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Optical Mode Properties of 2-D Deformed Microcavities

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1. Introduction

Optical microcavities have a wide range of applications and are used for fundamental studies, such as strong-coupling cavity quantum electrodynamics, enhancement and suppression of spontaneous emission, novel light sources, and dynamic filters in optical communication (Vahala, 2003). As a single-mode, ultralow threshold lasers, dielectric microdisks and microspheres have been studied, since they can support very high-Q whispering-gallery modes (WGMs) (McCall et al., 1992). Basically, strong light confinement of the WGMs in the dielectric microcavities is given by total internal reflection and only small evanescent leakage due to boundary curvature is a possible way of light loss. In a circular microcavity, the WGMs are characterized by two good mode/quantum number $(m,l)$, $m$ is the angular momentum quantum number and $l$ the radial quantum number. Light emission of WGMs is isotropic due to the rotational symmetry of the circle cavity, which is undesired when considered as a light source for device applications.

In their seminal paper (Nöckel & Stone, 1997), Nöckel and Stone have pointed out that a deformation of the cavity leads to partially chaotic ray dynamics and highly anisotropic emission. Optical modes in deformed microcavities do not have good mode/quantum numbers due to absence of rotational symmetry or non-integrability of system. It is also known that the deformed microcavity has rich physics related to quantum chaos, such as chaos-assisted tunnelling, dynamical localization. Only drawback of deforming microcavity would be degradation of Q-factor, but for a slight deformation the degree of degradation is not severe due to the existence of the Kolmogorov-Arnold-Moser (KAM) invariant tori/curves preventing ray diffusion toward the critical angle of total internal reflection from which rays can escape from the microcavity. Some of subsequent works have focused on search for optimal cavity shape supporting optical modes with good emission directionality such as unidirectional emission and directional emission with narrower divergence (Chern et al., 2003; Shang et al., 2008). Other works on microcavities have treated fundamental quantum chaotic features such as scarring phenomena, chaos-assisted tunnelling etc. (Lee, S.-B. et al., 2002; Podolskiy & Narimanov, 2005)

In principle, any microcavity with broken rotation symmetry has directional emission. For a very slight deformation, the emission is the result of tunnelling process, and the emission comes out tangentially at the boundary points with the highest curvature. When the deformation increases enough, the emission directionality is well explained by an ensemble...
of chaotic ray trajectories. Since a ray trajectory in chaotic dynamics diffuses following unstable manifolds, the structure of unstable manifolds near the critical line for total internal reflection is very important to understand emission directionality of optical modes. Based on this ray dynamical analysis, one can design the shape of microcavity for good emission directionality. Various shapes have been proposed and their optical modes are characterized in terms of the directionality and Q-factor.

On the other hand, the optical microcavity is an analogue of open quantum system. In a 2-D dielectric microcavity, the Maxwell equation governing electromagnetic waves can be reduced to the Helmholtz equation. And the Helmholtz equation is nothing but the Schrödinger equation for a particle moving under a constant potential. Therefore, a ray trajectory and electromagnetic field distributions in the 2-D dielectric microcavity correspond to the particle path and wave functions in billiard problems. In quantum chaos study, billiards have been a paradigm since the classical dynamics varies from regular to fully chaotic depending on the boundary shape and can be easily analyzed compared to other chaotic systems. Therefore, one can study, in the dielectric microcavities, various quantum chaotic features which have been important issues in billiards such as dynamical localization, scarring phenomena etc.

Another important aspect of the dielectric microcavity is its non-Hermitian features. The dielectric microcavities are open systems so that the field inside the microcavity decays with time, therefore, eigen-frequencies of each optical modes have negative imaginary part representing decay rate or spectral linewidth. A non-Hermitian system has complex eigenvalues in general. Therefore, the microcavity is a non-Hermitian system and possesses general non-Hermitian features. The degeneracy point in a non-Hermitian quantum system is called exceptional point (EP) at which two quantum states coalesce, i.e., only one eigenvalue and one eigenstate exist there. In mathematically, the EP is a square root branch point so that the energy surface near an EP has non-trivial topology and geometrical phase. These non-Hermitian features have been treated in dielectric microcavities (Lee, S.-Y. et al., 2008b; Schomerus, 2009).

In Sec. 2 and Sec. 3, several tools for ray and wave analysis on optical modes, such as the steady probability distribution, boundary element method, and Husimi functions, are introduced. Based on the methods, directional emissions from microcavities with various shapes are explained in Sec. 4 and the influence of openness on scarred and quasiscarred optical modes is discussed in Sec. 5. In Sec. 6 the mode-mode interaction along a deformation change and the exceptional point are reviewed. Finally, a brief summary will be given.

2. Ray dynamical analysis

To understand optical mode properties in deformed microcavities, such as internal field intensity pattern and emission direction, it is important to figure out underlying classical ray dynamics. There is a very useful tool to see if ray dynamics is regular or chaotic, it is the Poincaré surface of section (PSOS) that has been widely used in nonlinear dynamics community (Reichl, 1992). This PSOS can be extended to incorporate system's openness for describing open chaotic systems. A useful distribution in phase space made by the incorporation for fully chaotic case is the steady probability distribution (SPD) revealing long-lived ray trajectories in deformed microcavities.
2.1 PSOS and chaotic transition

Tracing a ray trajectory in a microcavity gives, in general, a useful information for optical mode pattern, e.g., consider a typical Fabry-Perot resonator or a circular microcavity where optical modes are easily matched to underlying ray trajectories. However, when the cavity is deformed, some trajectories are not simple, and they look too complicated to extract some useful information. The most effective way to see whether ray dynamics is regular or chaotic is to depict the Poincaré surface of section (PSOS). As an example, consider a particle trajectory in a 2-D billiard as shown in Fig. 1 (a) and (b). Complete description of the particle movement needs 4 dimensional phase space, e.g., its position \((x,y)\) and momentum \((p_x,p_y)\), but it is very difficult to imagine a trajectory in 4-D space for us living in 3-D spatial space. So, a useful method is to examine only sectional trace. Imagine a 2-D phase space and mark the intersecting points of the trajectory with a 2-D surface. In the billiard problem, the boundary length coordinate \((s)\) of a bounce position and the tangential component \((p = \sin \theta)\), \(\theta\) is the incident angle) is widely used, and they are canonical conjugate coordinates. Then, the trajectory can be represented by a set of points in phase space \((s, p)\) as shown in Fig. 1 (c).

In a circular cavity, a ray trajectory would give a straight line in phase space because the incident angle of the ray is invariant, so \(p\) is constant. This trajectory can be matched to a WGM. The WGM show a intensity localization similar to localization of the ray trajectory (see Fig. 2 (a)) with incident angle of \(\theta = \arcsin (m/ \text{Re}(k_{(m,l)}))\), \(k_{(m,l)}\) is the wavenumber of WGM\((m,l)\). As the microcavity is deformed, the rotational symmetry is broken so that the incident angle or angular momentum is not invariant any longer. Accordingly, the PSOS shows some wavy invariant lines and nonlinear resonances. The central points of nonlinear resonances corresponds to a stable periodic orbits and there are unstable periodic orbits between them, as shown in Fig. 2 (b). As the deformation increases more, the chaotic region is getting larger from the vicinity of the unstable periodic orbits. Even in this case, there exists the invariant curves dividing two chaotic regions. The trajectory on an invariant curve cannot go somewhere, i.e., lives forever on the invariant curve. This is important in ray
transport because rays in the chaotic region above this invariant curve cannot diffuse into below the curve. This invariant curve thus plays a role of some barrier in ray transport. With more stronger deformation, there are some stable islands and chaotic sea, see Fig. 2 (c). A ray trajectory lying in the islands cannot leave the islands and, on the other hand, a ray motion in chaotic sea shows a fast exponential diffusion in general over the chaotic sea. However the ray existing just outside the islands does not show such fast diffusion, instead, show some power-law diffusion due to the existence of complex small island chains (the red trajectory in Fig. 2 (c)). This kind of phase-space change is typical in non-linear system and is known as KAM scenario.

Fig. 2. Change of the PSOS in a chaotic transition. The billiard is quadrupole-deformed, \( r = r_0 (1 + \varepsilon \cos 2\phi) \). (a) \( \varepsilon = 0.00 \), the circular billiard showing regular dynamics. (b) \( \varepsilon = 0.03 \), almost regular with some nonlinear resonances. (c) \( \varepsilon = 0.09 \), some islands in chaotic sea represented by random dots. (d) \( \varepsilon = 0.18 \), almost chaotic with the robust islands for bouncing-ball trajectories. The blue, red, and green points in the PSOSs correspond to the trajectories shown in deformed billiards in the same color.

2.2 Steady Probability Distribution (SPD)
The PSOS for a fully chaotic system has no specific structure and only random chaotic sea exists in phase space. This implies that the PSOS itself does not give some useful information about optical modes of dielectric microcavities. However, if system’s openness is incorporated in the PSOS, one can find some specific structure in the PSOS, revealing both chaotic ray dynamics and openness character.
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Fig. 3. (a) The incident ray splits into reflected and transmitted rays at the dielectric interface when the incident angle $\theta$ is less than the critical angle $\theta_c$ (red arrows), and the incident ray would be totally reflected when $\theta$ is greater than $\theta_c$. (b) The transmission probability $T(p)$ for $n=1.5, 2, 3$. (c) The open region in phase space where the ray have non-zero probability to escape. (d) The exponential behaviour of the escape time distribution.

As well known in optics, a plane wave incident to an interface between dielectric media of refractive index $n$ and air ($n=1$) would split into reflecting wave and refracting (transmitting) wave. The reflection probability derived from Fresnel equations is given as

$$R = \left(\frac{n \cos \theta_i - \cos \theta_t}{n \cos \theta_i + \cos \theta_t}\right)^2$$

where $\theta_i$ and $\theta_t$ are the incident and refractive angles, respectively, and they are related by Snell’s law, $n \sin \theta_i = \sin \theta_t$. The transmission probability is then $T = 1 - R$. As illustrated in Fig. 3 (a), if the incident angle is greater than the critical angle, $\theta = \arcsin(1/n)$, the transmission is zero, i.e., total internal reflection takes place. The transmission with an incident angle less than $\theta$ depends on the refractive index $n$ and incident angle $\theta$. This dependence is shown in Fig. 3 (b). The transmission becomes maximum at the normal incident case, $\theta = 0$, and the small opening limit is given by $n \to \infty$. In phase space $(s, p)$, the non-zero transmission region is shown as the shaded in Fig. 3 (c).

Imagine a ray trajectory in a deformed microcavity with chaotic ray dynamics. If the incident angle of the ray at a bounce is greater than the critical angle, the ray would be totally reflected. In a chaotic microcavity the incident angle is not invariant, so it becomes soon less than the critical angle. Then, due to the above splitting at interface, part of ray would escape with the transmission probability $T$ and with the reflection probability $R$ the ray would be reflected from the boundary. This openness can be characterized by the exponential behaviour of escape time distribution for a fully chaotic microcavity, as shown.
for a stadium-shaped microcavity in Fig. 3 (d). This exponential behaviour implies that there are some ray distribution in phase space which are statistically invariant with time (Lee, S.-Y. et al., 2004). This distribution is called the steady probability distribution (SPD). The SPD contains both ray dynamical and open properties of the given chaotic microcavity, and shows the distribution of long-lived ray trajectories in phase space. It is emphasized that the structure of the SPD follows essentially unstable-manifold structure since all ray trajectories diffuse along unstable manifolds. In Fig. 4, three SPDs are shown for a stadium-shaped (a), a rounded triangular-shaped (b), and a spiral-shaped microcavities. In the spiral case, it is clear that only clockwise circulating ray can survive long due to no bouncing from the notch.

![SPD images](image)

**Fig. 4.** The steady probability distributions (SPD’s) of stadium-shaped (a), rounded triangular-shaped (b), and spiral-shaped (c) dielectric microcavities. The dashed lines denote the critical lines given by $1/n$.

### 3. Analysis of optical modes

In microcavity experiments, optical mode properties can be characterized by measuring spectra and far field pattern. However, it is very difficult to know some internal field distribution and the role of internal ray dynamics from the both measurements. This gap can be filled by numerical simulation for internal field distributions of optical modes. A popular method for the mode calculation is the boundary element method (BEM), first introduced by Wiersig to microcavity (Wiersig, 2003). The calculated optical modes can be transformed to some functions in phase space for comparing to the ray dynamics (PSOS or SPD). One of transformed functions is Husimi function which gives more intuitive explanation than another phase space representation, Wigner function.

#### 3.1 Boundary Element Method (BEM)

When a microcavity is slightly deformed, a perturbation method can be used to get optical mode frequencies and internal field distribution (Dubertrand et al., 2008). However, this is not effective for strongly deformed microcavities due to the need of enormous number of basis functions. The boundary element method (BEM) (Wiersig, 2003) is very effective for this strongly deformed case.
Maxwell equations governing electromagnetic waves in a 2-D microcavity can be reduced to the Helmholtz equation

\[
\left( \nabla^2 + n(\mathbf{r})^2 k^2 \right) \psi(\mathbf{r}) = 0.
\] (2)

The main idea of the BEM is to discretize a boundary integral equation derived from the Helmholtz equation. To obtain the boundary integral equation, the Green function, the solution of Helmholtz equation with a delta function source, and Green’s identity are used. The Green function is given by zeroth-order Hankel function of the first kind

\[
G(\mathbf{r}, \mathbf{r}'; nk) = \frac{i}{4} H_0^{(1)}(nk|\mathbf{r} - \mathbf{r}'|)
\] (3)

The resulting boundary integral equation can be written as

\[
c(\mathbf{r}) \psi(\mathbf{r}) = \oint_{\Gamma} \left[ \partial_n G(\mathbf{r}, \mathbf{s}; nk) \psi(\mathbf{s}) - G(\mathbf{r}, \mathbf{s}; nk) \partial_n \psi(\mathbf{s}) \right] \, ds
\] (4)

where \( c = 1 \) for \( \mathbf{r} \) inside the boundary \( \Gamma \), \( \frac{1}{2} \) for \( \mathbf{r} \) on the boundary, and zero otherwise, and \( \partial_n \) means the outward normal derivative at the boundary. This integral equation means that if one knows the boundary values of wave function \( \psi(\mathbf{s}) \) and its normal derivative \( \partial_n \psi(\mathbf{s}) \), one can then calculate the value of wave function \( \psi(\mathbf{r}) \) at any point inside the boundary. Note that the Green function inside the microcavity is different a little from the outside Green function due to the different refractive index \( n \).

The unknowns to be calculated are the boundary values of wavefunction \( \psi(\mathbf{s}) \) and its normal derivative \( \partial_n \psi(\mathbf{s}) \). By approaching \( \mathbf{r} \) to the boundary and discretizing whole boundary into \( m \) elements, one can obtain an equation containing \( 2m \) unknowns, \( \psi(\mathbf{s}_i) \) and \( \partial_n \psi(\mathbf{s}_i) \) \( (i = 1, \ldots, m) \). From Eq. (4) for inside microcavity, one can make \( m \) equations by pointing \( \mathbf{r} \) onto \( m \) boundary elements, and the other \( m \) equations can be obtained by considering outside microcavity where \( n = 1 \) so different argument of the Green function has to be used. Therefore, the discretization leads to a matrix equation and a non-trivial solution exists only when the determinant of the matrix is zero, which is the condition determining the complex wave number \( k \) of optical modes. Note that the Green function and its normal derivative are singular at the origin, so a careful treatment is needed to evaluate the diagonal elements of the matrix. The singularity of \( H_0^{(1)}(z) \) in \( \partial_n G \) can be compensated by a geometrical factor, and the corresponding diagonal terms can be expressed in terms of the curvature of the boundary. Also, the singularity of \( H_0^{(1)}(z) \) in \( G \) can be integrated by using the asymptotic expression.

Throughout this chapter we focus on TM (transverse magnetic field) polarization where both the wave function \( \psi(\mathbf{r}) \) and its normal derivative \( \partial_n \psi(\mathbf{r}) \) are continuous across the boundary, and the wavefunction \( \psi(\mathbf{r}) \) corresponds to the \( z \) component of the electric field when the 2-D microcavity lies on the \( x-y \) plane. In the case of TE (transverse electric field) polarization, the wavefunction \( \psi(\mathbf{r}) \) represents the \( z \) component of magnetic field, and the wavefunction \( \psi(\mathbf{r}) \) is continuous across the boundary, but its normal derivative is not, instead \( n^{-1} \partial_n \psi(\mathbf{r}) \) is continuous.
Since the boundary curvature appears in the diagonal elements in the matrix, non-analytic boundary points such as an angular corner, where the curvature is not defined, make a trouble in BEM. This can be overcome by rounding the angular corner by a part of circle. Here the radius of circle should be much less than the wave length of the optical mode, because the wave needs not to know the rounding process.

3.2 Husimi functions of optical modes

Sometimes phase-space representation of an optical mode is more useful than internal intensity distribution. If one wants to compare the optical mode to the classical PSOS or SPD in phase space, the transform from wave function in real space to some distribution in phase space is needed. The most popular one is the Husimi function. The Husimi function is determined by the overlap of the wave function with a coherent state of a minimum uncertainty. It constitutes a quasi-probability in phase space, which acquires an intuitive quasi-classical character in the semiclassical limit of short wavelength.

Recently, Hentschel et al. (Hentschel et al., 2003) have derived four Husimi functions applicable to the dielectric microcavity, two are for incident and reflected waves inside the microcavity and the other two are for those outside the microcavity. The incident and the reflective Husimi functions of inside waves are written as

\[
H^{inc,refl}(s, p) = \frac{k}{2\pi} \left| F h(s, p) + (-)^n \frac{in}{kF} h'(s, p) \right|^2
\]

(5)

where \( F = \sqrt{n(1 - p^2)} \) and the components of the Husimi function are given as

\[
h(s, p) = \int ds' \psi(s', s, p) \xi(s', s, p),
\]

(6)

\[
h'(s, p) = \int ds' \partial_n \psi(s', s, p) \xi(s', s, p).
\]

(7)

where the minimal-uncertainty wave packet is given by

\[
\xi(s', s, p) = \sum_j \frac{1}{\sqrt{\sigma \sqrt{\pi}}} \exp \left\{ -\frac{1}{2\sigma^2} (s' - s + Lj)^2 - ip(s' + Lj) \right\}
\]

(8)

where \( k \) and \( L \) are the wave number inside the cavity and the total length of the boundary, respectively. This packet corresponds to a coherent state centred at \((s, p)\). The aspect ratio factor is taken as \( \sigma = \sqrt{2}/k \).

From the BEM, one can obtain the boundary functions \( \psi(s), \partial_n \psi(s) \) of an optical mode \( \psi(j) \), and these can be used for calculating the Husimi function from the above equations. This Husimi function shows details of internal waves of the optical mode such as bouncing positions \((s')\) and incident angles \((p')\), \((s', p')\) are the phase-space positions of localization pattern of the Husimi function.

4. Emission properties of optical modes

From the viewpoint of applications, directionality of emission has been an important issue many authors have focused on. Various shapes of microcavity have been proposed for
optical modes with good directionality. The emission directionality can be understood by ray dynamics and Husimi functions mentioned above.

4.1 Directional emission

The PSOS of a circular microcavity shows simple invariant lines in phase space as shown in Fig. 2 (a), and its optical modes are WGMs with specific mode number \((m,l)\). The WGM\((m,l)\) has \(2m\) intensity spots along the perimeter of the circle, and \(l\) intensity spots along radial direction, so one can identify \((m,l)\) from the internal intensity distribution of the WGM. The Husimi function of the WGM\((m,l)\) reveals wave components with a definite incident angle, \(p=\pm \sin \theta = \pm m/ \Re \left[nk_{m,l}\right]\). If the incident angle is greater than the critical angle, the internal wave escapes through tunneling process. The WGMs showing the tunneling emission have high quality factor due to the small transmission probability. The tunneling interpretation comes from the fact that the radial equation of 2-D Helmholtz equation becomes 1-D potential barrier problem. Due to the rotational symmetry the tunneling rate is invariant about the polar angle, therefore the emission is isotropic. Speaking in detail, the emission is tangential because the incident wave with critical angle would be transmitted tangentially, and far field emission pattern is formed by interference of two tangential emissions tunnelled from oppositely circulating waves.

Fig. 5. (a) A WGM in a slightly deformed microcavity, \(\varepsilon=0.03\), and the arrows indicate the emission directions that are not clear in the intensity plot due to its negligible intensity compared to the internal one. (b) The far field pattern of emission. This is consistent with the arrows in (a). (c) The Husimi function of the WGM and the PSOS. One can expect that internal wave would tunnel out at the highest curvature points due to the shortest distance to the critical line (the orange line).

When the microcavity is slightly deformed, the tunneling emission is no longer isotropic. As shown in Fig. 2 (b) and Fig. 5 (c), the straight invariant lines become curved lines.
showing minimum at the boundary points with the highest curvature. This implies that the tunnelling rate would be maximum at the points as illustrated by arrows in Fig. 5 (c). Then, tunnelling emissions would appear tangential direction at the points with the highest curvature like the optical mode in Fig. 5 (a). This is confirmed by the far field pattern in Fig. 5 (b).

If the deformation increases a little more, the emission direction would have mode dependence because the optical mode can be localized on stable or unstable periodic orbit due to the perturbation originating external coupling (Unterhinninghofen et al., 2008). This mode dependency disappears and chaotic transport, including turnstile transport (Shim et al., 2008), become important when the deformation increases enough. The corresponding ray dynamics is chaotic and the rays diffuse along unstable manifolds, so the structure of unstable manifolds determines the emission directions, which will be discussed in detail when scarred optical modes are discussed.

4.2 Unidirectional emission

Optimal directionality would be unidirectional emission like usual Fabry-Perot laser with two mirrors, one has a perfect reflection and the other does not. In a microcavity, light confinement is not achieved by mirrors, but by the total internal reflection at the cavity boundary. Therefore, one cannot easily expect if the given shape of microcavity can support the uni-directional optical modes or not.

The unidirectional emission is first reported in a spiral-shaped microcavity by Chern et al. (Chern et al., 2003). The key of unidirectionality in the spiral-shaped microcavity is the existence of notch (see Fig. 6 (a)), because every periodic orbits have at least one bounce from the notch (Lee, S.-Y. et al., 2004). Since there are many chances of bounce from the notch, the spiral-shaped microcavity seems to be ideal to support optical modes showing unidirectional emission and the unidirectional lasing emission from the notch is confirmed by other experimental groups (Ben-Messaoud & Zyss, 2005; Tulek & Vardeny, 2007; Kim et al., 2008). However, it is interesting to note that the optical modes showing unidirectional emission have not been reported in BEM calculation.

A rounded triangle-shaped microcavity can support unidirectional optical modes (Kurdoglyan et al., 2004). Some clue of unidirectional emission can be found in the SPD shown in Fig. 4 (b), where the intensity of SPD corresponding to normal incident rays at right vertical boundary is rather strong. The unidirectional optical mode shows WGM-like pattern at the left circular part of boundary, which enables waves to turn their direction without emission to the left direction as illustrated in Fig. 7 (b). However, the unidirectional optical modes would not be high-Q mode, since the incident angles of waves at right vertical boundary are small, implying large transmission probability. Therefore, the unidirectional optical modes are rather leaky.

Some high-Q unidirectional optical modes has been found in an annular microcavity (see Fig. 7 (c)) (Wiersig & Hentschel, 2006). The annular microcavity can support WGM-like high-Q modes and leaky unidirectional modes. These two modes can be superposed under an avoided resonance crossing, and as a result of the superposition the high-Q optical mode has unidirectional emission without severe degradation of quality factor.

Robust high-Q optical mode with unidirectional emission has been found in Limaçon microcavity (Wiersig & Hentschel, 2008). As shown in Fig. 6 (d), initially glancing rays with large incident angle eventually escape at two boundary points with same direction. This can
be well explained by the unstable-manifold structure or SPD near the critical line. At the critical lines appreciable intensity of SPD appears only the two positions in phase space corresponding to the two escaping points.

![Fig. 6. Various shapes of dielectric microcavities supporting unidirectional optical modes.](image)

(a) Spiral-shaped microcavity. (b) Rounded triangle-shaped microcavity. (c) Annular microcavity in which high-Q WGM is coupled to a leaky unidirectional mode without severe degradation of Q factor. (d) Limaçon microcavity, \( r = r_0 (1 + \eta \cos \phi) \), \( \eta = 0.43 \). (e) Coupled asymmetric microdisks.

Recently, it is reported that a coupled asymmetric microdisks (see Fig. 6 (e)) can support unidirectional emission (Ryu et al., 2009). When the left disk in Fig. 6 (e) satisfies the resonance condition and the right disk does not, a WGM appears only on the left disk. In this case, the right disk plays a role of a defect on boundary, so through the contact point waves tunnel out and the circular boundary of right disk makes the tunnelled waves be collimated, resulting in a unidirectional emission.

4.3 Narrow divergent emission

Divergence of emission beam is another important property of optical modes. Narrow divergence of directional emission has been reported in a peanut-shaped ring cavity, made by two optical fibers coated by organic/inorganic hybrid materials doped by rhodamine B (Shang et al., 2008), the refractive indices of optical fiber and the coating medium are 1.46 and 1.52, respectively. The partial ray trajectories escaped are shown in Fig. 7 (a). The bounced rays at the circular part have incident angles greater than the critical angle so that there is no refractive emission from the circular part, and the rays begin to spread and are collimated to make a parallel beam output with rather large width. This is the key feature of the peanut-shaped microcavity for narrow divergence of output beam. Recall the single slit diffraction where the divergence angle is given by \( \theta \approx \lambda / a \), \( \lambda \) is the wavelength and \( a \) the opening size of the slit. The larger is the opening size of single slit, the smaller the divergence angle of diffraction. This can be applicable to the peanut-shaped microcavity where the large width of ray beam corresponds to the opening size of the single slit. The
corresponding optical mode can be found, it is shown in Fig. 7 (b). Four collimated emission beams come out from rather large part of circular boundary. The far field pattern of this mode shows very narrow divergence angle less than 10 degree (Fig. 8 (c)), comparing with the typical divergence angle, about 40 degree (Chern et al., 2003), of spiral-shaped microcavity. The SPD reveals this property in detail as shown in Fig. 8 (d). In the open region (-1/n < p < 1/n), stripe pattern exists and this pattern well overlaps with the red line in Fig 8 (d), indicating rays of parallel emission with far field angle 24 degree. The Husimi function of the optical mode shown Fig. 8 (d) also well overlaps with the red line.

Fig. 7. (a) The ray trajectories forming a collimated broad ray beam. The fish-shaped periodic orbit shows similar bouncing position on the circular part of boundary. (b) The intensity plot of an optical mode showing the narrow divergence. (c) The red line is the farfield pattern of the optical mode shown in (b), which show narrow divergence of less than 10 degree. The blue histogram is the result of SPD. (d) The Husimi function of the optical mode of (b) and SPD (background gray dots). The red line indicates the trajectories with 24 degree farfield angle. The dotted lines are the critical lines for total internal reflection.

5. Scarred and quasiscarred modes

In a stadium-shaped billiard, Heller has found the scarred eigenstate showing enhanced amplitude along a unstable periodic orbit (Heller, 1984). After his work, many authors have studied the scar phenomena in various chaotic systems. The reason of much attention on the scar phenomenon is its counter intuitive aspect. In common sense based on classical chaotic dynamics, showing evenly and randomly distributed PSOS, one would expect quite irregular intensity distribution of eigenfunction. In fact a typical eigenstate has this irregular eigenfunction and this is consistent with the prediction of random matrix theory. Although
Fig. 8. (a) The scarred optical modes associated with diamond and rectangle unstable periodic orbits in a stadium-shaped microcavity (Lee, S.-Y. et al., 2005). (b) Schematic diagrams of slight pattern rotations depending on the propagating way. (c) The Husimi function of the diamond optical mode with SPD in background. (d) The Husimi function of the rectangle optical mode with SPD in background. The red arrows denote the stable and unstable manifolds emanating from the rectangle periodic orbit. The dotted lines are the critical lines.

only small part of eigenfunctions are scarred in closed billiards, in open systems such as dielectric microcavity the scarred modes can be easily found because the openness suppresses the contribution of long trajectories. The scarred optical modes have been observed in dielectric microcavities (Lee, S.-B. et al, 2002; Rex et al., 2002)

5.1 Scarred modes

The scarred optical modes in a chaotic microcavity have different properties not seen in the scarred eigenfunctions of closed billiards (Lee, S.-Y. et al., 2005). First, consider a scarred optical mode with incident angles comparable to the critical angle. Figure 8 (a) shows scarred modes with diamond and rectangle patterns in a stadium-shaped microcavity with the parallel boundary length \(0.2 R\), \(R\) the radius of circular part. The refractive index is taken as \(\sqrt{2}\) so that the critical angle is 45 degree. Note that the incident angles of the diamond and rectangle periodic orbits are about 45 degree. In this case, the emission beams come out from the bouncing positions of the internal periodic orbit pattern.

Although the internal intensity plots of the scarred optical modes looks similar to scarred eigenfunctions of closed billiards, the detail is quite different. From the positions of maximum intensity of Husimi functions for the scarred modes (see Fig. 8 (c),(d)), one can see that the bouncing positions are slightly rotated and the way of rotation depends on the circulating way of internal waves. For example, in the diamond scarred mode the clockwise
circulating wave pattern is slightly rotated with positive angle shift and the counterclockwise one is done with negative angle shift as illustrated in Fig. 8 (b). More interestingly, in the rectangle case, the rotations appear in the opposite way (see Fig. 8 (b)). These intriguing angle shifts are certainly results of openness of the microcavity. At a bounce, partial waves are escaping refractively, and the rest of waves are reflected and then take part in resonance formation, which is known as Fresnel filtering effect (Rex et al., 2002). This is why the scarred patterns do not lie on the exact periodic orbits.

The difference of angle-shift way between diamond and rectangle scarred optical modes can be understood by comparing Husimi functions with SPD. This comparison is shown in Fig. 8 (c) and (d). The black dots indicate the exact positions of diamond and rectangle periodic orbits and the grey dots in the background represent the SPD. One can easily find the maximum intensities of Husimi functions locate on some points near the black dots and the deviations are toward dense part of the SPD. Note that the dense part in SPD corresponds to long-lived ray trajectories, surviving long time in the microcavity. Therefore, it is clear that only long-lived part of waves take part in the resonance formation, as a result, the angle shifts are determined by the structure of SPD near the critical lines.

![Fig. 9.](https://example.com/fig9.png)

Fig. 9. (a) A high-Q scarred optical mode in logarithmic scale. One can see the mismatch between the bouncing positions of internal pattern and emission positions. (b) The Husimi function of the optical mode shown in (a) with SPD in background. The emission positions coincide with the SPD structure near the (dotted) critical lines (Lee, S.-Y. et al., 2005).

Second, consider a high-Q scarred mode (see Fig. 9 (a)) whose Husimi function is localized far above the critical line due to its incident angles much greater than the critical angle. The emission mechanism in this high-Q case is somewhat different from the previous case. The emission points are not associated with the internal pattern, for example, the hexagonal scarred optical mode in Fig. 9 (a) has four emission points, which is clear only in logarithmic scale. This is also understood by the SPD structure reflecting unstable-manifold structure. The waves forming periodic orbit pattern cannot escape at the bouncing positions where the total internal reflections occur. Instead, the waves would diffuse toward the critical lines and eventually escape by refraction. The wave diffusion would follow underlying ray trajectories moving along the unstable manifolds. This mechanism is confirmed by Husimi function of the high-Q scarred mode. The structure of Husimi function, in logarithmic scale, below the critical line overlaps well on the SPD as shown in Fig. 9 (b).

This result is important because this mechanism can be applied to other high-Q modes in chaotic microcavity. The importance of unstable-manifold structure near the critical line has
been stressed by many authors (Schwefel et al., 2004; Altmann, 2009) and experimentally confirmed in liquid-jet microcavity (Lee, S.-B. et al., 2007).

5.3 Quasiscarred modes
Although the openness effect on scarred optical modes has been discussed in the previous subsection, more dramatic effect of openness has been found in the spiral-shaped microcavity (Lee, S.-Y. et al., 2004). As shown in Fig. 10 (a) and (b), scar-like optical modes exist in the microcavity, triangle- and star-shaped localizations are shown in cases of $n=2$ and $n=3$, respectively. These modes are named quasiscarred modes because, unlike scarred modes, there is no underlying unstable periodic orbit. It is then obvious that, even though there is no periodic ray orbit, the openness enables waves to make periodic propagation with constructive interference.

Fig. 10. (a), (b) The quasiscarred optical modes in a spiral-shaped dielectric microcavity (Lee, S.-Y. et al., 2004). The refractive index is 2 in (a) and 3 in (b). (c) A trajectory without bouncing from the notch. The distance between trajectory segment and the origin is getting larger for counter-clockwise circulating ray, $d_1<d_2<d_3$.

Non-existence of triangle- and star-shaped periodic orbits can be easily understood from the ray trajectory shown in Fig. 10 (c). For the counter-clockwise circulating ray without bouncing from the notch, the distance from the origin always increases, so it is impossible to make a periodic orbit without bouncing from the notch.

It is emphasized that many of optical modes at $n=2$ and $n=3$ are triangle- and star-shape quasiscarred, respectively (Lee, S.-Y. et al., 2008a). This is a direct evidence of crucial role of openness on the formation of quasiscarred optical modes, noting that the incident angle of internal waves is about the critical angle given by $\theta_c = \arcsin(1/n)$ (Lee, J. et al., 2008). It is noted that Goos-Hänchen shift, with which reflected wave beam comes out away from the incident beam position, and the Fresnel filtering effects become maximum when the incident angle of wave beam is about the critical angle (Lee, S.-Y. et al., 2005). These unique effects of dielectric microcavities should be important in the quasiscarred-mode formation, i.e., formation of constructive periodic wave propagation without underlying ray periodic orbit. The lasing of quasiscarred modes has been numerically studied (Kwon et al., 2006) using the Schrödinger-Bloch model (Harayama et al., 2003), and recently experimentally confirmed in spiral-shaped microcavity (Kim et al., 2009).

Although the quasiscarred optical modes have been found only in the spiral-shaped microcavity, there is no constraint preventing their appearance in other deformed microcavities.
with discrete symmetry. To find quasiscarred modes in general deformed microcavities one needs more quantitative criterion for the formation of quasiscarred modes.

6. Avoided resonance crossing and exceptional point

Eigenvalues of a closed quantum system show some variation under change of system’s parameter such as deformation of a billiard. An eigenvalue may increase or decrease depending on its mode property. When two eigenvalues encounter each other under a parameter change, they show level repulsion in general unless the two modes belong to different symmetry classes. This repulsive behaviour is called avoided level crossing (ALC) (Takami, 1992). This can be understood as non-zero interaction between two modes, i.e., non-zero off-diagonal terms in a two by two Hamiltonian matrix. Upon this ALC, their internal intensity patterns are exchanged and the gap of ALC is proportional to the interaction strength between two modes. Similar behaviour takes place in the microcavity system, an open system.

In dielectric microcavities, the eigenvalues are complex numbers, i.e., complex wave numbers or complex frequencies with negative imaginary part representing mode decay. Here, the repulsion behaviour called avoided resonance crossing (ARC) has similar property with ALC of closed systems. Upon an ARC in real part of wavenumbers the mode patterns are exchanged, implying the exchange of imaginary part of wavenumbers since the decay rates are characterized by internal mode patterns. At the point of an ARC, the internal mode patterns are given by superposition of interacting two modes like the ALC case. This feature of ARC explains the formation of scar-like localized mode pattern (Wiersig, 2006) and high-Q unidirectional mode in an annular microcavity, a hybrid mode of a high-Q WGM and a low-Q unidirectional modes at ARC (Wiersig & Hentschel, 2006).

The interaction source of ARC might be the classical nonlinearity such as chaotic ray trajectories like ALC in closed billiards. Additionally, in microcavities openness-mediated interaction, called as external coupling, is also possible to be the source of ARC. This external coupling can be observable when the internal coupling caused by classical nonlinear dynamics is almost negligible, like ellipse or rectangle cases, or when the cavity has angular corners at which strong diffraction occurs (Wiersig, 2006). In general chaotic microcavities with smooth boundary, it is thus difficult to find the external-coupling-dominant ARC.

Since the dielectric microcavity is an open system, one can find interesting non-Hermitian features in the eigenvalues and eigenfunctions. The most interesting one of the non-Hermitian properties is the existence of degeneracy point, called exceptional point (EP), where both eigenvalues and eigenfunctions of two interacting modes coalesce into one eigenvalue and one eigenfunction. The exceptional point is a singular point, a square root branch point, so the surface of real part of eigenvalues near the EP shows a non-trivial topology as shown in Fig. 11 (a). At the EP, the transition of ARC to resonance crossing (RC) takes place, and this results in interesting mode interchange during a cyclic parameter change encircling the EP in 2-D parameter space (see Fig. 11 (a)). Therefore, in order to get back to initial modes, one needs double cyclic parameter changes. This topology is equivalent with that of Möbius strip as shown in Fig. 11 (b) where two circulations are need to get back to the starting point. It is also known that the geometrical phase is \( \pi \) for the double cyclic parameter changes, as a result, four cyclic changes are actually required to recover the original mode at starting point (Mailybaev et al., 2005).
Fig. 11. (a) The topology of energy surface near an EP. (b) Möbius strip made by the difference of eigenvalues of interacting two modes.

In a stadium-shaped microcavity, the exceptional point has been found numerically near an ARC (Lee, S.-Y. et al., 2008b) in a parameter space spanned by a deformation and refractive index. Recently, ARC and RC in microcavity have been experimentally observed in a deformed microcavity made by liquid jet, where some discrete internal parameter, instead of the refractive index, is used and it is expected that the EP would be identified by observing the transition ARC to RC (Lee, S.-B. et al., 2009).

7. Summary

In this chapter, properties of optical modes in deformed dielectric microcavities have been reviewed. Although the ray dynamics in deformed cavity is complicated, through the PSOS one can easily identify the complexity of ray dynamics. The modified PSOS incorporating openness character of dielectric cavity can be characterized by the steady probability distribution (SPD) for fully chaotic case. This distribution reveals combination of unstable-manifold structure and openness character, and it plays a role of classical skeleton for understanding Husimi functions of optical modes supported by deformed microcavities. The directional emission from a strongly deformed microcavity can be well explained by the SPD. Influence of openness changes scarred optical modes to have opposite angular shift of scarred patterns depending on the way of wave circulation, and make it possible to form quasiscarred optical modes without underlying unstable periodic orbit. And the dielectric microcavity can be regarded as an example of non-Hermitian system with complex eigenvalues. The exceptional point (EP), degeneracy point in non-Hermitian systems, can be found in deformed microcavities.

Although much attention has been paid on the microcavity in the past decades and new understandings on optical modes have been achieved, there remain still many challenges. Multi-dimensional tunnelling appears in slightly deformed microcavities. However, there is no quantitative semiclassical theory to treat this tunnelling. Only a perturbation theory, for near integrable microcavity, explains tunnelling emissions (Creagh, 2007). Non-Hermitian properties of optical modes are also important due to their generality applicable to other open quantum systems. The Petermann excess noise factor, a measure of non-orthogonality
of eigenstates, is known to diverge at the EP, but its physical implications on the spontaneous emission rate and laser line width are not obvious so far. (Cheng, 2006; Lee, S.-Y. et al, 2008b; Schomerus, 2009)

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9. References


The title of this book, Advances in Optical and Photonic Devices, encompasses a broad range of theory and applications which are of interest for diverse classes of optical and photonic devices. Unquestionably, recent successful achievements in modern optical communications and multifunctional systems have been accomplished based on composing “building blocks” of a variety of optical and photonic devices. Thus, the grasp of current trends and needs in device technology would be useful for further development of such a range of relative applications. The book is going to be a collection of contemporary researches and developments of various devices and structures in the area of optics and photonics. It is composed of 17 excellent chapters covering fundamental theory, physical operation mechanisms, fabrication and measurement techniques, and application examples. Besides, it contains comprehensive reviews of recent trends and advancements in the field. First six chapters are especially focused on diverse aspects of recent developments of lasers and related technologies, while the later chapters deal with various optical and photonic devices including waveguides, filters, oscillators, isolators, photodiodes, photomultipliers, microcavities, and so on. Although the book is a collected edition of specific technological issues, I strongly believe that the readers can obtain generous and overall ideas and knowledge of the state-of-the-art technologies in optical and photonic devices. Lastly, special words of thanks should go to all the scientists and engineers who have devoted a great deal of time to writing excellent chapters in this book.

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