Optimum Maintenance Policy for Equipment over Changing of the Operation Environment

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Additional information is available at the end of the chapter

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Abstract

This chapter investigates optimization of maintenance policy of a repairable equipment whose lifetime distribution depends on the operating environment severity. The considered equipment is undergone to a maintenance policy which consists of repairing minimally at failure and maintaining after operating periods. The periodic maintenance is preventive maintenance (PM) and allows reducing consequently the equipment age but with higher cost than minimal repair. In addition, the equipment has to operate at least in two operating environments with different severity. Therefore, in this analysis, the equipment lifetime distribution function depends on the operating severity. Under these hypotheses, a mathematical modeling of the maintenance cost per unit of time is proposed and discussed. This cost is mathematically analyzed in order to derive optimal periods between preventive maintenance (PM) and the optimal condition under which these exist.

Keywords: minimal repair, preventive repair, repairable equipment, several operating environments

1. Introduction

To reduce the failure risk of production equipments, preventive maintenance or replacement activities should be performed in appropriate schedules. The search of these appropriate schedules has led to the development and implementation of maintenance optimization policies for stochastic degrading production equipments. Indeed, the literature on this matter is already extensive, growing rapidly and also very heterogeneous. Accordingly, this chapter focuses only to some relevant and fundamental works on the maintenance theory. Early in [1, 2], several models appeared on the optimization of replacement or maintenance policies on infinite time horizon. In these works, the authors mainly discussed about the optimality conditions of theses maintenance models. Subsequently to these works, many extensions of the previous models were proposed on finite time span [3, 4] and also on infinite time horizon in the literature. For survey, the reader may refer, for example, to [5–8] and the references therein. We note that in most cited works, the authors assumed that the equipment lifetime distribution is parametrically characterized and well known. However, Coolen and his coauthors [9, 10] showed that this assumption impacts clearly the optimal replacement age and its cost per unit of time when the equipments undergo an age replacement policy (ARP). Recently in [11], de Jonge et al. pointed out also the weakness of the assumption on knowing of the equipment lifetime distribution and proposed a parametric modeling of ARP for new equipment with an uncertainty on the parameters of the equipment lifetime distribution. In this work, de Jonge and his coauthors used Bayesian approach to model the uncertainty on the parameters and figured out that this uncertainty has effects on the optimal policy (age and cost) under ARP.

Another way, most existing models merely rely only on a classical assumption which states that the operating environment is steady and has not any effect on the equipment characteristic and its lifetime distribution. Roughly speaking, they assume that the degradation process is the same during the equipment's life cycle. This is a restrictive assumption in many industrial areas where production equipment may have experiences under different operating environments with their own severity degree that impacts the equipment performance. For example, the degradation process of the mining machinery is impacted by the severity level of the environment where the machinery is being exploited. Another example may be the engines used for oil extraction. The degradation process of such equipment depends on whether they are operated onshore or offshore. In some other industries, production equipments are first operated in a given environment and then moved to another location where this latter might be more or less severe than the first. In the same way, many companies operate their equipments at home for several years before shipping them to their subsidiaries in other countries where they would be subjected to more severe operating conditions. Therefore, suitable maintenance strategies, integrating the heterogeneous operating conditions, should be developed to assess the degradation of such equipments.

In this chapter, a preventive maintenance is investigated for such equipment subject to random failures. The equipments are assumed to have an experience under two operating environments. In fact, each operating environment is characterized by its own degree of severity, which impacts the equipment lifetime distribution. Therefore, the equipment lifetime distributions follow then a different distribution depending on the operating environments. To reduce the failure occurrence risk during operating under both operating environments, the equipment undergoes to an periodic preventive maintenance (PM). However, the equipment is subjected to minimal repair at failure. The objective consists then on evaluating the optimal age to perform periodic preventive repair in order to minimize the expected maintenance cost per unit of time. This expected cost is induced by the costs of minimal and preventive repairs. This policy was already discussed by Nakagawa in [12], in which Nakawaga considered that the equipment lifetime remains the same during the operation. Nakagawa analyzed mathematically the periodic and sequential maintenance policies. Therefore, our chapter can be considered as an extension of Nakawaga work.

The remainder parts of the chapter are organized as follows. The analyzed problem is briefly introduced in Section 2. This section proposes a mathematical formulation of the total maintenance

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cost. Section 3 focuses on the maintenance cost analysis in order to derive the optimal conditions which ensure minimal total cost per unit of time. In this same section, an heuristic is proposed to find the optimal number and period between preventive actions on both environments. Numerical experiments are conducted to illustrate the proposed approach on the one hand, and on the other hand, the accuracy and robustness of model are demonstrated through the simulation in Section 4. At the end, a conclusion and future works are drawn in the last section.

2. Mathematical formulation of the maintenance cost

In this section, modeling of the maintenance policy is going to be proposed. This modeling takes into account different hypotheses of our analysis. In fact, our equipment has to be used under two operating environments with different severities denoted by $j = 1$ and $j = 2$ which stand, respectively, for the first and second environments. Therefore, the equipment spends T1 and T2 respectively in operating environment 1 and 2. Therefore, the operation duration is the combination of both durations $T_1 + T_{2.}$ The equipment operates successively on both environments in order to perform its missions. During this operation, the equipment undergoes by two types of maintenance actions. Roughly speaking, the equipment is going to be repaired minimally at failure and preventively after some x_j operating periods. The minimal repair costs c_{mj} and allows that the equipment reaches the same reliability just before its failure. However, the preventive repair costs C_{pj} such as $C_p >> c_{mj}$. Therefore, this preventive repair impacts the equipment according to its age and its hazard function. First, the preventive action reduces the equipment age to zero. Second, the preventive action modifies the hazard function such as the hazard function after repair becomes higher than its hazard before. That involves that the wear-out process of the equipment degrades more after the preventive action than before **Figure 1**.

2.1. Preventive maintenance cost

During operation, the equipment undergoes by preventive action after each x_1 and x_2 unit of time, respectively, on the first and second environments. Each of these preventives actions costs C_{p1} on the first and C_{p2} on the second environment. In addition, the number of preventive actions is n_1 and $n_{2,}$ respectively, on the first and second environments. Therefore, the total preventive repair costs

$$C_{TP} = n_1{}^* c_{p1} + n_2{}^* c_{p2}, \tag{1}$$

during the length of operation $(T_1 + T_2) = (n_1{}^* x_1 + n_2{}^* x_2)$.

2.2. Minimal repair cost

The minimal repair is performed regardless of the preventive actions. The minimal repair is performed at failures in order that equipment reaches the same reliability just before failing. Each minimal repair costs c_{m1} and $c_{m2,}$ respectively, on the first and second environments.

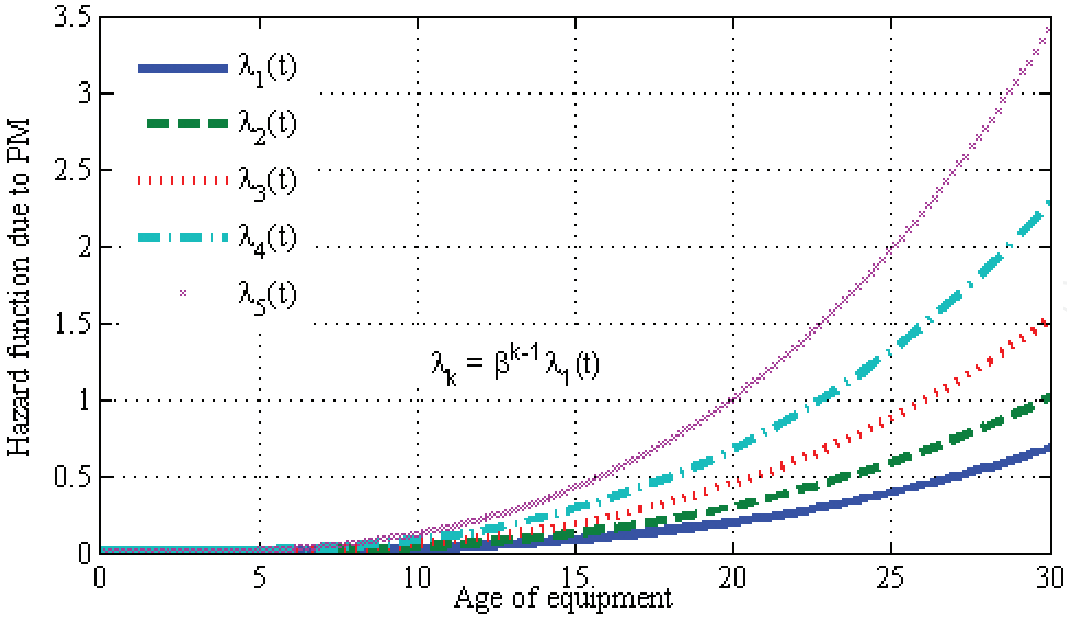

Figure 1. Evolution of hazard function due to preventive maintenance.

Therefore, the cost of minimal repair, on the k^{th} interval with a duration x_j, is product of expected number of failure by the cost of a minimal repair c_{mj}. From Thompson analysis [13], the expected number of renewal on the interval $[0; x_j]$ coincides with the integration of hazard function on $[0; x_j]$. Then, the minimal repair costs during the k^{th} interval are given by

$$C_{mj} = \int_0^{x_j} \lambda_{j,k}(t)dt, \tag{2}$$

$$= -c_{mj} \log R_{j,k}(x_j). \tag{3}$$

where $\lambda_{j,k}(t)$, and $R_{j,k}(t)$ stand for the hazard and the reliability functions of the equipment on the k^{th} and during the j^{th} environment. Therefore, the total minimal cost on the first environment is

$$Ct_{m1} = \sum_{k=1}^{n_1} \int_0^{x_1} \lambda_{1,k}(t)dt, \tag{4}$$

$$= -c_{m1} \sum_{k=1}^{n_1} \log R_{1,k}(x_1). \tag{5}$$

We also deduce the total minimal cost on the second environment as follows

$$Ct_{m2} = \sum_{k=n_1+2}^{n_1+n_2+1} \int_0^{x_2} c_{m2}\lambda_{2,k}(t)dt, \tag{6}$$

$$= -c_{m2} \sum_{k=n_1+2}^{n_1+n_2+1} \log R_{2,k}(x_2). \tag{7}$$

In addition, the operation on the $(n_1 + 1)^{th}$ period also implies a minimal cost. In fact, on this period, the equipment operates on both environments. On the first environment, the equipment operates on y units of time before moving on to the second environment such as $y < x_1$. The minimal cost during this operation is

$$C_{tmy} = c_{m1} \int_0^y \lambda_{1,\, n_1+1}(t) dt \tag{8}$$

$$= -c_{m1} \log R_{1,\, n_1+1}(y). \tag{9}$$

After, the equipment moves on to the second environment to operate between $[y; y + x_2]$. In addition, we point out that the second operating environment can be more or less severe than the first. Therefore, to ensure the continuity of reliability function between both operating environments, a transfer function $\phi(t)$ is introduced and defined such as:

$$\begin{aligned} R_{1,\, n_1+1}(t) &= R_{2,\, n_1+1}\big(\phi(t)\big), \\ \phi(0) &= 0. \end{aligned} \tag{10}$$

That involves a minimal cost on this period

$$C_\phi = c_{m2} \int_{\phi(y)}^{\phi(y)+x_2} \lambda_{2,\, n_1+1}(t) dt \tag{11}$$

$$= -c_{m2}\big(\log R_{2,\, n_1+1}\big(x_2 + \phi(y)\big) - \log R_{2,\, n_1+1}\big(\phi(y)\big)\big). \tag{12}$$

To reduce the complexity during computing, we assume that the duration $y = 0$. That involves a total minimal which clearly depends on the Eqs. (5), (7), (12). The total minimal cost on all operating duration is defined by addition

$$C_{Tm} = Ct_{m1} + Ct_{m2} + C_\phi. \tag{13}$$

Indeed, the hypothesis $y = 0$ also impacts the number of preventive actions. In fact, under this latter hypothesis, the number of preventive actions becomes $n_1 + n_2 - 1$ instead of $n_1 + n_2$ as we indicated in Eq. (1). The total preventive is going to cost

$$C_{TPh} = \begin{cases} n_1 C_{p1} + (n_2 - 1) C_{p2}, \\ (n_1 - 1) C_{p1} + n_2 C_{p2}. \end{cases} \tag{14}$$

Eq. (14) is equivalent to

$$C_{TPh,\gamma} = (n_1 - 1 + \gamma) C_{p1} + (n_2 - \gamma) C_{p2}, \tag{15}$$

where $\gamma = 1$ stands for the fact that at the end of n_1^{th} period the equipment is repaired before moving to the second environment, while $\gamma = 0$ corresponds to the reverse.

2.3. Total maintenance cost

From previous Eqs. (13) and (14), we deduce a mathematical formulation of the total maintenance cost according to the set of parameters (n_1, n_2, x_1, x_2) as follows:

$$C(n_1, n_2, x_1, x_2) = \frac{C_{Tm} + C_{TPh,\gamma}}{nx_1 + n_2 x_2}. \tag{16}$$

Based on the equation, the next section is going to analyze the optimality according to the different parameters such as the number and the duration between the preventive repairs.

3. Optimality analysis

Herein, the maintenance cost is rewritten in order to integrated the impacts of preventive maintenance (PM) on the equipment lifetime distribution. We assume that a preventive action allows to reduce the age of equipment to zero and increase the hazard function. **Figures 1** and **2** point out the impact of PM on the equipment hazard and reliability functions. The hazard function is defined after PM as follows

$$\lambda_{j,k}(t) = \beta_j \lambda_{j,k-1}(t), \tag{17}$$

where $j = 1, 2,$ and $\beta_j > 1$. Under these hypotheses, Eq. (5), which represents the total minimal cost on the first environment, is rewritten as

$$C_{tm1} = -c_{m1} \frac{1 - \beta_1^{n_1}}{1 - \beta_1} \log R_1(x_1), \tag{18}$$

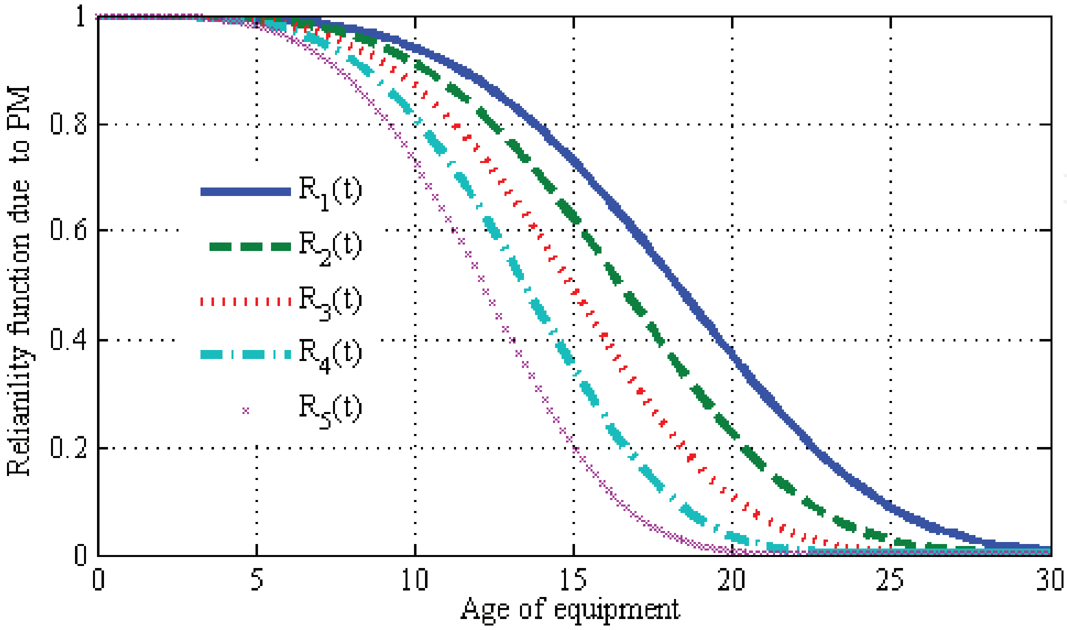

Figure 2. Evolution of reliability function due to preventive maintenance.

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with

$$R_1(x_1) = R_{1,1}(x_1). \tag{19}$$

In the second operating environment, the hazard function at $(n+1)^{th}$ is a consequence of $n_1 - 1 + \gamma$ PM in first and $1 - \gamma$ in the second environment.

$$\begin{aligned} \lambda_{2,n_1+1}(t) &= \beta_1^{n_1-1+\gamma}\beta_2^{1-\gamma}\lambda_2(t) \\ \lambda_{2,n_1+2}(t) &= \beta_1^{n_1-1+\gamma}\beta_2^{2-\gamma}\lambda_2(t) \\ \ldots &= \ldots \\ \lambda_{2,n_1+n_2}(t) &= \beta_1^{n_1-1+\gamma}\beta_2^{n_2-\gamma}\lambda_2(t) \end{aligned} \tag{20}$$

with

$$\lambda_2(t) = \lambda_{2,1}(t). \tag{21}$$

The total cost due to the minimal repair in the second environment becomes

$$C_{mt2} = -c_{m2}\beta_1^{n_1-1+\gamma}\beta_2^{1-\gamma}\frac{1-\beta_2^{n_2}}{1-\beta_2}\log R_2(x_2). \tag{22}$$

By considering Eqs. (15), (18), and (19), the total cost per unit of time is rewritten as follows

$$\begin{aligned} C(n_1,n_2,x_1,x_2) = &\frac{1}{n_1x_1+n_2x_2}\left((n_1-1+\gamma)C_{p1} + (n_2-\gamma)C_{p2}\right) \\ &-\frac{1}{n_1x_1+n_2x_2}\left(c_{m1}\frac{1-\beta_1^{n_1}}{1-\beta_1}\log R_1(x_1)\right) \\ &-\frac{1}{n_1x_1+n_2x_2}\left(c_{m2}\beta_1^{n_1-1+\gamma}\beta_2^{1-\gamma}\frac{1-\beta_2^{n_2}}{1-\beta_2}\log R_2(x_2)\right). \end{aligned} \tag{23}$$

3.1. Optimality according to n_1 and n_2

Let us assume that there is a pair (n_1, n_2) that provides the minimal cost per unit according to the Eq. (20) for given periods (x_1, x_2) between preventive repairs. Then, corresponding cost has to remain the unique lowest bound relative to other pairs of integer. This implies that cost at (n_1, n_2) must be better than the costs from the successive pairs $\{(n_1+1, n_2), (n_1-1, n_2)\}$; $\{(n_1, n_2+1), (n_1, n_2-1)\}$ and $\{(n_1+1, n_2+1), (n_1-1, n_2-1)\}$. The existence and uniqueness of the pairs are analyzed through some propositions.

3.1.1. Local optimality

The local optimality concerns the direct neighbors of the optimal pair such as $\{(n_1+1, n_2), (n_1-1, n_2)\}$ and $\{(n_1, n_2-1), (n_1, n_2+1)\}$. Let pose that

$$L_1(n_1|n_2) = C(n_1, n_2, x_1, x_2) - C(n_1 + 1, n_2, x_1, x_2), \tag{24}$$

$$L_2(n_2|n_1) = C(n_1, n_2, x_1, x_2) - C(n_1, n_2 + 1, x_1, x_2), \tag{25}$$

Proposition 1 *If the lifetime distribution functions are increasing failure rate (IFR) and* $L_1(1|n_2) > 0$, *then there exists a unique optimal number of PM* n_1 *in the first environments in which this* n_1 *ensures the minimal cost per unit time for a fixed pair* (x_1, x_2) *and* n_2.

Proof. As the maintenance cost per unit of time is minimal for (n_1, n_2), then we have

$$\begin{cases} C(n_1, n_2, x_1, x_2) \leq C(n_1 - 1, n_2, x_1, x_2), \\ C(n_1, n_2, x_1, x_2) < C(n_1 + 1, n_2, x_1, x_2). \end{cases} \tag{26}$$

This system is equivalent to

$$\begin{cases} L_1(n_1 - 1|n_2) \geq 0, \\ L_1(n_1|n_2) < 0. \end{cases} \tag{27}$$

with

$$\begin{aligned} L_1(n_1 - 1|n_2) &= -n_2 x_2 C_{p1} + x_1\big((\gamma - 1)C_{p1} + (n_2 - \gamma)C_{p2}\big) \\ &-x_1\left(c_{m_1}\frac{1-\beta^{n_1}}{1-\beta_1}\log R_1(x_1) + c_{m2}\beta_1^{n_1-1+\gamma}\beta_2^{1-\gamma}\frac{1-\beta_2^{n_2}}{1-\beta_2}\log R_2(x_2)\right) \\ &+(n_1x_1 + n_2x_2)\left(c_{m1}\beta_1^{n_1-1}\log R_1(x_1) + c_{m2}\beta_1^{n_1-2+\gamma}\beta_2^{1-\gamma}\frac{\beta_1 - 1}{1-\beta_2}\left(1-\beta_2^{n_2}\right)\log R_2(x_2)\right). \end{aligned}$$

In fact

$$\lim_{n_1 \leftarrow +\infty} L_1(n_1|n_2) = -\infty,$$

and

$$\begin{aligned} L_1(n_1|n_2) - L_1(n_1 - 1|n_2) &= (n_1x_1 + n_2x_2)\left(c_{m1}\beta_1^{n_1}\log R_1(x_1)\right) \\ &+(n_1x_1 + n_2x_2)\left(c_{m2}\beta_1^{n_1-1+\gamma}\beta_2^{1-\gamma}\frac{\beta_1 - 1}{1-\beta_2}\left(1-\beta_2^{n_2}\right)\log R_2(x_2)\right), \end{aligned}$$

The right-hand side of the previous equation shows that $L_1(n_1|n_2) - L_1(n_1 - 1|n_2) < 0$. This implies that $L_1(n_1|n_2)$ decreases with n_1. If $L_1(1|n_2) > 0$, then there exists a unique n_1 which verifies condition (23) and ensures the minimal cost per unit time for given n_2.

Proposition 2 *If the lifetime distribution function of equipment on both environments is IFR and* $L_2(1|n_1) > 0$, *then there exists a unique optimal number of PM* n_2 *in the second environment in which this number ensures the minimal cost per unit time for corresponding fixed pair* (x_1, x_2) *and* n_2.

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Proof. As the cost maintenance per unit time is minimal for (n_1, n_2), then we have

$$\begin{cases} C(n_1, n_2, x_1, x_2) \leq C(n_1, n_2 - 1, x_1, x_2), \\ C(n_1, n_2, x_1, x_2) < C(n_1, n_2 + 1, x_1, x_2). \end{cases} \tag{28}$$

This is equivalent to

$$\begin{cases} L_2(n_2 - 1|n_1) \geq 0, \\ L_2(n_2|n_1) < 0. \end{cases} \tag{29}$$

with

$$\begin{aligned} L_2(n_2 - 1|n_1) &= -n_1 x_1 C_{p2} + x_2\big((n_1 - 1 + \gamma)C_{p1} - \gamma C_{p2}\big) \\ &-x_2\left(c_{m1}\frac{1-\beta_1^{n_1}}{1-\beta_1}\log R_1(x_1) + c_{m2}\beta_1^{n_1-1+\gamma}\beta_2^{1-\gamma}\frac{1-\beta_2^{n_2}}{1-\beta_2}\log R_2(x_2)\right) \\ &+(n_1 x_1 + n_2 x_2)\left(c_{m2}\beta_1^{n_1-1+\gamma}\beta_2^{n_2-\gamma}\log R_2(x_2)\right). \end{aligned}$$

This equation implies

$$\lim_{n_2 \to \infty} L_2(n_2|n_1) = -\infty,$$

and

$$L_2(n_2|n_1) - L_2(n_2 - 1|n_1) = (n_1 x_1 + n_2 x_2)\left(c_{m2}\beta_1^{n_1-1+\gamma}\beta_2^{n_2-\gamma+1}\log R_2(x_2)\right) < 0.$$

Therefore, $L_2(n_2|n_1)$ decreases with and for $L_2(1|n_1) > 0$, we have a unique n_2 in which the total per unit of time is minimal for fixed n_1.

3.1.2. Global optimality

The global optimality compares the optimal pair to $\{(n_1 + 1, n_2 + 1), (n_1 - 1, n_2 - 1)\}$. Let us pose that

$$L_3(n_1, n_2) = C(n_1, n_2, x_1, x_2) - C(n_1 + 1, n_2 + 1, x_1, x_2). \tag{30}$$

Proposition 3 *If the lifetime distribution functions are IFR and $L_3(1, 1) > 0$, then there exists a unique optimal number of PM (n_1, n_2) in which this ensures the minimal cost per unit time for a fixed pair (x_1, x_2).*

Proof. As the cost is minimal for (n_1, n_2), then

$$\begin{cases} C(n_1, n_2, x_1, x_2) \leq C(n_1 - 1, n_2 - 1, x_1, x_2), \\ C(n_1, n_2, x_1, x_2) < C(n_1 + 1, n_2 + 1, x_1, x_2). \end{cases} \tag{31}$$

This is equivalent to

$$\begin{cases} L_3(n_1-1, n_2-1) \geq 0, \\ L_3(n_1, n_2) < 0. \end{cases} \tag{32}$$

with

$$\begin{aligned} L_3(n_1-1, n_2-1) &= (x_1+x_2)\big((\gamma-1)C_{p1} - \gamma C_{p2}\big) \\ &-(x_1+x_2)\left(c_{m1}\frac{1-\beta_1^{n_1}}{1-\beta_1}\log R_1(x_1) + c_{m2}\beta_1^{n_1-1}\beta_2^{1-\gamma}\frac{1-\beta_2^{n_2}}{1-\beta_2}\right) \\ &+(n_1x_1+n_2x_2)\left(c_{m1}\beta^{n_1-1}\log R_1(x_1) - c_{m2}\frac{1-\beta_1-\beta_2^{n_2-1}(1-\beta_2)}{1-\beta_2}\beta_1^{n_1-2}\beta_2^{1-\gamma}\right). \end{aligned}$$

With

$$L_3(+\infty, +\infty) = -\infty,$$

and

$$L_3(n_1, n_2) - L_3(n_1-1, n_2-1) < 0.$$

Therefore, $L_3(n_1, n_2)$ decreases with (n_1, n_2) and for $L_3(1,1) > 0$, we have a unique pair (n_1, n_2) in which the total per unit of time is minimal.

3.2. Optimality according to x_1 and x_2

For given number of preventive actions (n_1, n_2), the optimal durations (x_1, x_2) between preventive actions in both environments have to verify

$$\begin{cases} \dfrac{\partial}{\partial x_1} C(n_1, n_2, x_1, x_2) = 0 \\ \dfrac{\partial}{\partial x_2} C(n_1, n_2, x_1, x_2) = 0; \end{cases} \tag{33}$$

This implies

$$\begin{cases} c_{m1}\dfrac{1-\beta_1^{n_1}}{1-\beta_1}\lambda_1(x_1) & = n_1 C(n_1, n_2, x_1, x_2), \\ c_{m2}\beta_1^{n_1-1+\gamma}\beta_2^{1-\gamma}\dfrac{1-\beta_2^{n_2}}{1-\beta_2}\lambda_2(x_2) & = n_2 C(n_1, n_2, x_1, x_2). \end{cases} \tag{34}$$

By dividing, we obtain

$$\frac{\lambda_1(x_1)}{\lambda_2(x_2)} = \left(\beta_1^{n_1-1+\gamma}\beta_2^{1-\gamma}\right)\frac{n_1}{n_2}\frac{c_{m2}}{c_{m1}}\frac{1-\beta_2^{n_2}}{1-\beta_1^{n_1}}\frac{1-\beta_1}{1-\beta_2} \tag{35}$$

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Proposition 4 *If the lifetime functions of the equipment are Weibull-distributed in both environments with the same shape parameter b, then the optimal interval between PM is defined as*

$$\frac{x_1}{x_2} = Cste^* \left(\frac{n_1}{n_2}\right)^{1/(b-1)}. \tag{36}$$

Proof. As lifetime functions are Weibull-distributed with the same parameter b, then the hazard functions are defined as follows

$$\lambda_1(x_1) = \frac{b}{\eta_1}\left(\frac{x_1}{\eta_1}\right)^{b-1}, \tag{37}$$

$$\lambda_2(x_2) = \frac{b}{\eta_2}\left(\frac{x_2}{\eta_2}\right)^{b-1}, \tag{38}$$

and from Eq. (32), we deduce

$$\frac{x_1}{x_2} = \left(\frac{c_{m2}}{c_{m1}}\left(\beta_1^{n_1-1+\gamma}\beta_2^{1-\gamma}\right)\frac{1-\beta_2^{n_2}}{1-\beta_1^{n_1}}\frac{1-\beta_1}{1-\beta_2}\right)^{1/(b-1)}\frac{\eta_1}{\eta_2}\left(\frac{n_1}{n_2}\right)^{1/(b-1)}. \tag{39}$$

The uniqueness is tough to establish due to the number of parameters and the complexity of the proposed cost model here. To make the research of optimal solution easy, we propose a handy heuristic based on the optimal derived conditions in this chapter. The next section describes step by step the proposed heuristic which leads to a suitable solution for our optimization problem.

3.3. Numerical resolution of problem

Herein, an algorithm is drawn in order to find the optimal pairs for (n_1, n_2) and (x_1, x_2). The optimal pairs ensure the minimal cost per unit time defined by Eq. (20). Moreover, the existence of these optimal pairs is discussed in the previous sections. The proposed heuristic makes switching between the research of pairs ((n_1, n_2) and (x_1, x_2)). This algorithm converges surely toward the pair that ensures the minimal cost according to the conditions deduce from the Eq. (20). The next section presents an application of our approach. The algorithm is on the previous propositions and defined as follows.

Algorithm 1 Compute the optimal pairs of number (n_1, n_2) and periods (x_1, x_2) of PM.

Initialize the pair $(n_1, n_2)_0 = (1, 1)$.

Put $(n_1, n_2) = (n_1, n_2)_0$

STEP (A) Research optimal (x_1, x_2) for given (n_1, n_2).

Compute $L_1(1|n_2)$, $L_2(1|n_1)$ and $L_3(1, 1)$.

if $L_1(1|n_2) > 0$ **then**

Research $n_1(a)$ which verifies condition (24) is verified

$n_1(1) = n_1(a)$ and $n_2(1) = n_2$.

$C_1 = C(n_1(1), n_2(1))$

else

$\{L_1(1|n_2) < 0\}$

$C_1 = \infty$

if $L_2(1|n_1) > 0$ **then**

Research $n_2(b)$ which verifies conditions (26).

$n_1(2) = n_1; n_2(2) = n_2(b)$

$C_2 = C(n_1(2), n_2(2))$

else $\{L_2(1|n_1) < 0\}$

$C_2 = \infty$

if $L_3(1, 1) > 0$ **then**

Research $n_2(c)$ which verifies $L_3(n_1(c), n_2(c))$ (29).

$n_1(2) = n_1(c); n_2(2) = n_2(c)$

$C_3 = C(n_1(2), n_2(2))$

else

$\{L_3(1, 1) < 0\}$

$C_3 = \infty$

$C_{min} = \text{Min}\{C_1, C_2, C_3\}$

$m = m + 1$

$(n_1, n_2)_m = \{(n_1(i), n_2(i)|C_i = C_{min}\}$

if $(n_1, n_2)_m = (n_1, n_2)_{m-1}$ **then**

$(n_1, n_2) = (n_1, n_2)_m$

Keep corresponding (x_1, x_2) **else**

$\left\{(n_1, n_2)_m \neq (n_1, n_2)_{m-1}\right\}$

$(n_1, n_2) = (n_1, n_2)_m$

Go to step (A)

end if

End.

4. Numerical application

We consider an equipment whose lifetime distribution function is Weibull with the same shape parameter $b = 2.0$. The equipment has to be used on two environments with different severity. Their severity depends on the scale parameter, such as in first the scale is $\eta_1 = 20$, while $\eta_2 = 10$ stands for the scale parameter in the second environment. This implies that the second environment is twice more severe than first. To reduce the risk of equipment failure of the failure, the equipment undergoes periodic, preventive maintenance. The preventive maintenance costs $C_{p1} = 100$ and $C_{p2} = 150$, respectively, on the first and second. The preventive actions impact the lifetime distribution of equipment. The impact factors due to PM are equal to $\beta_1 = 1.85$ in first and $\beta_2 = 2.5$ in the second environment. In addition, the equipment is minimally repaired at failure. The costs of minimal repair are in both environments $c_{m1} = 80$ and $c_{m2} = 70$. Based on this information, we are going to solve the optimization problem in order to find the number and duration period between PM on each environment which ensure a minimal cost per unit of time. With these parameters, the minimal cost reaches 10.37. This minimal cost involves $n_1 = 1$ and $n_2 = 1$ preventive maintenance (PM) respectively in the first and second environments. The durations between each PM are $x_1 = 26.06$ and $x_2 = 3.03$.

5. Conclusion

This chapter shows how to solve Nakagawa maintenance policy problem for an equipment which operates simultaneously on two environments. Each environment impacts the lifetime distribution function of our equipment. Nakagawa's maintenance problem is modeled under lifetime distribution changing in operation. The proposed model is deeply analyzed in order to derive the conditions under which optimal pairs exist and are reachable. To reach these pairs, algorithm was proposed to find the optimal solution for the periodic preventive maintenance on infinite horizon. The model is handy and suitable for production equipments which have to experience under different operating environments with their own severity degree that impacts the equipment performance such as onshore or offshore.

For future work, we plan to propose a statistical modeling by ignoring the hypothesis on the knowledge of the equipment lifetime distribution and perform an extension of the analysis by considering an finite-time horizon/span.

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References

[1] Barlow R, Hunter L. Optimum preventive maintenance policies. Operations Research. 1960;**8**(1):90-100

[2] Barlow R, Proschan F. Mathematical Theory of Reliability. New York: John Wiley & Sons; 1965

[3] Nakagawa T, Mizutani S. A summary of maintenance policies for a finite interval. Reliability Engineering & System Safety. 2009;**94**(1):89-96

[4] Nakagawa T. Advanced Reliability Models and Maintenance Policies. Springer series in reliability engineering, London: Springer; 2008

[5] Cho D, Parlar M. A survey of maintenance models for multi-unit systems. European Journal of Operational Research. 1991;**51**(1):1-23

[6] Dekker R. Applications of maintenance optimization models: A review and analysis. Reliability Engineering & Systems Safety. 1996;**51**(3):229-240

[7] Jardine A, Tsang A. Maintenance, Replacement, and Reliability: Theory and Applications. Boca Raton: CRC/Taylor & Francis; 2006

[8] Lugtigheid D, Jardine A, Jiang X. Optimizing the performance of a repairable system under a maintenance and repair contract. Quality and Reliability Engineering International. 2007;**23**(8):943-960

[9] Coolen F, Coolen-Schrijner P, Yan K. Non-parametric predictive inference in reliability. Reliability Engineering & System Safety. 2002;**78**(2):185-193

[10] Coolen-Schrijner P, Coolen F. Non-parametric predictive inference for age replacement with a renewal argument. Quality and Reliability Engineering International. 2004;**20**(3): 203-215

[11] de Jonge B, Klingenberg W, Teunter R, Tinga T. Optimum maintenance strategy under uncertainty in the lifetime distribution. Reliability Engineering & System Safety. 2015;**133**: 59–67.

[12] Nakagawa T. Periodic and sequential preventive maintenance policies. Journal of Applied Probability. 1986;**23**(2):536-542

[13] Thompson WA. On the foundations of reliability. Technometrics. 1981;**23**(1):1-13