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Chapter 3

Multicriteria Support for Group Decision Making

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Abstract

This chapter presents the support method for group decision making. A group decision is when a group of people has to make one joint decision. Each member of the group has his own assessment of a joint decision. The decision making of a group decision is modeled as a multicriteria optimization problem where the respective evaluation functions are the assessment of a joint decision by each member. The interactive analysis that is based on the reference point method applied to the multicriteria problems allows to find effective solutions matching the group's preferences. Each member of the group is able to verify results of every decision. The chapter presents an example of an application of the support method in the selection of the group decision.

Keywords: multicriteria optimization problem, equitably efficient decision, scalarizing function, decision support systems

1. Introduction

The chapter presents the support method for group decision making—when a group of people who have different preferences want to make one joint decision.

The selection process of a group decision can be modeled with the use of game theory [1–3].

In this chapter, the choice of the group decision is modeled as a multicriteria problem. The individual coordinates of this optimization problem are functions to evaluate a joint decision by each person in the group. This allows one to take into account preferences of all members in the group. Decision support is an interactive process of proposals for subsequent decisions, by each member in the group and his evaluations. These proposals are parameters of the multicriteria optimization problem. The solution of this problem is assessed by members in

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the group. Each member can accept or refuse the solution. In the second case, a member gives his new proposal and the problem is resolved again.

2. Modeling of group decision making

The problem of choosing a group decision is as follows. There is a group of k members. There is given a set X_0—the feasible set. For each member i, $i = 1, 2, \ldots, k$, a decision evaluation function f_i is defined, which is an assessment of a joint decision. The assessment of the joint decision is to be made by all members in the group.

The problem of group decision making is modeled as multicriteria optimization problem:

$$\max_x \{ (f_1(x), \ldots, f_k(x)) : \quad x \in X_0 \}, \tag{1}$$

where $1, 2, \ldots, k$ are particular members, $X_0 \subset R^n$ is the feasible set, $x = (x_1, x_2, \ldots, x_n) \in X_0$ is a group decision, $f = (f_1, f_2, \ldots, f_k)$ is the vector function that maps the decision space $X_0 = R^n$ into the criteria space $Y_0 = R^k$, and specific coordinates $y_i = f_i(x),\ \ i = 1, 2, .., m$ represent the scalar evaluation functions—the result of a decision x $i - th$ member $i = 1, 2, \ldots, k$.

The purpose of the problem (1) is to support the decision process to make a decision that will be the most satisfactory for all members in the group.

Functions $f_1, \ldots, f_k$ introduce a certain order in the set of decision variables—preference relations:

$$x^1 \succ x^2 \Leftrightarrow f_1(x^1) \geq f_2(x^2), \ldots, f_k(x^1) \geq f_k(x^2) \quad \wedge \quad \exists j\, f_j(x^1) > f_j(x^2). \tag{2}$$

At point x^1, all functions have values greater than or equal to the value at point x^2, and at least one is greater.

The multicriteria optimization model (1) can be rewritten in the equivalent form in the space of evaluations. Consider the following problem:

$$\max_x \{ (y_1, \ldots, y_k) : \quad y \in Y_0 \}, \tag{3}$$

where $x \in X$ is a vector of decision variables, $y = (y_1, \ldots, y_k)$ is the evaluation vector and particular coordinates y_i represent the result of a decision x $i - th$ member $i = 1, 2, \ldots, k$, and $Y_0 = f(X_0)$ is the set of evaluation vectors.

The vector function $y = f(x)$ assigns to each vector of decision variables x an evaluation vector $y \in Y_0$ that measures the quality of decision x from the point of view of all members in the group. The set of results achieved Y_0 is given in the implicit form—through a set of feasible decisions X_0 and the mapping of a model $f = (f_1, f_2, \ldots, f_k)$. To determine the value y, the simulation of the model is necessary: $y = f(x)\ \ for \quad x \in X_0$.

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3. Equitably efficient decision

Group decision making is modeled as a special multicriteria optimization problem—the solution should have the feature of anonymity—no distinction is made between the results that differ in the orientation coordinates and the principle of transfers. This solution of the problem is named an equitably efficient decision. It is an efficient decision that satisfies the additional property-the property of preference relation anonymity and the principle of transfers.

Nondominated solutions (optimum Pareto) are defined with the use of preference relations which answer the question: which one of the given pair of evaluation vectors $y^1, y^2 \in R^k$ is better? This is the following relation:

$$y^1 \succ y^2 \Leftrightarrow y_i^1 \geq y_i^2 \forall i = 1, \dots, m \;\; \wedge \;\; \exists \mathrm{j} \;\; y_j^1 > y_j^2. \tag{4}$$

The vector of evaluation $\hat{y} \in Y_0$ is called the nondominated vector; if there is no such vector $y \in Y_0$, that $\hat{y}$ is dominated by y. Appropriate acceptable decisions are specified in the decision space. The decision $\hat{x} \in X_0$ is called efficient decision (Pareto efficient) if the corresponding vector of evaluations $\hat{y} = f(\hat{x})$ is a nondominated vector [4, 5].

In the multicriteria problem (1), which is used to make a group decision for a given set of the evaluation functions, only the set of the evaluation functions is important without taking into account which function is taking a specific value. No distinction is made between the results that differ in the arrangement. This requirement is formulated as the property of anonymity of preference relation.

The relation is called an anonymous (symmetric) relation if, for every vector $y = (y_1, y_2, \dots, y_k) \in R^k$ and for any permutation P of the set $\{1, \dots, k\}$, the following property holds:

$$\left(y_{P(1)}, y_{P(2)}, \dots, y_{P(k)}\right) \approx (y_1, y_2, \dots, y_k) \tag{5}$$

The relation of preferences that would satisfy the anonymity property is called symmetrical relation. Evaluation vectors having the same coordinates, but in a different order, are identified. A nondominated vector satisfying the anonymity property is called symmetrically nondominated vector.

Moreover, the preference model in group decision making should satisfy the principle of transfers. This principle states that the transfer of small amount from an evaluation vector to any relatively worse evaluation vector results in a more preferred evaluation vector. The relation of preferences satisfies the principle of transfers, if the following condition is satisfied:

for the evaluation vector $y = (y_1, y_2, \dots, y_k) \in R^k$:

$$y_{i'} > y_{i''} \Rightarrow y - \varepsilon \cdot e_{i'} + \varepsilon \cdot e_{i''} \succ y \;\; \text{for } 0 < y_{i'} - y_{i''} < \varepsilon \tag{6}$$

Equalizing transfer is a slight deterioration of a better coordinate of evaluation vector and, simultaneously, improvement of a poorer coordinate. The resulting evaluation vector is strictly preferred in comparison to the initial evaluation vector. This is a structure of equalizing—the evaluation vector with less diversity of coordinates is preferred in relation to the vector with the same sum of coordinates, but with their greater diversity.

A nondominated vector satisfying the anonymity property and the principle of transfers is called equitably nondominated vector. The set of equitably nondominated vectors is denoted by $\widehat{Y}_{0E}$. In the decision space, the equitably efficient decisions are specified. The decision $\widehat{x} \in X_0$ is called an equitably efficient decision, if the corresponding evaluation vector $\widehat{y} = f(\widehat{x})$ is an equitably nondominated vector. The set of equitably efficient decisions is denoted by $\widehat{X}_{0E}$ [2, 6, 7].

Equitable dominance can be expressed as the relation of inequality for cumulative, ordered evaluation vectors. This relation can be determined with the use of mapping $\overline{T} : R^k \rightarrow R^k$ that cumulates nonincreasing coordinates of evaluation vector.

The transformation $\overline{T} : R^k \rightarrow R^k$ is defined as follows:

$$\overline{T}_i(y) = \sum_{l=1}^{i} T_i(y) \quad \text{for } i = 1, 2, \ldots, k. \tag{7}$$

Define by $T(y)$ the vector with nonincreasing ordered coordinates of the vector y, i.e. $T(y) = (T_1(y), T_2(y), \ldots, T_k(y))$, where $T_1(y) \leq T_2(y) \leq \ldots \leq T_k(y)$ and there is a permutation P of the set $\{1, \ldots, k\}$, such that $T_i(y) = y_{P(i)}$ for $i = 1, .., k$.

The relation of equitable domination $\succ_e$ is a simple vector domination for evaluation vectors with cumulated nonincreasing coordinates of evaluation vector [6, 7].

The evaluation vector y^1 equitably dominates the vector y^2 if the following condition is satisfied:

$$y^1 \succ_e y^2 \Leftrightarrow \overline{T}\left(y^1\right) \geq \overline{T}\left(y^2\right) \tag{8}$$

The solution of choosing a group decision is to find the equitably efficient decision that best reflects the preferences of all members in the group.

4. Technique of generating equitably efficient decisions

Equitably efficient decisions for a multiple criteria problem (1) are obtained by solving a special problem in multicriteria optimization—a problem with the vector function of the cumulative, evaluation vectors arranged in a nonincreasing order. This is the following problem.

$$\max_y \left\{ \left(\overline{T}_1(y), \overline{T}_2(y), \ldots, \overline{T}_k(y)\right) : \quad y \in Y_0 \right. \tag{9}$$

where

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$y = (y_1, y_2, \ldots, y_k)$ is the evaluation vector, $\overline{T}(y) = (\overline{T}_1(y), \overline{T}_2(y), \ldots, \overline{T}_k(y))$ is the cumulative, ordered evaluation vector, and Y_0 is the set of achievable evaluation vectors.

The efficient solution of multicriteria optimization problem (9) is an equitably efficient solution of the multicriteria problem (1).

To determine the solution of a multicriteria problem (9), the scalaring of this problem with the scalaring function $s : Y_0 \times \Omega \to R^1$ is solved:

$$\max_x \{s(y, \overline{y}) : x \in X_o, \tag{10}$$

where $y = (y_1, y_2, \ldots, y_k)$ is the evaluation vector and $\overline{y} = (\overline{y}_1, \overline{y}_2, \ldots, \overline{y}_k)$ is the control parameter for individual evaluations.

It is the problem of single-objective optimization with specially created scalaring function of two variables—the evaluation vector $y \in Y$ and control parameter $\overline{y} \in \Omega \subset R^k$; we have thus $s : Y_0 \times \Omega \to R^1$. The parameter $\overline{y} = (\overline{y}_1, \overline{y}_2, \ldots, \overline{y}_k)$ is available to each member in the group that allows any member to review the set of equitably efficient solutions.

Complete and sufficient parameterization of the set of equitably efficient decision $\widehat{X}_{0E}$ can be achieved, using the method of the reference point for the problem (9). In this method the aspiration levels are applied as control parameters. Aspiration level is the value of the evaluation function that satisfies a given member.

The scalaring function defined in the method of reference point is as follows:

$$s(y, \overline{y}) = \min_{1 \le i \le k} \left(\overline{T}_i(y) - \overline{T}_i(\overline{y})_i\right) + \varepsilon \cdot \sum_{i=1}^{k} \left(\overline{T}_i(y) - \overline{T}_i(\overline{y})_i\right), \tag{11}$$

where $y = (y_1, y_2, \ldots, y_k)$ is the evaluation vector; $\overline{T}(y) = (\overline{T}_1(y), \overline{T}_2(y), \ldots, \overline{T}_k(y))$ is the cumulative, ordered evaluation vector; $\overline{y} = (\overline{y}_1, \overline{y}_2, \ldots, \overline{y}_k)$ is the vector of aspiration levels; $T(\overline{y}) = (T_1(\overline{y}), T_2(\overline{y}), \ldots, T_k(\overline{y}))$ is the cumulative, ordered vector of aspiration levels; and ε is the arbitrary, small, positive adjustment parameter.

This function is called a function of achievement. Maximizing this function with respect to y determines equitably nondominated vectors $\widehat{y}$ and the equitably efficient decision $\widehat{x}$. For any aspiration levels $\overline{y}$, each maximal point $\widehat{y}$ of this function is an equitably nondominated solution. Note, the equitably efficient solution $\widehat{x}$ depends on the aspiration levels $\overline{y}$. If the aspiration levels $\overline{y}$ are too high, then the maximum of this function is smaller than zero. If the aspiration levels $\overline{y}$ are too low, then the maximum of this function is larger than zero. This is the information for the group, whether a given aspiration level is reachable or not [4, 8].

A tool for searching the set of solutions is the function (11). Maximum of this function depends on the parameter $\overline{y}$, which is used by the members of the group to select a solution. The method for supporting selection of group decisions is as follows:

- Calculations—giving other equitably efficient decisions
- Interaction with the system—dialog with the members of the group, which is a source of additional information about the preferences of the group

The method of selecting group decision is presented in **Figure 1**.

The computer will not replace members of the group in the decision-making process; the whole process of selecting a decision is guided by all members in the group.

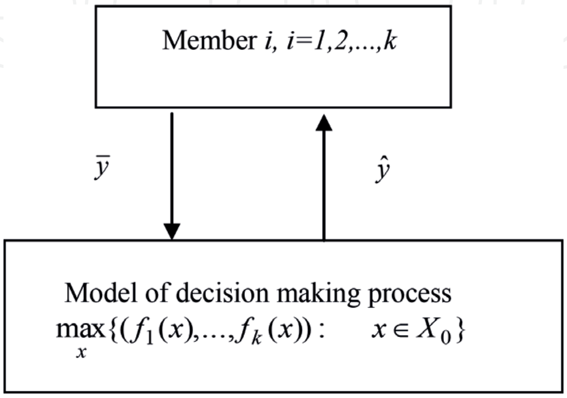

Figure 1. The method of selecting group decision.

5. Example

To illustrate the process of supporting group decision making, the following example is presented—selection of group decision by three members [8].

The problem of selecting the decision is the following:

1, 2, 3 are the members in the group.

$X_0 = x \in R^2: \; x_1 + 5 \cdot x_2 \leq 75, \; 3 \cdot x_1 + 5 \cdot x_2 \leq 95, x_1 + x_2 \leq 25, \; 5 \cdot x_1 + 2 \cdot x_2 \leq 110, \; x_1 \geq 0, x_2 \geq 0$ is the feasible set.

$x = (x_1, x_2) \in X_0$ is a group decision, belonging to the feasible set.

$f_1(x) = 10 \cdot x_1 + 60 \cdot x_2$ is the function of decision evaluation x by member 1.

$f_1(x) = 40 \cdot x_1 + 60 \cdot x_2$ is the function of decision evaluation x by member 2.

$f_1(x) = 60 \cdot x_1 + 20 \cdot x_2$ is the function of decision evaluation x by member 3.

The problem of selection of group decision is expressed in the form of multicriteria optimization problem with three evaluation functions:

$$\max_x \{(10 \cdot x_1 + 60 \cdot x_2, 40 \cdot x_1 + 60 \cdot x_2, 60 \cdot x_1 + 20 \cdot x_2) \; x \in X_0\}, \tag{12}$$

where X_0 is the feasible set and $x = (x_1, x_2) \in X_0$ is a group decision.

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A solution which is as satisfying as possible for all members in the group is searched for. All members in the problem of decision making in a group should be treated in the same way, no member should be favored. The decision-making model should have the anonymity properties of preference relation and satisfy the principle of transfers. The solution of the problem should be an equitably efficient decision of the problem (12).

For solving the problem (12) the method of reference point is used.

At the beginning of the analysis, a separate single-criterion optimization is carried out for each member in the group. In this way, the best results for each member are obtained separately. This is a utopia point of the multicriteria optimization problem. This also gives information about the conflict of evaluations of group members in the decision-making problem [9, 10].

When analyzing **Table 1**, it might be observed that the big selection possibilities have members 2 and 3 and lower member 1.

For each iteration, the price of fairness (POF) for each member is calculated [4]. It is the quotient of the difference between the utopia value of a solution and the value from the solution of the multicriteria problem, in relation to the utopia value.

$$POF = \frac{yiu - \hat{y}i}{yiu}, i = 1, 2, 3, \tag{13}$$

where yiu is the utopia value of a member i, $i = 1, 2, 3$, and yiu is the value from the solution of the multicriteria problems of a member, $i\ i = 1, 2, 3$.

The value of the POFs is a number between 0 and 1. POF values closer to zero are preferred by the members, as the solution is closer to a utopia solution. The more the values of the POFs of the members get closer to each other, the better the solution.

People in the group do control the process by means of aspiration levels. The multicriteria analysis is presented in **Table 2**.

At the beginning of the analysis (Iteration 1), members in the group define their preferences as aspiration levels equal to the values of utopia. The obtained effective leveling solution is ideal for member 2, while member 1 and member 3 would like to correct their solutions. In the next iteration, all members reduce their levels of aspiration. As a result (Iteration 2), the solution for

Optimization criterion	**Solution**		
	$\hat{y}1$	$\hat{y}2$	$\hat{y}3$
Member's evaluation 1 $y1$	900	900	300
Member's evaluation 2 $y2$	750	1200	1100
Member's evaluation 3 $y3$	220	880	1320
Utopia vector	900	1200	1320

Table 1. Matrix of goal realization with the utopia vector.

	Iteration	Member 1	Member 2	Member 3
		$\hat{y}_1$	$\hat{y}_2$	$\hat{y}_2$
1.	Aspiration levels $\overline{y}$	900	1200	1320
	Solution $\hat{y}$	750	1200	1100
	POF	0.166	0	0.153
2.	Aspiration levels $\overline{y}$	850	1000	1200
	Solution $\hat{y}$	800	1192	1007
	POF	0.111	0.006	0.224
3.	Aspiration levels $\overline{y}$	850	1000	1250
	Solution $\hat{y}$	775	1196	1053
	POF	0.138	0.003	0.189
4.	Aspiration levels $\overline{y}$	850	1000	1300
	Solution $\hat{y}$	750	1200	1100
	POF	0.166	0	0.153
5.	Aspiration levels $\overline{y}$	850	990	1300
	Solution $\hat{y}$	755	1199	1090
	POF	0.161	0.0006	0.160

Table 2. Interactive analysis of seeking a solution.

member 1 has improved, while the solution for member 2 and member 3 has deteriorated. The group now wishes to correct the solution for member 3 and increases the aspiration level for member 3, but does not change the aspiration levels for members 1 and 2. As a result (Iteration 3), the solution for member 2 and member 3 has improved, while the solution for member 1 has deteriorated. The group still wishes to correct the solution for member 3 and provides a higher value of the aspiration level for member 3, but does not change the aspiration levels for members 1 and 2. As a result (Iteration 4), the solution for member 2 and member 3 has improved, but the solution for member 1 has deteriorated. The group now wishes to correct the solution for member 1 and member 3 and reduces the aspiration level for member 2, but does not reduce the aspiration levels for members 1 and 3. As a result (Iteration 5), the solution for member 1 has improved, while the solution for members 2 and 3 has deteriorated. A further change to the value of the aspiration levels causes either an improvement in the solution for member 1 and at the same time a deterioration in the solution for member 3 or vice versa, as well as slight changes in the solution for member 2. Such a solution results from the specific nature of the examined problem—the solution for member 2 lies between solutions for members 1 and 3. The group decision for Iteration 5 is as follows: $x^5 = (14.81, \quad 10.12)$.

The final choice of a specific solution depends on the preferences of the members in the group. This example shows that the presented method allows the members to get to know their decision-making possibilities within interactive analysis and to search for a solution that would be satisfactory for the group.

6. Summary

The chapter presents the method of supporting group decision making. The choice is made by solving the problem of multicriteria optimization.

The decision support process is not a one-step act, but an iterative process, and it proceeds as follows:

- Each member of the group participates in the decision-making process.
- Then, each member determines the aspiration levels for particular results of decisions. These aspiration levels are determined adaptively in the learning process.
- The decision choice is not a single optimization act, but a dynamic process of searching for solutions in which each member may change his preferences.
- This process ends when the group finds a decision that makes it possible to achieve results meeting the member's aspirations or closest to these aspirations in a sense.

This method allows the group to verify the effects of each decision and helps find the decision which is the best for their aspiration levels. This procedure does not replace the group in decision-making process. The whole decision-making process is controlled by all the members in the group.

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