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The Approach of Probabilistic Risk Analysis and Rationale of Preventive Measures for Space Systems and Technologies

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Abstract

This chapter is devoted to the probabilistic risk analysis of collision of satellites with space debris. The uncertainty and random space-time characteristics of dynamic space objects are researched for the rationale of preventive measures for space systems and technologies. The proposed approach is illustrated by analyzing space debris and their distribution on satellite orbits. The actuality is confirmed by many dangerous convergences of controlled satellites and fragments of the old objects that have been discarded and transformed into uncontrolled debris. The research demonstrates a possibility of probabilistic modeling allows calculating preventive measures for avoiding collisions.

Keywords: probabilistic risk, calculating, space systems, control of space, modeling, control system, crucial situations

1. Introduction

Space systems are essential to human progress. They are an integral part of everyday life and the development of science. Now, there are thousands of satellites that provide remote sensing of the Earth in near-earth space, assist navigation, provide accurate weather forecasts, etc. The use of space systems involves the ability to control satellites. The control of the space objects includes the ability to change their orbit and orientation. One of the difficult tasks in objects control is the problem of evading uncontrolled objects or space debris, an example of which is shown in Figure 1 [1]. System of space monitoring is created in developed countries to solve these problems, and the main task is to track the trajectory of space objects and assess the risk of possible collisions. The technology of space control systems mostly reduced to the modeling...
and prediction of space objects motion. This chapter gives examples of probabilistic risk analysis of collisions with space debris and preventive measures to avoid such collisions.

Analysis of the complex systems for control in the space branch requires a large volume of simulation. The mathematical model of a complex system is created using the principles of the functional association of the models of elements and subsystems into a single program complex of the implemented algorithms. This complex accomplishes an imitation of the processes for the entire variety of input conditions and current states of the real system [2].

There are some questions that arise in the possible risk analysis of space objects collision, including collision with space debris. Methods of risk analysis and predictive preventive measures are based on probabilistic modeling. The questions of choosing the modeling method are the most determining while analyzing risks and substantiating security space systems.

The index of efficiency $R$ is the mathematical expectation of functional $Y$, which is determined on a set of functions

$$Z(t, \omega) \in Z$$

The output processes of a system in a single implementation characterize the function $Z(t, \omega')$ with a fixed value $\omega = \omega'$. A single implementation is a random interval of time $t \in [0, T]$ of system functioning. Assume that $\Omega$ is a space of elementary events $\omega$ with the possible measure $P(A)$, where $A$ is a random measurable subset $\Omega$. Then,

$$R = \int_{\Omega} \varphi(z(t, \omega))dP$$

(1)
Every complicated system realizes transformation of input signals into output ones. The system model makes the same.

Assume that \( X \) is a set of input signals \( x(t, \omega_X) \in X \), \( \Omega_X \) is a space of elementary events \( \omega_X \) with probabilistic measure \( P_\omega \) and that for every \( \omega_X \in \Omega_X \) there is an input signal \( x(t, \omega_X) \in X \).

The system properties are described with a random operator \( H(x(t, \omega_X), \omega') \). There is a probabilistic measure \( P_\omega \) in the set \( \Omega \). Elements of this set are \( \omega_H \in \Omega_H \).

By definition, a random operator \( H(x(t, \omega_X), \omega_H) \) is a set of nonrandom operators \( H(x(t, \omega'_X), \omega'_H) \), defined for each \( \omega'_H \in \Omega_H \). It implements the mapping of the set \( X \) to the set \( Z \) for all \( \omega_t \in \Omega_t \) in the set \( Z \). It means that every implementation of \( Z(t, \omega') \) is a result of transformation of an input signal \( x(t, \omega'_X) \) by a nonrandom operator \( H(x(t, \omega'_X), \omega'_H) \). In the operator form, the process of transformation is

\[
Z(t, \omega') = H(x(t, \omega'_X), \omega')
\]

(2)

It follows from Eq. (2) that for a known structure of a random operator \( H \), elements \( \omega \) from the set \( \Omega \) are generated by the elements \( \omega_X \in \Omega_X \) and \( \omega_H \in \Omega_H \), and for every \( (\omega_X, \omega_H) \), there is only one point \( \omega \) of space \( \Omega \). This correspondence allows the integral Eq. (1) to be written this way:

\[
R = \int_{\Omega = [\Omega_X \times \Omega_H]} \varphi[H(x(t, \omega_X), \omega_H)]dP_\omega dP_H
\]

(3)

Under real conditions, the structure of a random operator \( H \) and the probabilistic measures \( P_\omega, P_H \) are got by basis of a priori information \( I \) and information \( Z \), which are obtained in natural tests of elements and the whole system. An example of such tests is the archives of ballistic situations that were created by the space control systems of Russia and the US.

The problem of calculating \( R \) can be regarded as a statistical problem of the synthesis of decision rules \( W \) provided estimates \( \hat{R} \) with certain preassigned properties. The task determines the ultimate goals that include calculating indicators of efficiency of the complicated technical systems, risks of usage, and possible preventive measures.

In the operator form, the evaluation operation \( R \) can be written as:

\[
\hat{R} = W(Z, I, H)
\]

(4)

While choosing rules \( W \), it is required that the calculations related to finding \( \hat{R} \) by Eq. (4) are technically implementable, and the properties of the estimates \( \hat{R} \) satisfy the conditions of maximum achievable accuracy. In the problems of assessing the characteristics of complex technical systems, these requirements are usually decisive.

The features of the space systems tests consist in the fact that they can be carried out in the conditions of the regular functioning of the system in a very limited volume. Experiments on
the system in crucial situations and operating modes usually requires considerable effort and material costs, and it is sometimes associated with a risk of system failure, which is unacceptable for space systems. That is why while evaluating the system’s efficiency indicators, it is necessary to consider that the sample \( \tilde{Z}(t, \omega_1), ..., \tilde{Z}(t, \omega_n) \) reflects the results of the tasks performed by the system only under normal operating conditions. In other words, the direct use of the sample \( \tilde{Z}(t, \omega_1), ..., \tilde{Z}(t, \omega_n) \) to determine the characteristics of the system over a wide range of its operation, including critical modes, is practically impossible.

Calculation of system performance indicators for normal conditions of its operation is carried out in several stages:

a. a number of values \( \varphi(\tilde{Z}(t, \omega_1)), ..., \varphi(\tilde{Z}(t, \omega_n)) \) are calculated;

b. statistical properties of random variables \( \varphi(\tilde{Z}(t, \omega_i)) \) are determined, relying on known laws of distribution of measurement errors;

c. the a priori information about the value of \( R \) is expressed as the a priori density of the distribution \( P(R) \);

d. a criterion for the optimality of the \( W \) estimates \( \tilde{R} \) is chosen.

In the presence of the aforementioned information, it is possible to combine a priori information with the real information obtained in the process of carrying out field experiments to obtain estimates.

In calculating the integrals in Eq. (3) by simulation modeling methods, it is necessary:

a. to develop a model that allows the generation of processes \( \tilde{Z}(t, \omega) = H(x(t), \omega_X), \omega_H) \) for different \( (\omega_X, \omega_H) \);

b. to determine the method of setting up experiments on the model, provided that the probabilistic measures \( P_x, P_n \) are given;

c. to develop algorithms for processing simulation results;

d. to build a plan for conducting experiments and processing their results.

However, in modeling real systems, the estimated situation proves to be much more complicated, because the structure of the random operator \( H \) and the probability measures \( P_x, P_n \) are determined as a result of the complex processing of a priori information and information obtained from field testing of the elements and the entire system as a whole. The limitation of real statistics and a priori information usually leads to the fact that both the operator \( H \) and the measures \( P_x, P_n \) will contain errors, which in the general case are of a probabilistic nature.

It follows that in assessing the performance indicators of complex systems and their risks, there will be components due to errors in the definition of the operator \( H \) and errors in calculating the probability measures \( P_x \) and \( P_n \). In addition, modeling errors arise from inaccuracies in the implementation of the operator \( H \) on computational means and the limited amount of statistical data obtained by experimenting on the model.
Collectively, the aforementioned errors determine the total modeling error, which in general will be a random quantity consisting of deterministic and random components.

Further examples of probabilistic risk analysis and justification of preventive measures for space systems and technologies will be illustrated by using examples of the risk of collision of spacecraft with space debris.

The basis for assessing the risk of collisions in space is trajectory measurements.

2. Factors of uncertainty in processing trajectory measurements

In applied mathematics, the development of methods for processing trajectory measurements occupies a special place because of their complexity. General methodological problems in this area have not been solved in many respects due to the possibility of obtaining subjective conclusions or the use of excessive formalization, which makes it difficult to extract practically useful results.

An important role in solving problems associated with the development of methods for processing trajectory measurements is played by the concept of randomness. The concept of randomness is a certain type of uncertainty, characteristic of frequently observed events.

The methods used to solve problems related to the processing of trajectory measurements can be divided into sections corresponding to those deductive theories whose apparatus is used to solve problems from other fields of knowledge. If the studied problem, the solution of which provides the development of new methods for processing trajectory measurements, can be represented as a certain set of objects related by relations, then it is always possible either to find a suitable formalism to solve the problem or to create a new mathematical structure suitable for solving the problem.

It should be noted that the question of whether it is possible to single out objects forming a population in such a problem area as the development of new methods for processing trajectory measurements that allows them to be interpreted as sets connected by relations is a matter of not applied mathematics, but a part of the science of developing and testing of complex space systems.

The limited knowledge of the processes and phenomena studied does not allow the creation of absolutely accurate models of elements, including space systems and technologies. In addition, it is impossible to carry out an infinite number of experiments on the system model because of real limitations. For these reasons, the accuracy of the estimates obtained will be determined by the reliability of the information on the structure and parameters of the models being created, and also by the errors caused by the imperfection of the methods used for setting and processing the experiments carried out on the model.

If the accuracy of calculating the indicators will be sufficient for practical purposes, then the application of simulation methods can be considered justified. In practice, a series of control checks is carried out for this purpose, the main purpose of which is to establish a measure of
proximity between the real and simulated processes. If, according to the results of the comparison, it turns out that for all input conditions, the differences in the simulation of real processes do not exceed some given critical values, then the model is considered adequate for the system under analysis. Otherwise, the system model needs to be improved.

In the process of modeling, it is possible to use various assumptions and coarsening, as a result of which the mathematical formulation of the investigated problem can only be its approximate reflection. When carrying out formalizations, there always arises the question of the adequacy of the resulting mathematical description of problems associated with the development of methods for processing trajectory measurements. If the solution of the content problem is not complicated and requires a numerical solution, an adequacy check can be made by experimental calculations using the initial data for which the desired results are obtained experimentally. The calculated results should differ from the reference data by no more than the task of assessing the risks of collision of space vehicles or carrying out preventive measures eliminating the danger of such a collision allows.

2.1. Analysis of the risk of collisions of space appliances

The area where most artificial earth satellites operate is very extensive; its volume is about $10^{12}$–$10^{13}$ km$^3$.

The density of artificial Earth satellites and space debris can be estimated by numbers in the order of $10^{15}$, and this number is constantly increasing. For example, in 2007, China tested an anti-satellite missile, sending it to one of its old satellites, adding about 3500 extra fragments in the area between 160 and 2000 km above the surface. This is a very large amount of objects and they must somehow be taken into account. The general picture of the contamination of near-Earth space is clearly shown in Figure 2 [3]. The orbital information on more than 20,000 space objects (SO) of more than 10 cm in size has been fixed and regularly updated.

![Figure 2. Earth space.](image)
Simulation of the motion of such satellites reduces to obtaining, as a rule, systems of ordinary
differential equations of motion of an object and their integration by one or another method.
As a result, the dependence of the motion parameters on time is obtained for the given initial
conditions. These equations are a form of representation of the laws of dynamics and kinematics
and can be supplemented by equations of control.

By managing the catalogs of space objects (and such catalogs exist both in Russia and in the
US), one can assess the mutual position and carry out the forecast of their movement. In
particular, it is possible to assess the dangerous convergence and even collision of space
vehicles. The main method for determining the motion of space objects is modeling.

Simulation of the motion of controlled space objects can be performed with different goals
determined by the specific content of the tasks being solved. Among the tasks that involve the
use of motion models for controlled objects, let us dwell on the identification of parameters
and SO states based on the results of their measurements. Such a task is the basis for justifying
the control actions in order to evade the collisions of space vehicles.

Any of the models used in solving the problem presented earlier is a mathematical idealization
of real motion. Therefore, in modeling, of course, a special question arises about the adequacy
of the mathematical description of the real movement of an object. The adequacy of the model
is directly dependent on the degree of confidence in the a priori data, the completeness of their
accounting for modeling, and the accuracy of the model's reproduction on a computer.

Requirements for accuracy in modeling can be considered to be dictated by the content of the
problem being solved.

The a priori data on motion include the laws of kinematics and dynamics, the parameters (for
example, the traction force, the nature of its variation in time and the time of the engine, the
aerodynamic coefficients), the characteristics of the surrounding space (for example, the model
of the atmosphere, the gravitational potential), the characteristics of the interaction of the
object with the surrounding medium. Sometimes it becomes possible to use the data about
the software path (program parameters) of the movement of the object. The a priori informa-
tion may include the law governing the apparatus (for example, the method of parallel
approach) and the features of its implementation by the control system (lag, other manage-
ment errors). These data can include both deterministic and random parameters with known
distribution laws.

The most reliable are usually the laws of dynamics and kinematics of the motion of objects. The
validity of kinematic constraints such as “the linear velocity vector of the object is the first time
derivative of the vector of its position” is beyond doubt. At the same time, the use of the
dynamics relations should be carried out taking into account that they are sufficiently close to
the real movement only under the conditions of their formulation. In many practical applica-
tions, the object is idealized as a material point located at its center of mass.

If by the conditions of the solution of the problem this assumption is relatively rough, then the
degree of confidence in describing the trajectory of the movement of the object as a material
point can turn out to be low. If necessary, the object can be considered as a system of rigid
bodies and clarify the description. If it is required to take into account more subtle effects that have a noticeable effect on the components of the object’s motion that are of interest, for example, on orientation in space, then the movements of the fuel components in the supply lines and the like can be modeled.

The degree of confidence in the mathematical model is determined not only by the nature of the idealization of the object (material point, aggregate of rigidly bound bodies, etc.), but also careful consideration when modeling individual factors affecting the movement of the apparatus. The adequacy of the model also depends on the accuracy of knowledge of the parameters appearing in the formulas (ballistic coefficient, aerodynamic coefficients). These values are determined, as a rule, experimentally and are therefore known with some errors.

As criteria of comparison and estimation of models of movement of SO, it is expedient to apply those or other modeling functional errors.

Denote \( \lambda(t) \) is the real change in the motion parameter on the time interval, \([O, T]\), \( \lambda_M(t) \) are the process simulating this motion. Then, the absolute deviation of the model motion from the real is a function of time \( \Delta \lambda_M(t) = \lambda_M(t) - \lambda(t) \), describing the modeling errors in time. As the values that generalize such errors, extreme, averaged and confidence indicators can be used.

By extreme exponents, we mean the largest (least) values of some characteristics of a function \( \Delta \lambda_M(t) \), for example, a module \( \Delta \lambda_M(t) \):

\[
\alpha_1[\lambda_M(t)] = \max_{t \in [0, T]} |\Delta \lambda_M(t)|
\]

When the average is implemented, this or that characteristic of the function \( \Delta \lambda_M(t) \) is averaged over a time interval \([0, T]\). An example of averaged index of modeling quality is the average square error of modeling:

\[
\alpha_2[\lambda_M(t)] = \frac{1}{T} \int_0^T [\Delta \lambda_M(t)]^2 dt.
\]

The introduction of confidence indicators in quality is associated with the random nature of the modeling errors caused, for example, by the presence of random components in the movement of the object. The appearance of such components is due to fluctuations in the properties of the environment surrounding the SO, the parameters of various elements of the flight control system, and so on.

Confidence indicators are complex and combine a confidence interval and a confidence probability.

The confidence interval is the range of values of some characteristic \( \beta \) of the function \( \Delta \lambda_M(t) \): \( B = [\beta_{\min}, \beta_{\max}] \).

The confidence probability \( P \) is the probability that the calculated value of the characteristic \( \beta = \beta[\Delta \lambda_M(t)] \) in the confidence interval \( B \).
The confidence level of the modeling quality is the interval B, in which the error characteristic falls with a confidence probability.

In modeling, the influence of the atmosphere, the rotation of the earth, reactive forces, etc., can be taken into account to some extent. To simplify the model, a number of factors that determine the movement of the object, but are insignificant, are combined and replaced in the noise component model (“useful noise”). Useful noise is a random amount included in the model.

The resulting solution of the equations is the deterministic basis of the simulated motion. It can be supplemented by random specially imitated components, which are introduced additively, multiplicatively or additively-multiplicatively.

2.2. Differential equations of the spatial motion of space systems

Let us consider the system of equations of translational and rotational motions of SO. The first subgroup of equations characterizes the displacement of the center of mass, and the second characterizes the orientation of the object in space.

In doing so, we will use the normal terrestrial coordinate system. For simplicity, we assume that the acceleration of gravity is constant in magnitude and direction, Coriolis acceleration is absent; the curvature of the Earth is neglected, the wind is not taken into account.

The simplest way is that the velocity of the translational motion of the center of mass of the rocket relative to the Earth is described in the projections on the axis of the trajectory coordinate system $\Omega x_k y_k z_k$, since $v_{x_k} = v, v_{y_k} = v_{z_k} = 0$ in this case. Then,

$$
\dot{V} = \sum F_x / m; \dot{V}_{y_k} = \sum F_{y_k} / m; \dot{V}_{z_k} = - \sum F_{z_k} / m,
$$

where $\sum F_x, \sum F_{y_k}, \sum F_{z_k}$ are the projections on the axis of the specified coordinate system of the resultant force acting on the center of mass of the object; $v_{x_k}, v_{y_k}, v_{z_k}$ are the projections of the angular velocity of rotation of the trajectory coordinate system $\Omega x_k y_k z_k$ relatively fixed system $\Omega x g y g z g$ on the axis of the trajectory coordinate system. To determine them, we use the relations that reveal the relationship of these components of the angular velocity with the orientation angles of the trajectory coordinate system with respect to the normal one:

$$
\omega_{x_k} = \Psi \sin \theta \omega_{y_k} = \Psi \cos \theta \omega_{z_k} = \theta
$$

Here $\theta$ and $\Psi$ are the angles of the path and the slope of the trajectory. As a result of the substitution, we obtain a system of equations of the form

$$
\dot{V} = \sum F_{x_R} / m, \dot{V} = \sum F_{y_R} / m, \dot{V} \cos \theta = \sum F_{z_R} / m
$$

In the right-hand side of the equations, we include the components of the traction force, gravity, and control forces.
When obtaining an expression for the aerodynamic force, we use the velocity coordinate system \(\bar{x}_a\bar{y}_a\bar{z}_a\), and then from it, we proceed to the trajectory system. Projections of this force on the coordinate axis of the last system are represented in the form

\[
RAx_R = -x_R = -x_a;
\]

\[
RAy_R = y_R = y_a \cos \gamma_a - z_a \sin \gamma_a;
\]

\[
RAz_R = z_R = y_a \cos \gamma_a + z_a \sin \gamma_a
\]

where \(\gamma_a\) is an angle of rotation of the velocity system relative to the trajectory system.

Further equations that take into account the features of the motion of the SO will lead to a system of equations of motion.

Solving this system, we find all the characteristics of the motion of the rocket or SO:

\[V(t), \theta(t), \psi(t), x_g(t), y_g(t), r(t), \phi(t), \gamma(t), \alpha(t), \beta(t), \gamma_a(t), \sigma_x(t), \sigma_y(t), \sigma_z(t), m(t), z_g(t), \psi(t).\]

Naturally, the initial conditions for integration must be given.

With a rigorous theoretical approach to the solution of the problem of modeling, it is obviously impossible to separate the equations describing only the translational motion of the center of mass or only the rotational motion of the relative center of mass, and the equations of longitudinal and transverse motion. The relationship between translational and rotational movements is manifested through so-called cross-links.

The probability that the calculated values of the parameters of two SO movements \(\beta = \beta[\Delta \lambda_M(t)]\) in the confidence interval is estimated. In each of them, one can predict the risk of a dangerous convergence.

The same task is directly related to the definition of collision risk with space debris (CD). For probabilistic modeling of collision risks with space debris, special programs are used, for example:

Model SDPA [4] is a semi-analytic stochastic model for medium- and long-term forecasting of technogenic SG larger than 1 mm in low Earth orbits (LEOs) and geosynchronous Earth orbits (GEOs), for constructing the spatial distribution of concentration and velocity characteristics, as well as estimating collision risk [5].

The model uses summary data on SO of various sizes (including space debris without “binding” them to a specific source of pollution).

The measurement errors are estimated on the basis of averaging of the last measurements of the orbital parameters. In calculations of dangerous convergence, errors are calculated in several models, for example, in the orbital coordinate system, in the elements of the orbit, and in models of direct integration. The error matrix is used to calculate the collision probability.
If we consider the problem in posing the risk of collision of an uncontrolled SO with a controlled spacecraft (SC), then it is necessary to consider the process of mutual proximity in three-dimensional space and in the picture plane.

In this three-dimensional space, the spacecraft structure is considered as a sphere of a given radius. The region of possible position of the SO is represented in the form of an ellipsoid whose parameters and orientation are determined by the total error matrix

\[ C_{SC+SO} = C_{SC} + C_{SO} \]

In the picture plane, the SV and the SO region are represented as a circle and an ellipse, respectively.

The problem reduces to the search for the probability of hit of a random vector whose density is given by the ellipsoid of scattering errors into the sphere of a given cone in Figure 3, where \( T_{dc} \) is the time of dangerous convergence, and \( V \) and \( D_v \) are the speed.

For example, the approach of a SO to an ISS is considered safe if \( P_c < 10^{-5} \). For the value of \( P_c \) in the range between \( 10^{-4} \) and \( 10^{-5} \), the collision risk is high enough, therefore, when planning the control of the spacecraft, necessary maneuvers should be provided for the purpose of evasion.

To assess the characteristics of the collision risk, an archive of dangerous convergence (ADC) between all objects in the catalog is maintained in the Information and Analytical Center for Near-Earth Space Monitoring, taking into account the data of the catalog of the American USSSN; they are available on the Internet [6, 7].

The ADC gathers all potentially dangerous convergences between all cataloged SOs. “Potentially dangerous” means either convergence of two SOs to a distance less than a given distance.
Δ, or a closer distance to a greater probability with a collision probability \( p_c \) greater than the threshold \( p_{\text{min}} \). Such convergences are about \( \approx 15,000 \) per day. The archive is more than 20 years old. For each convergence, the following characteristics are stored in the ADC: the convergence time, trajectory and non-trajectory parameters of the objects convergence, the residuals at the moment of convergence and their probabilistic characteristics, the probability of collision. Briefly, the algorithm for the supporting ADC is described in the book [8]. There, a method for evaluating various risk characteristics using ADC is also described. For more detail, see Ref. [9].

2.3. Modeling of space debris

At present, there are several models of the objects fragmentation at hypersonic collision. Review of the probabilistic models of space debris is represented in monograph [5]. The main result of collision of two objects with masses \( M_1 \) and \( M_2 \) becomes formation of the large number of fragments of various shapes, masses, and dimensions. The following features are used to describe the effect of the collision:

- \( N_f(>m) \), \( N_f(>d) \) — the number of fragments with mass more than \( m \), or dimensions more than \( d \). This is one of the fundamental characteristics. Some assumption about fragments shape and weight are used to recalculate the mass values to dimension values;
- \( A/m(d) \) — the ratio of the square of the typical cross section to mass for fragments of different sizes. This parameter is related to the difference of shapes and materials of the colliding objects; it is necessary in the analysis of the evolution of SD to calculate the deceleration of fragments in the atmosphere;
- \( p(\Delta V) \) — the statistical distribution of the incremental speed of fragments by their size and direction. As a result of collision, some of the energy goes to changing speed of fragments, which leads to the spread of SD in some part of interplanetary space.

Experiments show that hundreds and even thousands of space debris objects formed in collisions with the satellites. In 2009, the collision of a communications satellite Iridium with Russian satellite Kosmos-2251 resulted in about 600 shards that flew at an altitude from 500 to 1300 km [10].

Space debris poses a great danger to functioning spacecrafts because of the large relative velocities of convergence (Table 1). In recent years, collisions with space debris have killed several spacecrafts.

Given the fact that simulation of motion of space debris is performed under conditions of essential indeterminacy of the input data and using the processing algorithms of random

<table>
<thead>
<tr>
<th>Kalashnikov</th>
<th>KAMAZ with cargo</th>
<th>Space debris</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bullet weight: 7.9 g</td>
<td>Mass: 14.5 t</td>
<td>Mass of fragment: 40 g</td>
</tr>
<tr>
<td>Muzzle velocity: 715 m/s</td>
<td>Velocity: 90 km/h</td>
<td>Relative speed: 15 km/s</td>
</tr>
<tr>
<td>Kinetic energy: 2 kJ</td>
<td>Kinetic energy: 4.5 MJ</td>
<td>Kinetic energy: 4.5 MJ</td>
</tr>
</tbody>
</table>

Table 1. Comparative analysis of the kinetic energy of objects [3].
events in large part, methods of probabilistic risk analysis and justification of preventive measures of damage reducing become most common for cosmic systems and technologies.

The movement of each element of the system of space objects can be divided into two components. First, the orbital object moves on a trajectory that can be represented in the elliptical form in the general case in the current time, which oriented in space in a certain way (osculating orbit). Second, the trajectory of the orbital object changes over time (generally, form and orientation are changing). Meanwhile, trajectories of motion of orbital objects change much slower than the position of orbital objects on these trajectories. Therefore, it is proposed to model changes of trajectories and identify the parts of the trajectories for the current moment in time, which are located from each other at a dangerous distance, from the point of view of possibility of collisions of orbital objects (nodes of mechanical conflicts). In other words, to simulate the nodes of mechanical conflicts, speed changing of which corresponds to speed changing of trajectories. For orbital objects, trajectories of which form a node conflict, it is necessary to determine the time intervals of their movement through the node conflicts without a significant investment of time (on the dangerous part of trajectory). Hence, the method of modeling system of orbital objects is based on the method of modeling the nodes of mechanical conflicts and the method of determining the time intervals of movement of the orbital object through a node of conflicts.

Tasks of the analysis of conflicts of orbital objects can be divided into two classes. In the first class, there are the tasks, where it is possible to analyze only the risk of collisions and not to predict specific orbital conflicts. Their solution is based on the consideration of the altitude-latitudinal density distribution of the orbital objects at a specific point in time.

In the second class, there are the tasks that demand prediction of orbital collisions. This prediction boils down to the prediction of convergence of pairs of orbital objects at a dangerous distance, from the point of view of their collision at possible deviations of objects from their calculated trajectories (these can be called dangerous or conflict convergences). In many cases, it is sufficient to predict only dangerous convergence of the orbital objects, and not to simulate the effects of conflicts, which change the trajectories of the colliding objects and form new orbital objects. Such tasks are solved when it is necessary to predict dangerous collisions for spacecraft, which can make the maneuver to avoid collisions. The task of prediction of dangerous convergence can be used as a base for models of near-Earth space contamination by orbital objects. The direct deterministic method is the most common. It is based on the formation of an archive of dangerous convergences of all possible pairs of orbital bodies at a specified time interval, which is included in the considered set of orbital objects (for each dangerous convergence, the passing time interval, the geometric characteristics of convergence and the probability of collision are determined).

The traditional method to predict dangerous convergence is based on modeling the movement of objects and analyzing the current distance between them. There is a difficulty in this method. The relative speed of orbital objects can be more than 10 km/s. Meanwhile, the convergence at a dangerous distance of several kilometers lasts less than 1 s. Therefore, the prediction of dangerous convergences requires modeling with a correspondingly small time step. At larger sizes of sets of orbital objects, it leads to significant time consumption.
An effective way to solving this problem is the implementation of the prediction of dangerous convergences in several stages. Each stage is the check of the possibility of dangerous convergence based on some rule or the simplified method of prediction.

There are three stages of checking the possibilities of dangerous convergence implemented for a given pair of orbital objects. In the first stage, the overlapping region of heights above the Earth’s surface, where their trajectories pass is checked. The second stage is based on the fact that the conflict between orbital objects is possible only when their trajectories intersect. It is assumed that the orbital object cannot deviate from the position on the calculated trajectory more than a certain distance. Hence, in each moment of time, the orbital object may be within a sphere, which has the center of the calculated trajectory, and radius is $R_{cr}$. A pair of sections of trajectories will be called a node of the mechanical conflicts, if that trajectories are located on the distance $L < L_{cr}$ from each other. If for a pair of trajectories of orbital objects there is defined a node of conflict, then the condition of the second stage is fulfilled. The third stage is based on the fact that the conflict convergence is possible during simultaneous motion of orbital objects on segments of trajectories, which form a node of conflict. In the third stage, the time intervals of orbital objects’ motion through the node of conflicts are defined. If these time intervals overlap each other, then dangerous convergence is possible, and its probability can be calculated.

Assume [11] that the set of the cataloged orbital objects is considered as a multi-element mechanical system. There are some quasiregular components in the movement of the elements of this system. Meanwhile, the interactions of the elements of the system are not taken into account. Such restrictions allow the allocation of the node of conflict at the current time, which is formed by the dangerous parts of the trajectories of orbital objects $k$ and $l$. This node of conflict restricts the dangerous part of the trajectory of each of these orbital objects. Considering the regularity of the motion parameters of the objects allows to simulate space debris as a combination of deterministic and probabilistic models.

### 3. Summary

Probabilistic modeling is an important method of risk analysis and justification of preventive measures for space systems and technologies.

Methods of calculating ballistic trajectories and assessment of collision risks with space debris are based on conversion of inaccurate source data, results of which are random variables. Therefore, the risk of collision can be specified as a probability measure.

Preventive measures for controlled spacecraft are reduced to change of trajectory, which could prevent or reduce this risk.

### 4. Conclusion

This section contains examples of the probabilistic risk analysis of collision of satellites with space debris.
It is shown how to predict the dangerous convergence between space objects and to justify preventive measures to reduce collision risk, solving tasks of modeling with random parameters.

The approach to constructing a probabilistic mathematical model of a complex system based on the principles of functional integration of the models of elements and subsystems in a single integrated software and implemented algorithms to perform the simulation processes for different input conditions and current state of the real system is given.

Modeling the motion of such satellites typically boils down to the obtainment of systems of ordinary differential equations of object motion and their integration by any method. The result is a dependence of the parameters of motion from time under given initial conditions. These equations are the form of representation of the laws of dynamics and kinematics, and can be supplemented with the equations of control.

Considering the catalogs of space objects (such catalogs exist in Russia and the United States), it is possible to estimate their relative position and to forecast their movement. In particular, it is possible to assess the threat of convergence and even the collision of spacecrafts.

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