We are IntechOpen, the world’s leading publisher of Open Access books
Built by scientists, for scientists

6,600
Open access books available

177,000
International authors and editors

195M
Downloads

154
Countries delivered to

TOP 1%
Our authors are among the most cited scientists

12.2%
Contributors from top 500 universities

WEB OF SCIENCE™
Selection of our books indexed in the Book Citation Index in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com
Chapter

Predicting Sets of Automata: Architecture, Evolution, Examples of Prognosis, and Applications

Sergey Kirillov, Aleksandr Kirillov, Vitalii Iakimkin, Michael Pecht and Yuri Kaganovich

Abstract

This chapter describes the sets of interacting automata constructed on the cascades of wavelet coefficients of input signal. The basic principles of the evolution of automata during the processing of incoming cascades and the vector of processes consisting of segments of cascades of constant length are described. The main principles of constructing the family of automata are determined from the internal symmetry of incoming cascades and the definition of symmetry groups of vector processes and their isotropy groups. The trajectories of states are defined on nontrivial topological spaces, the so-called degeneration spaces of the characteristic functional. The family of evolving automata with tunable communications architecture is designed to predict the state of engineering objects and identify predictors, early predictors, and hidden predictors of failure. This chapter provides examples of the work of predictive automata in various fields of engineering and medicine. It demonstrates the operation of the automaton in spaces with a nontrivial topology of input cascades, algorithms of the predictor search, and estimations. The family of evolving automata with reconstructing architecture of connections is designed to predict the state of engineering objects and medicine and identify predictors, early predictors, and hidden predictors of failure. The architecture and functional properties of automata are determined from the results and main conclusions.

Keywords: preventive monitoring, failure prognosis, remote calculating cluster, optimize drug therapy, Turing machine, maintenance optimization, preventive maintenance, remaining useful life

1. Introduction

This chapter is devoted to a detailed analysis of the structure and properties of predictive automata sets and the analysis of their ability to predict and to achieve accurate time estimates of the predicted events.

The creation of set of predictive automata was preceded by the construction of various models for prognosis of the state of technical devices; basically it was about rotational and reciprocating mechanisms [1]. The same interests of the authors attracted research in the field of prognosis in cardiology [2, 3], where, especially in the last decade, significant progress has been made in understanding the
mechanisms of the emergence of various kinds of arrhythmias and the mechanisms of so-called sudden cardiac death. In the field of technical devices, some progress is observed in the modeling of turbulent combustion of fuels.

As is known, the diagnosis and prognosis of combustion modes, their stability serves as the basis for the development of a causal prognosis, including for the mechanics of various types of engines and turbines, as well as for the analysis of hydrogasdynamic processes that support the combustion modes of fuel. Uniting all these systems is the fact that both the processes of propagation of action potential in the myocardium and the processes of combustion of fuels belong to the so-called reaction-diffusion systems. Regarding cardiological applications, it is also necessary to note the class of tasks for prognosis and management associated with implantable devices represented by the CRT and ICD devices. Here arises a problem of prognosis and developing a strategy for the management of devices in order to prevent such heart events as multiple births of sources of secondary waves in the myocardium, leading to fibrillation of the ventricles of the heart and to sudden cardiac death [4]. This chapter contains examples and comments on the operation of automata in various machines, without avoiding examples from cardiology. Demonstrating systems of automata sets for predicting objects of various natures (mechanisms and objects of biological nature), the authors sought to build automata on general principles that are universal for a wide range of objects. As an introduction to the subject matter, it is necessary to describe the basic conceptual constructions that precede the construction of set of recognizing automata and the description of their properties. We are talking about the substantive part of the hierarchy model [5] in the prognostic tasks. It is implied that the observed signals from various sensors and devices are represented as a continuous or discrete series of their wavelet coefficients [6]. All the observed signals from accelerometers, pressure sensors, and the sensors of the angle of rotation of the shaft, along with them the signals of the electrodes from the human body, have a quasi-periodic nature, i.e., their periods differ by a certain random value, for which when collecting statistics, one can use sequences of a series of signals with close periods. To simplify the presentation in the future, if there are no special reservations, only one class of wavelet coefficients generated by changing the number of the cycle or period will be considered while fixing all other indices of wavelet coefficients. Thus, each indexed cascade with numbers of cycles taken as discrete time corresponds to a cascade of wavelet coefficients with fixed indices of wavelet coefficients [6].

Further consideration of segments of a cascade of fixed length generates a cascade of vectors, where the growth of the cycle number is selected as the direction. The following construction of the task of prognosis is reduced to the representation of the task of wandering a vector along a multidimensional lattice or along its continual analog of a space of dimension $N^*$, where $N^*$ is the length of the selected segment. Further representation of the probability of transition from the initial vector to the final vector in the form of the Feynman integral along trajectories in $L$ steps leads in the continuum limit to the well-known evolution equations of the Fokker-Planck type for the probability density $P(R_0, R_L, L)$ [2, 7]. If the final vector is predetermined, then the problem of determining the number of steps that is necessary to achieve a given vector arises. The solution of such task gives the time to achieve a given vector; in other words, if the vector is specified on the border of the failure, then the remaining useful life is determined. If the solution of the evolution equation is known, for example, its solution in the moments, then further analytical calculation of the RUL is simple.

However, the presented approach to prognosis harbors many underwater reefs. The main difficulties are associated with the variability of properties of the observed signal and, therefore, the properties of the cascades. In terms of
wandering around the lattice or in its continuum analogs, these properties are associated with the presence of certain prohibitions, for example, wandering without self-intersections or taking into account the excluded volume [8] and more realistic taking into account the prohibitions on a part of admissible trajectories connected with the physical boundary of the failure. The listed and not only prohibitions generate a state space of $\mathbb{R}^N_*$ with forbidden states, which makes the state space a complex, multiply connected subset in $\mathbb{R}^N_*$ . In this case, the process under consideration may lose the properties of Markov process; the Chapman-Kolmogorov identity is not satisfied. Ultimately, the evolution equations become integro-differential, and moreover, there is a need to introduce many-particle densities of probability transition [7]. As a result the solution of evolution equations becomes difficult and hardly solvable for the general case.

In addition, most often, in practice, the probabilities of transition from nodes of a multidimensional lattice (elementary transitions) are unknown. The listed difficulties induce to the further development of the model free from the noted problems. For this, together with the main state space of the cascade vector, a symbolic space is constructed. Symbolic space appeals to the frequency representation of the process in the form of a histogram of a cascade vector built on each vector. In this case, elementary transitions are represented by an abelian subgroup of matrices. At the same time, multiplying such a matrix by a column from the frequencies in the representation of the affine space is reduced to adding or subtracting one in two fixed coordinates of the frequency vector. This fact allows us to consider the frequency histograms of the vectors of the cascade vector as the internal states of the Turing machine. Elementary transitions change the internal state of the Turing machine and correspond to the shift of the incoming tape by one. Some analogy with topological Markov chains is also possible, but the transition matrix contains both 1 and $\frac{1}{C_0}$. The representation of the vector column of the frequency histogram in Euclidean space of dimension equal to the number of components or the length of the column allows to represent the process of changing the internal states of the automaton as a walking in $\mathbb{R}^N_*$ space, or taking into account the obvious restriction on the value of the sum of the column components to represent the same process as a walking on a multidimensional simplex $\Sigma^{n-1}/C_0$ of the dimension $n-1$ [2].

The vector column in the process of evolution of the system is described by solving the basic kinetic or balance equations [2].

The constructed sets of automata allow to predict or determine one’s state at future times based on the set of states at previous moments, i.e., on the basis of knowledge of the state of the automaton and transition probabilities. Automata themselves receive this knowledge on the basis of a set of statistics after entering the stationary mode. The next step in the formalization and algorithmization of the prognostic tasks is to describe the set of admissible values of the state vectors. To this end, by analogy with the theory of topological defects of condensed media, a degeneration space $G/H$ and k-multiple loop space $\Omega^k(G/H)$ of degeneration space are introduced. Due to this step, the formalization of the prognosis model is completed, and further in the work, a lot of the automata for the prognosis are constructed.

2. Description of topological dynamics

The immediate goal of this section is to describe an automaton operating in a homogeneous space or space of degeneration. This section completes the construction of the set of interacting automata. To construct the set of automata defined for each cascade of wavelet coefficients of the observed signal, a certain symbolic space
was defined, which allows treating the change in the internal states of the automaton as a wandering point on an n-1-dimensional simplex. In parallel with this, the evolution of the states of the automaton was interpreted as the wandering of a point in the multidimensional homogeneous space or the degeneration space $\mathbb{G}_H$. The construction given in [6] without taking into account the topology of the space of degeneracy leads to evolution equations such as the balance equation or the master equation and, in more general cases, to the basic kinetic equation. The only restriction imposed on the probabilities of transitions from the quasi-stationarity condition was reduced to the fulfillment of the quasi-stationarity conditions, imposed on transition probabilities between histogram columns [2]:

$$\sum_k w_{k,m} = \sum_m w_{m,k}$$  \hspace{1cm} (1)

The transition from the initial state to the final state was determined by the product of elementary transitions, represented as matrices acting in affine space, and was reduced to subtracting one from one component of the vector to adding one to another component. Some analogy with topological Markov chains was noted.

This chapter is a direct continuation of the presentation of the models and algorithms given in the works [2, 3]. The basic model is reduced to the construction of two main spaces. The first of them is the state space, and it is defined as a subset in $R^N$ of all admissible values of the wavelet segments of the coefficients of the observed signal at the fixing all indices of wavelet coefficients $k_{Hist}W_{N,i,j}$, except the quasi-period index $N$.

Necessary definitions:

Cascade definition:

$$\{ k_{Hist}W_{N,i,j}^N : N = 1, 2, 3, \ldots, \infty \}$$  \hspace{1cm} (2)

Defining segments and defining states:

$$\{ k_{Hist}W_{N,i,j}^N : N = 1, 2, 3, \ldots, N^* \} \in R^{N^*}$$  \hspace{1cm} (3)

Definition of trajectories:

$$\{ R_k \} = \{ k_{Hist}W_{N,i,j}^N : kN^* \leq N \leq (k + 1)N^*, \forall k, R_k \in R^{N^*} \}$$  \hspace{1cm} (4)

On the cascades thus defined, segments of fixed length are defined. The segment length can be different, but the main requirement for the length is that at this length, the cascade goes into a quasi-stationary mode. That is, the following assumption is implicitly admitted: it follows from the construction that each cascade is indexed by the numbers of the columns of the histograms of quasi-periods, that is, each cascade corresponds to the numbers of the periods of almost constant length. For example, for rotary equipment, the set of revolutions of the shaft is indexed by the sequence numbers of revolutions with some fixed time of complete rotation. Thus, the set of revolutions is factorized by a histogram of the shaft revolution duration. For four-stroke internal combustion engines, the shaft turning time is fixed at $4\pi$. In cardiac applications, a histogram of beat-to-beat durations, etc. is determined. In part, this approach solves the synchronization problem by allowing the signal to be represented on a quasi-period as the implementation of some random process.
In this case, the assumption of the exit mode to quasi-stationarity in some cases, for example, if the process is diffusive, presupposes the existence of such a regime not only during the exit but also with the further evolution of the stochastic process. For example, in the number of additional conditions, the Lyapunov functional is determined and an analog of the $H$-theorem of Boltzmann is valid. In this case, the Lyapunov functional is entropy, which is a complete analog of the Kulbak entropy. At the same time, the nonstationary density of the distribution function asymptotically tends monotonically to the stationary one:

$$H = \int_{a}^{b} dx p(x,t) \ln \frac{p(x,t)}{p_s(x)}$$

The next step comes down to a more detailed description of the subset of states, i.e., sets of admissible states in $R^N$.

The basic principle of the construction of the marked set appeals to the construction of the degeneration spaces of the free energy functional in various condensed media [9]. For this, the symmetry group $G$ for the generating functional is determined, and then the isotropy group of the state vector is determined. The factorization of a symmetry group $G$ into isotropy subgroups $H$ gives the so-called degeneration space $G/H$. In essence, the symmetry group transitively acts where the states of the automaton defined in [2] are invariant with respect to the action of group $G$; the transition probability is constant during transformations of the state vector by multiplying the elements by the action of a symmetry group.

For further formalization of the prognosis task, all homotopy classes of the degeneration space $G/H$: $[M, G/H]$ are considered. Mapping $M \rightarrow G/H$ defines the set of system states. In particular, as $M$ spheres of various dimensions $S_i$ are considered. In this case

$$[S'_i, G/H] = \pi_i(G/H)$$

And $i = 1, \pi_1(G/H)$—a group, $i > 1, \pi_i(G/H)$—an Abelian group.

Some examples of the construction of degeneration spaces are described in this paper [10]. It is shown that under the assumptions made, the degeneration space for some set of segments of a fixed length is the so-called homogeneous space. The term “degeneration space” is often used in physics, which is more accurate, since with respect to the action of degeneration groups on vector states, they are invariant with respect to the characteristic functional of the process. In other words, it is assumed that the states of the predictive automaton introduced in the cited paper are invariant with respect to the discrete subgroup of the translation group. In this case, translation is carried out along a cascade with a step $N^*$. There is some analogy to the constructed objects with stochastic processes with a measure invariant with respect to the group of translations [11].

Since the properties of the signal are initially unknown, and under the assumption that the properties of the signal may change, it turns out that it is necessary to consider nested sequences by dimension:

$$R^{N_1} \subset R^{N_2} \subset R^{N_3} \subset \ldots \subset R^{N_k} \subset \ldots ; \frac{G_i}{H_i} \subset \frac{G_i}{H_i} \subset \ldots \subset \frac{G_k}{H_k} \subset \ldots$$

Nested sequences of groups and spaces of degeneracy, these sequences are constructed for each index of the wavelet coefficients; therefore, all trajectories generated by such embeddings as multitrajectories are considered.
The analog of the Turin machine defined in the work [2], hereinafter referred to as the recognizing automaton, moving along the cascade, changes its internal state. A change in the internal state generates a trajectory in the $n_k - 1$-dimensional simplex $\Sigma_{n_k-1}$. Accounting for all coefficients of the wavelet decomposition of the signal thus generates a set of predictive automata. Further, it is shown that the change in the internal state of the automaton is reduced to elementary steps on the $n_k - 1$-dimensional simplexes or the multidimensional space of frequency vectors. Each elementary step is determined by multiplying the frequency vector by the elementary jump matrix for each coordinate of the frequency vector [2, 3]. The product of the $N^*$-th number of such matrices in the space $R^{N^*}$ determines the transition from the initial vector to the final one in $N^*$ elementary steps.

The set of sequences of elementary matrices from $N^*$ from cofactors is also imposed by the restrictions arising from the condition of the quasi-stationarity of the state of the automaton (Eq. (1)). And following the above assumptions leading to the conditions of the H-theorem, the admissible set of matrices of elementary steps is also limited by the condition of convergence to a quasi-stationary state.

Thus, the interpretation of the model in terms of a random walk on a lattice or in a continuum undergoes significant changes. The meaning of them is as follows: topological restrictions, namely, the nontrivial homotopy type of degeneration space in state and trajectory spaces defines such configurations of states when, defined at the boundary of the some region, they cannot be continued to the interior of the region by continuity, which indicates the presence of discontinuities in attempts of continuous continuation. That is, there is a singularity inside such a region. The kernel of the singularity is a set with partially or completely broken symmetry, i.e., the degeneration space changes with its homotopy type. Thus, in the state spaces and spaces of k-multiple paths [2, 3], defined on these spaces, regions with broken symmetry appear, and these regions have a complex topological nature and are capable of various transformations within their homotopy class.

Examples: Let the space of degeneration is a homogeneous space

$$G/H = \frac{SO(3)}{SO(2) \times Z_2} = RP^2.$$  \hspace{1cm} (8)

$RP^2$ is projective space:

$$\pi_1(RP^2) = Z_2, \pi_2(RP^2) = Z$$ \hspace{1cm} (9)

To understand the main points of the model, further examples will be given for low-dimensional spaces of degeneracy. This demonstrates the basic principles of constructing automata that are intuitive in low dimensions. So, the nontriviality of the fundamental group $RP^2$ determines the presence of linear singularities that are closed at their ends by themselves or ending at the boundary of the region. The nontriviality of the second homotopy group, $\pi_2(RP^2)$, of the space of degeneracy, implies the existence of singularities in the spaces that are homotopy equivalent to a point. In the space of paths in the above case, (Eq.(10)) is used to interpret the singularities [12].

$$[W, \Omega X] \leftrightarrow [\Sigma W, X]$$ \hspace{1cm} (10)

From which it follows that

$$\pi_i(\Omega X) \cong \pi_{i+1}(X)$$ \hspace{1cm} (11)

Consequently,
According to the above formulas, one-dimensional singularities are not a separately taken state, but a path or loop, which is homotopy equivalent. The nontriviality of the homotopy groups of the space of degeneracy $G_{\mathcal{H}}$ and the presence of the group structure of homotopy classes give rise to a nontrivial interaction between singularities. In particular, the fusion of singularities is accompanied by the addition of elements of groups of homotopy groups (topological charges) generating singularity data. Other transformations are possible. For example, two linked circular singularities can be continuously transformed into the singularity shown on Figure 1.

The interaction of point singularities in the vicinity of a linear singularity can also be nontrivial. In particular, depending on the features of the topological dynamics in the vicinity of the linear singularity, the interacting point singularities may annihilate.

As already noted, the singularity kernel is a region of the state space, the trajectory space, where, in connection with symmetry breaking, trajectories with broken symmetry are realized. In particular, within the framework of the initial space, the degeneracies of the kernel of a singularity represent regions where the state of the automaton is not defined. In this case, the singularity in the original space of degeneracy is interpreted as a discontinuity in the standard topology of $\mathbb{R}^{N}$, generated by the metric of the Euclidean space. The gap in the trajectory space can be defined in various functional metrics. For example, as shown in Figure 2, two trajectories are on opposite sides of the singularity.

For simplicity, let it be a point singularity on a plane. The trajectories shown in Figure 2 are homotopically non-equivalent. Therefore, an attempt to continuously deform one trajectory into another cannot be continuous. If we are talking about a loop space, then the class of trajectories on both sides of the singularity will undergo the discontinuity in the topology defined in the loop space. That is, the kernel of the singularity contains a class of trajectories, not close to the class of trajectories. In practice, such trajectories surrounded by quasi-stationary trajectories are already non-quasi-stationary. Removing the degeneracy with respect to the time inversion leads to the appearance of trends in the states of the automaton in such trajectories. The conditions of quasi-stationarity are violated.

\[
\pi_1(\Omega(G_{\mathcal{H}})) = \pi_1(\Omega(SO(3)/SO(2)\times\mathbb{Z}_2)) = \pi_2(\mathbb{RP}^2) = \mathbb{Z} \quad (12)
\]

Figure 1
Transformation of two coupled singularities 1 and 2, with singularity 3 connecting them into a bunch of two one-dimensional spheres.

Figure 2
Two non-homotopic trajectories due to a point singularity between them.
At the same time, the range of permissible sequences in the product of matrices of elementary steps is reduced due to the need to take into account the chronological ordering on the set of products of matrices of elementary transitions. The listed changes do not allow the system to reach some previous states, in view of what new states and new trajectories appear in the system with necessity.

The set of admissible states of automata also changes. In particular, to overcome the prohibitions that have arisen on part of the trajectories, new cells of the automaton are born, that is, an increase in the dimension of the state space and the trajectory space occurs, i.e., there is an increase in the dimension of the state space and the trajectory space. As a result, such change increases the dimension of the symbolic state space represented as \( n - 1 \)-dimensional simplex \( \Sigma^{n-1} \). The increase in the simplex dimension and the dimension of the state space generates new types of singularities in the new dimensions and, on the other hand, allows us to again and expand the class of admissible trajectories by increasing the dimension.

In this case, the dimension in the evolution equation changes the conditions of quasi-stationarity and other prohibitions on the set of admissible states expressed as inequalities or equalities also change. An effect similar to the effect of singularity falling into the third dimension in nematic liquid crystals, for example, is observed. Only in the case under consideration is the system’s trajectory squeezed onto the verge of a simplex \( \Sigma^{K-1}, K > n \). The state of the automaton during such a process changes, and the empirical density of the distribution may become non-Gaussian. Thus, with a small movement of the trajectories of the previous class, the extruded path in higher dimensions has a gap in loop space.

Returning to the set of predictive automata, it should be noted that the described scenario of restructuring the internal state of the automaton is far from being the only one, and the topological model offers many different scenarios for the reconstruction of internal states. However, even in the above example, predictors or hidden predictors of reconstructions are identified. Since the task of the prognosis model is to predict exactly the reconstructions, the predictors detected by the automata are predictors of dysfunctions, failure, heart failure, etc. In the example analyzed above, the early predictors are the emergence of the process of birth and destruction of the cells of an automaton; the earlier predictors are associated with the violation of the limiting conditions for the transition probabilities. A change in the stationary conditions leads to an evolution equation that depends on time, respectively; in the state space, there is a change in the transition probabilities over time, which, in turn, determines the trajectories different from the stationary ones. Accordingly, the RUL estimates also change. Returning to the set of predictive automata, it should be noted that the described scenario of restructuring the internal state of the automaton is far from being the only one, and the topological model offers many different scenarios for the reconstruction of internal states. However, even in the above example, predictors or hidden predictors of reconstructions are identified. Since the task of the prognosis model is to predict exactly the reconstructions, the predictors detected by the automata are predictors of dysfunctions, failure, heart failure, etc. In the example analyzed above, the early predictors are the emergence of the process of birth and destruction of the cells of an automaton; the earlier predictors are associated with the violation of the limiting conditions for the transition probabilities. A change in the stationary conditions leads to an evolution equation that depends on time, respectively; in the state space, there is a change in the transition probabilities over time, which, in turn, determines the trajectories different from the stationary ones. Accordingly, the RUL estimates also change.
One of the conclusions arising from the symbolic model is associated with the prediction of hidden predictors. The essence of it is as follows. In the process of evolution, any material is subject to degradation. In the materials of which the mechanisms are composed, such as alloys, composite, and polymeric materials, changes occur that affect the functional properties of the material, for example, a local change in the composition of titanium alloys under thermal cycling conditions, grain growth in metals, the formation of dislocations and microcracks, etc. Partially such changes are reflected in the signals of the sensors, if, in addition, structural degradation affects the functional properties of the material. Biomaterials also degrade, for example, the appearance of scar tissue, which replaces the tissue in the myocardium, entailing changes in the ability of the material with the spread of action potential, etc.

With the degradation of the material, the number of class of permissible trajectories changes, as noted above. Taking into account the nontriviality of the homotopy type of degeneration space, cases are allowed when the forbidden trajectory is squeezed onto the verge of a simplex of higher dimension from the class of normal, not forbidden trajectories. Extrusion of a trajectory to higher dimensions leads to changes in the internal state of the automaton. In higher dimensions, there are changes in the number of kinetic or evolution equations, as a result, a change in the conditions of quasi-stationarity and other conditions that limit the set of trajectories, either represented as topological prohibitions, or as equalities or inequalities on the set of admissible states. This leads to the conclusion that in the general case, it is impossible to introduce a metric in the trajectory space in which the development of hidden predictors continuously migrates to the boundary of the heart event or to the boundary of failure. That is, the process of reaching the failure boundary may occur as jump. In this case, the jump is carried out from the class of permissible trajectories, against the background of the absence of dysfunctions by the mechanism of extrusion of the trajectory on the verge of a simplex of higher dimension. In this case, the state of the automaton is transformed in violation of the conditions of quasi-stationarity. Certain analogs of the described effect is noise-induced transitions, but in this case, the system does not require the additional conditions, under which such transitions occur, does not require a Markov property, the fulfillment of the Chapman-Kolmogorov identity, and the conditions imposed on the signal for the validity of the Fokker-Planck equations. In this case, the signal can be both stochastic and chaotic.

In conclusion, it is necessary to stop at the complexity model of the set of automata, begun in [3]. In this case, the interaction between automata is introduced, described in [3] using a metric based on the Radon-Nikodym derivative. However, a simple example should be given of the necessity of complicating automata in order to understand the ultimate goal of such complications. Let us consider an automaton with a two-dimensional cascade or its continual limit. Only such automata are combined into one automaton, for which it can be said that the corresponding stochastic processes are statistically dependent:

\[ P(X, Y) - P_1(X)P_2(Y) \neq 0 \]  \hspace{1cm} (13)

\( P, P_1, P_2 \) is the density of distribution function.

We now turn to the degeneration spaces for processes \((X, Y)\). In case of statistical independence

\[ g_{II}(X, Y) = g_{II}(X) \times g_{II}(Y) \]  \hspace{1cm} (14)

\( g_{II} \) is the density of distribution function.
\[ \pi_i(\mathcal{G}_H(X, Y)) = \pi_i(\mathcal{G}_H(X)) \times \pi_i(\mathcal{G}_H(Y)) \]  \hspace{1cm} (15)

The situation changes when there is a statistical dependence or some other variant of the interaction of processes. In this case, the formula (13, 14) is unfair. The situation is much more complicated. For example, if Eq. (13) holds true and

\[ \mathcal{G}_H(Y) \subset \mathcal{G}_H(X) \]  \hspace{1cm} (16)

That homotopy groups can be fundamentally different from the product of the homotopy groups of direct products of spaces of degeneration. In particular, the theorem on the exact sequence of a pair is valid [13]:

\[ \ldots \to \pi_i(\mathcal{G}_H(Y)) \to \pi_i(\mathcal{G}_H(X), \mathcal{G}_H(Y)) \to \pi_{i-1}(\mathcal{G}_H(Y)) \to \ldots \]

Example: \( \mathcal{G}_H(X) = S^3, \mathcal{G}_H(Y) = S^1 \)

\[ \ldots \to \pi_2(S^3) \to \pi_2(S^3, S^1) \to \pi_{1-1}(S^1) \to \ldots \]

\[ 0 \to \pi_2(S^3, S^1) \to \pi_1(S^1) \to 0 \]

\[ \pi_2(S^3, S^3) = \mathbb{Z} \] is the relative homotopy group.

For comparison: \( \pi_2(S^1) = 0, \pi_2(S^3) = 0. \)

Thus, in the interaction of automata, singularities appear, described in terms of relative homotopy groups, and these singularities in the general case are fundamentally different from the singularities of the direct product of degeneration spaces. Accordingly, interacting automata exhibit other scenarios of topological dynamics, other predictors and mechanisms for breaking symmetries or lifting degenerations.

The need for complicating the automata described above also determines a number of additional properties of the set of automata, namely, bundle of automata and hypernets.

For a fixed length of the partition of the physically permissible range of values of the sensor readings and in those cases when such partition is determined from the conditions of reducing computational costs or following the traditionally accepted in the general statistics, there is a risk of missing hidden predictors. In this case, bundle of the automaton occurs, which is determined by a shorter partition step. It turns out that each cell of the automaton is also an automaton. This stratification is also necessary in cases where the observed signal has several scale levels.

Formation of hypernets is associated with the formation in the network of interacting automata of a higher dimension. Such an association is not artificial, as was shown in this chapter. When merging automata into the higher-dimensional automaton, the degeneracy space of the finite automaton differs significantly from the degeneration spaces of each of the parent automata.

As a consequence, there is a change in the types of singularities, their topological dynamics, a change in the transformations of the states of automata in the process of their falling out into the faces of the higher dimensions of the simplex, changes in the numerical estimates of the achievement of predicted indicators or limits of failure, heart events, etc.

Ultimately, the architecture and functionality of the predictive monitoring system is as follows. Telemetry enters the computing kernels with automata and preprocessing programs, which includes the wavelet transformation, cleaning the signal by the wavelet filters, and then fed to the recognizing automata. As already noted, the task of predictive automata is reduced to determining the class of the trajectory, determining trends and early predictors based on an analysis of
transition probabilities, the number of permissible transition matrices in the product (omega), and also to determine the evolution of this number and the qualitative structure of the set of transition probabilities. It also checks the fulfillment of the stationarity conditions, the behavior of the Kulbak entropy with deviations from the stationarity. Taking into account the revealed evolution of transition probabilities, the trajectory of a walk on the faces of a simplex is determined (predicted). This happens on all the set of automata, including automata with high dimension. Moreover, a causal graph is determined on a subset of high-dimensional automata. Automata that had previously reacted by changing their state to subsequent predictors of later automata are allocated to a separate class. It should be noted here that the selected class is also subject to change, that is, the automata can leave this class or, on the contrary, appear in this class.

The reason for this behavior is as follows:

a. Taking into account the individual characteristics of the mechanism.

b. Individual changes on the sets of transition probabilities and, therefore, on the entire trajectory of states.

c. The emergence of new types of singularities due to a change in the structure of the space of degeneration.

In a certain class, the earliest automata can be partially placed on the computing power located on the mechanism itself, for example, on the onboard computer, the computing power of the microprocessor of a wearable medical device, etc. Such automata acquire some additional functions; in particular, these automata begin to control the computing kernels of the remote cluster. And, on the contrary, when changing in the whole class, the computing cluster replaces onboard automata with other automata with the same functions. As a result, it becomes possible to optimize the calculations in the system “automata on-board - automata of the computing kernels of remote cluster” up to the advent of opportunities to go offline.

However, periodically online mode is required for inspection. At the same time, the onboard automata connect the online mode based on the determination of their own state, for the most part, with the appearance of birth-death processes, changes in the structure of the set of transition probabilities, the appearance of trends, or rearrangements.

The described situation reflects a more general property of the family of automata, namely, their ability to differentiate. It has already been noted that automata possess the properties of differentiation; a similar property of automata is necessary in cases where an explicit failure predictor is formed. In this case, several automata proceed only to the predictor analysis, for example, when signs of the beginning of a trend in the QT interval are predicted. In this case, besides the set of intervals that capture the cascades corresponding to the QT interval of the ECG, an additional automaton is added to them, processing only the length of the QT interval. The task of the selected family is in this case an estimate of the time to reach critical values of the QT interval. All of the above applies to any traditional geometric characteristics of the ECG as a whole and intervals, complex, and teeth.

3. Return on investment

The question about ROI, more precisely, about optimizing the maintenance strategies, was discussed on the basis of the automata model and the hierarchical
prognosis model in the works [5, 14]. In the most general formulation, the optimization task is reduced to carrying out the necessary maintenance and variation of the management parameters in order to maximize the stay of the process trajectory in a given homotopy class. The preventive monitoring cluster architecture described above allows reducing the computation time on a remote cluster by transferring a small part of the automata to the monitored object using onboard computing power.

In the field of medical applications (cardiology), we can take as an example the transfer of automaton of the R-R interval to the processor of portable ECG recorder for multi-day monitoring. In addition to this automaton and depending on the general analysis of the evolution of states, automata of a specific purpose are also installed. For example, an automaton for the QT interval, which can autonomously predict a change in the QT interval with an estimate of the time, it takes to reach critical values of a given ECG interval. All of the above is also true for the definition and evolution of such ECG indicators as R-R pause length, QT interval dispersion, and other significant ECG predictors of cardiac events. It should be recalled that the onboard automata can also go offline and manage the computing kernels of the remote cluster. In the above example, such transition is carried out on the basis of the prognosis obtained by the onboard automatic devices. In other cases, an automaton on one of the leads may be sufficient, and a specific cascade corresponding to, for example, the average value of one of the ECG teeth, is monitored.

The described manipulation with automata allows minimizing the cost of the traffic of all ECG leads, as well as the costs associated with the abundance of computations by the entire set of cluster automata.

Of course, the described optimization of calculations increases the comfort of using wearable ECG gadgets, bracelets, and mHealth platforms.

However, the general goal in this case is prognosis of the life-threatening cardiac events, taking preventive measures to eliminate them or minimize the consequences. An example should be the procedure for optimizing drug therapy for atrial fibrillation. The well-known fact that antiarrhythmic drugs used in the treatment of arrhythmias can provoke the appearance of life-threatening arrhythmias. Timely adjustment of the dose of the drug, the rejection of the drug, and its replacement with another drug is one of the actual problems during treatment. In this situation, predictive automata solve the problem of optimizing drug therapy, determining the ineffectiveness of prescribed drugs, thereby reducing the time they are taken. On the other hand, automata predict with an estimate of the probability and time to achieve proarrhythmic effects, which allows the doctor to take steps in advance to correct the treatment.

Consider the field of monitoring engineering for mobile objects, in particular, in transport and directly monitoring for internal combustion engines, hybrid engines. Here, as onboard signaling automata, it offers an automatic device that processes signals from onboard standard sensors, for example, a crankshaft angle sensor or a pressure sensor on common rail systems, etc. With an onboard computer and graphics processors, the number of automata can be increased by covering most of the standard sensors that give an analog or digital signal, for example, the automata processing of the so-called uneven stroke. The evolution of the states of automata that process uneven stroke based on the crankshaft angle sensor reflects the evolution of the cylinder-piston group and, therefore, predicts the development of the most dangerous predictors of failure associated with changes in the combustion mode of the fuel mixture. Adding an accelerometer to the family of standard sensors for analyzing the vibrations of the engine body allows to expand the prediction capabilities for the main friction pairs in the engine mechanics and analyze the status of injectors and high pressure fuel pump (diesel engines). The calculated
RUL estimate allows to calculate an optimizing maintenance strategy in real time. This can also be attributed to railway transport where there are many problems with the analysis of friction pairs, for example, bearings, rotors, etc.

The following fact also contributes to cost reduction and, accordingly, a change in ROI. At the beginning of the operation of automata, the entire set of automata participates in the processing of sensor signals. In the course of time and with the increase in statistical data, the increase in the number of monitored objects of the same type on the set of mechanisms, its clustering becomes possible, that is, the division of mechanisms into classes determined by approximately the same set of predictors necessary for prognosis. In the case of clustering, an additional automaton is introduced, the purpose of which is to determine whether the mechanism belongs to a certain class of the cluster. At the same time, the cumulative chronological database as individual objects or the entire class can significantly reduce the cost of remote computing resource, particularly in cases where onboard computing resource is sufficient for placing on it a few key automata.

The effect of optimization of the computing resource is also provided with the development of self-maintenance or self-recovery systems. As an example of self-recovery systems, there are such failures as coking of nozzles or a piston group of an engine, which is especially common when operating engines subjected to tuning, in violation of the speeds of operation, etc. In such cases, an early prognosis using an automaton on the rotation sensor of the engine and accelerometer allows to determine the initial stages, or rather, the change in the mode of combustion of the fuel mixture at the stage of nucleation of coking centers. However, feeding on several motorcycles of the depleted mixture leads to the dissolution of the germs of growth of the new phase, in this case, the centers of growth of coke deposits. At the same time, substantial savings are quite obvious, since in the more advanced stages of coking, dismantling the nozzle is necessary for its cleaning, processing in an ultrasonic bath, checking at the stand, etc. The same problem is relevant in aviation, where coking of the inner surface of turbine compressor blades is often detected under the conditions of a repair aircraft factory. In this case, expensive blades are simply replaced by new ones. Thus, self-recovery or maintenance of the system or modes can be very effective, especially in the case of early prediction of dysfunctions and at the stage where the group of time inversion enters the degeneration space, which turns the degeneration space into a projective space. In other words, the trajectories in this case are invariant with respect to the time inversion. The presence of such symmetry and, moreover, reversibility is some good approximation of the process at low diffusion velocities in evolution equations of the Fokker-Planck type, for example. It should be noted that the maintenance management task formalized in the previous chapter is most effectively solved exactly in the described class of processes, i.e., in the presence of marked symmetry.

4. Conclusions

The prognostic models considered in this chapter are built on the assumption that each observed signal to a certain extent reflects the internal state of the object being studied. In engineering, in medicine, the observed signal is at least indirectly due to the processes occurring in the mechanisms and bioobjects. The task of a complete set of measured parameters as a whole is hardly solvable; one way or another, when studying any object, a model of the object itself is needed, capable of reflecting not only the instantaneous state but also the evolution of the object over time. The observed signal, as a rule, is not obtained directly from an evolving object, because it is impossible to place an accelerometer or a pressure sensor directly into the boiler of
a nuclear reactor, glue the patient’s heart with electrodes and install a pressure sensor in the working part of the aircraft turbine. Thus, in the end, there is a signal from only ten electrodes from the surface of the human body; the sensors of the machinery are installed far enough from the combustion zone in engines and turbines. Everything briefly listed above leads to the appearance of artifacts of various natures in the signal, and these artifacts must be taken into account in order to isolate the useful part of the signal. In addition to all the above, signals from the so-called distributed systems are usually analyzed. Strong heterogeneity, complex geometry, and complex processes supporting the functionality of these systems are not restored and not reproduced by a set of a dozen sensors. Therefore, by virtue of the above, physical models of objects and their evolution are needed, using the so-called concise description, where, often, an infinite set of measured parameters necessary for modeling the system is replaced by a much smaller number of parameters. The simplest example and classical descriptive compression is the equilibrium thermodynamics, when coordinates and momenta of all particles are reduced in thermodynamic approximation to such quantities as temperature, pressure, etc.

In the cases under consideration, the main emphasis in constructing the prognostic model for evolving distributed systems is to take into account the internal symmetry of the observed signal, without discussing the relationship between the internal signal symmetry and the symmetry inherent in the state of the object and the processes in it.

So, internal symmetry allows representing a signal as a product of slowly varying amplitude and a rapidly oscillating phase cofactor. The phase cofactor has symmetry groups, which acts transitively in pseudo-phase space. Ultimately, the state of the system is described as a vector in multidimensional space, and when taking into account the space of degeneration of a point in the space of degeneration, the evolution of the system is a trajectory of states in the space of degeneration or a multiple space of loops. Further analogies with the theory of topological defects of condensed media make it possible to formalize the evolution of the prediction problem in the form of the evolution of the set of trajectories, their transformations, and a change in the types of trajectories with various kinds of lifting degenerations.

The transition to symbolic space, and in fact to the space of probability measures, offers different interpretations of the evolution of a measure in time, in particular, in the form of a complex-arranged walk of the point in a multidimensional simplex. This, in turn, allows to construct families of predictive automata, the evolution of which internal states takes into account the topological nontriviality of the space of degeneracy. Considering the topological nontriviality and, as a result, the topological nontriviality of the set of trajectories determines the complex topological dynamics of the trajectories and the dynamics of the interaction of topological singularities and, most importantly, provides an opportunity for algorithmic construction of the search and definition of predictors of various events associated with changes in the characteristics of the trajectory. Finally, it becomes clear that any change in the characteristic features of the trajectory is associated with the violation of the corresponding symmetry, the removal of the degeneracy in groups or subgroups of the isotropy of the space of degeneracy.

The resulting set of automata has a number of properties that must be mentioned in the context of their further development:

1. Differentiation of automata
2. Bundle of automata
3. Formation of hypernets
Ultimately, the predicting set of automata will have the ability to adapt, i.e., restructuring of its entire structure with changes in the properties of the observed signal, adding signals from new sensors, and changing the operating conditions of engineering.

Thus, the family of automata is capable of analyzing a multitude of signals from various sensors, which makes it possible to use them in all types of engineering and medicine. In the initial stage of operation of the automata, one should limit ourselves to standard onboard diagnostic sensors, which allows one to obtain the necessary data on the need for additional sensors, which, in turn, provides further calculation of the ROI for construction of optimizing maintenance strategies, calculating optimal operating conditions, etc.
References


