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Kinematic Design of Manipulators

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1. Introduction

Robots can be considered as the most advanced automatic systems and robotics, as a technique and scientific discipline, can be considered as the evolution of automation with interdisciplinary integration with other technological fields.

A robot can be defined as a system which is able to perform several manipulative tasks with objects, tools, and even its extremity (end-effector) with the capability of being re-programmed for several types of operations. There is an integration of mechanical and control counterparts, but it even includes additional equipment and components, concerned with sensorial capabilities and artificial intelligence. Therefore, the simultaneous operation and design integration of all the above-mentioned systems will provide a robotic system, as illustrated in Fig. 1, (Ceccarelli 2004).

In fact, more than in automatic systems, robots can be characterized as having simultaneously mechanical and re-programming capabilities. The mechanical capability is concerned with versatile characteristics in manipulative tasks due to the mechanical counterparts, and re-programming capabilities concerned with flexible characteristics in control abilities due to the electric-electronics-informatics counterparts. Therefore, a robot can be considered as a complex system that is composed of several systems and devices to give:

- mechanical capabilities (motion and force);
- sensorial capabilities (similar to human beings and/or specific others);
- intellectual capabilities (for control, decision, and memory).

Initially, industrial robots were developed in order to facilitate industrial processes by substituting human operators in dangerous and repetitive operations, and in unhealthy environments. Today, additional needs motivate further use of robots, even from pure technical viewpoints, such as productivity increase and product quality improvements. Thus, the first robots have been evolved to complex systems with additional capabilities. Nevertheless, referring to Fig. 1, an industrial robot can be thought of as composed of:

- a mechanical system or manipulator arm (mechanical structure), whose purpose consists of performing manipulative operation and/or interactions with the environment;
- sensorial equipment (internal and external sensors) that is inside or outside the mechanical system, and whose aim is to obtain information on the robot state and scenario, which is in the robot area;
• a control unit (controller), which provides elaboration of the information from the sensorial equipment for the regulation of the overall systems and gives the actuation signals for the robot operation and execution of desired tasks;
• a power unit, which provides the required energy for the system and its suitable transformation in nature and magnitude as required for the robot components;
• computer facilities, which are required to enlarge the computation capability of the control unit and even to provide the capability of artificial intelligence.

![Figure 1. Components of an industrial robot](https://www.intechopen.com)

Thus, the above-mentioned combination of sub-systems gives the three fundamental simultaneous attitudes to a robot, i.e. mechanical action, data elaboration, and re-programmability.

Consequently, the fundamental capability of robotic systems can be recognized in:
• mechanical versatility;
• re-programmability.

Mechanical versatility of a robot can be understood as the capability to perform a variety of tasks because of the kinematic and mechanical design of its manipulator arm. Re-programmability of a robot can be understood as the flexibility to perform a variety of task operations because of the capability of its controller and computer facilities.

These basic performances give a relevant flexibility for the execution of several different tasks in a similar or better way than human arms. In fact, nowadays robots are well-established equipment in industrial automation since they substitute human operators in operations and situations.

The mechanical capability of a robot is due to the mechanical sub-system that generally is identified and denominated as the 'manipulator', since its aim is the manipulative task.

The term manipulation refers to several operations, which include:
• grasping and releasing of objects;
• interaction with the environment and/or with objects not related with the robot;
• movement and transportation of objects and/or robot extremity.

Consequently, the mechanical sub-system gives mechanical versatility to a robot through kinematic and dynamic capability during its operation. Manipulators can be classified
according to the kinematic chain of their architectures as:

- serial manipulators, when they can be modeled as open kinematic chains in which the links are jointed successively by binary joints;
- parallel manipulators, when they can be modeled as closed kinematic chains in which the links are jointed to each other so that polygonal loops can be determined.

In addition, the kinematic chains of manipulators can be planar or spatial depending on which space they operate. Most industrial robotic manipulators are of the serial type, although recently parallel manipulators have aroused great interest and are even applied in industrial applications.

In general, in order to perform similar manipulative tasks as human operators, a manipulator is composed of the following mechanical sub-systems:

- an arm, which is devoted to performing large movements, mainly as translations;
- a wrist, whose aim is to orientate the extremity;
- an end-effector, which is the manipulator extremity that interacts with the environment.

Several different architectures have been designed for each of the above-mentioned manipulator sub-systems as a function of required specific capabilities and characteristics of specific mechanical designs. It is worthy of note that although the mechanical design of a manipulator is based on common mechanical components, such as all kinds of transmissions, the peculiarity of a robot design and operation requires advanced design of those components in terms of materials, dimensions, and designs because of the need for extreme lightness, compactness, and reliability.

The sensing capability of a robot is obtained by using sensors suitable for knowing the status of the robot itself and surrounding environment. The sensors for robot status are of fundamental importance since they allow the regulation of the operation of the manipulator. Therefore, they are usually installed on the manipulator itself with the aim of monitoring basic characteristics of manipulations, such as position, velocity, and force. Additionally, an industrial robot can be equipped with specific and/or advanced sensors, which give human-like or better sensing capability. Therefore, a great variety of sensors can be used, to which the reader is suggested to refer to in specific literature.

The control unit is of fundamental importance since it gives capability for autonomous and intelligent operation to the robot and it performs the following aims:

- regulation of the manipulator motion as a function of current and desired values of main kinematic and dynamic variables by means of suitable computations and programming;
- acquisition and elaboration of sensor signals from the manipulator and surrounding environment;
- capability of computation and memory, which is needed for the above-mentioned purposes and robot re-programmability.

In particular, an intelligence capability has been added to some robotic systems concerned mainly with decision capability and memory of past experiences by using the means and techniques of expert systems and artificial intelligence. Nevertheless, most of the current industrial robots have no intelligent capability since the control unit properly operates for the given tasks within industrial environments. Nowadays industrial robots are usually equipped with minicomputers, since the evolution of low-cost PCs has determined the wide use of PCs in robotics so that sequencers, which are going to be restricted to PLC units only, will be used mainly in rigid automation or low-flexible systems.
Generally, the term manipulator refers specifically to the arm design, but it can also include the wrist when attention is addressed to the overall manipulation characteristics of a robot. A kinematic study of robots deals with the determination of configuration and motion of manipulators by looking at the geometry during the motion, but without considering the actions that generate or limit the manipulator motion. Therefore, a kinematic study makes it possible to determine and design the motion characteristics of a manipulator but independently from the mechanical design details and actuator’s capability.

This aim requires the determination of a model that can be deduced by abstraction from the mechanical design of a manipulator and by stressing the fundamental kinematic parameters. The mobility of a manipulator is due to the degrees of freedom (d.o.f.s) of the joints in the kinematic chain of the manipulator, when the links are assumed to be rigid bodies.

A kinematic chain can be of open architecture, when referring to serial connected manipulators, or closed architecture, when referring to parallel manipulators, as in the examples shown in Fig. 2.

![Figure 2. Planar examples of kinematic chains of manipulators: a) serial chain as open type; b) parallel chain as closed type](image)

Of course, it is also possible to design mixed chains for so-called hybrid manipulators. Regarding the joints, although there are several designs both from theoretical and practical viewpoints, usually the joint types in robots are related to prismatic and revolute pairs with one degree of freedom. They can be modeled as shown in Fig. 3.

However, most of the manipulators are designed by using revolute joints, which have the advantage of simple design, long durability, and easy operation and maintenance. But the revolute joints also allow a kinematic chain and then a mechanical design with small size, since a manipulator does not need a large frame link and additionally its structure can be of small size in a work-cell.

In addition, it is possible to also obtain operation of other kinematic pairs with revolute joints only, when they are assembled in a proper way and sequence. For example, three revolute joints can obtain a spherical joint and depending on the assembling sequence they may give different practical spherical joints.

In general the multidisciplinarity aspects of structure and operation of robots will require a
complex design procedure with a mechatronic approach of integration of all constraints and requirements of the different natures of the robot components. In Fig. 4 a general scheme is reported as referring to a procedure, which is based on step by step design approach for the different aspects but by considering and integrating them from each other. Nevertheless it is stressed the fundamentals of the design of the manipulator structure which will affect and will be affected from the other components of a robot. Indeed, each component will affect the design and operation of other part of a robot when a design and operation is conceived with full exploit of the capability of each component. The design of a manipulator can be considered as a starting point of an iterative process in which each aspect will contribute and will affect the previous and next solution to a mechatronic integrated solution of the robot system. Similarly important are the characteristics and requirements of the task and application to which the robot is devoted. Thus an so-called optimal design of a robot will be achieved only after a reiteration of design process both for the components and the whole systems, by looking at each component separately and integrated approach. Thus, even the design of the manipulator can be considered at the same time as starting and final point of the design process.

Figure 4. A general scheme for a design procedure of robots

Kinematic design of manipulators refers to the determination of the dimensional parameters of a kinematic chain, i.e. link lengths and link angles. Once the kinematic architecture of a
A robot manipulator is sized by means of a kinematic design, a manipulator can be completely defined by means of a mechanical design that specifies all the sizes and details for a physical construction. Indeed, kinematic design is a fundamental step in a design procedure of any mechanical system and its accuracy will affect strongly the basic properties of a mechanical systems. In the case of manipulators the kinematic design is of a particular importance since the manipulator tasks can be performed when the kinematic architecture has been properly conceived or chosen and specifically synthesized (i.e. kinematic design).

Several approaches have been formulated for the kinematic design of mechanisms and many of them have been specialized for robotic manipulators. General procedures and specific algorithms both for general kinematic architectures and specific designs of manipulators have been proposed in a very rich literature. A limited list of references is reported with the aim to give to the readers basic sources and suggestions for further reading on the topic.

In this chapter a survey of current issues is presented by using basic concepts and formulations in order to emphasize on problem formulation and computational efforts. Indeed, a great attention is still addressed to kinematic design of manipulators by robot designers and researchers mainly with the aim to improve computational efficiency, generality and optimality of the algorithms, even with respect to new and new requirements for robotic manipulations. In addition, theoretical and numerical works are usually validated by the same investigators through tests with prototypes and experimental activity on performance characteristics.

This survey has overviewed the currently available procedures for kinematic design of manipulators that can be grouped in three main approaches, namely extension of Synthesis of Mechanisms (for example: Precision Point Techniques, Workspace Design, Inversion algorithms, Optimization formulation), application of Screw Theory, application of 3D Kinematics/Geometry (for example: Lie Group Theory, Dual Numbers, Quaternions, Grassmann Geometry).

A kinematic design procedure is aimed to obtain closed-form formulation and/or numerical algorithms, which can be used not only for design purposes but even to investigate effects of design parameters on design characteristics and operation performance of manipulators.

Usually, there is a distinction between open-chain serial manipulators and closed-chain parallel manipulators. This distinction is also considered as a constraint for the kinematic design of manipulators and in fact different procedures and formulation have been proposed to take into account the peculiar differences in their kinematic design. Nevertheless, recently, attempts have been made to formulate a unique view for kinematic design both of serial and parallel manipulators, mainly with an approach using optimization problems.

Future challenges in the field of robot design can be recognized mainly in the aspects for computational efficiency and in conceiving new manipulator architectures with a fully insight of design degeneracy both of the kinematic possibilities and proposed numerical algorithms.

2. The design problem

The manipulator architecture of a robot is composed of an arm mostly for translation movements, a wrist for orientation movement, and an end-effector for interaction with the environment and/or external objects, as shown in Fig. 1. Generally, the term manipulator
refers specifically to the arm design, but it can also include the wrist when attention is addressed to the overall manipulation characteristics of a robot.

A kinematic study of robots deals with the determination of configuration and motion of manipulators by looking at the geometry during the motion, but without considering the actions that generate or limit the manipulator motion. Therefore, a kinematic study makes possible to determine and design the motion characteristics of a manipulator but independently from the mechanical design details and actuator capability.

A kinematic chain can be of open architecture, when referring to serial connected manipulators, or closed architecture, when referring to parallel manipulators, as in the example in Fig. 5b).

The kinematic model of a manipulator can be obtained in the form of a kinematic chain or mechanism by using schemes for joints and rigid links through essential dimensional sizes for connections between two joints. The mobility of a manipulator is due to the degrees of freedom (d.o.f.s) of the joints in the kinematic chain, when the links are assumed to be rigid bodies. In order to determine the geometrical sizes and kinematic parameters of open-chain general manipulators, one can usually refer to a scheme like that in Fig. 5a) by using a H–D notation, in agreement with a procedure that was proposed by Hartenberg and Denavit in 1955.

The kinematic parameters of a manipulator can be defined according to the H–D notation in Fig. 5a) as:

- $a_j$, link length that is measured as the distance between the $Z_j$ and $Z_{j+1}$ axes along $X_j$;
- $\alpha_j$, twist angle that is measured as the angle between the $Z_j$ and $Z_{j+1}$ axes about $X_j$;
- $d_{j+1}$, link offset that is measured as the distance between the $X_j$ and $X_{j+1}$ axes along $Z_{j+1}$.

Figure 5. A kinematic scheme for manipulator link parameters: a) according to H-D notation; b) for parallel architectures

This scheme gives the minimum number of parameters that are needed to describe the geometry of a link between two joints, but also indicates the joint variables. The joints in Fig. 5a) are indicated as big black points in order to stress attention to the link geometry and H-D parameters. In particular, referring to Fig. 5a) for j-link, the j-frame $X_jY_jZ_j$ is assumed as fixed to j-link, with the $Z_j$ axis coinciding with the joint axis, with the $X_j$ axis lying on the common normal between $Z_j$ and $Z_{j+1}$ and pointing to $Z_{j+1}$.
• \( \theta_{j+1} \), joint angle that is measured as the angle between the \( X_j \) and \( X_{j+1} \) axes about \( Z_{j+1} \).

When a joint can be modelled as a rotation pair, the angle \( \theta_{j+1} \) is the corresponding kinematic variable. When a joint is a prismatic pair, the distance \( d_{j+1} \) is the corresponding kinematic variable. Other H–D parameters can be considered as dimensional parameters of the links.

The H–D notation is very useful for the formulation of the position problems of manipulators through the so-called transformation matrix by using matrix algebra.

The position problem of manipulators, both with serial and parallel architectures, consists of determining the position and orientation of the end-effector as a function of the manipulator configuration that is given by the link position that is defined by the joint variables.

In general, the position problem can be considered from different viewpoints depending on the unknowns that one can solve in the following formulations:

• Kinematic Direct Problem in which the dimensions of a manipulator are given through the dimensional H–D parameters of the links but the position and orientation of the end-effector are determined as a function of the values of the joint variables;

• Kinematic Inverse Problem in which the position and orientation of the end-effector of a given manipulator are given, and the configuration of the manipulator chain is determined by computing the values of the joint variables.

A third kinematic problem can be formulated as:

• Kinematic Indirect Problem (properly ‘Kinematic Design Problem’) in which a certain number of positions and orientations of the end-effector are given but the type of manipulator chain and its dimensions are the unknowns of the problem.

Although general concepts are common both for serial and parallel manipulators, peculiarities must be considered for parallel architectures chains.

In parallel manipulators one can consider as generalized coordinates the position coordinates of the center point \( P \) of the moving platform with respect to a fixed frame \( (X_o, Y_o, Z_o) \), Fig. 5b), and the direction is described by Euler angles defining the orientation of the moving platform with respect to a fixed frame. A matrix \( R \) defines the orthogonal \( 3 \times 3 \) rotation matrix defined by the Euler angles, which describes the orientation of the frame attached to the moving platform with respect to the fixed frame, Fig. 5b). Let \( A_i \) and \( B_i \) be the attachment points at the base and moving platform, respectively, and \( d_i \) the leg lengths. Let \( a_i \) and \( b_i \) be the position vectors of points \( A_i \) and \( B_i \) in the fixed and moving coordinate frames, respectively. Thus, for parallel manipulators the Inverse Kinematics Problem can be solved by using

\[
A_i B_i = p + R b_i - a_i
\]

(1)

to extract the joint variables from leg lengths. The length of the \( i \)-th leg can be obtained by taking the dot product of vector \( A_i B_i \) with itself, for \( i:1,\ldots,6 \) in the form

\[
d_i^2 = [p + R b_i - a_i] [p + R b_i - a_i]
\]

(2)

The Direct Kinematics Problem describes the mapping from the joint coordinates to the generalized coordinates. The problem for parallel manipulators is quite difficult since it involves the solution of a system of nonlinear coupled algebraic equations (1), and has many solutions that refer to assembly modes. For a general case of Gough-Stewart Platform with
planar base and platform, the Direct Kinematics Problem may have up to 40 solutions. A 20-th degree polynomial has been derived leading to 40 mutually symmetric assembly modes.

3. Algorithms for kinematic design of manipulators

Synthesis deals with reverse problems of Analysis. Thus, Synthesis of mechanisms and manipulators deals with design of the kinematic chain as function of manipulative tasks. Characteristic manipulative tasks of manipulators concern with manipulation of objects as movement and orientation of grasped objects or end-effector itself during a suitably programmed motion of a manipulator. But manipulation includes also other aspects of functional and operation characteristics, and nowadays mechatronic approaches are also used to consider those other aspects in fully integrated approaches.

Design calculation of kinematic chain of mechanisms and manipulators is usually attached through three problems, namely type synthesis, number synthesis, and dimensional synthesis. Number synthesis concerns with the determination of the number of links and joints in the chain, which are useful or necessary to obtain a desired mobility and manipulation capability of a manipulator mechanism. Similarly, type synthesis concerns with the determination of the structure of the kinematic chain, i.e. the type of joints and kinematic architecture, that are useful or necessary to obtain a desired mobility and manipulation capability of a manipulator mechanism. Finally, dimensional synthesis (i.e. kinematic design) concerns with the calculation of the link sizes and range mobility of joints that are useful or necessary to obtain a desired mobility and manipulation capability of a manipulator mechanism.

Type synthesis and number synthesis are related to morphologies of manipulator architectures and today they are approached with designer’s own experience or through complex design procedures that most of the time can be understood as data bases in informatics expert systems.

The traditional design activity on manipulators is still recognized in the problem of the dimensional design of a manipulator when its kinematic architecture is given. This is the problem that is surveyed in this paper.

The manipulative tasks that are used as design data and constraints, are related mainly to kinematic features such as workspace, path planning, Static accuracy; but other aspects can be also considered within the Mechanics of robots, such as singularities, stiffness behaviour, dynamic response. One aspect of relevant significance for manipulator design is the workspace analysis that is often used as design means yet, beside as a criteria for evaluating the quality of designed solutions. Positioning and orientation capability can be evaluated by computing position and orientation workspaces that give the reachable regions by the manipulator extremity or end-effector as function of the mobility range of the manipulator joints. Position workspace refers to reachable points by a reference point on manipulator extremity, and orientation workspace describes the angles that can be swept by reference axes on manipulator extremity. Once the workspace points (both in position and orientation) are determined, one can use them to perform an evaluation of workspace characteristics and a feasibility evaluation of kinematic design solutions. In particular, a cross-section area can be determined by selecting from the computed workspace points those that lay on a cross-section plane under examination. Thus, the shape can be illustrated by the result in a plot form. The computation of the value of a cross-section area can be obtained by using a grid evaluation or an algebraic formula.
By referring to the scheme of Fig. 6a) for a grid evaluation, one can calculate the area measure $A$ as a sum of the scanning resolution rectangles over the scanned area as

$$A = \sum_{i=1}^{I} \sum_{j=1}^{J} \left( A_{ij} \Delta i \Delta j \right)$$

(3)

by using the $A_{ij}$ entries of a binary matrix that are related to the cross-section plane for $A$.

Figure 6. General schemes for an evaluation of manipulator workspace: a) through binary representation; b) through geometric properties for algebraic formulation

Alternatively, one can use the workspace points of the boundary contour of a cross-section area that can be determined from an algebraic formulation or using the entries of the binary matrix. Thus, referring to the scheme of Fig. 6b) and by assuming as computed the coordinates of the cross-section contour points as an ordinate set $(r_j, z_j)$ of the contour points $H_j$ with $j=1, \ldots, N$, the area measure $A$ can be computed as

$$A = \sum_{j=1}^{N} \left( z_{t+1,j} + z_{t,j} \right) \left( r_{t+1,j} - r_{t,j} \right)$$

(4)

By extending the above-mentioned procedures, the workspace volume $V$ can be computed by using the grid scanning procedure in a general form as

$$V = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \left[ P_{ijk} \Delta i \Delta j \Delta k \right]$$

(5)

in which $P_{ijk}$ is the entry of a binary representation in a 3D grid.

When the workspace volume is a solid of revolution, by using the boundary contour points through the Pappus-Guldinus Theorem the workspace volume $V$ can be computed within the binary mapping procedure, Fig.6, but yet in the form as

$$V = 2\pi \sum_{i=1}^{I} \sum_{j=1}^{J} \left[ P_{ij} \Delta i \Delta j \left( i \Delta i + \frac{\Delta j}{2} \right) \right]$$

(6)
or within the algebraic formulation in the form

\[ V = \frac{\pi}{2} \sum_{j=1}^{N} \left( z_{1,j+1} + z_{1,j} \sqrt{n_{1,j}^2 - n_{1,j+1}^2} \right) \]

(7)

Therefore, it is evident that the formula of Eq. (6) has a general application, while Eqs. (6) and (7) are restricted to serial open-chain manipulators with revolute joints. Those approaches and formulation can be proposed and used for a numerical evaluation of workspace characteristics of parallel manipulators too.

Similarly, hole and void regions, as unreachable regions, can be numerically evaluated by using the formulas of Eqs (3) to (7) to obtain the value of their cross-sections and volumes, once they have been preliminarily determined.

Orientation workspace can be similarly evaluated by considering the angles in a Cartesian frame representation.

A design problem for manipulators can be formulated as a set of equations, which give the position and orientation of a manipulator in term of its extremity (such as workspace formulation) together with additional expressions for required performance in term of suitable criteria evaluations.

### 3.1 Synthesis procedures for mechanisms

Since manipulators can be treated as spatial mechanisms, the traditional techniques for mechanism design can be used once suitable adaptations are formulated to consider the peculiarity of the open chain architecture.

Two ways can be approached as referring to general model for closure equations:

- elaboration of closure equations for the open polygon either by adding a fictitious link with its joints either by using the coordinates of the manipulator extremity, (Duffy 1980), as shown in the illustrative example of Fig.7.
- In any case, traditional techniques for mechanisms are used by considering the manipulator extremity/end-effector as a coupler link whose kinematics is the purpose of the formulation. Thus, Direct and Inverse Kinematics can be formulated and Synthesis problems can be attached by using Precision Points as those points (i.e. poses in general) at which the pose and/or other performances are prescribed as to be reached and/or fulfilled exactly.

![Figure 7. Closing kinematic chain of a3R manipulator by adding a fictitious link and spherical joint or by looking at coordinates of point Q](www.intechopen.com)
This can be expressed in a general form as
\[ F_i = F(X_i) \]  
(8)
in which \( F_i \) is the performance evaluation at \( i \)-th precision pose whose coordinates \( X_i \) are function of the mechanism configuration that can be obtained by solving closure equations through any traditional methods for mechanism analysis. Thus, design requirements and design equations can be formulated for the Precision Points, whose maximum number for a mathematical defined problem can be determined by the number of variables. But, the pose accuracy and path planning as well as the performance value away from the Precision Points will determine errors in the behaviour of the manipulator motion, whose evaluation can be still computed by using the design equations in proper procedures for optimization purposes.

Precision Points techniques for mechanisms have been developed for path positions, but for manipulator design the concept has been extended to performance criteria such as workspace boundary points and singularities. Thus, new specific algorithms have been developed for manipulator design by using approaches from Mechanism Design but with specific formulation of the peculiar manipulative tasks. Approaches such as Newton-Raphson numerical techniques, dyad elimination, Graph Theory modeling, mobility analysis, Instantaneous kinematic invariants have been developed for manipulator architectures as extension of those basic properties of planar mechanisms that have been investigated and used for design purposes since the second half of 19-th century. Of course, the complexity of 3D architectures have requested development of new more efficient calculation means, such as a suitable use of Matrix Algebra, 3D Geometry considerations, and Screw Theory formulation.

3.2 Application of 3D geometry and Screw Theory
Three dimensional Geometry of spatial manipulators has required and requires specific consideration and investigation on the 3D characteristics of a general motion. Thus, different mathematizations can be used by taking into account of generality of 3D motion. Dual numbers and quaternions have been introduced in the last decades to study Mechanism Design and they are specifically applied to study 3D properties of rigid body motion in manipulator architectures.

The structure of mathematical properties of rigid body motion has been also addressed for developing or applying new Algebra theories for analysis and design purposes of spatial mechanisms and manipulators. Recently Lie Group Theory and Grassman Geometry have been adapted and successfully applied to develop new calculation means for designing new solutions and characterizing manipulator design in general frames. A group \( G \) is a non-empty set endowed with a closed product operation in the set satisfying some definition conditions. A subset \( H \) of elements of a group \( G \) is called a subgroup of \( G \) if the subset \( H \) constitutes a group which has common group operation with the group \( G \). Furthermore, a group \( G \) is called a Lie group if \( G \) is an analytic manifold and the mapping \( G \times G \) to \( G \) is analytic. The set \( |D| \) of rigid body motions or displacements is a 6-dimensional Lie group of transformations, which acts on the points of the 3-dimensional Euclidean affine space. The Lie subgroups of \( |D| \) play a key role in the mobility analysis and synthesis of mechanisms. Therefore using the mathematics of this algebra is possible to describe general
features in a synthetic form that allows also fairly easy investigation of new particular conditions.

For example in Fig.8, (Lee and Hervè 2004), a hybrid spherical-spherical spatial 7R mechanism is a combination of two trivial spherical chains. Both chains are the spherical four-revolute chains A-B-C-D and G-F-E-D with the apexes O₁ and O₂ respectively. The mechanical bond [L(4,7)] between links 4 and 7 as the intersection set of two subsets [G₁] and [G₂] is given by

\[ [G₁] = [R(O₁,u₂₁)] [R(O₂,u₂₂)] [R(O₂,u₂₃)] [R(O₂,u₂₄)] [G₂] = [R(O₂,u₂₇)] [R(O₂,u₂₈)] [R(O₁,u₂₅)] \]

where a mechanical bond is a mechanical connection between rigid bodies and it can be described by a mathematical bond, i.e. connected subset of the displacement group.

Hence, the relative motion between links 4 and 7 is depicted by

\[ L(4,7) = [G₁] \cap [G₂] \]

In general, \([R(O₁,u₂₁)] [R(O₂,u₂₂)] [R(O₂,u₂₃)] [R(O₂,u₂₄)] [R(O₂,u₂₅)]\) is a 6-dimensional kinematic bond and generates the displacement group \(D\). Therefore, \([R(O₁,u₂₁)] [R(O₂,u₂₂)] [R(O₂,u₂₃)] [R(O₂,u₂₄)] [R(O₂,u₂₅)] \cap [R(O₂,u₂₇)] = D \cap [R(O₂,u₂₇)] = [R(O₂,u₂₇)]\). This yields that the A-G-B-F-C-E-D 7R chain has one dof when all kinematic pairs move and consequently \([L(4,7)]\) includes a 1-dimensional manifold denoted by \([L(1/D)(4,7)]\). If all the pairs move and joint axes do not intersect again, any possible mobility characterized by this geometric condition stops occurring and we have \([L(4,7)] \supseteq [L(1/D)(4,7)]\). Summarizing, the kinematic chain works like a general spatial 7R chain whose general mobility is with three dofs, but with the above-mentioned condition is constrained to one dof, since it acts like a spherical four-revolute A-B-C-D chain with one dof, or a spherical four-revolute G-F-E-D chain with one dof.

Grassman Geometry and further developments have been used to describe the Line Geometry that can be associated with spatial motion. Plucker coordinates and suitable algebra of vectors are used in Grassman Geometry to generalize properties of motion of a line that can be fixed on any link of a manipulator, but mainly on its extremity.

Figure 8. Hybrid spherical-spherical discontinuously movable 7R mechanism, (Lee and Hervè 2004)
Screw Theory was developed to investigate the general motion of rigid bodies in its form of helicoidal (screw) motion in 3D space. A screw entity was defined to describe the motion and to perform computation still through vector approaches.

A unit screw is a quantity associated with a line in the three-dimensional space and a scalar called pitch, which can be represented by a $6 \times 1$ vector $\begin{bmatrix} s \, r \times s + \lambda s \end{bmatrix}^T$ where $s$ is a unit vector pointing along the direction of the screw axis, $r$ is the position vector of any point on the screw axis with respect to a reference frame and $\lambda$ is the pitch of the screw. A screw of intensity $\rho$ is represented by $S = \rho \, s$. When a screw is used to describe the motion state of a rigid body, it is often called a twist, represented by a $6 \times 1$ vector $\begin{bmatrix} \omega \, v \end{bmatrix}^T$, where $\omega$ represents the instant angular velocity and $v$ represents the linear velocity of a point $O$ which belongs to the body and is coincident with the origin of the coordinate system.

Screw Theory has been applied to manipulator design by using suitable models of manipulator chains, both with serial and parallel architectures, in which the joint mobility is represented by corresponding screws, (Davidson and Hunt 2005). Thus, screw systems describe the motion capability of manipulator chains and therefore they can be used still with a Precision Point approach to formulate design equations and characteristics of the architectures. In Fig. 9 an illustrative example is reported as based on the fundamental so-called Screw Triangle model for efficient computational purposes, even to deduce closed-form design expressions.

3.3 Optimization problem design

The duality between serial and parallel manipulators is not anymore understood as a competition between the two kinematic architectures. The intrinsic characteristics of each architecture make each architecture as devoted to some manipulative tasks more than an alternative to the counterpart. The complementarities of operation performance of serial and parallel manipulators make them as a complete solution set for manipulative operations.
The differences but complementarities in their performance have given the possibility in the past to treat them separately, mainly for design purposes. In the last two decades several analysis results and design procedures have been proposed in a very rich literature with the aim to characterize and design separately the two manipulator architectures. Manipulators are said useful to substitute/help human beings in manipulative operations and therefore their basic characteristics are usually referred and compared to human manipulation performance aspects. A well-trained person is usually characterized for manipulation purpose mainly in terms of positioning skill, arm mobility, arm power, movement velocity, and fatigue limits. Similarly, robotic manipulators are designed and selected for manipulative tasks by looking mainly to workspace volume, payload capacity, velocity performance, and stiffness. Therefore, it is quite reasonable to consider those aspects as fundamental criteria for manipulator design. But generally since they can give contradictory results in design algorithms, a formulation as multi-objective optimization problem can be convenient in order to consider them simultaneously. Thus, an optimum design of manipulators can be formulated as

$$\min \mathbf{F}(\mathbf{X}) = \min \left[ f_1(\mathbf{X}), f_2(\mathbf{X}), \ldots, f_N(\mathbf{X}) \right]^T$$

subjected to

$$\mathbf{G}(\mathbf{X}) < 0$$

$$\mathbf{H}(\mathbf{X}) = 0$$

where $T$ is the transpose operator; $\mathbf{X}$ is the vector of design variables; $\mathbf{F}(\mathbf{X})$ is the vector of objective functions $f_i$ that express the optimality criteria, $\mathbf{G}(\mathbf{X})$ is the vector of constraint functions that describes limiting conditions, and $\mathbf{H}(\mathbf{X})$ is the vector of constraint functions that describes design prescriptions.

There is a number of alternative methods to solve numerically a multi-objective optimization problem. In particular, in the example of Fig. 10 the proposed multi-objective optimization design problem has been solved by considering the min-max technique of the Matlab Optimization Toolbox that makes use of a scalar function of the vector function $\mathbf{F}(\mathbf{X})$ to minimize the worst case values among the objective function components $f_i$.

The problem for achieving optimal results from the formulated multi-objective optimization problem consists mainly in two aspects, namely to choose a proper numerical solving technique and to formulate the optimality criteria with computational efficiency. Indeed, the solving technique can be selected among the many available ones, even in commercial software packages, by looking at a proper fit and/or possible adjustments to the formulated problem in terms of number of unknowns, non-linearity type, and involved computations for the optimality criteria and constraints. On the other hand, the formulation and computations for the optimality criteria and design constraints can be deduced and performed by looking also at the peculiarity of the numerical solving technique. Those two aspects can be very helpful in achieving an optimal design procedure that can give solutions with no great computational efforts and with possibility of engineering interpretation and guide.

Since the formulated design problem is intrinsically high no-linear, the solution can be obtained when the numerical evolution of the tentative solutions due to the iterative process converges to a solution that can be considered optimal within the explored range. Therefore
a solution can be considered an optimal design but as a local optimum in general terms. This last remark makes clear once more the influence of suitable formulation with computational efficiency for the involved criteria and constraints in order to have a design procedure, which is significant from engineering viewpoint and numerically efficient.

![Flowchart for design procedure](chart.png)

Figure 10. A general scheme for optimum design procedure by using multi-objective optimization problem solvable by commercial software

### 4. Experimental validation of manipulators

Engineering approach for kinematic design is completed by experimental activity for validation of theories and numerical algorithms and for validation and evaluation of prototypes and their performance as last design phase. Experimental activity can be carried out at several levels depending on the aims and development sequence:

- by checking mechanical design and assembly problems for manipulators and test-beds;
- by looking at operation characteristics of tasks and manipulator architectures;
- by simulating manipulators both in terms of kinematic capability and dynamic actions;
- by validating prototype performance in term of evaluation of errors from expected behavior.

Construction activity is aimed to check the feasibility of practical implementation of designed manipulators. Assembly possibilities are investigated also by looking at alternative
components. The need to obtain quickly a validation of the prototypes as well as of novel architectures has developed techniques of rapid prototyping that facilitate this activity both in term of cost and time. Test-beds are developed by using or adjusting specific prototypes or specific manipulator architectures. Once a physical system is available, it can be used both to characterize performance of built prototypes and to further investigate on operation characteristics for optimality criteria and validation purposes. At this stage a prototype can be used as a test-bed or even can be evolved to a test-bed for future studies. This activity can be carried out as an experimental simulation of built prototypes both for functionality and feasibility in novel applications. From mechanical engineering viewpoint, experimental activity is understood as carried out with built systems with considerable experiments for verifying operation efficiency and mechanical design feasibility. Recently experimental activity is understood even only through numerical simulations by using sophisticated simulation codes (like for example ADAMS).

The above mentioned activity can be also considered as completing or being preliminary to a rigorous experimental validation, which is carried out through evaluation of performance and task operation both in qualitative and quantitative terms by using previously developed experimental procedures.

5. Experiences at LARM in Cassino

As an example of the above-mentioned aspects illustrative cases of study are reported from the activity of LARM: Laboratory of Robotics and Mechatronics in Cassino in Figs. 11-19.

Since the beginning of 1990s at LARM in Cassino, a research line has been dedicated to the development of analysis formulation and experimental activity for manipulator design and performance characterization. More details and further references can be found in the LARM webpage http://webuser.unicas.it/weblarm/larmindex.htm.

Workspace has been analyzed to characterize its manifold and to formulate efficient evaluation algorithms. Scanning procedure and algebraic formulation for workspace boundary have been proposed. Results can be obtained likewise in the illustrative examples in Fig. 11.

![Illustrative examples of results of workspace determination through: a) binary representation in scanning procedure; b) algebraic formulation of workspace boundary](http://www.intechopen.com)
A design algorithm has been proposed as an inversion of the algebraic formulation to give all possible solutions like for the reported case of 3R manipulator in Fig. 12.

Further study has been carried out to characterize the geometry of ring (internal) voids as outlined in Fig. 13.

A workspace characterization has been completed by looking at design constraints for solvable workspace in the form of the so-called Feasible Workspace Regions. The case of 2R manipulators has been formulated and general topology has been determined for design purposes, as reported in Fig. 14.

Singularity analysis and stiffness evaluation have been approached to obtain formulation and procedure that are useful also for experimental identification, operation validation, and performance testing. Singularity analysis has been approached by using arguments of Descriptive Geometry to represent singularity conditions for parallel manipulators through suitable formulation of Jacobians via Cayley-Grassman determinates or domain analysis. Figure 15 shows examples how using tetrahedron geometry in 3-2-1-parallel manipulators has determined straightforward the shown singular configurations.

Figure 12. Design solutions for 3R manipulators by inverting algebraic formulation for workspace boundary when boundary points are given: a) all possible solutions; b) feasible workspace designs

Figure 13. Manifolds for ring void of 3R manipulators
Recently, optimal design procedures have been formulated and experienced by using multi-criteria optimization problem when Precision Points equations have been combined with suitable numerical evaluation of performances. An attempt has been proposed to obtain a unique design procedure both for serial and parallel manipulators through the objective formulation

\[ f_1(X) = 1 - \left( \frac{V_{\text{pos}}}{V_{\text{pos}}} \right) \]
\[ f_2(X) = 1 - \left( \frac{V_{\text{cor}}}{V_{\text{cor}}} \right) \]
\[ f_3(X) = -\frac{\min(\det J)}{\det J_0} \]
\[ f_4(X) = 1 - \left[ \frac{\Delta U_d}{\Delta U_g} \right] \]
\[ f_5(X) = 1 - \left[ \frac{\Delta Y_d}{\Delta Y_g} \right] \]

Figure 14. General geometry of Feasible Workspace Regions for 2R manipulators depicted as grey area

Figure 15. Determination of singularity configuration of a wire 3-2-1 parallel manipulator by looking at the descriptive geometry of the manipulator architecture
where $V_{pos}$ and $V_{or}$ values correspond to computed position and orientation workspace volume $V$ and prime values describe prescribed data; $J$ is the manipulator Jacobian with respect to a prescribed one $J_o$; $\Delta U_d$ and $\Delta U_g$ are compliant displacements along $X$, $Y$, and $Z$-axes, $\Delta Y_d$ and $\Delta Y_g$ are compliant rotations about $\phi$, $\theta$ and $\psi$; $d$ and $g$ stand for design and given values, respectively. Illustrative example results are reported in Figs.16 and 17 as referring to a PUMA-like manipulator and a CAPAMAN (Cassino Parallel Manipulator) design.

Experimental activity has been particularly focused on construction and functionality validation of prototypes of parallel manipulators that have been developed at LARM under the acronym CAPAMAN (Cassino Parallel Manipulator). Figures 18 and 19 shows examples of experimental layouts and results that have been obtained for characterizing design performance and application feasibility of CAPAMAN design.

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**Figure 16.** Evolution of the function $F$ and its components versus number of iterations in an optimal design procedure: a) for a PUMA-like robot; b) a CAPAMAN design in Fig.15a). (position workspace volume as $f_1$; orientation workspace volume as $f_2$; singularity condition as $f_3$; compliant translations and rotations as $f_d$ and $f_g$)

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**Figure 17.** Evolution of design parameters versus number of iterations for: a) PUMA-like robot in Fig.16a); CAPAMAN in Fig.16b)
6. Future challenges

The topic of kinematic design of manipulators, both for robots and multi-body systems, addresses and will address yet attention for research and practical purposes in order to achieve better design solutions but even more efficient computational design algorithms. An additional aspect that cannot be considered of secondary importance, can be advised in the necessity of updating design procedures and algorithms for implementation in modern current means from Informatics Technology (hardware and software) that is still evolving very fast.

Thus, future challenges for the development of the field of kinematic design of manipulators and multi-body systems at large, can be recognized, beside the investigation for new design solutions, in:

- more exhaustive design procedures, even including mechatronic approaches;
- updated implementation of traditional and new theories of Kinematics into new Informatics frames.
Research activity is often directed to new solutions but because the reached highs in the field mainly from theoretical viewpoints, manipulator design still needs a wide application in practical engineering. This requires better understanding of the theories at level of practicing engineers and user-oriented formulation of theories, even by using experimental activity. Thus, the above-mentioned challenges can be included in a unique frame, which is oriented to a transfer of research results to practical applications of design solutions and procedures.

Mechatronic approaches are needed to achieve better practical design solutions by taking into account the construction complexity and integration of current solutions and by considering that future systems will be overwhelmed by many sub-systems of different natures other than mechanical counterpart. Although the mechanical aspects of manipulation will be always fundamental because of the mechanical nature of manipulative tasks, the design and operation of manipulators and multi-body systems at large will be more and more influenced by the design and operation of the other sub-systems for sensors, control, artificial intelligence, and programming through a multidisciplinary approach/integration. This aspect is completed by the fact that the Informatics Technology provides day by day new potentialities both in software and hardware for computational purposes but even for technical supports of other technologies. This pushes to re-elaborate design procedures and algorithms in suitable formulation and logics that can be used/adapted for implementation in the evolving Informatics. Additional efforts are requested by system users and practitioner engineers to operate with calculation means (codes and procedures in commercial software packages) that are more and more efficient in term of computation time and computational results (numerical accuracy and generality of solutions) as well as more and more user-oriented design formulation in term of understand ability of design process and its theory. This is a great challenge: since while more exhaustive algorithms and new procedures (with mechatronic approaches) are requested, nevertheless the success of future developments of the field strongly depends on the capability of the researchers of expressing the research result that will be more and more specialist (and sophisticated) products, in a language (both for calculation and explanatory purposes) that should not need a very sophisticate expertise.

7. Conclusion

Since the beginning of Robotics the complexity of the kinematic design of manipulators has been solved with a variety of approaches that are based on Theory of Mechanisms, Screw Theory, or Kinematics Geometry. Algorithms and design procedures have evolved and still address research attention with the aim to improve the computational efficiency and generality of formulation in order to obtain all possible solutions for a given manipulation problem, even by taking into account other features in a mechatronic approach. Theoretical and numerical approaches can be successfully completed by experimental activity, which is still needed for performance characterization and feasibility tests of prototypes and design algorithms

8. References

The reference list is limited to main works for further reading and to author’s main experiences. Citation of references has not included in the text since the subjects refer to a very reach literature that has not included for space limits.


In this book we have grouped contributions in 28 chapters from several authors all around the world on the several aspects and challenges of research and applications of robots with the aim to show the recent advances and problems that still need to be considered for future improvements of robot success in worldwide frames. Each chapter addresses a specific area of modeling, design, and application of robots but with an eye to give an integrated view of what make a robot a unique modern system for many different uses and future potential applications. Main attention has been focused on design issues as thought challenging for improving capabilities and further possibilities of robots for new and old applications, as seen from today technologies and research programs. Thus, great attention has been addressed to control aspects that are strongly evolving also as function of the improvements in robot modeling, sensors, servo-power systems, and informatics. But even other aspects are considered as of fundamental challenge both in design and use of robots with improved performance and capabilities, like for example kinematic design, dynamics, vision integration.

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