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Chapter

The Importance of Spatial Reasoning in Early Childhood Mathematics

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Abstract

It is important to recognize the critical role spatial reasoning, relational thinking, and mathematical modeling play in the overall development of students' central understanding of mathematics. Spatial reasoning predicts students' later success in higher levels of mathematics, such as proportional thinking and algebraic reasoning. The National Research Council report implores educators to recognize the importance of developing spatial reasoning skills with students across all areas of mathematics. This chapter describes a study that used the Primary Math Assessment—Screener and Diagnostic to assess students' spatial reasoning and relational thinking. The results highlighted curricular resources to improve students' understanding of mathematics. Students' mathematical spatial reasoning improved significantly.

Keywords: spatial reasoning, relational thinking, early childhood, mathematics, achievement, DMTI

1. Introduction

It is important for educators to recognize the critical role spatial reasoning along with mathematical modeling plays in the overall development of mathematical skills and understanding. It is a fundamental bridge to algebraic thinking and conceptual understanding. The National Research Council report [1] urges educators to recognize the importance of developing these skills with students across all areas of mathematics.

Bruner's [2] modes of representation describe the process of enriching students' understanding by working through enactive, iconic, and symbolic (EIS) models. The enactive (physical) and iconic (visual) models are critical to help students develop connections to a task and allows for better recall of mathematical ideas. It is critical for teachers to expose students to different methods of modeling relationships with multiple representations. Students will have a better opportunity to generalize and build on existing foundational knowledge of equivalence throughout their mathematical careers.

Many students have difficulty in understanding concepts without being able to first observe a pictorial image of an idea in their mind [3]. Mathematics curricula loaded with symbolic representation require students to memorize procedures, denying the student an opportunity to utilize their visual thinking modality in the

process of building conceptual understanding. On the other hand, curricula that embed more iconic models may allow for students to deepen their understanding of the mathematics and improve their skill levels [4]. Thus, we wanted to investigate whether there was a significant difference in first grade students' performance in spatial reasoning after being introduced to mathematics that included a plethora of iconic modeling.

2. Spatial reasoning

Spatial reasoning is strongly correlated with achievement in mathematics [5–7]. Students who perform better on spatial tasks also perform better on tests of mathematical ability [8–10]. Spatial reasoning involves (a) composing and decomposing shapes and figures, (b) visualization, or the ability to mentally manipulate, rotate, twist, or invert pictures or objects, (c) spatial orientation, or the ability to recognize an object even when the object's orientation changes, and (d) spatial relations, or the ability to recognize spatial patterns, to understand spatial hierarchies, and to imagine maps from verbal descriptions [10, 11]. Recent evidence indicates that spatial reasoning training can have transfer effects on mathematics achievement, particularly on missing term problems (e.g., 7 + __ = 15), which are important in developing algebraic understanding [8].

In addition, spatial reasoning skills and mathematical competency are directly related to each other [12–15]. Learning with specific spatial reasoning tasks improves students' abilities in the Science, Technology, Engineering, and Mathematics (STEM) fields [16, 17]. And there is a strong link between spatial reasoning ability and geometry where strong visuospatial skills predict how well students will complete 3D geometry tasks [18–20]. As educators become more aware of the need for spatial reasoning tasks, it is important to recognize the critical role mathematical modeling plays in the overall development of mathematical thinking.

The National Research Council report [1] urges educators to recognize the importance of developing spatial reasoning skills with students across all areas of mathematics. And the National Council of Teachers of Mathematics [21] suggests more spatial reasoning be integrated into the elementary mathematics curriculum to promote relational thinking skills. Mathematical modeling may be a key component to help students explain their thinking when representing algebraic concepts.

Mix and Cheng [22] found that students with strong spatial reasoning skills do well in mathematics [23]. Spatial reasoning is a critical element for developing ways students think about equations. Given the opportunity, students' spatial reasoning skills can increase when practice is integrated and supported throughout mathematics instruction [24]. By the time students reach kindergarten, their spatial reasoning skills predict their overall mathematical success [25]. Therefore, students' educational experience in elementary school should have an intentional focus on improving spatial reasoning skills.

The focus of the next section is to highlight the connection between spatial reasoning and spatial orientation on a number line, gesture, visualization, and mental rotation. For instance, a crucial component to understanding ordinality (the position of a number in relation to its location on a number line) and magnitude (the size of a number) is the development of a spatial representation of numbers in connection to the symbolic representations [26]. The number line has been shown in cognitive studies to be important for the development of numerical knowledge [27–29]. Ramani and Siegler [30] report that students who play board games such as Chutes and Ladders increase rote counting skills, number identification, and the

conceptual understanding of numerical magnitude. Additionally, activities which include puzzles, video games, and blocks with significant connections to spatial reasoning skills and mathematical competency improve accuracy of symbolically representing a number line [31].

Problem-solving tasks regarding orientation, transformations, and movement of shapes create an opportunity among students and the teacher to engage in rich, mathematical discourse. As students discuss their thinking, they will use their hands to gesture while attempting to convey their thoughts surrounding the task. Gesturing allows students to explain the visual imagery taking place inside their head as they work on problem-solving specific tasks [32]. Students' gestures represent the movement of the transformation and create an avenue for their thinking to emerge through the discussion. Alibali and Nathan [33] found gestures to be an excellent tool for teaching students how to solve spatial transformation tasks by placing an emphasis on the importance of moving the pieces without the actual physical movement. In essence, they used their hands to gesture what their mind was creating and conveying mathematical thinking.

The ability to gesture what the mind is thinking is dependent upon students' ability to visualize mathematical transformations [34]. The ability to think relationally requires students to visualize how numbers can be manipulated and rearranged in an equation [35]. Therefore, visualization is a key component across mathematical topics [34]. Spatial visualization tasks require students to create an image in their mind, hold the image, and then mentally transform or manipulate that image to be different. Some examples of these types of tasks include composing and decomposing pattern blocks to determine a new composed image, imagining transformations and perspectives of a three-dimensional cube, or activities that involve mentally folding a two-dimensional shape to form a new three-dimensional shape. In addition to spatial visualization, mental rotation has also been shown to increase student performance in mathematics [8].

Students who are allotted time to practice mental rotation have demonstrated the ability to solve a series of multi-step word problems [36]. Mental rotation consists of the ability to look at an object or picture of an object and visualize what it might look like when rotated in 2D or 3D space. The most recent study of spatial training with mental rotation was conducted with young students developing number sense, counting sequence, fact fluency, and missing term problems [8, 22]. Although the other areas showed improvement with the spatial training, missing term problems such as 2 + __ = 6 indicated the most significant effect size. Much like the relational skills needed to find the most efficient way to solve missing term problems, the completion of mental rotation tasks during spatial training helped to strengthen students' ability to visualize the necessary transformations of numbers within equations for simpler computation [8].

It is important to note that mental rotation and spatial visualization are both subsets to spatial reasoning and much of their characteristics overlap [34]. Developing both skills is a powerful way to connect back to the bigger idea of conceptual understanding for relational thinking, spatial reasoning, and equivalence [37, 38].

3. Relational thinking

In addition to spatial reasoning, relational thinking or early algebraic reasoning is critical for long-term success in mathematics. Students need time to develop relational thinking, with practice designed to explicitly examine the way in which numbers relate, and ways that those relations can generalize to other

areas of mathematics [39–42]. One way to improve conceptual understanding is to increase the exposure of problem-solving tasks involving nontraditional equations. It has been shown that students as young as kindergarten and first grade have informal knowledge of number relations; however, the mathematics presented in traditional textbooks do not explicitly draw out these relations, allow time for the relations to organically emerge, or instruct students to determine how the ideas can be generalized (Blanton and Kaput [40]). Consequently, there is a need for mathematics instruction to incorporate more than just the traditional format of equations into daily lessons and include ways to represent relational equivalence [43, 44].

One aspect of relational thinking is equal sign. Most elementary students begin to develop their awareness of the equal sign's functionality at an operational level, where the equal sign acts as a symbol to perform a calculation or action [42]. When the bulk of instruction is focused on procedures and computing facts, many elementary students develop a shallow understanding of the equal sign and consider it an operational symbol [45, 46]. For instance, students with an operational view of the equal sign will reject any equations presented outside of the traditional format, $a + b = c$, and will define the purpose of the equal sign as a cue to perform the calculations on the left side of the equal sign to get an answer [47]. However, given more exposure to a variety of equations, students can become more flexible with their thinking and progress to different levels of understanding [40]. Mathematics instruction for early elementary classrooms should foster relational thinking by including tasks designed to draw attention to how numbers relate to one another and develop the flexibility to think of numbers in a variety of ways to establish the idea of equivalence [8, 48].

Matthews et al. [49] developed a construct map based on the research of Carpenter [47] and Hunter [50] to explain the continuum of relational thinking for students' thinking. The first level of student understanding is called rigid operational. Students at this level are calculating traditional or missing term equations. Traditional equations, written $a + b = c$, place the equal sign as a function for solving the addition problem $a + b$ to produce an answer. This traditional format instills an operational view of the equal sign [51]. With exposure to nontraditional equations, such as $a = b + c$, students become more flexible in their determination of a correctly written equation. However, their view of the equal sign still remains as a cue for calculation. As students move into the basic relational stage, their flexibility to solve equations written with operations on both sides of the equals sign increases. However, it is not until the final stage, comparative relational when students consider the number relations on each side of the equal sign to determine equivalency and their need to calculate diminishes. This level of relational thinking demonstrates students' knowledge about how the equal sign relates to the entire equation, where they are looking for relatable numbers in the equation prior to solving the problem [52]. Identifying these relationships in equations and their connections with the numbers is a critical component of mathematical understanding. Developing and applying the knowledge of relational thinking to solve mathematical equivalence problems will increase early algebraic understanding [41, 44, 46, 53]. Students who think at the comparative relational level have a strong understanding of the equal sign and a deeper connection to algebraic reasoning [47, 50].

The natural tendency for students as young as kindergarten is to demonstrate an operational view of the equal sign; however, they do have the capabilities to think relationally if given the opportunity [45]. Therefore, relational thinking skills should be explicitly taught at an early age to avoid a deep-rooted set of operational skills [54]. Relational thinking involves flexible thinking to determine how numbers can be manipulated before answering a problem. Using relational thinking to

solve an algebraic equation requires the conceptual understanding that each time a number is manipulated the equation remains equivalent.

Providing students with a progression of nontraditional number sentences focused on numerical relationships and patterns will develop relational thinking. As a starting point for young students reversing the order of the number sentence to begin with the answer such as 3 = 2 + 1 presses students to accept that the answer does not always need to be after the operation [49, 55]. Next, students develop their understanding of the term equal as they begin to recognize that both sides of the equation compute to the same quantity through exposure to nontraditional equations written with the operations on both sides of the equal sign [47]. Students who possess the conceptual knowledge of equivalence recognize transformations can occur by adding the same number to both sides of the equal sign without changing the structure of the equation. For example, when asked whether the equation 18 + 3 = 16 + 5 is true or false, students who are taught to think about the relationship between 18 and 16, notice that 18 is 2 more than 16, and reason that it must be true because 5 is 2 more than 3. Unfortunately, if students are not taught to look at equations relationally, then the transformations between 18 and 16 simply become proceduralized and learned as memorized rules [52]. This strategy shows a level of relational thinking in which students use number relations to make the problem more manageable. Thinking relationally, therefore, is different from applying a collection of memorized mathematical rules and procedures [56]. Students who think relationally identify number relations and reason about which transformations make sense in a particular problem [42].

Providing students with true or false equations can be another way to press students to think about number relationships. Equations such as 14 + 18 = 13 + 17 are more compatible with instructing students to see number relationships because a numerical answer is not required. Engaging students in a discussion of how the numbers relate to each other to determine whether the equation is true or false strengthens their conceptual understandings of equivalence (Carpenter [47]). Students with sufficient conceptual knowledge of how these number properties are applied have the understanding to transfer their procedural knowledge of mathematical equations to algebraic thinking [48]. Meaningful discussions about number relationships and the transferability of those ideas helps students make more mathematical generalizations [39].

4. Enactive, iconic, and symbolic representations

Bruner's modes of representations begin with the enactive, which includes manipulatives, or concrete, physical objects. The second representation is iconic, which represents any visual representations like diagrams, number lines, bar models, and graphs. The third representation is symbolic, which are abstract symbols like equations and algorithms. According to Bruner [2], students access their background knowledge of the representations to help make connections when the abstract symbols are isolated from other contexts. Concrete materials provide an opportunity for students to build background knowledge with iconic images depicting the meaning of the abstract symbols. When new abstract symbols are introduced, students can use their visual background knowledge as a retrieval mechanism to help remind them of the relevant concepts.

Instructional tasks heavily focused on abstract symbols tend to draw out the use of rote, memorized skill practice, which has been shown to compete with the development of spatial reasoning skills [57]. One way to help students make connections between numbers and symbols is to incorporate concrete materials

for students to manipulate during their practice and application [58]. Including concrete manipulatives for mathematical tasks has been shown to improve student understanding and retention of the practiced concept [59]. The use of concrete materials in isolation does not always guarantee that students will flexibly transfer the concrete representation to the symbolic representations [60]. Solving problems strictly in symbolic form leads to inefficient solution strategies, entrenchment of operational procedures, and inconsistent errors [42, 54, 57]. As a whole, mathematics instruction that isolates the symbolic representations leads students to manipulate symbols without conceptual understanding and a weakened ability to solve problems outside of their procedural understandings [61]. Alternatively, instruction designed to include a progression of representations beginning with an enactive or physical model to then an iconic or visual representation to a symbolic form can support a deep understanding of the mathematics [2, 62, 63].

Many students have difficulty in understanding concepts without being able to first visualize an idea in their mind [3]. Visualization helps students use figures or shapes in their mind to recall, understand, make connections, clarify, and remember new information [64]. Mathematics curricula loaded with only symbolic representations require students to memorize procedures, denying the student an opportunity to utilize their visual thinking in the process of building conceptual understanding. However, including visual representations into daily mathematics lessons can support the learning process and increase conceptual understandings [65].

Strong visualization and spatial reasoning skills contribute greatly to students' ability to organize the structure of equations and mathematics [66]. Mathematical models can be a way to connect one's visualization to their understandings of the problem [67]. The model connects the visualization into the spatial layout of an equation so students can devise a solution to solve the problem [68]. As students visualize the problem, they flexibly decode the context into the spatial layout of an equation [69].

When given the opportunity, students can develop the necessary spatial skills to visualize mathematics. Gesturing assists students to communicate their thinking. Mental rotation and spatial visualization can strengthen students' ability to solve nontraditional equations. Therefore, promoting spatial reasoning and modeling (EIS) early on in students' learning can promote mathematical competency and algebraic thinking.

5. Developing mathematical thinking

Curriculum should include ways to promote spatial reasoning through mathematical modeling to develop students' conceptual understandings [47, 70]. Mathematical tasks should include both traditional and nontraditional equations [44, 46]. The use of mathematical modeling should connect through a progression of enactive models, iconic models, and formal, symbolic models. Iconic models are one way to introduce spatial reasoning tasks and can be integrated throughout the instructional year to increase students' flexibility with the structure of equations and mathematical competency [8, 34].

The Developing Mathematical Thinking Institute (DMTI) offers a comprehensive curriculum designed to encompass all of these components for students to develop procedural and conceptual understandings. The DMTI curriculum is an alternative to the typical curriculum for teaching mathematics to help teachers develop a different approach to how mathematics is taught [71].

The DMT framework consists of five key elements for teachers to reflect upon as they plan, prepare, and instruct mathematics lessons: taking student's ideas

seriously, encouraging multiple solution strategies and models, pressing students conceptually, addressing misconceptions, and maintaining a focus on the structure of the mathematics [72, 73]. Using students' informal strategies values their thinking and gives the teacher insight as to the level of understanding each student has. Teachers use the five elements of the DMT to develop more efficient strategies and multiple models for solutions to mathematical problems. Students are encouraged to talk with others about their thinking, compare solutions, and make corrections to their errors. One of the most critical components of the framework is to draw attention to the structural components in mathematics.

One of the ways the DMTI curriculum builds student thinking is through the inclusion of Bruner's [2] enactive, iconic, and symbolic models. Each module is comprised of lessons with tasks centered on the EIS framework to develop a strong foundation for the development of conceptual understanding and for solving problems [72]. For example, students in first grade are given a contextual problem about 10 children playing in sandbox, where they need to determine whether six of the children are boys, and then how many children are girls? Students first demonstrate their thinking using unifix cubes, followed by drawing an iconic bar model to match their unifix cubes model. The symbolic representation of the numbers is then attached with labels. For example, to highlight the variety of ways to represent the number 10, students are asked to demonstrate the other possible representations for making 10 following the EIS progression. Modeling all of the possible combinations for 10 emphasizes the idea of equivalence, and using the EIS progression helps all students to visualize how the numbers relate to one another. **Figure 1** provides a sample solution for the students to use as a model.

As students become fluent with facts within 10, they are introduced to the variety of ways to compose the teen numbers using units of tens and ones. For example, one task is to represent each teen number using units of one. Eventually, students begin to recognize the inefficiency of counting each unit of one. At that point, the teacher introduces a more efficient way of building the teen numbers by using a unit of 10. Over time, students independently build efficient models for larger numbers based on their previous experiences building with units of one. Once again, tasks such as these expose students to relational thinking and highlight the structure of equivalence through the use of mathematical modeling.

The DMTI curriculum encourages students to represent solutions to contextual problems, explain their solutions, and then generalize their understandings to other concepts (see **Figure 2**). An example of this is with contextual compare problems presented in Module 3 where students represent the number of blocks used to build two different towers. The task states that one tower is eight blocks tall, and another

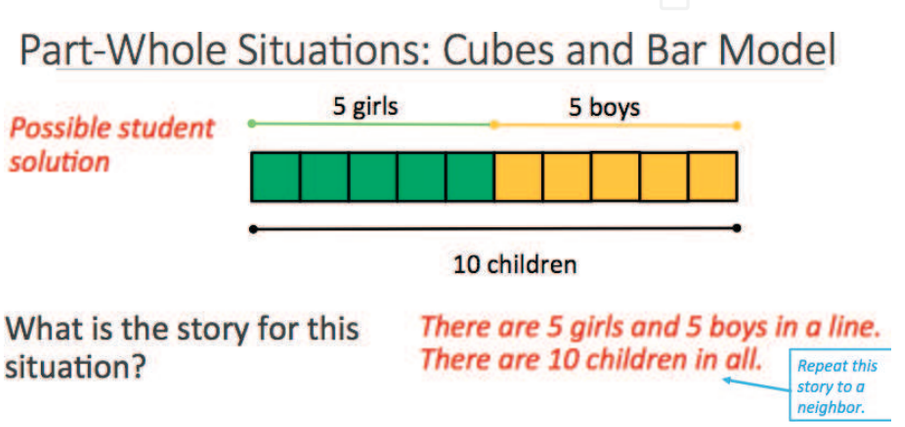

Figure 1.
Sample solution for making 10 in Module 3 of the DMTI curriculum.

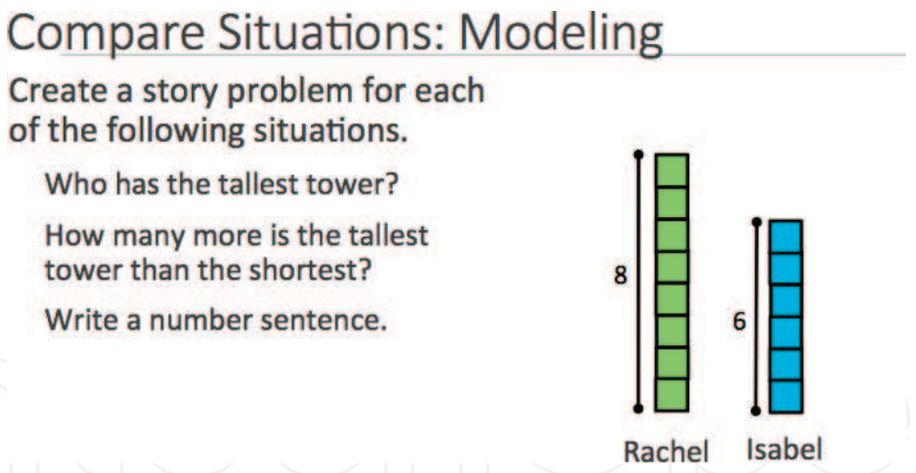

Figure 2.
Example of student work mat from Module 3 of the DMTI curriculum.

tower is six blocks tall. Students are asked to represent both towers using unifix cubes and determine whose tower is tallest and by how much. Next, students draw an iconic representation of the towers, paying attention to the spatial relationship between the number seven and four. The drawing should depict that one tower is taller than the other, and the enactive model is used to determine the difference between the numbers seven and four. Last, students connect their understandings of the relationship between the two towers back to the symbolic representation by notating 8–6 = 2. As students fluently build models to represent the context, they are then asked to look at a given set of numbers, build the models with unifix cubes to match, draw an iconic representation of the models, and create their own story to match their model. Students work in partners to listen to the story, but then also explain the relationships between the two towers. With this activity, students often times gesture with their hands to explain how many more blocks are in one tower than the other tower.

As suggested by NCTM [21], the DMTI curriculum intentionally focuses on building students' conceptual understandings of mathematical concepts through spatial reasoning tasks. Each task presents students with meaningful problem-solving situations where they are encouraged to begin to represent their thinking through enactive mathematical modeling, followed by an iconic representation depicting their thinking, and lastly with a connection to the symbolic representation of the problem. Students are encouraged to communicate their thinking with partners to check for understanding or assessing any misconceptions that may arise. The structural components are intentionally highlighted within each lesson to foster deep conceptual understanding and help students generalize their knowledge to other tasks throughout the year. Overall, the DMT framework delivers a comprehensive curriculum designed to increase students' mathematical understanding and improve spatial reasoning.

6. Summary

Most elementary students begin to develop their awareness of the equal sign's functionality at an operational level, where the equal sign acts as a symbol to perform a calculation or action [42]. When the bulk of instruction is focused on procedures and computing facts, many elementary students develop a shallow understanding of the equal sign and consider it an operational symbol [45, 46]. Mathematics instruction for early elementary classrooms should foster relational

thinking by including tasks designed to draw attention to how numbers relate to one another and develop the flexibility to think of numbers in a variety of ways to establish the idea of equivalence [8, 48]. Mathematical tasks should include both traditional and nontraditional equations [44, 46].

As educators become more aware of the need for relational thinking tasks, it is important to recognize the critical role spatial reasoning and mathematical modeling play in the overall development of algebraic thinking and the equal sign. The National Research Council report [1] and the National Council of Teachers of Mathematics [21] suggest more spatial reasoning be integrated into the elementary mathematics curriculum to promote relational thinking skills. Spatial visualization, gesturing, and mental rotation have been shown to increase student performance in mathematics [8].

Mathematical modeling gives students a visual representation to explain their mathematical thinking [74]. The use of mathematical modeling should connect through a progression of concrete representations, visual or iconic representations to more formal, and abstract representations [62]. We will examine whether curriculum that supports students' conceptual understandings through the integration of relational thinking, spatial reasoning, and mathematical models by incorporating Bruner's EIS framework improves students' spatial reasoning and relational thinking.

7. Overview of the study

This study was conducted to investigate whether there was a significant difference in first grade students' performance in spatial reasoning when they learn to construct and compare numbers using iconic modeling. The study examined spatial reasoning for first grade students whose teachers either received a curriculum built on the use of enactive, iconic, and symbolic representations (EIS group) and an adopted traditional curriculum (traditional group). Students in both groups were tested using the Primary Mathematics Assessment Screener [75] in September, prior to the mathematics instruction, and again mid-May after the mathematics instruction. Student performance was compared across time. Thus, this study used a 2 (EIS group versus comparison group) × 2 (pretest versus posttest) design. The dependent variable was the students' knowledge of spatial reasoning measured with the PMA-S. The goal of this study was to determine whether student achievement on the PMA-S differed between the EIS and traditional groups and whether achievement differed across time. The following research question was investigated: What is the effect of integrating iconic representations through student drawings in conjunction with the enactive, iconic, and symbolic teaching methodology into mathematics instruction on first grade students' spatial reasoning and relational thinking performance?

The study consisted of first grade classrooms from five school districts. Two of the school districts serve between 15,650 and 26,240 students, and three of the districts serve between 600 and 1725 students. There were over 2600 students with Limited English Proficiency (LEP) comprising approximately 8% of the total districts. In these districts, the student demographics were 79.3% white, 10.3% Hispanic/Latino, 5.9% Asian, 3.3% black, 0.9% Native American, and 0.8% Pacific Islander. First, grade classrooms were chosen on the basis of similarly matched demographics related to students who received free and reduced lunch assistance. There were 10 teachers in the EIS treatment group and 12 teachers in the traditional comparison group. The treatment group used the DMTI curriculum [76], and the comparison group used Bridges in Mathematics [77], and Math in Focus, Singapore Math [78].

The Primary Mathematics Assessment [75] is a formative assessment that includes a screener and six diagnostic measures. The PMA-Screener (PMA-S) builds

a profile of students' strengths and weaknesses for six dimensions: number sense and sequencing, number facts, contextual problems, relational thinking, measurement, and spatial reasoning.

One of the diagnostics includes a series of questions for shape composition. There are three subsections which include—shape composition without the need to rotate, composing a figure requiring overlapping of pieces during translations, and composing a figure by filling in a missing space.

A two-way design was used to explore the main effects on the different treatments, EIS instruction and Traditional instruction and their interactions under different conditions, pretest and posttest. The research question was analyzed using a 2 × 2 analysis of variance (ANOVA) to explore whether scores on the pre and posttest was dependent upon the type of instruction. Repeated measure analysis of variance (ANOVA) allows a look at change over time using the PMA-S given two times over 9 months of instruction with different conditions (EIS and traditional instruction). Main effects and interactions were analyzed on the independent variables (EIS and traditional instruction and time) from the dependent variable PMA-S scores.

8. Findings

A two-way repeated measure ANOVA was conducted to determine whether there was a significant difference in growth between the EIS group and the traditional group for relational thinking and spatial reasoning. The PMA-S screened four other subset dimensions, facts, context, sequence, and measurement, which were not included in the design of the study.

For the relational thinking subtest, there was a main effect for TIME with a statistically significant difference for both groups (EIS and traditional)—scores increase from pretest to posttest, $F(1, 449) = 105.2$, $MSe = 0.9$, $p < 0.001$. There is also a main effect for groups with a statistically significant difference between EIS and traditional, $F(1, 449) = 5.6$, $MSe = 1.2$, $p = 0.019$.

There was a statistically significant interaction between both groups and time on relational thinking, $F(1, 449) = 13.2$, $MSe = 0.9$, $p < 0.001$, $\eta^2 = 0.03$. This indicates that the difference between the change in students' knowledge of relational thinking in the EIS and traditional groups was dependent upon the type of mathematical instruction. Based on the profile plots of estimated marginal means of relational thinking in **Figure 3**, EIS (group 1) and traditional (group 2), EIS and traditional groups' trajectories indicate different patterns of mean scores over time. The p-value for the two-way interaction effect is <0.001, indicating mean relational thinking changed differently over time depending on whether students were in EIS or traditional.

To better understand the interaction, tests of simple effects were conducted. These results showed for the EIS group, scores on the relational thinking scale increased significantly from pretest to posttest, $t(242) = 10.2$, $p < 0.001$. For the traditional group, scores on the relational thinking scale also increased significantly from pretest to posttest, $t(242) = 4.6$, $p < 0.001$. Thus, for both groups, scores increased from pretest to posttest. The EIS and traditional groups were also compared separately on the pretest and then on the posttest. These results showed that for the pretest, the groups differed significantly, $t(449) = 4.5$, $p < 0.001$. For the posttest, the groups were not significantly different, $t(449) = 0.53$, $p = 0.6$. For the pretest, scores were greater for the traditional group than for the EIS group.

Taken all together, the results of these analyses show that scores on the relational thinking subtest scores did not differ across groups. However, significant

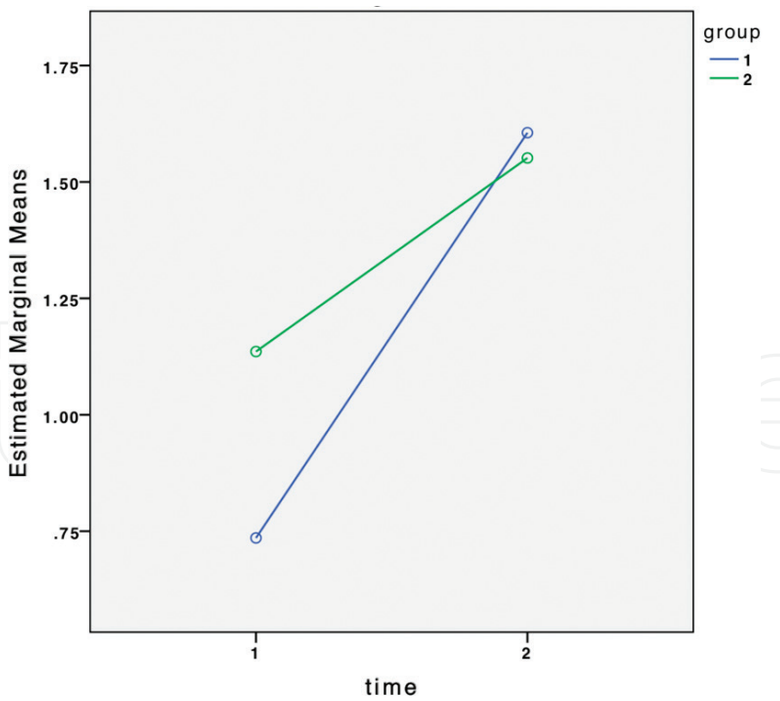

Figure 3.
Estimated marginal means of relational thinking.

interaction suggests that the change from pretest to posttest was not the same for the two groups. As seen in **Table 1**, the change was greater for the EIS group than for the traditional group. The EIS group began the study with significantly lower scores on the relational thinking subtests. The EIS group shows statistically higher gains than the traditional, thus confirming EIS has an effect.

For the spatial reasoning diagnostic, there was a main effect for TIME with a statistically significant difference for both groups (EIS and traditional)—scores increased from pretest to posttest, $F(1, 449) = 85.2$, $MSe = 0.6$, $p < 0.001$. There was also a main effect for groups with a statistically significant difference between EIS and traditional, $F(1, 449) = 3.9$, $MSe = 0.9$, $p = 0.05$.

There was a marginal significant interaction between both groups and time on spatial reasoning, $F(1, 449) = 3.3$, $MSe = 0.6$, $p < 0.071$, $\eta^2 = 0.01$. This indicates that the difference between the change in students' knowledge of spatial reasoning in the EIS and traditional groups was dependent upon the type of mathematical instruction. Based on the profile plots of estimated marginal means of spatial reasoning (**Figure 4**), EIS and traditional groups' trajectories indicate slightly different patterns of mean scores over time.

To better understand the interaction, tests of simple effects were conducted. These results showed for the EIS group, scores on the spatial reasoning scale increased significantly from pretest to posttest, $t(207) = 7.4$, $p < 0.001$. For

Relational thinking				
	Pretest		**Posttest**	
Group	**Mean**	**SD**	**Mean**	**SD**
EIS	0.74	0.77	1.61	1.2
Traditional	1.14	1.1	1.55	1.1

Table 1.
Relational thinking descriptive statistics.

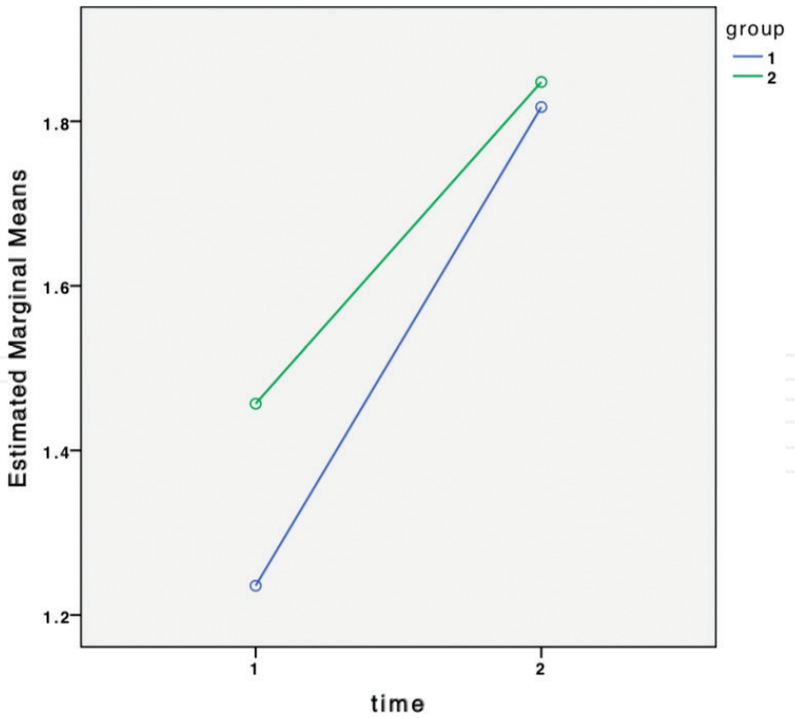

Figure 4.
Estimated marginal means of spatial reasoning.

the traditional group, scores on the spatial reasoning scale also increased significantly from pretest to posttest, $t(242) = 5.5$, $p < 0.001$. Thus, for both groups, scores increased from pretest to posttest. The EIS and traditional groups were also compared separately on the pretest and then on the posttest. These results showed that for the pretest, the groups differed significantly, $t(449) = 2.8$, $p < 0.01$. For the posttest, the groups were not significantly different, $t(449) = 0.36$, $p = 0.72$. For the pretest, scores were greater for the traditional than for the EIS group, and on the posttest, scores were the same across both groups.

Taken together, the results of these analyses show that scores on the spatial reasoning subtest were equal on the posttest across both groups. However, the marginally significant interaction suggests that the change from pretest to posttest was not the same for the two groups. As seen in **Table 2**, the change was greater for the EIS group than for the traditional group. The EIS group began the study with significantly lower scores on the spatial reasoning subtests. The EIS group shows statistically higher gains than the traditional, thus confirming EIS has an effect.

In summary, the instructional method (EIS vs. traditional) did have a significant effect on first grade students' spatial reasoning. The study demonstrated statistical significance between the treatment groups who implemented the EIS instruction and comparison group who used traditional mathematics instruction. The next

Spatial reasoning				
	Pretest		**Posttest**	
Group	**Mean**	**SD**	**Mean**	**SD**
EIS	1.24	0.803	1.82	0.871
Traditional	1.46	0.905	1.85	0.912

Table 2.
Spatial reasoning descriptive statistics.

section will provide details of the interpretation of findings, practical implications for educators, and recommendations for further study.

9. Interpretation of findings

The primary focus of the study was to look at the effects on students' conceptual understandings of relational thinking and spatial reasoning when integrating the EIS representations into first grade mathematics lessons. As Cheng and Mix [8] revealed through their research, the need to integrate spatial reasoning tasks is critical for the development of students' conceptual knowledge. Similar claims can be made based on the results of this study.

The EIS group performed statistically higher in relational thinking than the traditional group, doubling mean scores from pretest (0.74) to posttest (1.27). Previous work has shown students who are instructed to solve equations strictly in symbolic form struggle with algebraic thinking [79]. Integrating EIS representation into first grade mathematics lessons with a balanced set of equations has shown to be effective at developing students' relational thinking and spatial reasoning.

As Cheng and Mix [8] revealed through their research, the need to integrate spatial reasoning tasks is critical for the development of students' conceptual knowledge. Similar claims can be made based on the results from this study. We conclude that the integration of spatial reasoning had positive effects on first grade students' spatial reasoning skills, relational thinking, the development of conceptual understanding, and mathematical competency.

The findings support the notion that the integration of EIS representation into mathematics lessons offers students sufficient conceptual knowledge to develop number operations and mathematical competency [48]. Gain scores in facts and context are found to be consistent with earlier works from Carbonneau and colleagues [61], who suggests mathematics instruction should refrain from isolated skill and procedural practice in lieu of the development of conceptual understanding. Curriculum designed to include a progression of enactive, iconic, and symbolic models supports students' conceptual understanding [2, 62, 63]. Students in the EIS group were instructed to enactively build and iconically represent their math facts simultaneously. In doing so, they increased their conceptual understanding of the mathematics. K-12 reform has included an integration of meaningful lessons designed to enhance algebraic thinking across all mathematical domains, and altering the curriculum to include spatial reasoning tasks has shown to improve mathematical performance [54]. Our investigation has demonstrated a positive effect on students' spatial reasoning, relational thinking, and overall mathematical competency when first grade mathematics lessons integrate EIS representations.

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