W}^T\tilde{W}\right\} = -e^T\tilde{W}\sigma(x(t-\tau)) \tag{25}$$

Then Eq. (24) is reduced to

$$\begin{aligned} aD_t^\alpha V = & \frac{-1}{a}(\lambda - 1 + K_p)e^T e + \frac{e^T}{a}\hat{W}\varnothing_\sigma(t-\tau) \\ & + \left(1 + \frac{K_i}{a}\right)e^T w - \frac{\gamma}{a}\left(\frac{1}{2} + \frac{1}{2}\|\hat{W}\|^2 L_\phi^2\right)e^T e \\ & + \frac{1}{a}\left[\varnothing_\sigma^T(t)\hat{W}^T\hat{W}\varnothing_\sigma(t) - \varnothing_\sigma^T(t-\tau)\hat{W}^T\hat{W}\varnothing_\sigma(t-\tau)\right] \end{aligned} \tag{26}$$

Next, let us consider the following inequality proved in [12]

$$X^T Y + Y^T X \le X^T \Lambda X + Y^T \Lambda^{-1} Y \tag{27}$$

Which holds for all matrices $X, Y \in \mathbb{R}^{nxk}$and $\Lambda \in \mathbb{R}^{nxn}$ with $\Lambda = \Lambda^T > 0$. Applying (27) with $\Lambda = I$ to the term $\frac{e^T}{a}\hat{W}\varnothing_\sigma(t-\tau)$from Eq. (26), where

$$\frac{e^T}{a}\hat{W}\varnothing_\sigma(t-\tau) \le \frac{1}{a}\left[e^T e + \varnothing_\sigma^T(t-\tau)\hat{W}^T\hat{W}\varnothing_\sigma(t-\tau)\right]$$

we get

$$\begin{aligned} aD_t^\alpha V \le & \frac{-1}{a}(\lambda - 1 + K_p)e^T e + \frac{1}{a}\left(\frac{e^T e}{2} + \frac{1}{2}\|\hat{W}\|^2 L_\phi^2\right)e^T e \\ & + \left(1 + \frac{K_i}{a}\right)e^T w - \frac{\gamma}{a}\left(\frac{1}{2} + \frac{1}{2}\|\hat{W}\|^2 L_\phi^2\right)e^T e \end{aligned} \tag{28}$$

Here, we select $\left(1 + \frac{K_i}{a}\right) = 0$ and $K_v = K_i + 1$, with $K_v \ge 0$ then $K_i \ge -1$, with this selection of the parameters from Eq. (28) is reduced to:

$$aD_t^\alpha V \le \frac{-1}{a}(\lambda - 1 + K_p)e^T e - \frac{(\gamma - 1)}{a}\left(\frac{1}{2} + \frac{1}{2}\|\hat{W}\|^2 L_\phi^2\right)e^T \tag{29}$$

From the previous inequality, we need to guarantee that Eq. (29) is less than zero, for which we select,

$\lambda - 1 + K_p > 0$, $a > 0$, $(\gamma - 1) > 0$, so that: $aD_t^\alpha V \le 0, \forall e, w, \hat{W} \ne 0, e \ne 0$, is wanted to be demonstrate.

The control law is given by Eq. (30)

$$\begin{aligned} u_n = & \Omega^\dagger\left[-\hat{W}\Gamma\left(z(x_n(t-\tau)) - z\left(x_p(t-\tau)\right)\right)\right. \\ & -(A + I)\left(x - x_p\right) + K_p e + K_i aD_t^{-\alpha} e + K_v aD_t^\alpha e \\ & -\Gamma\left(\frac{1}{2} + \frac{1}{2}\|\hat{W}\|^2 L_\phi^2\right)e_n + f_r(x_r, u_r) - Ax_r \\ & \left. -\hat{W}\Gamma_z(x_r(t-\tau)) - x_r + x_p\right] \end{aligned} \tag{30}$$

Theorem: The control law Eq. (30) and the neuronal adaptation law given by Eq. (25) guarantee that the fractional tracking error converges to zero, by which the tracking of trajectories of the non-linear system is guaranteed Eq. (5).

Corollary 2: *If* $aD_t^\alpha V \le \frac{-1}{a}(\lambda - 1 + K_p)(e_n{}^T)(e_n) - \frac{(\gamma-1)}{a}\left(\frac{1}{2} + \frac{1}{2}\|\hat{W}\|^2 L_\phi^2\right)(e_n{}^T)(e_n)$ $< 0, \forall e \ne 0, \forall \hat{W}$, where V is decreasing and bounded from below by $V(0)$, and:

$$V = \frac{1}{2}\left(e_n{}^T, w^T\right)(e_n, w)^T + \frac{1}{2a} tr\left\{\tilde{W}^T\tilde{W}\right\} + \frac{1}{a}\int_{t-\tau}^{t}\left[\varnothing_\sigma^T(s)\hat{W}^T\hat{W}\varnothing_\sigma(s)\right]ds,$$

then we conclude that $e, \tilde{W} \in L_1$; this means that the weights remain bounded.

5. Modeling of the time-delay adaptive neural network and the delayed plant

The nonlinear delayed unknown plant and the neural network are given as:

$$aD_t^\alpha x_p = \mathrm{f_p}\left(x_p(t-\tau)\right) + \mathrm{g_p}\left(x_p(t)\right)\mathrm{u_p},$$

$$aD_t^\alpha x_n = \mathrm{A}(x) + \mathrm{W}^*\Gamma_\mathrm{z}[x(t-\tau)] + w_{per} + \Omega_\mathrm{u}$$

where $x_p, f_p \in \mathbb{R}^n, u \in \mathbb{R}^m, g_p \in \mathbb{R}^{nxn}$. And f_p, is unknown and $g_p = I, A = -\lambda I$, with Γ Lypschitz function, W^* are the fixed weigths but unknown from the neural networks, which minimize the modeling error.

Theorem: We will show that $e_p$and e_n tend to zero and therefore e tends to zero, that is, the neural network follows the plant.

For this proposal, we first define the modeling error between the neural network and the plant: $e_p = x_p - x_n$, whose derivative in the time is

$$aD_t^\alpha e_p = aD_t^\alpha x_p - aD_t^\alpha x_n \tag{31}$$

Adding and subtracting, to the right hand side from (34) the terms $\hat{W}\Gamma_\mathrm{z}(x_r x(t-\tau))$, $\alpha_p\left(\mathrm{t}, \hat{W}\right)$

$$aD_t^\alpha e_p = \mathrm{A}\left(e_p\right) - \tilde{W}\Gamma_\mathrm{z}[x_n(t-\tau)] + \hat{W}\varnothing\left(e_p(t-\tau)\right) + \tilde{u_p} \tag{32}$$

6. Identification of the unknown non-linear system by the neural network

First, it is easy to see that $\left(e, \hat{W}\right) = 0$ is a state of equilibrium (equilibrium point). Previous, so we propose to demonstrate that this point of equilibrium is asymptotically stable; for this, be:

$$\tilde{u_p} = -\gamma\left(\frac{1}{2} + \frac{1}{2}\left\|\hat{W}\right\|^2 L_\phi^2\right)e_p \tag{33}$$

We will show, the feedback system is asymptotically stable. Replacing (36) in (35)

$$aD_t^\alpha e_p = \mathrm{A}\left(e_p\right) - \tilde{W}\Gamma_\mathrm{z}[x_n(t-\tau)] + \hat{W}\varnothing\left(e_p(t-\tau)\right) - \gamma\left(\frac{1}{2} + \frac{1}{2}\left\|\hat{W}\right\|^2 L_\phi^2\right)e_p \tag{34}$$

We will show, the new state e_p is asymptotically stable, and the equilibrium point is $e_p \to 0$, when $\hat{W}\sigma(x_n(t-\tau)) = 0$, as an external disturbance.

Let V be, the next candidate Lyapunov function as

$$V = \frac{1}{2}\left(e_p{}^T, w^T\right)\left(e_p, w\right)^T + \frac{1}{2a} tr\left\{\tilde{W}^T \tilde{W}\right\} + \frac{1}{a}\int_{t-\tau}^{t} \left[\varnothing_\sigma^T(s)\hat{W}^T \hat{W}\varnothing_\sigma(s)\right] ds \tag{35}$$

Then, (35) is reduced to

$$aD_t^\alpha V \leq \frac{-1}{a}\left(\lambda - 1 + K_p\right)\left(e_p{}^T\right)\left(e_p\right) - \frac{(\gamma - 1)}{a}\left(\frac{1}{2} + \frac{1}{2}\left\|\hat{W}\right\|^2 L_\phi^2\right)\left(e_p{}^T\right)\left(e_p\right) < 0 \tag{36}$$

The previous inequality guarantees that the identification of the non-linear system is satisfied, that is, the approach error converges to zero asymptotically

$$\begin{aligned} u_p = \Omega^\dagger \big[&\hat{W}\Gamma z(x_r(t-\tau)) - \hat{W}\Gamma\left(z(x_n(t-\tau)) - z\left(x_p(t-\tau)\right)\right) \\ &-(A+I)\left(x - x_p\right) + K_p e + K_i a D_t^{-\alpha} e + K_v a D_t^\alpha e \\ &-\Gamma\left(\frac{1}{2} + \frac{1}{2}\left\|\hat{W}\right\|^2 L_\phi^2\right) e_n - \Gamma\left(\frac{1}{2} + \frac{1}{2}\left\|\hat{W}\right\|^2 L_\phi^2\right) e_p + f_r(x_r, u_r) \\ &-f_p\left(x_p\right) + A x_p - A x_r + \hat{W}\Gamma_z\left(x_p\right) - x_r + x_p\big] \end{aligned} \tag{37}$$

7. Simulation

The mathematical model, which describes the movement dynamics of the bipedal robot, is obtained using the Euler-Lagrange equations [1, 13] (**Figure 2**).

$$D(q(t)\ddot{q}(t) + C(q(t), \dot{q}(t))\dot{q}(t) + G(q(t)) = B\tau(t)$$

where $q(t) = \left[q_{31}(t)q_{32}(t)q_{41}(t)q_{42}(t)\right]^T$, is the generalized coordinates vector. As usual, $D(q(t))$ is the inertia matrix, bounded and positive definite, and $C(q(t), \dot{q}(t))$ is the matrix of Coriolis and centripetal forces. $G(q(t))$ represents a matrix of gravitational effects and B defines the input matrix. The vector $\tau(t) = [\tau_{31}(t)\tau_{32}(t)\tau_{41}(t)\tau_{42}(t)]^T$, defines the applied joint torques of the robot.

To illustrate the theoretical results obtained, we propose an example, which, as can be seen in the simulations, trajectory tracking is guaranteed.

The neural network is described by:

$aD_t^\alpha x_p = \mathrm{A}(x) + \mathrm{W}^*\Gamma_z(x(t-\tau)) + \Omega_u$, with τ = 25 s, $A = -20I, I \in \mathbb{R}^{4x4}$, and, W^* is estimated using the learning law given in (28).

$\Gamma_z(x(t-\tau)) = (\tanh(x_1(t-\tau)), \tanh(x_2(t-\tau)), \ldots, \tanh(x_n(t-\tau)))^T, \Omega =$

$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^T$ and the u is obtained using (33).

and the reference signal that they have to follow, both the non-linear system and the neural network is given by the Duffing equation [14].

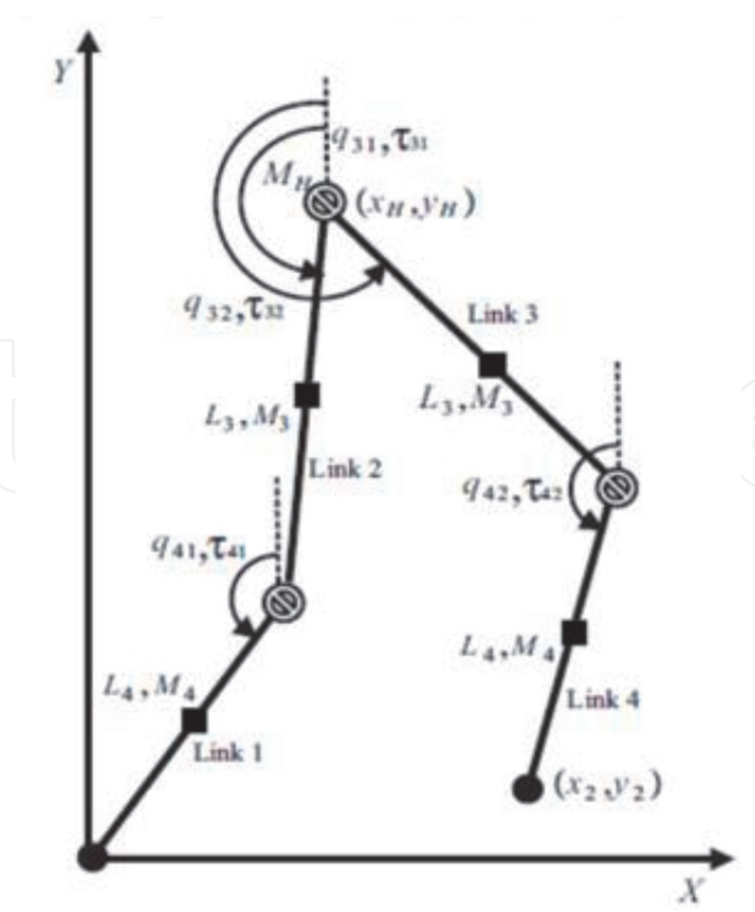

Figure 2.
Dynamic model of biped robot.

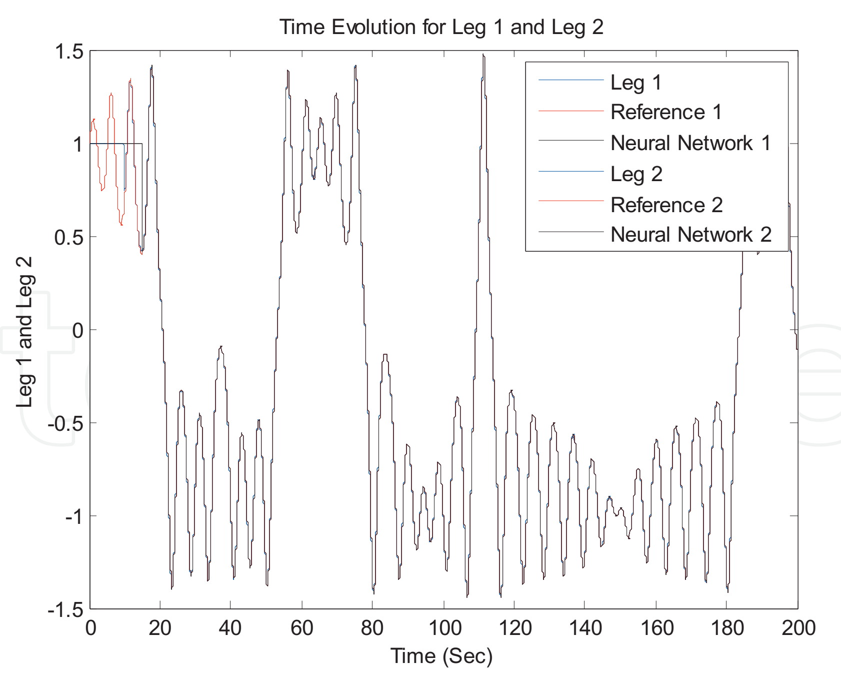

Figure 3.
Time evolution for the angular position Leg 1 and Leg 2 (rad) of link 1.

$$\ddot{x} - x + x^3 = 0.114\cos(1.1t) : x(0) = 1, \dot{x}(0) = 0.114$$

$$\frac{x(t)}{dt} = y(t)$$

$$\frac{y(t)}{dt} = x(t) - x^3(t) - \alpha y(t) + \delta\cos(\omega t)$$

Here, the conventional derivatives are replaced by the fractional derivatives as follows:

$$aD_t^{\alpha}x(t) = y(t)$$
$$aD_t^{\alpha}x(t) = x(t) - x^3(t) - \alpha y(t) + \delta \cos(\omega t)$$

where α, ω, δ, are the parameters of the Duffing differential equation, which we will use as a reference trajectory, that the non-linear system and the neural network have to follow.

As can be seen in **Figures 3–6**, the tracking of trajectories in the states of the system are performed with satisfaction, **Figure** 7 shows the phase plane of the

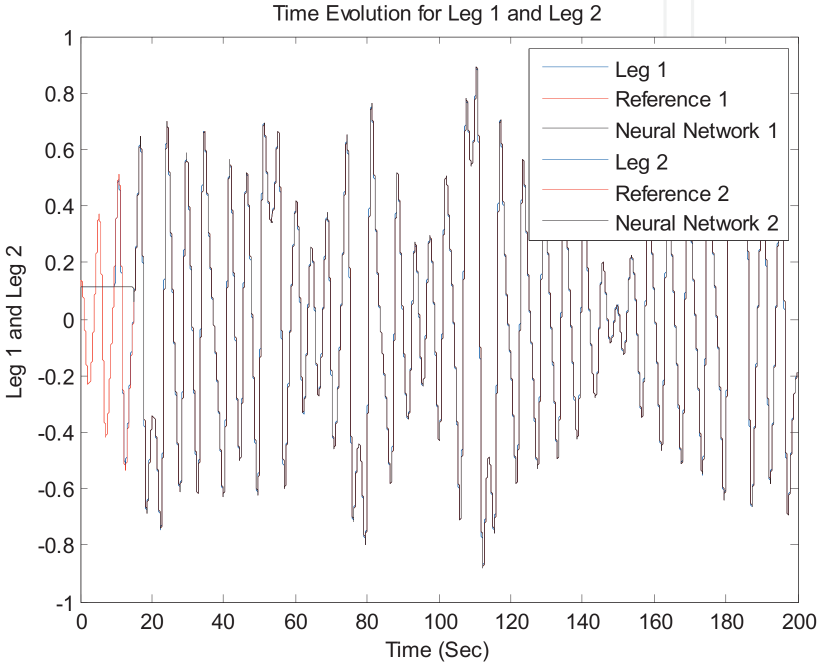

Figure 4.
Time evolution for the angular position Leg 1 and Leg 2 (rad) of link 2.

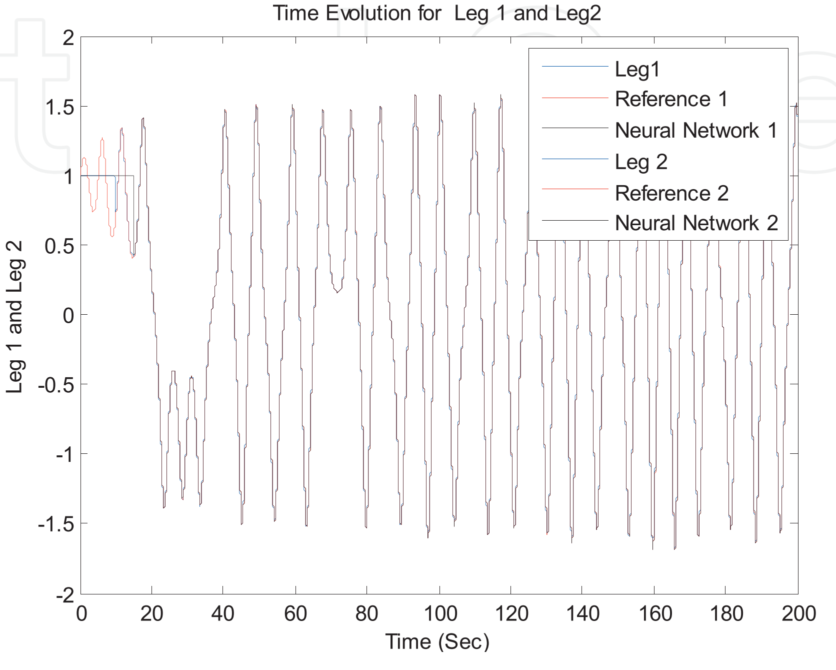

Figure 5.
Time evolution for the angular position Leg 1 and Leg 2 (rad) of link 1.

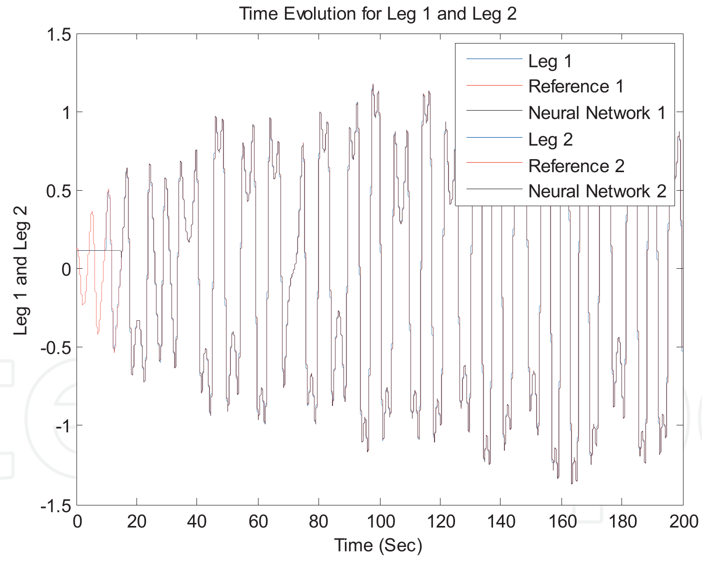

Figure 6.
Time evolution for the angular position Leg 1 and Leg 2 (rad) of link 2.

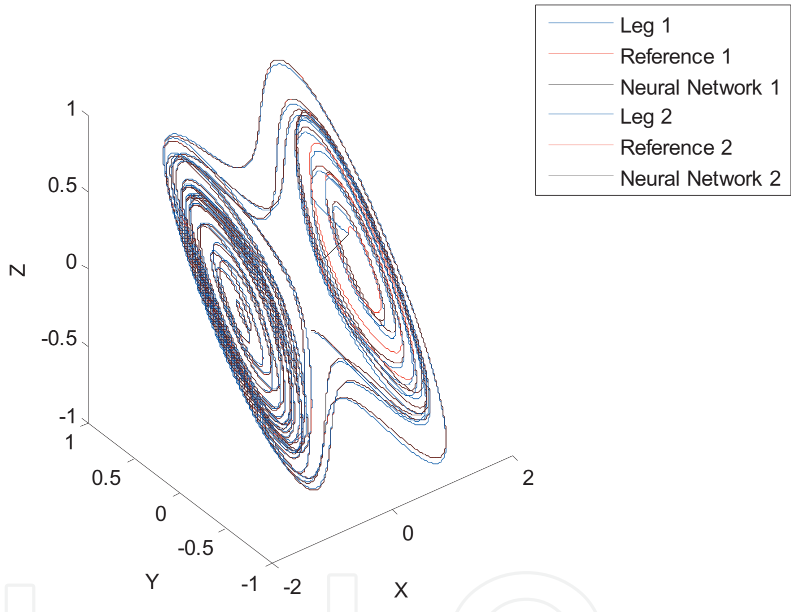

Figure 7.
A phase space trajectory of Duffing equation.

Duffing equation, while **Figure 8** shows the plane phase of the same fractional order differential equation.

Figures 9–12 show the torques applied to the ends of the bipedal robot.

Parameter values of the fractional order, alpha (0.001) and beta (0.0001) are included.

$$\alpha = 1, \quad \beta = 1$$
$$\alpha = 0.001, \quad \beta = 0.001$$

8. Conclusions

In this chapter we study the mathematical model and control of non-linear systems, which are modeled by differential equations of fractional order, where it is

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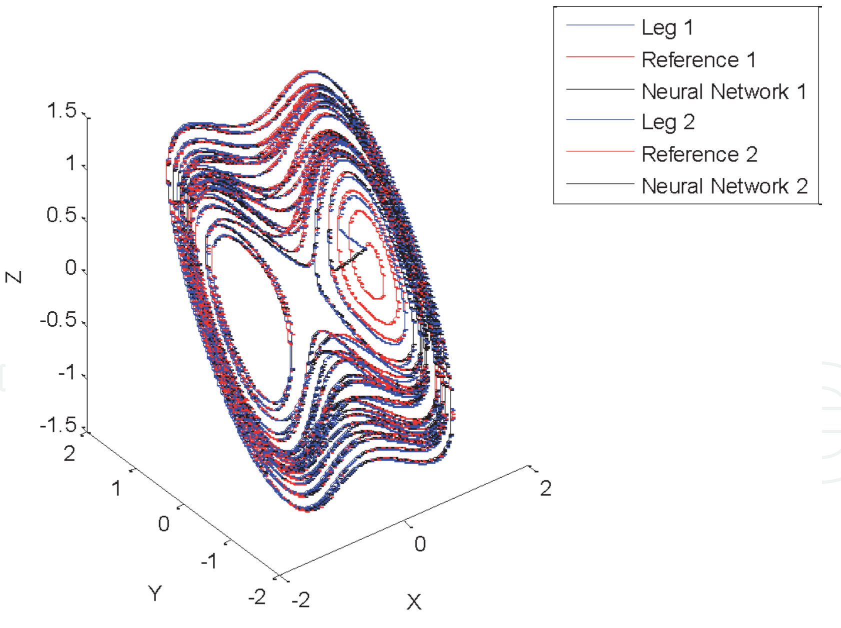

Figure 8.
A phase space trajectory of Duffing equation.

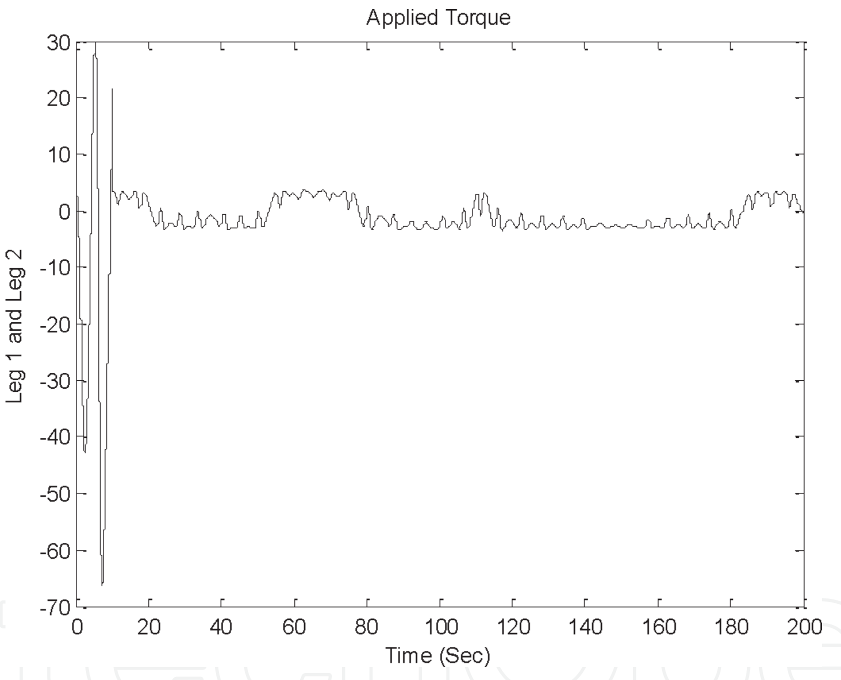

Figure 9.
Torque (Nm) applied to Leg 1 and Leg 2 of link 1.

observed that these systems have a better performance than the systems modeled by ordinary differential equations, those of fractional order they produce responses, solutions at simulation level, softer, by varying the order of the derivative.

The magnitude of the fractional order systems are smaller than the responses of the systems of ordinary differential equations, and with smaller control signals, which implies, less energy in the control process.

In this research work, conditions have been obtained in the parameters of the adaptive recurrent neural network, as well as laws of control and laws of neuronal adaptation, which, together, guarantee that the tracking error of trajectories between the non-linear system and the reference signal converges asymptotically to zero, so that trajectory tracking is develops with satisfaction.

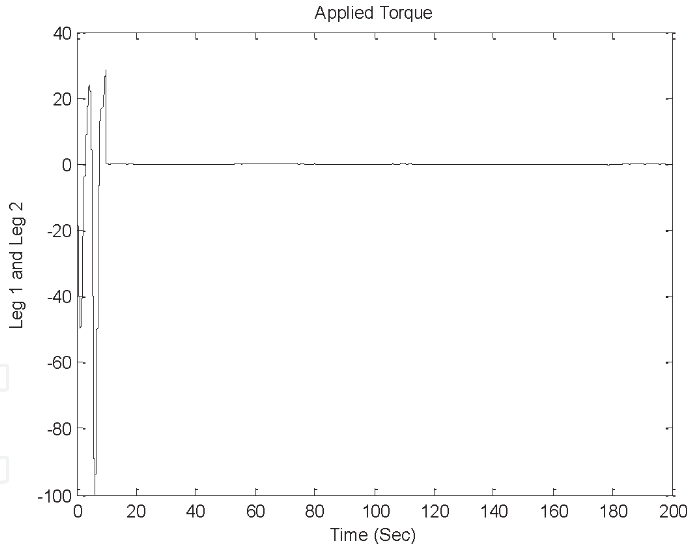

Figure 10.
Torque (Nm) applied to Leg 1 and Leg 2 of link 2.

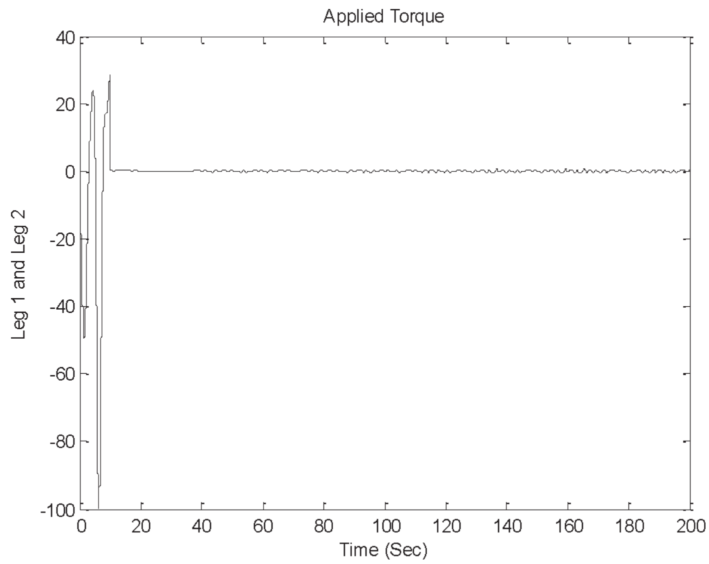

Figure 11.
Torque (Nm) applied to Leg 1 and Leg 2 of link 1.

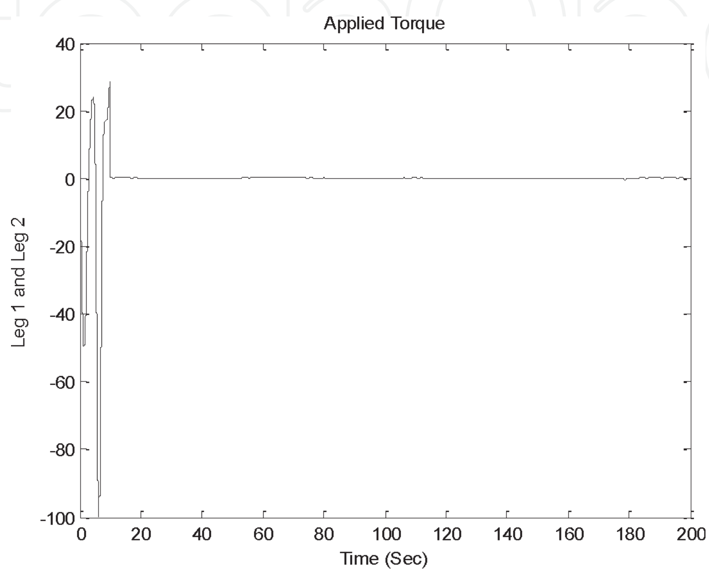

Figure 12.
Torque (Nm) applied to Leg and Leg 2 of link 2.

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Conflict of interest

The first author of the reference manuscript, in his name and that of all authors, declares that there is no potential conflict of interest related to the article.

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