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Nonlinear Radiative Heat Transfer of Cu-Water Nanoparticles over an Unsteady Rotating Flow under the Influence of Particle Shape

K. Ganesh Kumar, B.J. Gireesha and S. Manjunatha

Abstract

A 3D study on Cu-water-rotating nanofluid over a permeable surface in the presence of nonlinear radiation is presented. Particle shape and thermophysical properties are considered in this study. The governing equations in partial forms are reduced to a system of nonlinear ordinary differential equations using suitable similarity transformations. An effective Runge-Kutta-Fehlberg fourth-fifth order method along with shooting technique is applied to attain the solution. The effects of flow parameters on the flow field and heat transfer characteristics were obtained and are tabulated. Useful discussions were carried out with the help of plotted graphs and tables. It is found that the rate of heat transfer is more enhanced in column-shaped nanoparticles when compared to tetrahedron- and sphere-shaped nanoparticles. Higher values of rotating parameter enhance the velocity profile and corresponding boundary layer thickness. It has quite the opposite behavior in angular velocity profile. Further, unsteady parameter increases the velocity profile and corresponding boundary layer thickness.

Keywords: particle shape effect, nonlinear radiation, Cu-water nanoparticles, unsteady rotating flow

1. Introduction

The interaction of thermal radiation has increased greatly during the last decade due to its importance in many practical applications. We know that the radiation effect is important under many isothermal and nonisothermal situations. If the entire system involving the polymer extrusion process is placed in a thermally controlled environment, then radiation
could become important. The knowledge of radiation heat transfer in the system can, perhaps, lead to a desired product with a sought characteristic. Magnetohydrodynamic 3D flow of viscoelastic nanofluid in the presence of nonlinear thermal radiation has been examined by Hayat et al. [1]. Shehzad et al. [2] proposed the nonlinear thermal radiation in 3D flow of Jeffrey nanofluid. Refs. [3–11] are some of the works associated with stretching sheet problem of thermal radiation.

Nanotechnology has been widely utilized in the industries since materials with the size of nanometers possess distinctive physical and chemical properties. Nanofluids are literally a homogeneous mixture of base fluid and the nanoparticles. Common base fluids embody water, organic liquids, oil and lubricants, biofluids, polymeric solution and other common liquids. Nanoparticles are created from totally different materials, like oxides, nitrides, carbide, ceramics metals, carbons in varied (e.g., diamond, graphite, carbon nanotubes, fullerene) and functionalized nanoparticles. Nanofluids have novel properties that are potentially helpful in several applications in heat transfer, as well as microelectronics, pharmaceutical processes, heat exchanger, hybrid-powered engines, domestic refrigerator, fuel cells, cooling/vehicle thermal management, nuclear reactor agent, in grinding, in space technology, ships and in boiler flue gas temperature reduction. Choi [12] was the first who composed the analysis on nanoparticles in 1995. Later, Maiga et al. [13] initiated the heat transfer enhancement by using nanofluids in forced convective flows. The laminar fluid flow which results from the stretching of a flat surface in a nanofluid has been investigated by Khan and Pop [14]. Recently, a number of researchers are concentrating on nanofluid with different geometries; see [15–19].

Experimental studies have shown that the thermal conductivity of nanofluids is determined by the parameters related to: nanoparticles, concentration, size, spherical and nonspherical shapes, agglomeration (fractal-like shapes), surface charge and thermal conductivity, base fluids (e.g., thermal conductivity and viscosity), nanofluids (e.g., temperature), the interfacial chemical/physical effect or interaction between the particles and base fluid and others. For more details, readers are referred to the studies [20–25].

Impact of nonlinear thermal radiation on 3D flow and heat transfer of Cu-water nanoliquid over unsteady rotating flow have been considered. The heat transfer characteristics are studied in the presence of different particle shapes, thermophysical properties and nonlinear thermal radiation. The principal equations of continuity, momentum, energy and mass equations are transferred into a set of nonlinear similarity equalities by applying the appropriate transformations. The condensed equalities are solved numerically, and the impacts of relevant parameters are discussed through plotted graphs and tables.

2. Mathematical formulation

An unsteady laminar flow over a permeable surface in a rotating nanofluid is considered in this study. The copper-water motion is 3D due to Coriolis force in the present problem. The Cartesian coordinates are $x, y$ and $z$ where the rotation of the nanofluid is at an angular velocity $\Omega(t)$ about the $z$-axis, and time is denoted as $t$. Let $u_w(x, t) = \frac{\beta w}{1 + \beta}$ and $v_w(x, t)$ represent...
the surface velocity in x and y directions, respectively, and \( w_w(x, t) \) is the wall mass flux velocity in the z-direction as represented in Figure 1. Under these conditions, the governing equations can be written as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + 2\Omega v = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial z^2}, \tag{2}
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + 2\Omega v = - \frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 v}{\partial z^2}, \tag{3}
\]

\[
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = \frac{1}{\rho_{nf}} \frac{\partial q_r}{\partial z} + \frac{\alpha_{nf}}{\rho_{nf} c_p} \frac{\partial^2 T}{\partial z^2} \tag{4}
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha_{nf} \frac{\partial^2 T}{\partial z^2} + \frac{1}{\rho_{nf} c_p} \frac{\partial q_r}{\partial z}. \tag{5}
\]

Boundary conditions for the problem are,

\[
u = u_w(x, t) = v = 0, w = 0, T = T_w \text{ at } z = 0,
\]

\[
u \rightarrow 0, v \rightarrow 0, w \rightarrow 0, T \rightarrow T_\infty \text{ as } z \rightarrow \infty \tag{6}
\]

where velocity components in x, y and z directions are \( u, v \) and \( w \), constant angular velocity of the Nano fluid is \(\Omega\), dynamic viscosity of the Nano fluid is \(\mu_{nf}\), density of the nanofluid is \(\rho_{nf}\), thermal diffusivity of the nanofluid is \(\alpha_{nf}\), \( T \) is temperature of nanofluid and wall temperature is \( T_w \), \( T_\infty \) denotes temperature outside the surface (Table 1).

Figure 1. Influence of \( \Omega \) on \( f'(\eta) \).
The radiative heat flux expression in Eq. (5) is given by:

\[ q_r = -\frac{16\sigma^*}{3k^*}T^3\frac{\partial T}{\partial z} \]  

(7)

where \( \sigma^* \) and \( k^* \) are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively, and in view to Eq. (7), Eq. (4) reduces to:

\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{\partial}{\partial y} \left[ \left( \alpha_{nf} + \frac{16\sigma^* T^3}{3(\rho c_p)_nf k^*} \right) \frac{\partial T}{\partial y} \right]. \]

(8)

Parameters \( \mu_{nf}, \rho_{nf} \) and \( \alpha_{nf} \) are interrelated with nanoparticle volume fraction \( \phi \) and can be defined as:

\[ \rho_{nf} = \rho_f \left( 1 - \phi + \phi \left( \frac{\rho_s}{\rho_f} \right) \right), \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{\frac{2}{3}}}, \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_nf} \]

\[ (\rho c_p)_nf = (\rho c_p)_f \left( 1 - \phi + \phi \left( \frac{c_p s}{c_p f} \right) \right), k_{nf} = \frac{k_s + (m-1)k_f - (m-1)\phi(k_f - k_s)}{k_s + (m-1)k_f + \phi(k_f - k_s)} \]

(9)

where volumetric heat capacity of the solid nanoparticles is \( (\rho c_p)_s \), volumetric heat capacity of the base fluid is \( (\rho c_p)_f \), volumetric heat capacity of the nanofluid is \( (\rho c_p)_nf \), thermal conductivity of the nanofluid is \( k_{nf} \), thermal conductivity of the base fluid is \( k_f \), thermal conductivity of the solid nanoparticles is \( k_s \), nanoparticle volume fraction is \( \phi \), density viscosity of the base fluid is \( \mu_f \) and dynamic viscosity of the base fluid is \( \mu_f \) (Table 2).

Now, we introduce similarity transformations:

\[ u = \frac{bx}{1 - \delta t} f(\eta), \quad v = \frac{bx}{1 - \delta t} g(\eta), \quad w = -\sqrt{\frac{bx}{1 - \delta t}} f(\eta), \quad \eta = \sqrt{\frac{bx}{v(1 - \delta t)}} z, \quad \theta(\eta) = \frac{T - T_w}{T - T_w} \]

(10)

with \( T = T_w(1 + (\theta_w - 1)\theta) \) and \( \theta_w = \frac{T_w}{T} \), \( \theta_w > 1 \) is the temperature ratio parameter.

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( \rho_f )</th>
<th>( k_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper (Cu)</td>
<td>385</td>
<td>8933</td>
</tr>
<tr>
<td>Water</td>
<td>997.1</td>
<td>4179</td>
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Table 1. Thermophysical properties of water and nanoparticles.

<table>
<thead>
<tr>
<th>Particle shapes</th>
<th>Sphere</th>
<th>Tetrahedron</th>
<th>Column</th>
</tr>
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<tbody>
<tr>
<td>( m )</td>
<td>3</td>
<td>4.0613</td>
<td>6.3698</td>
</tr>
</tbody>
</table>

Table 2. Values of the empirical shape factor for different particle shapes.
Using Eqs. (2)–(6) and (10), we can have

\[
\frac{1}{(1 - \phi)^{2.5} (1 - \phi + \phi \left( \frac{\partial}{\partial \eta} \right)^n)} f'' - \left[ f'^2 - ff' - 2\Omega g + \lambda \left( \frac{\partial^2 f}{\partial \eta^2} \right) \right] = 0, \quad (11)
\]

\[
\frac{1}{(1 - \phi)^{2.5} (1 - \phi + \phi \left( \frac{\partial}{\partial \eta} \right)^n)} g'' - \left[ fg' - f g' - 2\Omega f + \lambda \left( \frac{\partial^2 f}{\partial \eta^2} \right) \right] = 0, \quad (12)
\]

\[
\frac{k_f}{k_f (1 - \phi + \phi \left( \frac{\partial}{\partial \eta} \right)^n)} \left[ 1 + R(1 + (\theta_w - 1)\theta) + \left( \frac{\partial^2 \theta}{\partial \eta^2} \right) \right] = 0, \quad (13)
\]

The transformed boundary conditions are as follows:

\[
f(0) = 0, f'(0) = 0, g(0) = 0, \theta(0) = 1 \quad \text{at} \quad \eta = 0
\]

\[
f'(\eta) \to 0, g(\eta) \to 0, \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty
\]

where \( \Omega = \frac{\omega}{\eta} \) is rotation rate, \( \lambda = \frac{\delta}{\eta} \) is unsteadiness parameter \( R = \frac{16\sigma^3 T^3}{\nu^3} \) is radiation parameter, \( Pr = \frac{a_n}{c_n} \) is Prandtl number and primes denote the differentiation with respect to \( \eta \).

Rosca et al. [11] mentioned that the pressure term \( p \) can be integrated from Eq. (4); thus, we obtain:

\[
p = \nu p \frac{\partial w}{\partial \zeta} - \frac{\rho w^2}{2} + c
\]

The physical quantities of interest in this problem are the skin friction coefficients in \( x \) and \( y \) directions, \( C_{fx} \) and \( C_{fy} \) as well as the local Nusselt number \( Nu_x \) which are defined as:

\[
C_{fx} = \frac{\tau_{ux}}{\rho u_w^2}, \quad C_{fy} = \frac{\tau_{uy}}{\rho u_w^2}, \quad Nu_x = \frac{\chi q_w}{k_f (T_w - T_{\infty})}
\]

Surface shear stress \( \tau_{ux}, \tau_{uy} \) and surface heat flux \( q_w \) are defined as:

\[
\tau_{ux} = \mu_{uf} \left( \frac{\partial u}{\partial \zeta} \right)_{z=0}, \quad \tau_{uy} = \mu_{uf} \left( \frac{\partial v}{\partial \zeta} \right)_{z=0}, \quad q_w = -k_f \left( \frac{\partial T}{\partial \zeta} \right)_{z=0}
\]

Using Eqs. (16) and (17), we obtain

\[
\sqrt{Re_x} C_{fx} = \frac{1}{(1 - \phi)^{2.5} f'(0)}, \quad \sqrt{Re_x} C_{fy} = \frac{1}{(1 - \phi)^{2.5} g'(0)} \quad \text{and} \quad Nu_x = \frac{k_f}{k_f} \left( \frac{1}{\left[ 1 + R \theta_w^3 \right] \theta' (0)} \right),
\]

where \( Re_x = \frac{u_w \eta}{v} \) is local Reynolds number.
3. Numerical method

Numerical solutions of nonlinear coupled differential Eqs. (11)–(13) subject to the boundary conditions (14) constitute a two-point boundary value problem. Due to coupled and highly nonlinear nature, which are not amenable to closed-form solutions; therefore, we resorted to numerical solutions. In order to solve these equations numerically, we follow most efficient fourth-fifth order Runge-Kutta-Fehlberg integration scheme along with shooting technique. In this method, it is most important to choose the appropriate finite values of $\eta_\infty$. The asymptotic boundary conditions at $\eta_\infty$ were replaced by those at $\eta_8$ in accordance with standard practice in the boundary layer analysis.

4. Result and discussion

To get a clear insight into the physical situation of the present problem, numerical values for velocity and temperature profile are computed for different values of dimensionless parameters using the method described in the previous section. The numerical results for the local Nusselt number are presented for different values of the governing parameters in Table 3.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\Omega$</th>
<th>$\text{Pr}$</th>
<th>$R$</th>
<th>$\theta_w$</th>
<th>$\phi$</th>
<th>Nusselt number</th>
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Table 3. Numerical values of Nusselt number for different physical parameters.
Figure 1 portrays the effect of $\Omega$ on velocity profile $f'(\eta)$. The velocity profile and corresponding thickness of the boundary layer enhance with larger values of $\Omega$. This is because the larger value of $\Omega$ parameter leads to higher rotation rate as compared to stretching rate. Therefore, the larger rotation effect enhances velocity field. **Figure 2** shows the impact of $\Omega$ on angular profile $g(\eta)$. From this figure, one can see that $g(\eta)$ reduces for larger values of $\Omega$. Further, it is noticed that rate of heat transfer is larger in column-shaped nanoparticles when compared to tetrahedron- and sphere-shaped nanoparticles.

**Figures 3** and **4** depict the effect of $\lambda$ on the $f'(\eta)$ and $g(\eta)$ profile. It is clear from both the figures that an increase in $\lambda$ decreases the momentum boundary layer thickness resulting in velocity decrease. It is also noted that $g(\eta)$ decreases smoothly with the increase in the
unsteadiness parameter. This shows an important fact that the rate of cooling is much faster for higher values of $\lambda$, whereas it may take a longer time in steady flows.

Influence of the solid volume fraction parameter $\phi$ on temperature profiles $\theta(\eta)$ can be visualized in Figure 5. It is observed that the temperature profile increases by increasing values of the solid volume fraction parameter. This is due to the fact that the volume occupied by the dust particles per unit volume of mixture is higher so that it raises the rate of heat transfer. It was noticed that the development in the temperature profiles of column-shaped nanoparticles is high when compared to temperature profiles of sphere- and tetrahedron-shaped nanoparticles due to the increase in volume fraction of nanoparticles.

The effect of temperature ratio parameter ($\theta_w$) on temperature profile is shown in Figure 6. The influence of temperature ratio parameter enriches the temperature profile and corresponding boundary layer thickness. This may happen due to the fact that the fluid temperature is much
higher than the ambient temperature for increasing values of $\theta_w$, which increases the thermal state of the fluid. It is also observed that the rate of heat transfer is higher in the column-shaped nanoparticles than that of tetrahedron- and sphere-shaped nanoparticles.

**Figure 7** demonstrates the effect of the Prandtl number $Pr$ on temperature profiles of $\theta(\eta)$. The above mentioned graph elucidate that the temperature profile and corresponding thermal boundary layer thickness decrease rapidly with increasing values of $Pr$. Physically, the Prandtl number is the ratio of momentum diffusivity to thermal diffusivity. In fact, the larger Prandtl number means that the lower thermal diffusivity. A decrease in the thermal diffusivity leads to a decrease in the temperature and its associated boundary layer thickness.

The temperature distribution $\theta(\eta)$ for various values of radiation parameter $R$ is shown in **Figure 8**. This figure reveals that the larger values of radiation parameter increase the temperature profile and thermal boundary layer thickness. Generally, higher values of
radiation parameter produce additional heat to the operating fluid that shows associate enhancement within the temperature field. We have noticed an improvement within the temperature profile because of increase in the radiation parameter. Moreover, the rate of heat transfer at the wall is less in case of the sphere-shaped particles when compared to the tetrahedron- and column-shaped nanoparticles.

Figure 9 shows the effect of $\theta_w$ and $R$ on the skin friction coefficient. Here, we observed that the skin friction coefficient decreases for larger values of $\theta_w$ and $R$. Figure 10 delineates the influence of $\phi$ and $Ec$ on Nusselt number. One can observe from the figure that Nusselt number decreases for larger values of $\phi$ and $Ec$. It is also perceived from these figures that the maximum decrease in the rate of heat transfer of nanofluid is motivated by the column-shaped, followed by
tetrahedron- and sphere-shaped nanoparticles, respectively. It is just because of the nanofluid which contains column-shaped nanoparticles having maximum thermal conductivity than nanofluids containing tetrahedron- and sphere-shaped nanoparticles. Table 3 presents the numerical values of Nusselt number for various values physical parameter values. It is observed that Nusselt number increases with increasing Pr. Further, from Table 3, we observe that Nusselt number decreases with increasing values of $θ_w$, $R$, $ϕ$ and $λ$.

5. Conclusions

In the present analysis, impact nonlinear radiative heat transfer of Cu-water nanoparticles over an unsteady rotating flow under the influence of particle shape is considered. Effects of various parameters are studied graphically. The main points of the present simulations are listed as follows:

- The highlight of this study is that temperature profile is more enhanced in column-shaped nanoparticles when compared to tetrahedron- and sphere-shaped nanoparticles.
- Temperature profile and thermal boundary layer thickness increase with increasing values of $R$ and $θ_w$.
- The thermal boundary layer thickness and temperature profile enhance with increasing values of $ϕ$.
- Higher values of rotating parameter enhance the velocity profile and corresponding boundary layer thickness. It has quite opposite behavior in angular velocity profile.
- Unsteady parameter increases the velocity profile and corresponding boundary layer thickness.
Author details

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References


