Chapter 1

Modification of the Electromagnetic Field in the Photonic Crystal Medium and New Ways of Applying the Photonic Band Gap Materials

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Additional information is available at the end of the chapter

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Abstract

Photonic crystals (PCs) are periodic systems that consist of dielectrics with different refractive indices. Photonic crystals have many potential technological applications. These applications are mainly based on the photonic bang gap effect. However the band gap is not only effect that follows from the periodic changing of the refractive index in the photonic crystal. The periodic change of the photon-matter interaction in photonic crystal medium gives rise to the fact that the mass of an electron in the photonic crystal must differ from its mass in vacuum. Anisotropy of a photonic crystal results in the dependence of the electromagnetic mass correction on the orientation of the electron momentum in a photonic crystal. This orientation dependence in turn gives rise to the significant correction to the transition frequencies in an atom placed in air voids of a photonic crystal. These corrections are shown to be comparable to the atomic optical frequencies. This effect allows one to control the structure of the atomic energy levels and hence to control resonance processes. It can serve as the basis for new line spectrum sources. The effect provides new ways of realization of quantum interference between decay channels that can be important for quantum information science.

Keywords: photonic crystals, electron mass, anisotropic vacuum, electromagnetic field, Lamb shift

1. Introduction

Photonic crystals (PCs) are a major field of research having many potential applications [1–15]. These applications are mainly based on the photonic bang gap effect in the photonic crystal. In Ref. [16], it has been shown that a strong modification of the electromagnetic interaction in photonic crystals results in the fact that the electron mass changes its value. Actually in this

case, we deal with a quantum electrodynamical (QED) effect that does not manifest itself in the free space. In fact, the interaction of an electron with its own radiation field gives rise to a contribution to its physical mass m_{ph} known as the electromagnetic mass of the electron m_{em}. Nonrenormalizable ultraviolet divergences do not allow one to calculate the electron electromagnetic mass. However, fortunately, only physical mass m_{ph} is observable, and hence m_{em} can be included into it. On the other hand, the modification of the electromagnetic interaction in PC medium gives rise to a correction to the electromagnetic mass m_{em}. This correction δm_{pc} cannot be hidden in the physical mass of the electron and hence is an observable. Thus in PC medium, the novel observable δm_{pc} comes into play. A remarkable feature of δm_{pc} is its dependence on the orientation of the electron momentum in a PC, and this dependence gives rise to significant corrections to the transition frequencies in an atom placed in air voids of a photonic crystal, being comparable to the ordinary atomic frequencies. Such an effect is a consequence of the fact that in the case of atoms in the PC medium, the most contribution comes from the self-energy of electrons associated with mass correction m_{em}^{pc} rather than from the self-energy of atoms associated with the Lamb shift being the QED corrections to the nucleus-electrons coupling. In this chapter, we discuss the origins of the effect of the change in the electron mass caused by the modification of the electromagnetic interaction in a PC and its possible applications.

2. Lamb shift in hydrogen atom in the free space

The processes of the interaction of charged particles with their own radiation field play the important role in the modern physics. These processes give rise to the fact that actually we deal with the particles dressed by a cloud consisting of virtual particles (photons, electron-positron pairs, and so on). In the case of electrons or muons bound to an atomic nucleus, the self-interaction results in the Lamb shift of the atomic energy levels. The results of the recent measurements of the Lamb shift in muonic hydrogen [17, 18] have allowed to determine the value of the root-mean-square charge radius of the proton r_p which is 4% smaller than the radius determined by electron-proton experiments [19, 20] and precision spectroscopy of the ordinary atomic hydrogen [21–27]. This discrepancy known as the "proton radius puzzle" has not been explained yet. Solving the puzzle may require new insights into the problem of the description of the self-energy of the electron and the Lamb shift.

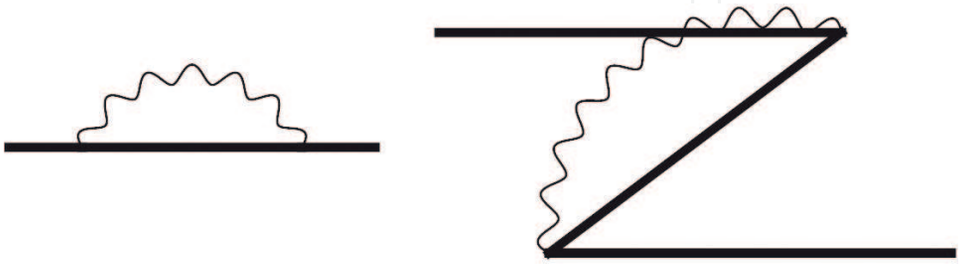

Figure 1. The time-ordered diagrams describing the dominant contribution to the Lamb shift. The thick line denotes the electron (positron) propagating in the Coulomb field; the wavy line denotes emission and reabsorption of a virtual photon.

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The Lamb shift consists of the self-energy and vacuum polarization contributions. The modification of the vacuum polarization contribution in the PC medium is negligible, and for this reason, we will focus only on the self-energy one. At leading order self-energy of the electron, which is bound in a hydrogen-like atom, is defined by the process in which a photon is emitted and then is reabsorbed by the electron or positron. This process is described by the time-ordered diagrams in **Figure 1**.

In quantum electrodynamics the corresponding contribution to the Lamb shift in hydrogen-like atoms is given by the term that appears in the second-order perturbation theory and in the Furry picture can be written as

$$\Delta E_{L,n} = \langle n|H_I \frac{1}{E_n^{(0)} - H_0^F} H_I|n\rangle, \tag{1}$$

where H_0^F is the unperturbed Dirac-Coulomb Hamiltonian in the Furry picture ($H_0^F|n\rangle = E_n^{(0)}|n\rangle$), $|n\rangle$ is an atomic state, and

$$H = H_0 + \int d^3x H_I(t=0,\mathbf{x}), \tag{2}$$

with $H_I(t,\mathbf{x})$ being the interaction Hamiltonian density:

$$H_I(t,\mathbf{x}) = \frac{e}{2} A_\mu(t,\mathbf{x}) \left[\overline{\Psi}(t,\mathbf{x}), \gamma^\mu \Psi(t,\mathbf{x})\right]. \tag{3}$$

Here $\Psi(x)$ is the Dirac field in the Furry picture. Usually the contributions to the Lamb shift (1) are separated into the low and high energy parts. For the reasons explained bellow, we will focus on the low-energy part of the shift [28]:

$$\Delta E_{L,n}^{<} = \frac{2\pi\alpha}{3m_e^2} \int_0^\Lambda \frac{d^3k}{2|\mathbf{k}|(2\pi)^3} \sum_m \frac{|\langle n|\mathbf{p}|m\rangle|^2}{E_n - |\mathbf{k}| - E_m}, \tag{4}$$

where $\mathbf{p}$ is the operator of the electron momentum and the cutoff Λ limits the energies of virtual photons in the processes of their emission and reabsorption. The cutoff must be much less than typical electron momenta but much larger than the atomic binding energies:

$$(Z\alpha)^2 m_e << \Lambda << (Z\alpha) m_e. \tag{5}$$

Here and below the natural unit system is used, where $\hbar = c = \varepsilon_0 = 1$. This is the reason why one can use the nonrelativistic Hamiltonian:

$$H = \frac{1}{2m_e}[\mathbf{p} - e\mathbf{A}]^2 \tag{6}$$

instead of the Hamiltonian defined in Eqs. (2) and (3). Eq. (4) can be rewritten in the form:

$$\Delta E_{L,n} = -\frac{\Delta m_e^{<}}{2m_e^2}\langle n|\mathbf{p}^2|n\rangle + \Delta E_L^{<}, \tag{7}$$

where

$$\Delta m_e^{<} = \frac{\alpha}{p^2\pi^2}\sum_{\lambda=1}^{2}\int_0^{\Lambda}\frac{d^3k}{2|\mathbf{k}|^2}|\mathbf{p}\cdot\varepsilon_\lambda(\mathbf{k})|^2 \tag{8}$$

is the low-energy electron mass correction caused by its self-interaction [29]. It should be noted that $\Delta E_{L,n}$ does not contain a term describing the electromagnetic correction to the electron mass. This is the result of making use of the nonrelativistic Hamiltonian (6), and for this reason, the mass correction is extracted from the first term on the right-hand part of Eq. (7) describing the correction to the kinetic energy. Thus, in this case the electromagnetic mass correction is regarded to be included into the physical mass of the electron. The first term on the right-hand part of Eq. (7) must be also included into the physical mass. In this way we arrive at the ordinary expression for the low-energy Lamb shift in hydrogen-like atoms:

$$\Delta E_{L,m}^{<} = \frac{\alpha}{6\pi^2 m_e^2}\sum_m\int_0^{\Lambda}\frac{d^3k}{2|\mathbf{k}|^2}\,\frac{|\langle n|\mathbf{p}|m\rangle|^2}{E_n - |\mathbf{k}| - E_m}(E_n - E_m). \tag{9}$$

Adding to $\Delta E_L^{<}$ the high energy contribution [28]:

$$\Delta E_L^{>} = \frac{4\alpha}{3}\frac{Z\alpha}{m^2}\left|\Psi_{nlmj}(0)\right|^2\left(\ln\frac{m_e}{2\Lambda} + \frac{11}{24} - \frac{1}{5}\right), \tag{10}$$

where n, l, m, j, and $\Psi_{nlmj}(x)$ being, respectively, the main quantum number, orbital quantum number, magnetic quantum number, inner quantum number, and the wave function, we get the expression to the total Lamb shift of the energies of the states of the hydrogen-like atoms. In the S-state it reads

$$\Delta E_{L,n} = \frac{4\alpha(Z\alpha)^4}{3\pi n^2}\left(\ln\frac{m_e}{2\overline{E}} + \frac{11}{24} - \frac{1}{5}\right)m_e + o(Z\alpha)^4, \tag{11}$$

where $\overline{E} = \alpha^2 m_e$.

3. The Lamb shift in atoms placed in a PC

Investigation of the Lamb shift in hydrogen atom placed in a PC attracts much attention for a long time since the Lamb shift is (historically and in practice) the most important phenomenon of quantum electrodynamics. Interestingly, the calculation results obtained in different works differed strongly in order of magnitude, and the significance of interaction with vacuum, depending on which model of the dispersion of a photon in a photonic crystal, was used.

The first attempt was made by John and Wang [4] by using the solution of the scalar wave equation in one dimension. Thus, the photon dispersion relation was chosen to be isotropic and satisfy the transcendental equation:

$$4n\cos(kL) = (1+n)^2\cos[(2na+b)\omega_k] - (1-n)^2\cos[(2na-b)\omega_k]. \tag{12}$$

Using this dispersion relation, the authors predicted anomalous Lamb shift affecting the odd-parity $2P_{1/2}$ state and not the even-parity $2S_{1/2}$. Magnitude of the effect makes it detectable using microwave. The fact that the anomalous Lamb shift of the $2P_{1/2}$ state is larger than the ordinary Lamb shift of the $2S_{1/2}$ state originates from the dimension of the phase space occupied by band edge photons of vanishing group velocity. John and Wang overestimated this phase space by assuming that $d\omega_k/dk$ vanishes over the entire sphere $|\mathbf{k}| = \pi/L$. At the same time for the case of real photonic crystals, the shift was expected to be comparable to the ordinary Lamb shift of the $2S_{1/2}$ level.

The authors of work [30] noted that a real photonic crystal in general has an anisotropic structure in momentum space and a three-dimensional dispersion relation is required because the density of states (DOS) in isotropic or one-dimensional case has a singularity near band edge. In this study the atomic transition frequency ω is assumed to be near the band edge ω_c and the dispersion relation was approximated by the expression.

$$\omega_k = \omega_c + A\left|\mathbf{k} - \mathbf{k}_0^i\right|^2, \tag{13}$$

where A is a model-dependent constant and $\mathbf{k}_0^i$ is a finite set of symmetrically placed points leading to a three-dimensional band structure. Using this model the Schrödinger equation was solved, and analytical expression for the Lamb shift was obtained. The value of the Lamb shift turned out to be smaller than that for a hydrogen atom in an ordinary vacuum. Authors explained this result by the fact that the DOS in the photonic crystals with three-dimensional dispersion relations is much lower than that in the ordinary vacuum. This result is also very different from that from the one-dimensional case where DOS has a singularity or from the two-dimensional case where DOS has a sudden jump.

In paper [31] all previous approaches to calculate Lamb shift in photonic crystal were criticized, because they are basically scalar. Authors of this work demonstrated the rigorous solution of the problem of calculation of the Lamb shift in atomic hydrogen in a 3D photonic crystal and showed that the presence of a photonic band gap (PBG) at optical wavelengths can hardly change the Lamb shift. The correction to the energy of electronic state $|m\rangle$ was calculated in the second order of perturbation theory. The quantization of EM fields in a 3D photonic crystal was made by expanding the EM fields in a set of eigenmodes (Bloch states). These states can be solved numerically by means of a plane-wave expansion method. Finally, it was given an expression for the energy shift containing the local density of states (LDOS):

$$\Delta\mathrm{E} = \frac{e^2\hbar}{u_0^2 m_e^2}\sum_n E_{nm}\left|p_{mn}\right|^2\int_0^\infty d\omega\,\frac{\rho(\omega,\mathbf{r})}{\omega^3(E_{nm}+\hbar\omega)}, \tag{14}$$

with $\rho(\omega, \mathbf{r})$ being LDOS:

$$\rho(\omega, \mathbf{r}) = \frac{u_0^2 c^2}{2\hbar\varepsilon_0 (2\pi)^3 \varepsilon^2(\mathbf{r})} \sum_n \int_{BZ} d^3k \frac{|\nabla \times \mathbf{H}_{n\mathbf{k}}(\mathbf{r})|^2}{3\omega_{n\mathbf{k}}} \delta(\omega - \omega_{n\mathbf{k}}), \tag{15}$$

where u_0 is dipole moment, $\varepsilon(\mathbf{r})$ is dielectric constant function, and $\mathbf{H}_{n\mathbf{k}}(\mathbf{r})$ is magnetic field distribution of the Bloch states with energy $\hbar\omega_{n\mathbf{k}}$. The authors estimated the magnitude of the Lamb shift and concluded that PBG at optical wavelengths will not cause an appreciable variation to the energy-level shift induced by self-interaction for different atom positions and different variations of the LDOS.

Vats with colleagues used the anisotropic band edge model and pseudogap model to calculate the Lamb shift in an atom placed in photonic crystal [32]. In the first case near the band edge, dispersion relation (13) was used and corresponding DOS derived. Calculated Lamb shift was an order of magnitude larger than the free space Lamb shift. Then authors treated the case of a pseudogap, for which the stop band does not extend over all propagation directions, thus resulting in a suppression of the DOS rather than the formation of a full PBG:

$$N(\omega) = \omega^2 \left[1 - h\exp\left(-\frac{(\omega - \omega_0)^2}{\Gamma^2} \right) \right]. \tag{16}$$

Here, h and Γ are parameters describing the depth and width of the pseudogap, respectively, and ω_0 is the central frequency of the pseudogap. Vats with coworkers concluded that for a sufficiently strong pseudogap, the maximal value of Lamb shift may be on the order of 15% of the free space value.

The authors of work [33] using method of Green functions developed a general formalism for calculating the Lamb shift in multilevel atoms. The radiative correction to the bound level l is determined by the expression

$$\omega - \omega_l = \sum_j \frac{\alpha_{lj}}{2\pi} (\omega - \omega_j) \beta(\mathbf{r}, \omega - \omega_j), \tag{17}$$

where

$$\alpha_{lj} = \frac{e^2 \left| \mathbf{p}_{lj} \right|^2}{3\pi m_e^2 \varepsilon_0 \hbar c^3} \tag{18}$$

is the relative linewidth of the atomic radiation from the l state to the j state in vacuum

$$\beta(\mathbf{r}, \omega - \omega_j) = P \int_0^{m_e c^2/\hbar} d\omega' \frac{g(\mathbf{r}, \omega')}{(\omega - \omega_j - \omega')\omega'}. \tag{19}$$

The function $g(\mathbf{r}, \omega)$ is the local spectral response function (LSRF) proportional to the photon LDOS:

$$g(\mathbf{r},\omega)=\frac{c^3V_{pc}}{2\pi\omega}\sum_n\int_{BZ}d^3k|\mathbf{E}_{n\mathbf{k}}(\mathbf{r})|^2\delta(\omega-\omega_{n\mathbf{k}}) \tag{20}$$

with V_{pc} being the PC volume and $\mathbf{E}_{n\mathbf{k}}(\mathbf{r})$ being the electromagnetic eigenmodes. Authors revealed that in a 3D PC, real photons make a dominant contribution to the value of the Lamb shift, while the contribution from interaction with virtual photons is small. This differs significantly from the free space case. It was shown that the PC structure can lead to a giant Lamb shift, that is, up to two orders of magnitude larger than that for an ordinary vacuum [34]. The Lamb shift is sensitive to both the position of an atom in PCs and the transition frequency of the related excited level.

4. Photonic crystal medium corrections to the electron rest mass

For a long time in investigations of QED effects in the PC medium, researches focused on study of the Lamb shift in hydrogen atom placed in a PC. In all the listed studies, the subtraction of the modified by PC medium self-energy of the free electron from the modified self-energy of the bound electron was used. This procedure was correct, if this self-energy could be included into the electron physical mass. However this is not the case, because the electromagnetic mass of the electron in a PC differs from that in the free space and cannot be hidden in the physical mass. In fact

$$m_{em}^{pc}=m_{em}+\delta m_{pc} \tag{21}$$

and hence the total electron mass m_e^{pc} in a PC is

$$m_e^{pc}=m_e+\delta m_{pc}. \tag{22}$$

Thus, the modification of the interaction of the electron with its own radiation field in the PC medium results in the change in its mass. Let us now determine the mass correction δm_{pc}. For this we have to generalize our analysis of the electron self-energy to the case where it is in the PC medium. It is natural to start from determining of a quantized vector potential of electromagnetic field inside PC. It could be made by taking into account that photon states in periodic dielectric media have Bloch structure. Photonic Bloch states $|\mathbf{k}n\rangle$ can be obtained by means of the plane-wave expansion method [35]. By introducing the operators $\widehat{a}_{\mathbf{k}n}^{+}$ and $\widehat{a}_{\mathbf{k}n}$ that describe the creation and annihilation of the photon in the state $|\mathbf{k}n\rangle$, respectively ($\widehat{a}^{\mathbf{k}n+}|0\rangle=|\mathbf{k}n\rangle$ and $\widehat{a}_{\mathbf{k}n}|\mathbf{k}n\rangle=|0\rangle$), we can construct a modified vector potential:

$$\mathbf{A}_{pc}(\mathbf{r},t)=\sum_{\mathbf{k}n}\left[\mathbf{A}_{\mathbf{k}n}(\mathbf{r})\widehat{a}_{\mathbf{k}n}e^{-i\omega_{\mathbf{k}n}t}+\mathbf{A}_{\mathbf{k}n}^{*}(\mathbf{r})\widehat{a}_{\mathbf{k}n}^{+}e^{i\omega_{\mathbf{k}n}t}\right], \tag{23}$$

where $\mathbf{A}_{\mathbf{k}n}(\mathbf{r})=\sqrt{1/V\omega_{\mathbf{k}n}}\mathbf{E}_{\mathbf{k}n}(\mathbf{r})$ with $\mathbf{E}_{\mathbf{k}n}(\mathbf{r})$ being the Bloch eigenfunctions satisfying the following orthonormality condition:

$$\int_V d^3r\varepsilon(\mathbf{r})\mathbf{E}_{\mathbf{k}n}(\mathbf{r})\mathbf{E}^*_{\mathbf{k}'n'}(\mathbf{r}) = V\delta_{\mathbf{k}\mathbf{k}'}\delta_{nn'}. \tag{24}$$

Using vector potential (23) we can define nonrelativistic interaction Hamiltonian in the form

$$H_I^{pc} = -\frac{e}{m_e}\mathbf{p}\cdot\mathbf{A}_{pc}. \tag{25}$$

The matrix element $\langle \mathbf{p}'; \mathbf{k}, n|H_I^{pc}|\mathbf{p}\rangle$ of this Hamiltonian can be represented in the form

$$\langle \mathbf{p}'; \mathbf{k}, n|H_I^{pc}|\mathbf{p}\rangle = -\frac{e}{m_e}\int d^3r\Psi^*_{\mathbf{p}'}(\mathbf{r})(-i\nabla_{\mathbf{r}}\mathbf{A}_{\mathbf{k}n}(\mathbf{r}))\Psi_{\mathbf{p}}(\mathbf{r}) = \frac{e}{m_e V^{3/2}\sqrt{\omega_{\mathbf{k}n}}}\int d^3re^{-i\mathbf{p}'\mathbf{r}}(i\nabla_{\mathbf{r}}\mathbf{E}_{\mathbf{k}n}(\mathbf{r}))e^{i\mathbf{p}\mathbf{r}} \tag{26}$$

with $\Psi_{\mathbf{p}}(\mathbf{r})$ being the normalized wave function of the electron state $\Psi_{\mathbf{p}}(\mathbf{r}) = \langle \mathbf{r}|\,\mathbf{p}\rangle$. Here we have taken into account that $\Psi_{\mathbf{p}} = e^{i\mathbf{p}\mathbf{r}}/\sqrt{V}$ for $\mathbf{r}\in V$ and $\Psi_{\mathbf{p}} = 0$ for $\mathbf{r}\notin V$. Taking also into account that $\mathbf{E}_{\mathbf{k}n}(\mathbf{r})$ can be expanded as

$$\mathbf{E}_{\mathbf{k}n}(\mathbf{r}) = \sum_{\mathbf{G}}\mathbf{E}_{\mathbf{k}n}(\mathbf{G})e^{i(\mathbf{k}+\mathbf{G})\cdot\mathbf{r}} \tag{27}$$

with $\mathbf{G}$ being the reciprocal lattice vector of the photonic crystal ($\mathbf{G} = N_1\mathbf{b}_1 + N_2\mathbf{b}_2 + N_3\mathbf{b}_3$ where $\mathbf{b}_i$ is the basis vector of a reciprocal lattice), for $\langle \mathbf{p}'; \mathbf{k}, n|H_I^{pc}|\mathbf{p}\rangle$ we get

$$\langle \mathbf{p}'; \mathbf{k}, n|H_I^{pc}|\mathbf{p}\rangle = -\frac{e}{m_e}\frac{1}{\sqrt{V\omega_{\mathbf{k}n}}}\sum_{\mathbf{G}}\mathbf{p}\cdot\mathbf{E}_{\mathbf{k}n}(\mathbf{G})\delta_{\mathbf{p},\mathbf{q}} \tag{28}$$

with $\mathbf{q} = \mathbf{p}' + \mathbf{k} + \mathbf{G}$. For $\langle \mathbf{p}|H_I^{pc}|\mathbf{p}'; \mathbf{k}, n\rangle$ we find

$$\langle \mathbf{p}|H_I^{pc}|\mathbf{p}'; \mathbf{k}, n\rangle = -\frac{e}{m_e}\frac{1}{\sqrt{V\omega_{\mathbf{k}n}}}\sum_{\mathbf{G}}\mathbf{p}\cdot\mathbf{E}^*_{\mathbf{k}n}(\mathbf{G})\delta_{\mathbf{p},\mathbf{q}}. \tag{29}$$

Using these matrix elements, we can determine the mass correction δm_{pc} as a difference of the electromagnetic masses in PC and free space:

$$\delta m_{pc} = -\frac{2e^2}{\mathbf{p}^2V}\left(\sum_{\mathbf{G}}\sum_{\mathbf{k}n}\frac{1}{\omega_{\mathbf{k}n}}\frac{|\mathbf{p}\cdot\mathbf{E}_{\mathbf{k}n}(\mathbf{G})|^2}{\frac{\mathbf{p}^2}{2m_e} - \frac{(\mathbf{p}-\mathbf{k}-\mathbf{G})^2}{2m_e} - \omega_{\mathbf{k}n}} - \sum_{\mathbf{k}}\sum_{\lambda=1}^{2}\frac{1}{2|\mathbf{k}|}\frac{|\mathbf{p}\cdot\varepsilon_\lambda(\mathbf{k})|^2}{\frac{\mathbf{p}^2}{2m_e} - \frac{(\mathbf{p}-\mathbf{k})^2}{2m_e} - |\mathbf{k}|}\right). \tag{30}$$

It should be noted that this expression has a natural cutoff because dielectric constant vanishes at higher optical energies. Taking into account that electron momentum is much higher that photon momentum, Eq. (30) can be rewritten in the form

$$\delta m_{pc} = \frac{2e^2}{\mathbf{p}^2V}\left(\sum_{\mathbf{G}}\sum_{\mathbf{k}n}\frac{|\mathbf{p}\cdot\mathbf{E}_{\mathbf{k}n}(\mathbf{G})|^2}{\omega^2_{\mathbf{k}n}} - \sum_{\mathbf{k}}\sum_{\lambda=1}^{2}\frac{|\mathbf{p}\cdot\varepsilon_\lambda(\mathbf{k})|^2}{2\mathbf{k}^2}\right). \tag{31}$$

Now in the expression of δm_{pc}, we can replace the discreet sums by integrals:

$$\int d^3k \sum_{\mathbf{k}n} \to \frac{V}{(2\pi)^3} \sum_n \int d^3k, \quad \sum_{\mathbf{k}} \to \frac{V}{(2\pi)^3} \int d^3k. \tag{32}$$

In this way we get

$$\delta m_{pc} = \frac{\alpha}{\pi^2} \left[\sum_n \int\limits_{FBZ} \frac{d^3k}{\omega_{\mathbf{k}n}^2} \sum_{\mathbf{G}} \left| \frac{\mathbf{p}}{|\mathbf{p}|} \cdot \mathbf{E}_{\mathbf{k}n}(\mathbf{G}) \right|^2 - \int \frac{d^3k}{2\mathbf{k}^2} \sum_{\lambda=1}^{2} \left| \frac{\mathbf{p}}{|\mathbf{p}|} \cdot \varepsilon_\lambda(\mathbf{k}) \right|^2 \right]. \tag{33}$$

Accounting for the effect under study for the energy of an electron in the PC medium, we get

$$E_p = m_e + \delta m_e \left(\widehat{\mathbf{p}/|\mathbf{p}|} \right) + \frac{\mathbf{p}^2}{2m_e} \delta m_{pc} \left(\widehat{\mathbf{p}/|\mathbf{p}|} \right) + o\left(\frac{|\mathbf{p}|^4}{m_e^4} \right) m_e. \tag{34}$$

In dealing with an atomic electron, we have also to take into account that its momentum should be described by the momentum operator $\widehat{\mathbf{p}}$ and hence δm_{pc} should be described by the corresponding operator $\delta m_{pc} \left(\widehat{\mathbf{p}/|\mathbf{p}|} \right)$. In this way we arrive at the following expression for the mass correction ΔE_i^{mc} to energies of the states of a hydrogen-like atom:

$$\Delta E_i^{mc} = \left\langle i \left| \delta m_e \left(\widehat{\mathbf{p}/|\mathbf{p}|} \right) \right| i \right\rangle + \left\langle i \left| \frac{|\mathbf{p}|^2}{2m_e} \delta m_e \left(\widehat{\mathbf{p}/|\mathbf{p}|} \right) \right| i \right\rangle + o(\cdots) m_e. \tag{35}$$

In the ground S-state $|S\rangle$, the mean value of the operator $\delta m_{pc} \left(\widehat{\mathbf{p}/|\mathbf{p}|} \right)$ is

$$\left\langle \delta m_{pc} \right\rangle_S = \frac{4\alpha}{3\pi} \int d\omega \frac{N(\omega) - \omega^2}{\omega^2}, \tag{36}$$

where $N(\omega) = N_{DOS}(\omega) D(\omega)$ and $N_{DOS}(\omega)$ is the photon density of states

$$N_{DOS}(\omega) = \frac{1}{4\pi} \sum_n \int\limits_{FBZ} d^3k \delta(\omega - \omega_{\mathbf{k}n}) \tag{37}$$

and

$$D(\omega) = \sum_{\mathbf{G}} |\mathbf{E}_{\mathbf{k}n}(\mathbf{G})|^2_{|\omega_{\mathbf{k}n} = \omega.}} \tag{38}$$

The function $N(\omega)$ is closely associated with DOS of the PC. The exact calculation of this function is challenging for 3D PC; therefore we will use a model having the form

$$N(\omega) = \omega^2 n_{eff}^3 \left[1 - h \exp\left(-\frac{(\omega - \omega_0)^2}{\sigma^2} \right) \right] F(\omega), \tag{39}$$

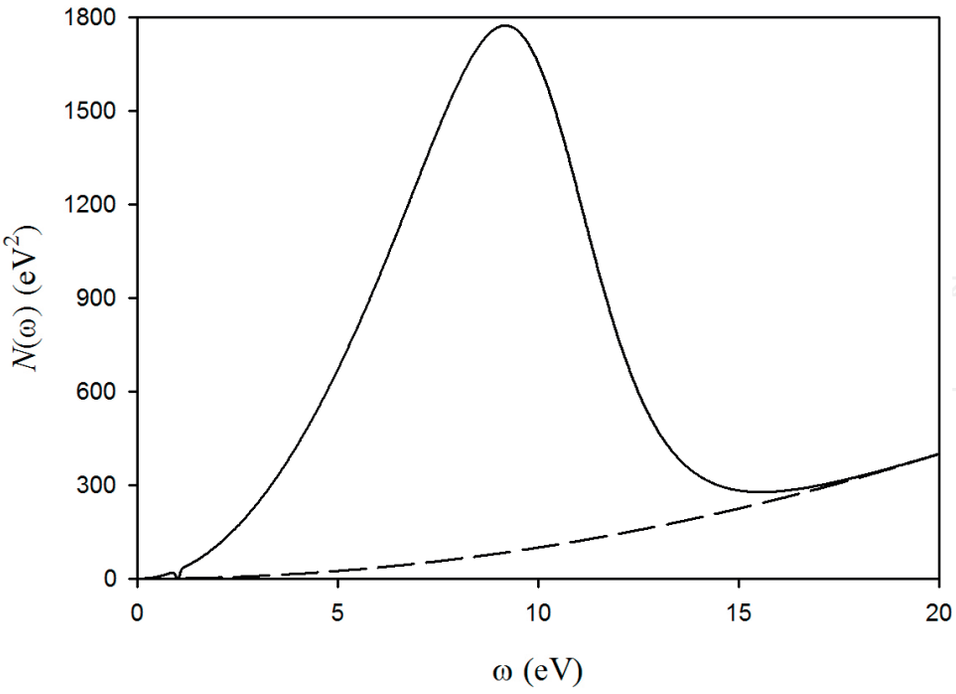

Figure 2. The model $N(\omega)$ determined by the Eq. (39) with $n_{eff}=3$, $h=0.96$, $\sigma=0.07$ eV, $\mu=15$ eV, $\tau=0.01$ eV, and $\omega_0=1$ eV. Dashed line denotes the free space DOS.

where the factor $F(\omega)=n_{eff}^{-3}+\left(1-n_{eff}^{-3}\right)/\left(\exp\left\{(\omega-\mu)/\tau\right\}+1\right)$ with $n_{eff}\equiv\sqrt{\overline{\varepsilon}}$. $\overline{\varepsilon}=\varepsilon\cdot f+(1-f)$ is an average dielectric constant with ε being the dielectric constant of the host material and f being the dielectric fraction in the PC. This model can recapture the existence of photonic band gap, optical density of dielectric host of PC sample, and the fact that at high enough photon energies, $N(\omega)$ must approach the free space DOS (**Figure 2**). For the parameters which were used in **Figure 2**, our calculations have given $\langle\delta m_{pc}\rangle_S=2.4\cdot10^{-6}m_e$.

Let us now consider the effect of the change in the electron mass on the energies of the atomic states and the transition frequencies. Here we will restrict ourselves to the hydrogen-like atoms. In the free space, the energy of the atoms in the state $|a\rangle=|n,j,l,m\rangle$ is the sum of the energy derived from the solution of the Dirac equation $E^D=m_eR_{nj}$ and the Lamb shift of the energy in this state:

$$E_{njl}=m_eR_{nj}+\Delta E_{L,a}, \tag{40}$$

where

$$R_{nj}=\left[1+\left(\frac{Z\alpha}{n-(j+1/2)+\sqrt{(j+1/2)^2-\alpha^2}}\right)^2\right]^{-1/2} \tag{41}$$

and $\Delta E_{L,a}$ is the Lamb shift of the energy of the state $|a\rangle$. The transition frequency between this state and the state $|b\rangle=|n',j',l',m\rangle$ is given by

$$\omega_{ab} = m_e\left(R_{nj} - R_{n'j'}\right) + (\Delta E_{L,a} - \Delta E_{L,b}). \tag{42}$$

When the atom is placed in the void of a PC, the transition frequencies ω_{ab}^{PC} are modified as follows:

$$\omega_{ab}^{PC} = \left(m_e + \left\langle a\middle|\delta m_e^{PC}\left(\widehat{\mathbf{p}/|\mathbf{p}|}\right)\middle|a\right\rangle\right)R_{nj} + \Delta E_{L,a}^{PC} - \left(m_e + \left\langle b\middle|\delta m_e^{PC}\left(\widehat{\mathbf{p}/|\mathbf{p}|}\right)\middle|b\right\rangle\right)R_{n'j'} - \Delta E_{L,b}^{PC}. \tag{43}$$

In the case when the atom is light, Eq. (43) is reduced to the following expression:

$$\begin{aligned}\omega_{ab}^{PC} &= \left\langle a\middle|\delta m_e^{PC}\left(\widehat{\mathbf{p}/|\mathbf{p}|}\right)\middle|a\right\rangle\left(1 - \frac{Z^2\alpha^2}{2n^2}\right) - \left\langle b\middle|\delta m_e^{PC}\left(\widehat{\mathbf{p}/|\mathbf{p}|}\right)\middle|b\right\rangle^2\left(1 - \frac{Z^2\alpha^2}{2n^2}\right)\\ &+ \frac{m_e Z\alpha^2}{2}\left(\frac{1}{n^2} - \frac{1}{(n')^2}\right) + \Delta E_{L,a}^{PC} - \Delta E_{L,b}^{PC} = \frac{m_e Z\alpha^2}{2}\left(\frac{1}{n^2} - \frac{1}{(n')^2}\right) + \Delta\omega_{ab}^{PC} + o\left(Z^2\alpha^4\right)m_e,\end{aligned} \tag{44}$$

where $\Delta\omega_{ab}^{PC}$ is the correction to the transition frequency in the PC medium given by

$$\Delta\omega_{ab}^{PC} = \left\langle a\middle|\delta m_e^{PC}\left(\widehat{\mathbf{p}/|\mathbf{p}|}\right)\middle|a\right\rangle - \left\langle b\middle|\delta m_e^{PC}\left(\widehat{\mathbf{p}/|\mathbf{p}|}\right)\middle|b\right\rangle. \tag{45}$$

As we have shown, the values of the mass corrections $\left\langle i\middle|\delta m_{PC}\left(\widehat{\mathbf{p}/|\mathbf{p}|}\right)\middle|i\right\rangle$ may be of order $10^{-6}m_e$, and hence the corrections to the transition frequencies are comparable to the atomic optical frequencies.

5. Experimental observation

Since spectra remain discrete when the PC medium affects interaction between atoms and their own emission fields, it would be logical to conduct an experiment in which we could observe this effect. This could be accomplished by observing the classical spectra of the atoms in the gas phase, pumped into PC cavities. From a theoretical point of view, it would be best to conduct the experiment with hydrogen atoms, since they are the simplest physical system. However, the handling of atomic hydrogen creates a number of technical difficulties; from a practical point of view, the best candidates for the role of such atoms are those of the noble gases, for example, helium. With respect to the requirements for a PC sample, it is first of all obvious that it should have cavities that are sufficiently interconnected to ensure the possibility of pumping gas. Second, the material of the PC sample should have the largest possible refractive index in the widest possible range of energies, since the effect depends strongly on the optical contrast [36]. Finally, the larger the amount of material filling the PC volume, the greater the effect. At the same time, the cavities must remain large enough to meet the condition that the atoms are free to move. It should be noted that an increase in the relative shift of the lines $\delta\omega/\omega$, along with an increase in the main quantum number n, is unequivocal confirmation of the effect, since the predicted shift of the lines does not depend on it.

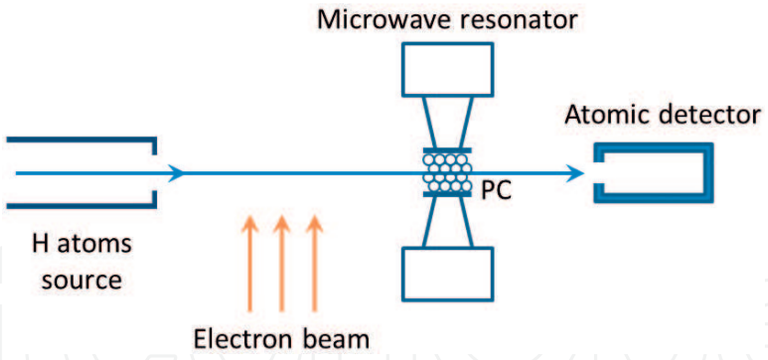

Figure 3. Scheme of modified Lamb shift experiment.

As a simple and natural way to confirm the considered effect, we propose to use a modified experiment to measure Lamb shift in hydrogen atom placed in the voids of photonic crystal (**Figure 3**). In the experiment the hydrogen atoms are exposed to electromagnetic radiation of a certain frequency, and if this frequency corresponds to the difference between the $2S_{1/2}$ and $2P_{1/2}$ energy levels (~1058 MHz without PC medium), no excited atoms will reach the detector. However taking into account the influence of the photonic crystal on the energy levels of atoms the Lamb shift will differ from 1058 MHz, the excited atoms will appear on the detector which will confirm the effect. Then we can measure new Lamb shift by adjusting the frequency of electromagnetic radiation.

There are a number of technical issues which need to be resolved. First, all exposed atoms must be within the photonic crystal, that is, electromagnetic radiation should be concentrated in a relatively small volume of a photonic crystal using antennas or waveguides. Second, as already noted, there are many requirements to the sample of photonic crystal, including the quality of the structure and possibility of free passage of hydrogen atoms through the PC medium. To solve the last one, we propose to use photonic crystals with inverted opal structure [37], the volume fraction of air voids which is approximately 74%. Such structures are fabricated from synthetic opals by filling voids between spherical particles with any desired material. After that initial particles are removed leaving a framework with spherical air voids. However, the resulting structures have a large number of defects and have significant limitations in linear dimensions.

6. Prospects of applications of the effect

The most surprising feature of the effect under study is that the electromagnetic mass of the electron comes into play when an atom is placed in the voids of a PC. There are no analogs of such QED effect in the free space. The correction to the electromagnetic mass caused by the modification of the electromagnetic interaction strongly changes the character of processes of the spontaneous emission and the absorption of atoms placed in the PC medium, and this can open up new possibilities for applying PCs. For the first time, one can change the transitions on the value comparable to the ordinary atomic transition frequencies. This effect becomes possible due to the dependence of the electromagnetic mass correction on the orientation of the

electron momentum in the PC medium. This provides a way to control the structure of the atomic energy levels. In this way, in particular, light sources with the line spectrum of a new type could be developed.

The line spectrum sources such as He-Ne laser play an important role in physics and technologies. However, the corresponding transition frequencies in the optical range are limited. The mass-change effect under study opens possibilities to tune the energy levels of He and Ne and, as a consequence, to increase the slope efficiency. It allows one to create the new He-Ne-like lasers.

One of the most perspective applications of the effect is a realization of quantum interference. Quantum interference among different decay channels caused by the anisotropic vacuum is the major field of research. Several ways have been proposed to create the anisotropy and to provide interference between atomic levels in such materials as negative-index materials [38–43], metasurfaces [44], hyperbolic metamaterials [45], metallic nanostructures [46, 47], topological insulators [48], and external fields [49–51]. The possibility for making use of anisotropy in the PC medium for these purposes has been investigated in Refs. [52–55]. The authors of the listed papers based themselves on the idea voiced by Agarwal [56] who pointed that the anisotropy of the vacuum can cause the quantum interference between nearest energy levels (e.g., Zeeman sublevels) having orthogonal dipole moments. The effect of the change in the electron mass in a PC provides new possibilities to create conditions at which quantum interference becomes possible via nonradiative transitions between atomic levels with breaking the strict selection rules.

7. Conclusion

The QED effects on which we focused play an important role in the physics of PCs. The Lamb shift in atoms that is one of the most important phenomena of the QED becomes larger in the case when the atom is placed in the air voids of PCs. But what is especially important is that in the case where an atom is placed in the artificial PC medium, we face a phenomenon that does not manifest itself in vacuum. This phenomenon consists in the fact that the part of the electromagnetic mass m_{em} of the electron that together with the bare mass m_0 constitutes the physical mass $m_{ph}=m_0+m_{em}$ becomes observable. In vacuum only m_{ph} is observable. This fact is used in the renormalization theory that is of the central importance in QED. The renormalization procedure implies that the terms describing the self-energy of the free electron should be removed from any expressions describing the processes in which the electron takes place. This is an explanation of the fact that for long time, this subtraction procedure was used in describing the Lamb shift in atoms placed in PCs despite that the electromagnetic interaction in the PC medium is significantly modified. The correction $\delta m_{pc}=m_{em}^{pc}-m_{em}$ to the electromagnetic mass of the electron caused by this modification cannot be hidden in the physical mass of the electron and for this reason is observable. Thus, in the case of the artificial PC medium, the electromagnetic mass (more precisely its part δm_{pc}) comes into play. In contrast to the Lamb shift that is relatively small correction to the atomic energy levels, the

electromagnetic mass correction δm_{pc} can have a significant effect not only on the energy levels of atoms placed in the PC medium but also on the physical processes in these atoms. The key point is that δm_{pc} depends on the orientation of the electron momentum in a PC and actually is an operator $\delta m_{pc}\left(\widehat{\mathbf{p}/|\mathbf{p}|}\right)$ whose diagonal matrix elements determine the corrections to the transition frequencies that are comparable to the atomic frequencies in the free space. The nondiagonal matrix elements determine nonradiative transitions between the states with breaking the strict selection rules. These transitions give rise to the quantum interference between the different decay channels. The possibility of controlling these quantum-interference processes can be important for quantum information science.

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