Chapter 3

Fuzzy Interpolation Systems and Applications

Longzhi Yang, Zheming Zuo, Fei Chao and
Yanpeng Qu

Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/68051

Abstract

Fuzzy inference systems provide a simple yet effective solution to complex non-linear problems, which have been applied to numerous real-world applications with great success. However, conventional fuzzy inference systems may suffer from either too sparse, too complex or imbalanced rule bases, given that the data may be unevenly distributed in the problem space regardless of its volume. Fuzzy interpolation addresses this. It enables fuzzy inferences with sparse rule bases when the sparse rule base does not cover a given input, and it simplifies very dense rule bases by approximating certain rules with their neighbouring ones. This chapter systematically reviews different types of fuzzy interpolation approaches and their variations, in terms of both the interpolation mechanism (inference engine) and sparse rule base generation. Representative applications of fuzzy interpolation in the field of control are also revisited in this chapter, which not only validate fuzzy interpolation approaches but also demonstrate its efficacy and potential for wider applications.

Keywords: fuzzy inference systems, fuzzy interpolation, adaptive fuzzy interpolation, sparse rule bases, fuzzy control

1. Introduction

Fuzzy logic and fuzzy sets have been used successfully as tools to manage the uncertainty of fuzziness since their introduction in the 1960s, which have been applied to many fields, including [1–6]. The most widely used fuzzy systems are fuzzy rule-based inference systems, each comprising of a rule base and an inference engine. Different inference engines were invented to support different situations, such as the Mamdani inference engine [7] and the TSK inference engine [8]. The rule bases are usually extracted from expert knowledge or learned from data. The TSK model produces crisp outputs due to its polynomial rule consequences in TSK-style rule bases, while the Mamdani model is more appealing in handling inferences based on human natural language due to its fuzzy rule consequences. Despite of the wide applications, these conventional fuzzy inference mechanisms are only workable with dense rule bases which fully cover the entire input domain.

Fuzzy interpolation systems (FISs) were proposed to address the above issue [9], and they also help in complexity reduction for fuzzy models with too complex (dense) rule bases. If there is only a spare rule base available and a given input does not overlap with any rule antecedent, conventional fuzzy inference systems will not be applicable. However, FISs are still able to generate a conclusion by means of fuzzy interpolation in such situations, thus enhancing the applicability of conventional fuzzy inference systems. FISs can also improve the efficiency of complex fuzzy inference systems by excluding those rules that can be accurately interpolated or extrapolated using other rules in a complex rule base. Various fuzzy interpolation methods based on Mamdani-style rule bases have been proposed in the literature such as Refs. [9–20], with successful applications in the fields of decision-making support, prediction and control, amongst others.

FISs have also been developed to support TSK-style sparse fuzzy rule bases by extending the traditional TSK fuzzy inference system [21]. This approach was developed based on a modified similarity degree measure that enables the effective utilisation of all rules during inference process to generate a global result. In particular, the modified similarity measure guarantees that the similarity degree between any given input and any rule antecedent is greater than 0 even when they do not overlap at all. Therefore, all the rules in the rule base can be fired to certain degrees such that they all contribute to the final result to some extents and consequently a conclusion still can be generated even when no rule antecedent is overlapped with the given observation. The extended TSK fuzzy model enjoys the advantages of both TSK model and fuzzy interpolation, which is able to obtain crisp inference results from either sparse, dense or unevenly distributed (including dense parts and spare parts) TSK-style fuzzy rule bases.

FISs have been successfully applied to real-world problems. In some real world scenarios, neither complete expert knowledge nor complete data set is available or readily obtainable to generate evenly distributed dense rule bases. FISs therefore have been applied in such situations. For instance, a FIS has been applied to building evaluation in the work of Molnárka et al. [22] in an effort to help estate agencies making decisions for residential building maintenance, when some necessary relevant data have been lost. In Ref. [23], a FIS system was applied successfully to reduce the complexity and improve the efficiency of a fuzzy home heating control system. The work of Bai et al. [24] applied a FIS to calibrate parallel machine tools for industry use. A behaviour-based fuzzy control system is introduced in Ref. [25], which applied a FIS to make decisions when only incomplete knowledge base has been provided or available. Most recently, FISs have also been used to support network quality of service [26] and network intrusion detection [27].

The remainder of this chapter is organised as follows. Section 2 reviews the theoretical underpinnings of conventional fuzzy inference systems, that is, the Mamdani inference system and the TSK inference system. Section 3 discusses different fuzzy interpolation approaches to support sparse Mamdani-style rule bases. Section 4 presents the extension of the conventional TSK inference system in supporting sparse TSK-style rule bases. Section 5 reports two representative examples of fuzzy interpolation systems in the field of system control. Section 6 concludes the chapter and points out the directions for future work.

2. Fuzzy inference systems

The process of fuzzy inference is basically an iteration of computer paradigm based on fuzzy set theory, fuzzy-if-then-rules and fuzzy reasoning. Each iteration takes an input which can be an observation or a previously inferred result, crisp or fuzzy. Then, these inputs are used to fire the rules in a given rule base, and the output is the aggregation of the inferred results from all the fired rules. There are generally two primary ways to construct a rule base for a given problem. The first way is directly translating expert knowledge to rules, and the fuzzy inference systems with such rule bases are usually called fuzzy expert systems or fuzzy controllers [28]. In this case, rules are fuzzy representations of expert knowledge, and the resultant rule base offers a high semantic level and a good generalisation capability. The difficulty of building rule bases for complex problems has resulted in the development of another approach of rule base construction, which is driven by data, that is, fuzzy rules are obtained from data by employing machine learning techniques rather than expert knowledge [29, 30]. In contrast, the rule bases built in this way lack comprehensibility and transparency. There are two types of rule bases depending on the expression of the consequences of the fuzzy rules composing the rule base. Mamdani-style fuzzy rules consider fuzzy terms or linguistic values in the consequence, while TSK-style fuzzy rules represent the consequences as polynomial functions of crisp inputs.

2.1. Inference with Mamdani-style rule bases

There are a number of fuzzy inference mechanisms that can be utilised to derive a consequence from a given observation using a Mamdani rule base. The two most significant modes are the compositional rule of inference (CRI) [31] and analogy-based reasoning [24, 33], which are introduced below.

2.1.1. Compositional rule of inference

The introduction of CRI marks the era of fuzzy inference [31]. Given a rule 'IF x is A, THEN y is B'' and an observation 'x is A^*', the conclusion B* can be generated through CRI as:

$$\mu_{B^*}(v) = sup_{u \in U_x} T\left(\mu_{A^*}(u), \mu_R(u, v)\right), \tag{1}$$

where T is a triangular norm, *sup* represents supremum, and R is the relationship between variables x and y. Essentially, CRI is a fuzzy extension of classical modus ponens which can be viewed from two perspectives. Firstly, classical modus ponens only supports predicates concerning singleton elements, but CRI is able to deal with predicates which concern a set of elements in the variable domain. This is achieved by representing a fuzzy rule as a fuzzy relation over the Cartesian product of the domains of the antecedent and consequent variables. Various fuzzy implication relations have been proposed [7, 32–34], each of which may have its own properties and therefore is suitable for a certain group of applications. Secondly, classical modus ponens only supports Boolean logic, but CRI supports multi-value logic. That is, CRI is able to deal with predicates with partial truth values, which are implemented by a compositional operator sup T, where T represents a t-norm [35].

A number of existing fuzzy reasoning methods based on CRI have been developed [36, 37], including the first successful practical approach, that is, the Mamdani inference [28]. This approach is also the most commonly seen fuzzy methodology in physical control systems thus far. It was originally proposed as an attempt to control a steam engine and boiler combination by synthesising a set of linguistic control rules obtained from experienced human operators. Mamdani inference implements CRI using *minimum* as the t-norm operator due to its simplicity. In particular, the inferred result from each fired rule is a fuzzy set which is transformed from the rule consequence by restricting the membership of those elements whose memberships are greater than the firing strength. The firing strength is also sometimes termed the satisfaction degree, which is the supremum within the variable domain of the minimum of the rule antecedent and the given observation. A defuzzification process is needed when crisp outputs are required.

2.1.2. Analogy-based fuzzy inference

Despite the success of CRI in various fuzzy system applications, it suffers various criticisms including its complexity and vague underlying semantics [34, 38]. This has led to another group of fuzzy reasoning approaches which are based on similarity degree, usually called analogy-based fuzzy reasoning [38–41]. Similarity considerations play a major role in human cognitive processes [42], so do they in approximate reasoning. It is intuitive that if a given observation is similar to the antecedent of a rule, the conclusion from the observation should also be similar to the consequence of the rule. Different to CRI-based fuzzy reasoning, analogy-based fuzzy reasoning does not require the construction of a fuzzy relation. Instead, it is based on the degree of similarity (given a certain similarity metric) between the given observation and the antecedent of a rule. Utilising the computed similarity degree, the consequence of the fired rule can be modified to the consequence of the given observation.

Approximate analogical reasoning schema is a typical analogy-based fuzzy inference approach [34, 38]. In this method, rules are fired according to the similarity degrees between a given observation and the antecedents of rules. If the degree of similarity between the given observation and the antecedent of a rule is greater than a predefined threshold value, the rule will be fired and the consequence of the observation is deduced from the rule consequence by a given modification procedure. Another analogy-based fuzzy inference approach was proposed in Refs. [39, 40], which particularly targets medical diagnostic problems. This approach is based on the cosine angle between the two vectors that represent the actual and the user's specified values of the antecedent variable. Several modification procedures can be found in Refs. [43, 44]. Particularly, a fuzzy reasoning method which employs similarity measures based on the degree of subsethood between the propositions in the antecedent and a given observation is proposed in Ref. [45]. This method has also been extended to consider the weights of the propositions in the antecedent [46]. Analogy-based fuzzy inference approaches usually arrive at solutions with more natural appeal than those introduced in the last section.

2.2. Inference with TSK-style rule bases

The TSK fuzzy inference system was proposed for the direct generation of crisp outputs [8]. In difference with the Mamdani-style fuzzy rule bases, TSK-style rule bases are usually generated from data using a clustering algorithm such as K-Means and an algorithm to determine the number of clusters such as Ref. [47]. Also, the consequence of a TSK fuzzy rule is a polynomial function rather than a fuzzy set. A typical TSK fuzzy rule can be defined as:

$$\text{IF } x_1 \text{ is } A_1 \wedge \ldots \wedge x_m \text{ is } A_m \text{ THEN } z = f(x_1, \ldots, x_m), \tag{2}$$

where $A_1,\ldots A_m$ are fuzzy values with regard to antecedent variables $x_1, \ldots, x_m$ respectively, and $f(x_1,\ldots,x_m)$ is a crisp polynomial function of crisp inputs determining the crisp output value. The rule consequent polynomial functions $f(x_{1,\ldots,}x_m)$ are usually zero order or first order. For simplicity, suppose that a TSK-style rule base is formed by two-antecedent rules as follows:

$$\begin{aligned} R_i &: \text{IF } x \text{ is } A_i \wedge y \text{ is } B_i \text{ THEN } z = f_i(x, y) \\ R_j &: \text{IF } x \text{ is } A_j \wedge y \text{ is } B_j \text{ THEN } z = f_j(x, y). \end{aligned} \tag{3}$$

Suppose that (x_0,y_0) is the crisp input pair, then the inference process can be shown in **Figure 1**. As the input values overlap with both rule antecedents, both rules are fired. Using rules R_i and R_j, the given input then leads to system outputs $f_i(x_0,x_0)$ and $f_j(x_0,x_0)$, respectively. The consequences from both rules are then integrated using weighted average function, where

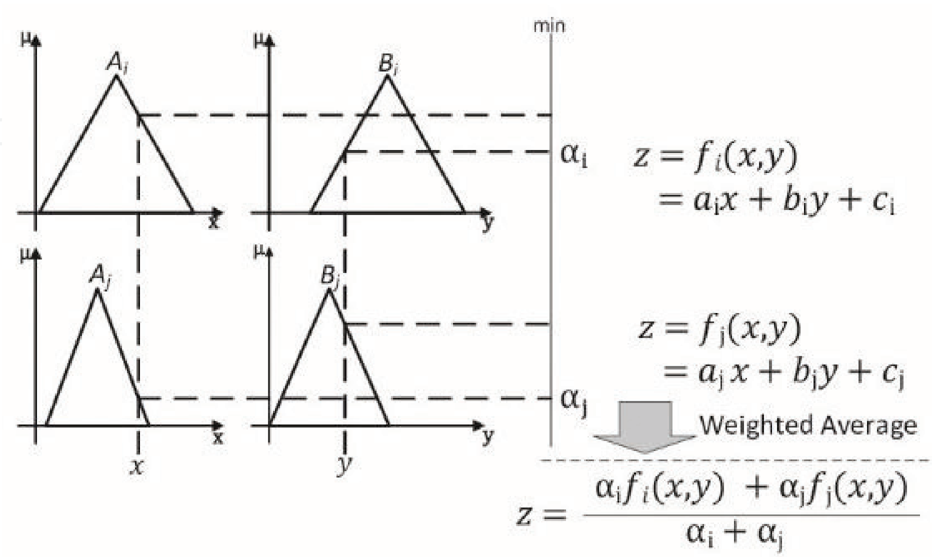

Figure 1. TSK fuzzy inference [21].

the values of weights represent the matching degrees between the given input and the rule antecedents (often referred to as firing strengths). Assume that $\mu_{A_i}(x_0)$ and $\mu_{B_i}(y_0)$ are the matching degree between inputs (x_0 and y_0) and rule antecedents (A_i and B_i), respectively. The firing strength of rule R_i, denoted as α_i, is calculated as:

$$\alpha_i = \mu_{A_i}(x_0) \wedge \mu_{B_i}(y_0), \tag{4}$$

where $\wedge$ stands for a t-norm operator. Different implementations can be used for the t-norm operator, with the minimum operator being used most widely. Of course, if another system input (x_1, y_1) is presented and it is not covered by the rule base, the matching degrees between this new input and rule antecedents of R_i and R_j are equal to 0. In this case, no rule will be fired, and thus traditional TSK is not applicable. In this case, fuzzy interpolation is required, which is introduced in Section 4.

3. Fuzzy interpolation with sparse Mamdani-style rule bases

FISs based on Mamdani-style rule bases can be categorised into two classes. One group of approaches were developed based on the decomposition and resolution principle, termed as 'resolution principle-base interpolation'. In particular, the approach represents each fuzzy set as a series of α-cuts ($\alpha \in (0,1]$), and the α-cut of the conclusion is computed from the α-cuts of the observation and the α-cuts of rules. The final fuzzy set is assembled from all the α-cut consequences using the resolution principle [48–50]. The other group of fuzzy interpolation approaches were developed using the analogy reasoning system, thus termed as 'analogy-based fuzzy interpolation'. This group of approaches firstly generates an intermediate rule whose antecedent maximally overlaps with the given observation, then the system output is produced from the observation using the intermediate rule. Two representative approaches of the two classes, the KH approach [10] and the scale and move transformation-based approach [9, 51, 52], are discussed in this section based on simple rule bases with two antecedent rules. Despite of the simple examples used herein, both of these approaches have been extended to work with multiple multi-antecedent rules.

3.1. Resolution principle-based interpolation

Single step interpolation approaches are computationally efficient, such as the KH approach proposed in Refs. [9, 10, 53]. Following these approaches, all variables involved in the reasoning process must satisfy a partial ordering, denoted as $\prec$ [31]. According to the decomposition principle, a normal and convex fuzzy set A can be represented by a series of α-cut intervals, each denoted as A_α, $\alpha \in (0,1)$. Given fuzzy sets A_i and A_j which are associated with the same variable, the partial ordering $A_i \prec A_j$ is defined as:

$$\inf\{A_{i\alpha}\} < \inf\{A_{j\alpha}\} \text{ and } \sup\{A_{i\alpha}\} < \sup\{A_{j\alpha}\}, \quad \forall \alpha \in (0, 1], \tag{5}$$

where $\inf\{A_{i\alpha}\}$ and $\sup\{A_{i\alpha}\}$ denote the infimum and supremum of $A_{i\alpha}$, respectively.

http://dx.doi.org/10.5772/68051

Take the KH approach as an example here. For simplicity, suppose there are two fuzzy rules: If x is A_i, then y is B_i, and If x is A_j then y is B_j, shorten as $A_i \Longrightarrow B_i$ and $A_j \Longrightarrow B_j$, respectively. Also, suppose that these two rules are adjacent, in other words, there is no rule $A \Longrightarrow B$ existing such that $A_i \prec A \prec A_j$ or $A_j \prec A \prec A_i$. Given an observation A^* which satisfies $A_i \prec A^* \prec A_j$ or $A_j \prec A^* \prec A_i$, a conclusion B^* can be computed as:

$$\frac{D(A_{i\alpha}, A^*_\alpha)}{D(A^*_\alpha, A_{j\alpha})} = \frac{D(B_{i\alpha}, B^*_\alpha)}{D(B^*_\alpha, B_{j\alpha})}, \tag{6}$$

where given any $0 < \alpha \leq 1$, the distance $D(A_{i\alpha}, A_{j\alpha})$ between the α-cuts $A_{i\alpha}$ and $A_{j\alpha}$ is defined by the interval $[D^L(A_{i\alpha}, A_{j\alpha}), D^U(A_{i\alpha}, A_{j\alpha})]$ with:

$$D^L(A_{i\alpha}, A_{j\alpha}) = inf\{A_{j\alpha}\} - inf\{A_{i\alpha}\}, D^U(A_{i\alpha}, A_{j\alpha}) = sup\{A_{j\alpha}\} - sup\{A_{i\alpha}\}. \tag{7}$$

Following Eqs. (4) and (5), the following is resulted:

$$\begin{cases} min\{B^*_\alpha\} = \dfrac{\dfrac{inf(B_{i\alpha})}{D^L(A_{i\alpha}, A^*_\alpha)} + \dfrac{inf(B_{j\alpha})}{D^L(A^*_\alpha, A_{j\alpha})}}{\dfrac{1}{D^L(A_{i\alpha}, A^*_\alpha)} + \dfrac{1}{D^L(A^*_\alpha, A_{j\alpha})}} \\ max\{B^*_\alpha\} = \dfrac{\dfrac{sup(B_{i\alpha})}{D^U(A_{i\alpha}, A^*_\alpha)} + \dfrac{sup(B_{j\alpha})}{D^U(A^*_\alpha, A_{j\alpha})}}{\dfrac{1}{D^U(A_{i\alpha}, A^*_\alpha)} + \dfrac{1}{D^U(A^*_\alpha, A_{j\alpha})}} \end{cases} \tag{8}$$

For simplicity, let

$$\begin{cases} \Lambda^L_\alpha = \dfrac{\inf\{A^*_\alpha\} - \inf\{A_{i\alpha}\}}{\inf\{A_{j\alpha}\} - \inf\{A_{i\alpha}\}} \\ \Lambda^U_\alpha = \dfrac{\sup\{A^*_\alpha\} - \sup\{A_{i\alpha}\}}{\sup\{A_{j\alpha}\} - \sup\{A_{i\alpha}\}} \end{cases} \tag{9}$$

Also, denote $\Lambda = [\Lambda^L_\alpha, \Lambda^U_\alpha]$ hereafter. From this, Eq. (8) can be re-written as:

$$\begin{cases} \min\{B^*_\alpha\} = (1 - \Lambda^L_\alpha)\inf\{B_{i\alpha}\} + \Lambda^L_\alpha \inf\{B_{j\alpha}\} \\ \max\{B^*_\alpha\} = (1 - \Lambda^U_\alpha)\sup\{B_{i\alpha}\} + \Lambda^U_\alpha \sup\{B_{j\alpha}\} \end{cases} \tag{10}$$

This means $B^*_\alpha = [\min\{B^*_\alpha\}, \max\{B^*_\alpha\}]$ is generated. The final consequence B^* is then reassembled as:

$$B^* = U_{\alpha \in (0,1]} \alpha B^*_\alpha. \tag{11}$$

The KH approach may generate invalid interpolated results [54], which is usually called 'the abnormal problem'. To eliminate this deficiency, a number of modifications or improvements have been proposed, including Refs. [9, 10, 13, 14, 18, 53, 55–60]. Approaches such as Refs. [15, 16, 61–63] also belong to this group.

3.2. Analogy-based interpolation

The scale and move transformation-based fuzzy interpolation [51, 52, 64] is a representative approach in the analogy-based interpolation group. For simplicity, following the same assumption of a simple rule base containing two rules with two antecedents, the transformation-based approach is shown in **Figure 2** and outlined as follows.

Given neighbouring rules If x is A_i, then y is B_i, and If x is A_j then y is B_j and observation A^*, this method first maps fuzzy sets A_i, A_j and A^* to real numbers a_i, a_j and a^* (named as *representative values*) respectively, using real function f_1. Then, the location relationship between A^* and rule antecedents (A_i and A_j) is computed. This is achieved by another mapping function f_2, which results in the *relative placement factor* λ. In contrast to the resolution-based interpolation approaches, the generated *relative placement factor* in analogy-based fuzzy interpolation approach is a crisp real number. Finally, linear interpolation is implemented using mapping function f_3 of λ, which leads to the intermediate rule $A^{*'} \Rightarrow B^{*'}$.

Note that the *representative value* of intermediate rule antecedent $A^{*'}$ equals to that of A^* (the given observation), although $A^{*'}$ and A^* are not identical for most of the situations. In the scale and move transformation-based fuzzy interpolation approach, the similarity degree between two fuzzy sets A^* and $A^{*\prime}$ with the same representative value is expressed as the scale rate s, scale ratio $\mathbb{S}$ and move rate $\mathbb{M}$, which is obtained by real function f_4. From this, the consequence B^* is calculated from $B^{*'}$ using a transformation function f_5 which imposes the similarity degree between A^* and $A^{*\prime}$. Different approaches have been developed for intermediate rule generation and final conclusion production from the intermediate rule [17, 55, 63, 65].

3.3. Adaptive fuzzy interpolation

Fuzzy interpolation strengthens the power of fuzzy inference by enhancing the robustness of fuzzy systems and reducing the systems' complexity. Common to both classes of fuzzy

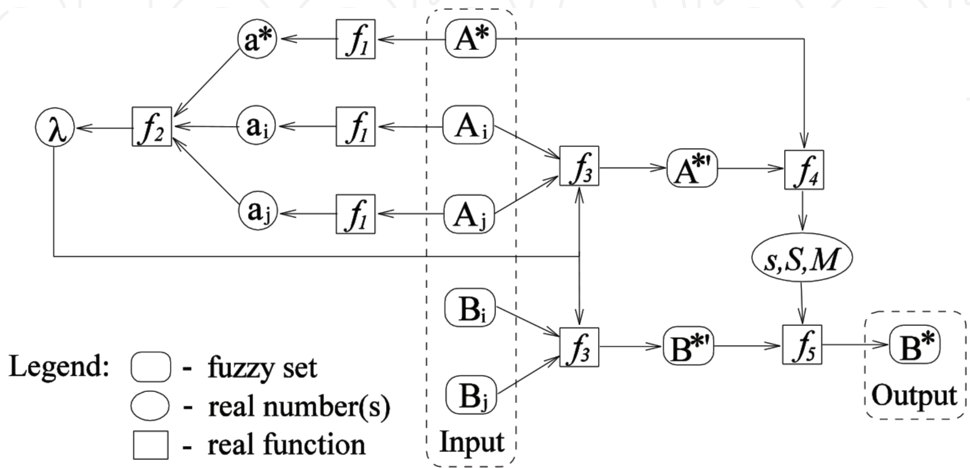

Figure 2. Transformation-based fuzzy interpolation [12].

interpolation approaches discussed above is the fact that interpolation is carried out in a linear manner. This may conflict with the nature of some realistic problems and consequently this may lead to inconsistencies during rule interpolation processes. Adaptive fuzzy interpolation was proposed to address this [12, 66–68]. It was developed upon FIS approaches, which detects inconsistencies, locates possible fault candidates and modifies the candidates in order to remove all the inconsistencies.

Each pair of neighbouring rules is defined as a *fuzzy reasoning component* in adaptive fuzzy interpolation. Each fuzzy reasoning component takes a fuzzy value as input and produces another as output. The process of adaptive interpolation is summarised in **Figure 3**. Firstly, the interpolator carries out interpolation and passes the interpolated results to the truth maintenance system (ATMS) [69, 70], which records the dependencies between an interpolated value (including any contradiction) and its proceeding interpolation components. Then, the ATMS relays any β_0-contradictions (i.e. inconsistency between two different values for a common variable at least to the degree of a given threshold β_0 $(0 \leq \beta_0 \leq 1)$) as well as their dependent fuzzy reasoning components to the general diagnostic engine (GDE) [71] which diagnoses the problem and generates all possible component candidates. After that, a modification process takes place to correct a certain candidate to restore consistency by modifying the original linear interpolation to become first-order piecewise linear.

The adaptive approach has been further generalised [11, 72, 73], which allows the identification and modification of observations and rules, in addition to that of interpolation procedures that were addressed in Ref. [12]. This is supported by introducing extra information of certainty degrees associated with such basic elements of FIS. The work also allows for all candidates for modification to be prioritised, based on the extent to which a candidate is likely to lead to all the detected contradictions, by extending the classic ATMS and GDE. This study has significantly improved the efficiency of the work in Ref. [12] by exploiting more information during both the diagnosis and modification processes. Another alternative implementation of the adaptive approach has also been reported in Ref. [74].

3.4. Sparse rule base generation

A Mamdani-style fuzzy rule base is usually implemented through either a data-driven approach [75] or a knowledge-driven approach [76]. The data-driven approach using artificial intelligence approach extracts rules from data sets, while the knowledge-driven approach generates rules by human expert. Due to the limited availability of expert knowledge, data-driven approaches have been increasingly widely applied. However, the application of such

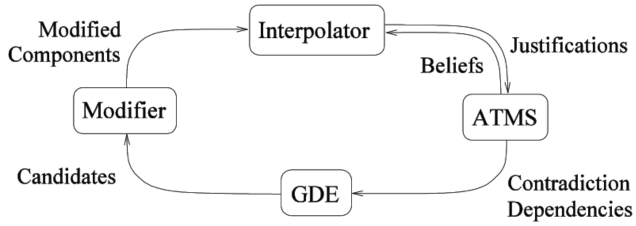

Figure 3. Adaptive fuzzy interpolation [12].

approaches usually requires a large amount of training data, and it often leads to dense rule bases to support conventional fuzzy inference systems, despite of the availability of rule simplification approaches such as Refs. [77, 78].

A recent development or rule base generation has been reported with compact sparse rule bases targeted [79]. This approach firstly partitions the problem domain into a number of sub-regions and each sub-region is expressed as a fuzzy rule. Then, the importance of each sub-region is analysed using curvature value by artificially treating the problem space as a geography object (and high-dimensional problem space is represented as a collection of sub-three-dimensional spaces). Briefly, the profile curvature of a surface expresses the extent to which the geometric object deviates from being 'flat' or 'straight', the curvature values of the sub-regions are then calculated to represent how important they are in terms of linear interpolation. Given a predefined threshold, important sub-regions can be identified, and their corresponding rules are selected to generate a raw sparse rule base. The generated raw rule base can then be optimised by fine-tuning the membership functions using an optimisation algorithm. Generic algorithm has been widely used for various optimisation problems, such as Ref. [80], which has also been used in the work of Ref. [79].

Compared to most of the existing rule base generation approaches, the above approach differs in its utilisation of the curvature value in rule selection. Mathematically, curvature is the second derivate of a surface or the slope of slope. The profile curvature [81] is traditionally used in geography to represent the rate at which a surface slope changes whilst moving in the direction, which represents the steepest downward gradient for the given direction. Given a sub-region $f(x, y)$ and a certain direction, the curvature value is calculated as the directional derivative which refers to the rate at which any given scalar field is changing. The overall linearity of a sub-region can thus be accurately represented as the maximum profile curvature value on all directions. From this, those rules corresponding to sub-regions with higher profile curvature values (with respect to a given threshold) are selected, which jointly form the sparse rule base to support fuzzy rule interpolation.

FISs relax the requirement of complete expert knowledge or large data sets covering the entire input domain from the conventional fuzzy inference systems. However, it is still difficult for some real-world applications to obtain sufficient data or expert knowledge for rule base generation to support FISs. In addition, the generated rule resulted from most of the existing rule base generation approaches are fixed and cannot support changing situations. An experience-based rule base generation and adaptation approach for FISs has therefore been proposed for control problems [82]. Briefly, the approach initialises the rule base with very limited rules first. Then, the initialised rule base is revised by adding accurate interpolated rules and removing out-of-date rules guided by the performance index from a feedback mechanism and the performance experiences of rules.

4. Fuzzy interpolation with sparse TSK-style rule base

The traditional TSK inference system has been extended to work with sparse TSK fuzzy rule base [21]. This approach, in the same time, also enjoys the benefit from its original version,

http://dx.doi.org/10.5772/68051

which directly generates crisp outputs. The extended TSK inference approach is built upon a modified similarity measure which always generates greater than zero similarity degrees between observations and rule antecedents even when they do not overlap at all. Thanks to this property, a global consequence can always be generated by integrating the results from all rules in the rule base.

4.1. Rule firing strength

The modified similarity measure is developed from the work described in Ref. [83]. Suppose there are two fuzzy sets A and A' in a normalised variable domain. Without loss generality, a fuzzy set with any membership can be approximated by a polygonal fuzzy membership function with n odd points. Therefore, A and A' can be represented as $A = (a_1, a_2, \ldots a_n)$ and $A' = (a'_1, a'_2, \ldots a'_{n'})$, as shown in **Figure 4**. The similarity degree $S(A,A')$ between A and A' is computed as:

$$S(A, A^{'}) = \left(1 - \frac{\sum_{i=1}^{n} |a_i - a^{'}_i|}{n}\right)(DF)^{\tilde{B}(supp_{A'}, supp_{A'})} \frac{\min\left(\mu(C_A), \mu(C_{A'})\right)}{\max\left(\mu(C_A), \mu(C_{A'})\right)}, \tag{12}$$

where c_A is the centre of gravity of fuzzy sets A, and $\mu(c_A)$ is the membership of the centre of gravity of fuzzy set A; DF represents a distance factor which is a function of the distance between two concerned fuzzy sets, and $B(supp_{A'}, supp_{A'})$ is defined as follows:

$$B(supp_{A'}, supp_{A'}) = \begin{cases} 1, & if supp_A + supp_{A'} \neq 0, \\ 0, & if supp_A + supp_{A'} = 0, \end{cases} \tag{13}$$

where $supp_A$ and $supp_{A'}$ are the supports of A and A', respectively.

In Eq. (13), $B(supp_{A'}, supp_{A'})$ is used to determine whether distance factor is considered. That is, if both A and A' are of crisp values, the distance factor DF will not take into consideration during the calculation of the similarity degree; otherwise, DF will be considered. The centre of gravity of a fuzzy set is commonly approximated as the average of its odd points. That is:

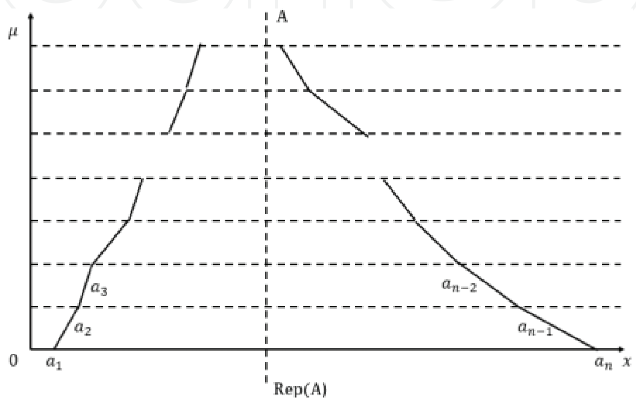

Figure 4. An arbitrary fuzzy set with n odd points.

$$c_A = \frac{a_1 + a_2 + \ldots + a_n}{n}, \tag{14}$$

$$\mu(c_A) = \frac{\mu(a_1) + \mu(a_2) + \ldots + \mu(a_n)}{n}. \tag{15}$$

The distance factor *DF* is represented as:

$$DF = 1 - \frac{1}{1 + e^{-hd+5}} \tag{16}$$

where d is the distance between the two fuzzy sets, and $h(h > 0)$ is a sensitivity factor. The smaller the value of h is, the more sensitive the similarity degree to their distance is. The value of h is usually within the range of (20, 60), but the exact value is problem specific.

4.2. Fuzzy interpolation

Using the modified similarity measure as traduced above, the similarity between any given observation and a rule antecedent is always greater than zero. This means that all the rules in the rule base are fired for inference. Therefore, if only a sparse rule base is available and a given observation is not covered by the sparse rule base, a consequence still can be generated by firing all the rules in the rule base. The inference process is summarised as below:

1. Calculate the matching degree $S(A^*, A_i)$ and $S(B^*, B_i)$ between each pair of rule antecedent (A_i, B_i) and the input values (A^*, B^*) based on Eq. (12).
2. Determine the firing strength of each rule by integrating the matching degrees between the input items and rule antecedents as calculated in Step 1:

$$\alpha_i = S(A^*, A_i) \wedge S(B^*, B_i). \tag{17}$$

3. Compute the consequence of each rule in line with the given input and the polynomial function in rule consequent:

$$f_i(A^*, A_i) = \alpha_i \cdot c_{A^*} + b_i \cdot c_{B^*} + c_i. \tag{18}$$

4. Obtain the final result z by integrating the sub-consequences from all m rules in the rule base:

$$z = \frac{\sum_{i=1}^{n} \alpha_i f_i(A^*, B^*)}{\sum_{i=1}^{n} \alpha_i}. \tag{19}$$

5. Applications of fuzzy interpolation

Fuzzy interpolation systems have been successfully applied to a number of real-world problems including Refs. [23, 22, 25, 52, 57], two of which are reviewed in the section below.

5.1. Truck backer-upper control

Backing a trailer truck to a loading dock is a challenging task for all yet the most skilled truck drivers. Due to the difficulties, this challenge has been used as a control benchmark problem with various solutions proposed [75, 84, 85]. For instance, an artificial neural network has been applied to this problem, but a large amount of training data is required [84]. An adaptive fuzzy control system was also proposed for this problem, but the generation of the rule base is computationally expensive. Another solution combines empirical knowledge and data [85]. That is, a combined fuzzy rule base is generated by joining the previously generated rules (data-driven) and linguistic rules (expert knowledge-driven). More recently, a supervisory control system was proposed with fewer number of state variables required due to its capability to the decomposition of the control task, thus relieving the curse of dimensionality [86].

Fuzzy interpolation system has also been applied to the trailer truck backer-upper problem [52] to further reduce the system complexity. The problem can be formally formulated as $\theta = f(x, y, \varnothing)$. Variables x and y represent the coordinate values corresponding to horizontal and vertical axes; $\varnothing$ refers to the azimuth angle between the truck's onward direction and the horizontal axis; and θ is the steering angle of the truck. Given that enough clearance is present between the truck and loading lock in most cases, variable y can be safely omitted and hence results in a simplified formula to $\theta = f(x, \varnothing)$. By evenly partitioning each variable domain into three fuzzy sets, nine (i.e. 3*3) fuzzy rules were generated using FISMAT [87] and each of which is denoted as IF x is A AND $\varnothing$ is B THEN θ is C, where A, B and C are three linguistic values. Noting that domain partitions appear to be symmetrical in some sense, the three rules which are flanked by other rule pairs were removed from the rule base resulting a more compact rule base with only six fuzzy rules.

If the traditional fuzzy inference system were applied, the sparse rule base would cause a sudden break of the truck for some situations as no rule would be fired when the truck is in the position that can be represented by the omitted rules. In this case, fuzzy interpolation is naturally applied and the sudden break problem can be avoided. In addition, thanks to the great generalisation ability of the fuzzy interpolation systems, smooth performance is also demonstrated compared to the conventional fuzzy inference approaches. This study clearly demonstrates that fuzzy interpolation systems are able to simplify rule bases and support inferences with sparse rule bases.

5.2. Heating system control

The demotic energy waste contributes a large part of CO^2 emissions in the UK, and about 60% of the household energy has been used for space heating. Various heating controllers have been developed to reduce the waste of energy on heating unoccupied properties, which are usually programmable and developed using a number of sensors. These systems are able to successfully switch off heating systems when a property is unoccupied [88–92], but they cannot intelligently preheat the properties by warming the property before users return home without manual inputs or leaving the heating systems on unnecessarily for longer time. A smart home heating controller has been developed using a FIS, which allows efficient home heating by accurately predicting the users' home time using users' historic and current location data obtained from portable devices [23].

The overall flow chart of the smart home heating system is shown in **Figure 5**. The controller first extracts the resident's location and moving information. There are four types of residents' location and moving information that need to be considered: At Home, Way Back Home, Leaving Home and Static (i.e. at Special Location). The user's current location and moving states are obtained effectively using the GPS information provided by user's portable devices. From this, if the resident's current state is At Home, the algorithm terminates; and if the residents' current state is Leaving Home, that is the residents are moving away from home,

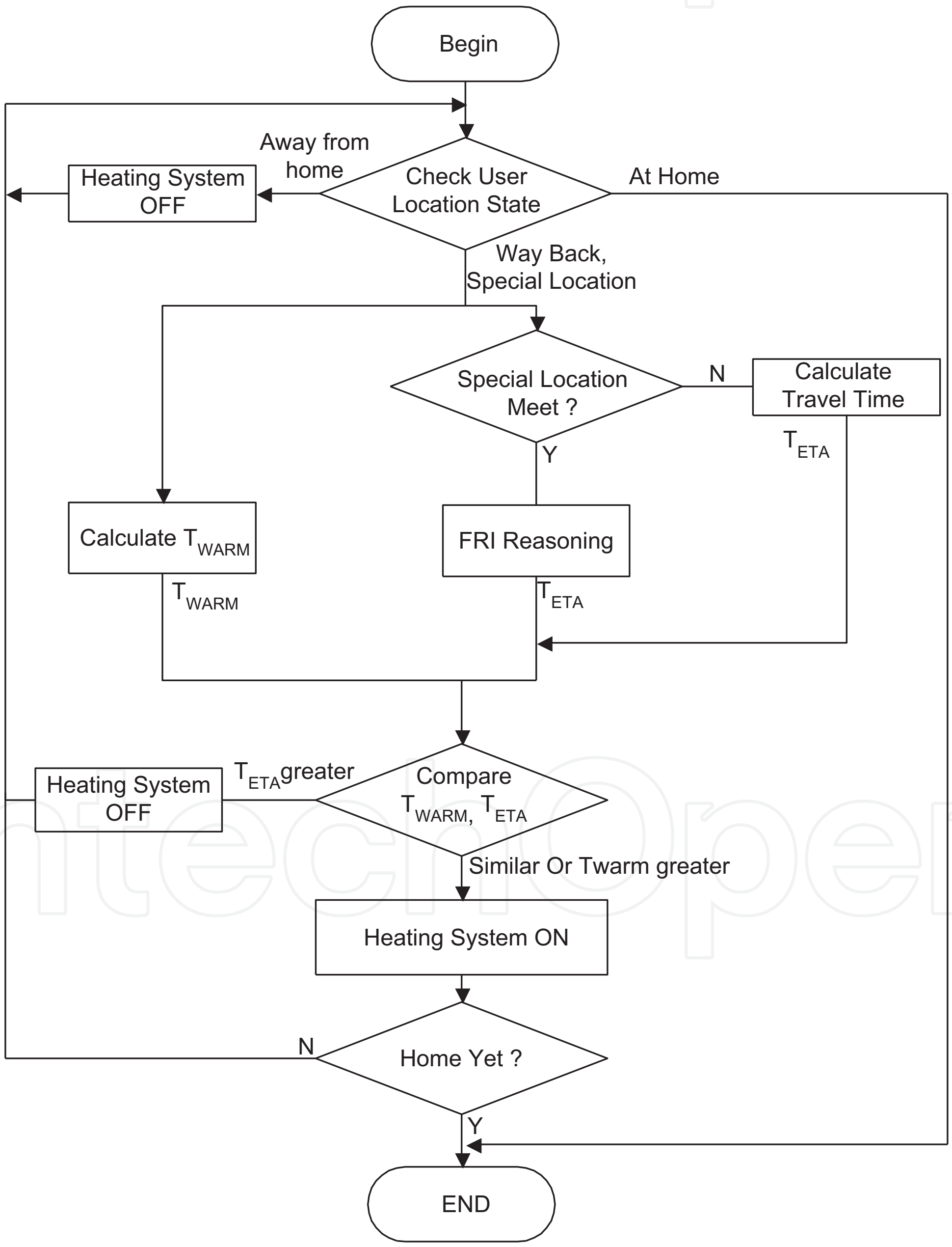

Figure 5. The flow chart of the heating controller [23].

the boiler is off and the system will check the resident's location and moving information again in a certain period of time. Otherwise, the time to arriving home (denoted as TAH) is predicted and the time to preheat the home to a comfortable temperature (denoted as TPH) is also calculated, based on the resident's current situation and the current environment around home.

The user's current travel modes (i.e. driving, walking or bicycling) can be detected by employing a naïve Bayes classifier [93] using the GPS information. Then the travel distance and time between the current location and home can be estimated using Google Distance Matrix API. Note that the time spent on different locations may vary significantly, and also different residents usually spend different amount of times at the same special location as people have their own living styles. The time that the residents spent at the current location is therefore estimated using fuzzy interpolation systems, thanks to the complexity of the problem. In particular, the fuzzy interpolation engine takes five fuzzy inputs and produces one fuzzy output which is the estimate of the time to getting home. The five inputs are the current location, the day of the week, the time of the day, the time already spent at the current location and the estimated travel between the current location and home.

If each input domain is fuzzy partitioned by 5 to 13 fuzzy, tens of thousands of rules will be resulted which requires significant resources during inferences. The proposed system, however, has selected the most important 72 rules forming a sparse rule base to support fuzzy rule interpolation, which significantly improve the system performance. Once the home time is calculated, the home can then be accurately preheated based on a heating gain table developed based on the particular situation and environment of a concerned property [91]. This work has been applied to a four-bedroom detached house with a total hearing space of $100\ \mathrm{m}^2(floor\ area) \times 2.4\ \mathrm{m}(hight)$. The house is heated by a 15 kW heating boiler. The study has shown that the controller developed using fuzzy inference has successfully reduced the burning time of the boiler for heating and more accurately preheat the home.

Despite of the success of the applications introduced above, there is a potential for FISs to be applied to more and larger scales real-world problems, especially in the field of system control. Note that robotics has taken the centre in the control field to perform tasks from basic robot calligraphy system [94] to complex tasks which require hand-eye (camera) coordination [95]. FISs can also be applied to such advanced areas in the field of robotics, which require further investigation.

6. Conclusions

This chapter reviewed fuzzy interpolation systems and their applications in the field of control. There are basically two groups of fuzzy interpolation approaches using the two most common types of fuzzy rule bases (i.e. Mamdani-style rule bases and TSK-style rule bases) to supplement the two groups of widely used fuzzy inference approaches (i.e. the Mamdani inference and the TSK inference). The applications of fuzzy interpolation systems have also been discussed in the chapter which demonstrate the power of the approaches. FISs can be further improved despite of its promising performance. Firstly, type-2 FISs have already been proposed in the literature, but how type-2 FISs can be applied in real-world applications requires further investigation. Also, more theoretical analysis for FISs is needed to mathematically prove the convergence property of the approaches. In addition, most of the existing fuzzy interpolation approaches are proposed as a supplementary of the existing fuzzy inference models. It is interesting to investigate the development of a united platform which integrates both the existing fuzzy models and fuzzy inference systems such that the new system can benefit from both approaches.

Acknowledgements

This work was jointly supported by the National Natural Science Foundation of China (No. 61502068) and the China Postdoctoral Science Foundation (Nos. 2013M541213 and 2015T80239).

Author details

Longzhi Yang[1]*, Zheming Zuo[1], Fei Chao[2] and Yanpeng Qu[3]

*Address all correspondence to: longzhi.yang@northumbria.ac.uk

1 Department of Computer and Information Sciences, Northumbria University, Newcastle, UK

2 School of Information Science and Engineering, Xiamen University, Xiamen, PR China

3 Information Science and Technology College, Dalian Maritime University, Dalian, PR China

References

[1] Fu X, Zeng XJ, Wang D, Xu D, Yang L. Fuzzy system approaches to negotiation pricing decision support. Journal of Intelligent & Fuzzy Systems. 2015;**29**(2):685–699

[2] Yang L, Neagu D. Integration strategies for toxicity data from an empirical perspective. In: 2014 14th UK Workshop on Computational Intelligence (UKCI). 2014. pp. 1–8

[3] Yang L, Neagu D. Toxicity risk assessment from heterogeneous uncertain data with possibility-probability distribution. In: 2013 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). 2013. pp. 1–8

[4] Yang, Neagu D, Cronin MTD, Hewitt M, Enoch SJ, Madden JC, Przybylak K. Towards a fuzzy expert system on toxicological data quality assessment. Molecular Informatics. 2012;**32**(1):65–78

[5] Yang L, Neagu D. Towards the integration of heterogeneous uncertain data. In: 2012 IEEE 13th International Conference on Information Reuse & Integration (IRI). 2012. pp. 295–302

[6] Guo Q, Qu Y, Deng A, Yang L. A new fuzzy-rough feature selection algorithm for mammographic risk analysis. In: 2016 12th International Conference on Natural Computation, Fuzzy Systems and Knowledge Discovery (ICNC-FSKD). 2016. pp. 934–939

[7] Mamdani EH. Application of fuzzy logic to approximate reasoning using linguistic synthesis. IEEE Transactions on Computers. 1976;**26**(12):1182–1191

[8] Takagi T, Sugeno M. Fuzzy identification of systems and its applications to modeling and control. IEEE Transactions on Systems, Man, and Cybernetics. 1985;**1**:116–132

[9] Kóczy L, Hirota K. Approximate reasoning by linear rule interpolation and general approximation. International Journal of Approximate Reasoning. 1993;**9**(3): 197–225

[10] Kóczy L, Hirota K. Interpolative reasoning with insufficient evidence in sparse fuzzy rule bases. Information Sciences. 1993;**71**(1–2):169–201

[11] Yang L, Chao F, Shen Q. Generalised adaptive fuzzy rule interpolation. IEEE Transactions on Fuzzy Systems. 2016. DOI: 10.1109/TFUZZ.2016.2582526

[12] Yang L, Shen Q. Adaptive Fuzzy Interpolation. IEEE Transactions on Fuzzy Systems. 2011;**19**(6):1107–1126

[13] Yang L, Shen Q. Closed form fuzzy interpolation. Fuzzy Sets and Systems. 2013;**225**:1–22

[14] Tikk D, Baranyi P. Comprehensive analysis of a new fuzzy rule interpolation method. IEEE Transactions on Fuzzy Systems. 2000;**8**(3):281–296

[15] Kovács S. Extending the fuzzy rule interpolation "FIVE" by fuzzy observation. In: Computational Intelligence, Theory and Applications. Springer; 2006. pp. 485–497

[16] Johanyák ZC, Kovács S. Fuzzy rule interpolation based on polar cuts. In: Computational Intelligence, Theory and Applications. Springer; 2006. pp. 499–511

[17] Baranyi P, Kóczy LT, Gedeon TD. A generalized concept for fuzzy rule interpolation. IEEE Transactions on Fuzzy Systems. 2004;**12**(6):820–837

[18] Dubois D, Prade H. On fuzzy interpolation. International Journal of General System. 1999;**28**(2–3):103–114

[19] Jin S, Diao R, Quek C, Shen Q. Backward fuzzy rule interpolation. IEEE Transactions on Fuzzy Systems. 2014;**22**(6):1682–1698

[20] Chen C, Parthaláin NM, Li Y, Price C, Quek C, Shen Q. Rough-fuzzy rule interpolation. Information Sciences. 2016;**351**:1–17

[21] Li J, Qu Y, Shum HP, Yang L. TSK inference with sparse rule bases. In: Advances in Computational Intelligence Systems. 2017. pp. 107–123

[22] Molnárka GI, Kovács S, Kóczy LT. Fuzzy rule interpolation based fuzzy signature structure in building condition evaluation. In: 2014 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). 2014. pp. 2214–2221

[23] Li J, Yang L, Shum HP, Sexton G, Tan Y. Intelligent home heating controller using fuzzy rule interpolation. In: 2015 15th UK Workshop on Computational Intelligence (UKCI). 2015

[24] Bai Y, Zhuang H, Wang D. Apply fuzzy interpolation method to calibrate parallel machine tools. The International Journal of Advanced Manufacturing Technology. 2012;**60**(5–8):553–560

[25] Kovács S, Koczy LT. Application of interpolation-based fuzzy logic reasoning in behaviour-based control structures. In: 2004 IEEE International Conference on Fuzzy Systems. 2004

[26] Li J, Yang L, Fu X, Chao F, Qu Y. Dynamic QoS solution for enterprise networks using TSK fuzzy interpolation. In: 2017 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). 2017

[27] Yang L, Li J, Fehringer G, Barraclough P, Sexton G, Cao Y. Intrusion detection system by fuzzy interpolation. In: 2017 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). 2017

[28] Mamdani EH, Assilian S. An experiment in linguistic synthesis with a fuzzy logic controller. International Journal of Man-Machine Studies. 1975;**7**(1):1–13

[29] Castro JL, Zurita JM. An inductive learning algorithm in fuzzy systems. Fuzzy Sets and Systems. 1997;**89**(2):193–203

[30] Chen SM, Lee SH, and Lee CH. A new method for generating fuzzy rules from numerical data for handling classification problems. Applied Artificial Intelligence. 2001;**15**(7):645–664

[31] Zedeh LA. Outline of a new approach to the analysis of complex systems and decision processes. IEEE Transactions on Systems, Man, and Cybernetic. 1973;**3**:28–44

[32] Bandler W, Kohout L. Fuzzy power sets and fuzzy implication operators. Fuzzy Sets and Systems. 1980;**4**(1):13–30

[33] Turksen IB. Four methods of approximate reasoning with interval-valued fuzzy sets. International Journal of Approximate Reasoning. 1989;**3**(2):121–142

[34] Turksen IB, Zhong Z. An approximate analogical reasoning approach based on similarity measures. IEEE Transactions on Systems, Man, and Cybernetics. 1988;**18**(6):1049–1056

[35] Klement EP, Mesiar R, Pap E. Triangular norms. Position paper I: Basic analytical and algebraic properties. Fuzzy Sets and Systems. 2004;**143**(1):5–26

[36] Mizumoto M, Zimmermann HJ. Comparison of fuzzy reasoning methods. Fuzzy Sets and Systems. 1982;**8**(3):253–283

[37] Nakanishi H, Turksen IB, Sugeno M. A review and comparison of six reasoning methods. Fuzzy Sets and Systems. 1993;**57**(3):257–294

[38] Turksen IB, Zhong Z. An approximate analogical reasoning schema based on similarity measures and interval-valued fuzzy sets. Fuzzy Sets and Systems. 1990;**34**(3): 323–346

[39] Chen SM. A new approach to handling fuzzy decision-making problems. IEEE Transactions on Systems, Man, and Cybernetics. 1988;**18**(6):1012–1016

[40] Chen SM. A Weighted fuzzy reasoning algorithm for medical diagnosis. Decision Support Systems. 1994;**11**(1):37–43

[41] Yeung DS, Tsang ECC. A comparative study on similarity-based fuzzy reasoning methods. IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics). 1997;**27**(2):216–227

[42] Tversky A. Features of similarity. Psychological Review. 1977;**84**(4):327

[43] Yeung DS, Tsang ECC. Fuzzy knowledge representation and reasoning using petri nets. Expert Systems with Applications. 1994;**7**(2):281–289

[44] Yeung DS, Tsang ECC. Improved Fuzzy Knowledge Representation and Rule Evaluation Using Fuzzy Petri Nets and Degree of Subsethood. International Journal of Intelligent Systems. 1994;**9**(12):1083–1100

[45] Kosko B. Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence. New Jersey: Prentice-Hall; 1992

[46] Yeung DS, Ysang ECC. A multilevel weighted fuzzy reasoning algorithm for expert systems. IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans. 1998;**28**(2):149–158

[47] Thorndike RL. Who belongs in the family? Psychometrika. 1953;**18**(4):267–276

[48] Klir GJ, Yuan B. Fuzzy Sets and Fuzzy Logic: Theory and Applications. Prentice Hall PTR; 1995

[49] Robinson JA. A machine-oriented logic based on the resolution principle. Journal of the ACM (JACM). 1965;**12**(1):23–41

[50] Zadeh LA. Quantitative fuzzy semantics. Information Sciences. 1971;**3**(2):159–176

[51] Huang Z, Shen Q. Fuzzy interpolative reasoning via scale and move transformations. IEEE Transactions on Fuzzy Systems. 2006;**14**(2):340–359

[52] Huang Z, Shen Q. Fuzzy interpolation and extrapolation: A practical approach. IEEE Transactions on Fuzzy Systems. 2008;**16**(1):13–28

[53] Kóczy LT, Hirota K. Size reduction by interpolation in fuzzy rule bases. IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics). 1997;**27**(1):14–25

[54] Yan S, Mizumoto M, Qiao WZ. Reasoning conditions on Koczy's interpolative reasoning method in sparse fuzzy rule bases. Fuzzy Sets and Systems. 1995;**75**(1):63–71

[55] Chen SM, Ko YK. Fuzzy interpolative reasoning for sparse fuzzy rule-based systems based on ?-cuts and transformations techniques. IEEE Transactions on Fuzzy Systems. 2008;**16**(6):1626–1648

[56] Hsiao WH, Chen SM, Lee CH. A new interpolative reasoning method in sparse rule-based systems. Fuzzy Sets and Systems. 1998;**93**(1):17–22

[57] Wong KW, Tikk D, Dedeon TD, Kóczy LT. Fuzzy rule interpolation for multidimensional input spaces with applications: A case study. IEEE Transactions on Fuzzy Systems. 2005;**13**(6):809–819

[58] Yam Y, Kóczy LT. Representing membership functions as points in high-dimensional spaces for fuzzy interpolation and extrapolation. IEEE Transactions on Fuzzy Systems. 2000;**8**(6):761–772

[59] Yeung Y, Wong ML, Baranyi P. Interpolation with function space representation of membership functions. IEEE Transactions on Fuzzy Systems. 2006;**14**(3):398–411

[60] Yang L, Chen C, Jin N, Fu X, Shen Q. Closed form fuzzy interpolation with interval type-2 fuzzy sets. In: 2014 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). 2014. pp. 2184–2191

[61] Chang YC, Chen SM, Liau CJ. Fuzzy interpolative reasoning for sparse fuzzy-rule-based systems based on the areas of fuzzy sets. IEEE Transactions on Fuzzy Systems. 2008;**16**(5):1285–1301

[62] Tikk D, Joó I, Kóczy L, Várlaki P, Moser B, Gedeon TD. Stability of interpolative fuzzy KH controllers. Fuzzy Sets and Systems. 2002;**125**(1):105–119

[63] Ughetto L, Dubois D, Prade H. Fuzzy interpolation by convex completion of sparse rule bases. In: 2000 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). 2000. pp. 465–470

[64] Shen Q, Yang L. Generalisation of scale and move transformation-based fuzzy interpolation. Journal of Advanced Computational Intelligence and Intelligent Informatics. 2011;**15**(3):288–298

[65] Jenei S, Klement EP, Konzel R. Interpolation and extrapolation of fuzzy quantities-The multiple-dimensional case. Soft Computing. 2002;**6**(3–4):258–270

[66] Yang L, Shen Q. Towards adaptive interpolative reasoning. In: 2009 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). 2009. pp. 542–549

[67] Yang L, Shen Q. Extending adaptive interpolation: From triangular to trapezoidal. In: 2009 9th UK Workshop on Computational Intelligence (UKCI). 2009. pp. 25–30

[68] Yang L, Shen Q. Adaptive fuzzy interpolation and extrapolation with multiple-antecedent rules. In: 2010 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). 2010. pp. 1–8

[69] Kleer JD. An assumption-based TMS. Artificial Intelligence. 1986;**28**(2):127–162

[70] Kleer JD. Extending the ATMS. Artificial Intelligence. 1986;**28**(2):163–196

[71] Kleer JD, Williams BC. Diagnosing multiple faults. Artificial Intelligence. 1987;**32**(1):97–130

[72] Yang L, Shen Q. Adaptive fuzzy interpolation with prioritized component candidates. In: 2011 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). 2011. pp. 428–435

[73] Yang L, Shen Q. Adaptive fuzzy interpolation with uncertain observations and rule base. In 2011 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). 2011. pp. 471–478

[74] Cheng SH, Chen SM, Chen CL. Adaptive fuzzy interpolation based on ranking values of polygonal fuzzy sets and similarity measures between polygonal fuzzy sets. Information Sciences. 2016;**342**:176–190

[75] Wang LX, Mendel JM. Generating fuzzy rules by learning from examples. IEEE Transactions on Systems, Man, and Cybernetics. 1992;**22**(6):1414–1427

[76] Johanyák ZC, Kovács S. Sparse fuzzy system generation by rule base extension. In: 2007 11th IEEE International Conference on Intelligent Engineering Systems. 2007. pp. 99–104

[77] Bellaaj H, Ketata R, Chtourou M. A new method for fuzzy rule base reduction. Journal of Intelligent & Fuzzy Systems. 2013;**25**(3):605–613

[78] Tao CW. A reduction approach for fuzzy rule bases of fuzzy controllers. IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics). 2002;**32**(5):668–675

[79] Tan Y, Li J, Wonders M, Chao F, Shum HP, Yang L. Towards sparse rule base generation for fuzzy rule interpolation. In: 2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). 2016. pp. 110–117

[80] Cowton J, Yang L. A smart calendar system using multiple search techniques. In: 2015 15th UK Workshop on Computational Intelligence (UKCI). 2015

[81] Peckham SD. Profile, plan and streamline curvature: A simple derivation and applications. In: Proceedings of Geomorphometry. 2011. pp. 27–30

[82] Li J, Shum PH, Fu X, Sexton G, Yang L. Experience-based rule base generation and adaptation for fuzzy interpolation. In: 2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). 2016. pp. 102–109

[83] Chen SJ, Chen SM. Fuzzy risk analysis based on similarity measures of generalized fuzzy numbers. IEEE Transactions on Fuzzy Systems. 2003;**11**(1):45–56

[84] Nguyen D, Widrow B. The truck backer-upper: An example of self-learning in neural networks. In: Proceedings of the International Joint Conference on Neural Networks (IJCNN). 1989;**2**:357–363

[85] Kong SG, Kosko B. Adaptive fuzzy systems for backing up a truck-and-trailer. IEEE Transactions on Neural Networks. 1992;**3**(2):211–223

[86] Riid A, Rustern E. Fuzzy logic in control: Truck backer-upper problem revisited. In: 2001 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). 2001;**1**:513–516

[87] Lotfi A. Fuzzy Inference Systems Toolbox for Matlab (FISMAT). 2000

[88] Haissig C. Adaptive fuzzy temperature control for hydronic heating systems. IEEE Control Systems. 2000;**20**(2):39–48

[89] Lu J, Sookoor T, Srinivasan V, Gao G, Holben B, Stankovic J, Field E, Whitehouse K. The Smart Thermostat: Using Occupancy Sensors to Save Energy in Homes. In: Proceedings of the 8th ACM Conference on Embedded Networked Sensor Systems. 2010. pp. 211–224

[90] Nevius M, Pigg S. Programmable thermostats that go berserk: Taking a social perspective on space heating in Wisconsin. In: Proceedings of the 2000 ACEEE Summer Study on Energy Efficiency in Buildings. 2000;8:e44

[91] Scott, J, Bernheim Brush AJ, Krumm J, Meyers B, Hazas M, Hodges S, Villar N. PreHeat: Controlling home heating using occupancy prediction. In: Proceedings of the 13th International conference on Ubiquitous Computing. 2011 pp. 281–290

[92] Von Altrock, C, Arend HO, Krausse B, Steffens C. Customer-adaptive fuzzy logic control of home heating system. In: 1994 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). 1994;**3**:1713–1718

[93] Friedman N, Geiger D, Goldszmidt M. Bayesian network classifiers. Machine Learning. 1997;**29**(2–3):131–163

[94] Chao F, Huang Y, Zhang X, Shang C, Yang L, Zhou C, Hu H, Lin CM. A robot calligraphy system: From simple to complex writing by human gestures. Engineering Applications of Artificial Intelligence. 2017;**59**:1–14

[95] Chao F, Zhu Z, Lin CM, Hu H, Yang L, Shang C, Zhou C. Enhanced robotic hand-eye coordination inspired from human-like behavioral patterns. IEEE Transactions on Cognitive and Developmental Systems. 2017. DOI: 10.1109/TCDS.2016.2620156