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Abstract

Direct-contact heat transfer involves the exchange of heat between two immiscible fluids by bringing them into contact at different temperatures. There are two basic bubbling regimes in direct-contact heat exchanger: homogeneous and heterogeneous. Industrially, however, the homogeneous bubbling regime is less likely to prevail, owing to the high gas flow rates employed. The mixture homogeneity and the non-homogeneity of the mixture can be characterized by the Betti numbers and the mixing time can be estimated relying on image analysis and statistics in a direct-contact heat exchanger. To accurately investigate the space-time features of the mixing process in a direct contact heat exchanger, the uniformity coefficient method based on discrepancy theory for assessing the mixing time of bubbles behind the viewing windows is effective. Hence, the complexity of the bubble swarm patterns can be reduced and their mechanisms clarified, and the heat transfer performance in a direct-contact heat exchanger can be elucidated.

Keywords: direct-contact heat transfer, flow pattern, Betti numbers, discrepancy, mixing uniformity

1. Introduction

1.1. Direct-contact heat exchanger

Direct transfer involves two immiscible fluids under different temperatures in contact for heat exchange [1]. Compared with the traditional direct-contact heat exchanger, heat transfer means has more advantages due to a more simple design, low temperature driving force and higher heat transfer efficiency [2, 3]. Direct-contact heat exchangers (DCHEs) make use of gas-liquid phase change heat exchanger within the working fluid. That is to say, DCHEs put to use heat transfer between two kinds of fluid in the absence of a partition. A direct contact heat exchanger can be used for seawater desalination, heat recovery, ocean thermal energy conversion, thermal energy storage systems, etc.[4, 5]. In addition, DCHEs have been applied...
to give a good solution in harnessing the solar energy [6] and provide a better understanding of ice formation, growth and detachment from the droplets producing ice slurry [7].

1.2. Mixing efficiency assessment

Mixing plays a fundamental role in many industrial applications, such as chemical engineering, metallurgical process, printing process, medical and bio-medical industries, and has a decisive impact on the overall performance of reaction processes. The purpose of mixing is to obtain a homogeneous mixture; however, many researchers have pointed out that the local mixing and the flow pattern has significant effects on the properties of the final products [8]. There is an increased want for measuring and comparing mixing performance. An efficient evaluation of mixing effects is required in those various fields, but as a result of its intricacy, theoretical methods are very limited. Monitoring or measuring the mixing appropriately is of much concern from the practical point of view and for the confirmation of theoretical models as much [9]. The existence of a second phase that makes the continuous phase flow and mixing process more complicated, especially for a direct contact with the boiling heat transfer process. The boiling heat transfer process, in which mixing efficiency assessment is common, is one of the most efficient kinds of heat transfer processes widely used in numerous engineering systems. Hence, the work of characterizing the homogeneous bubbling regimes in a DCHE is one of the most useful and instructive topics in DCHE.

1.3. Bubbling regimes

There are two basic types of bubbling regimes in DCHE: homogeneous and heterogeneous. In the homogeneous bubbling regime, there are few diversifications in the size of the bubbles, and breakage and coalescence phenomena are inappreciable [10–12]. Industrially, nevertheless, the homogeneous bubbling regime is not likely to prevail, thanks to the high gas flow rates used. This is good for the heterogeneous bubbling regime, characterized by a widespread of bubble sizes and crucial frequencies of breakage and coalescence [13]. For an air-water system, Ribeiro and Lage [13] employed transient experimental measuring of the temperature of the liquid, bubbling height, evaporation rate, gas volume fraction and bubble size distributions in a direct-contact evaporator for four surface gas velocities including operation in both homogeneous and heterogeneous bubbling regimes. Ribeiro et al. [14] also analysed the photographs of homogeneous and heterogeneous bubbling regimes using different liquids in a DCHE handling with a perforated-plate sparger. Le Coënt et al. [15] studied the compounding of two staves and a viscous liquid in a classical reactor. He found that there was an alleged “pseudo-homogeneous” state before it was mixed completely homogeneously. In reality, a pseudo-homogeneity was achieved much more quickly (<40 s), but subsequent images revealed that polymers still remained in the reactor. The time of the pseudo-homogeneous state begins is called the pseudo-homogeneous time. In our DCHE, we found that there was a comparatively stable state in the completely heterogeneous bubbling regimes also. Consequently, we defined this completely stable state as pseudo-homogeneous. Peyghambarzadeh et al. [16] found that bubble growth was a considerably complicated process, and detecting distinguishable bubbles was scarcely possible at high heat fluxes, while in this experiment, we have captured the rough sketch of bubbles.
1.4. Image analysis

A literature survey showed that image analysis has been used in transparent laboratory vessels to circumvent the drawback of subjectivity of measurement interpretation. Fortunately, the image processing technology has been widely used for feature extraction in medical and chemical industries. Thus, just that technology of image intensification, these bubble images can be computed with the following methods. Bubble growth is severely a function of flow of heat and liquid flow rate \([16]\). If the flow rate is lower, larger bubbles are observed at constant heat fluxes. This may be due to the fact that the growth of bubbles weakens with the time which is necessary at the velocity of flow is higher. Hence, the bubbles are smaller than those observed at higher high velocity. Similarly, according to the results of Ref. \([16]\), the effect of heat flux is more meaningful than that of flow velocity. Many small bubbles are created on the heat transfer surface, inventing high turbulence flow at high heat fluxes. Consequently, heterogeneous and pseudo-homogeneous bubbling regimes are necessary and worth learning in a DCHE. At the meantime, it is one of the most challenging tasks of direct-contact heat transfer. The current commonly used method is to do with image processing techniques to acquire the features of bubbling regimes.

1.5. Betti numbers

In 1995, Hyde et al. \([17]\) recommended the topological invariant features the topology penetration structure complexity, in the number of micro-structure processing is one of the two material phases. From the perspective of theory, Betti numbers are the number of handles a special case of a topological invariants in a micro-structure \([18]\). Algebraic topology provides measurable information on complex objects, and Betti numbers are rough measures of this information. Gameiro et al. \([18]\) came up with a method using the Betti numbers to describe the geometry of the fine-grained and snake-like micro-structures created in the process of spinodal decomposition. The zeroth Betti numbers \(\beta_0\) figure the number of connected components (pieces) in the space \(\Omega\). More accurately, if \(\beta_0 = k\), then \(\Omega\) has exactly \(k\) components. The first Betti numbers \(\beta_1\) state a measure of the number of tunnel structure. In a two-dimensional field, tunnels are decreased to loops. It is a remarkable fact that the size and the shape of the component and the loops do not affect the number of Betti numbers. Friedrich \([19]\) proposed the same chemical group which used the structure descriptors to distinguish un-related chemical group of chondrite and the application of Betti numbers to research the difference in rock chondrite meteorites. A multiphase mixture usually shows a macroscopic homogeneity consisting scattered fine pieces. With an increase in dispersity of the pieces, the homogeneity of the mixture was increased. In the cracks of the pieces are the blowholes, and a polymerization of blowholes gets an aggregation. The more frequently the pores appear, the more likely agglomeration is to occur at the surface. Moreover, increase in the number of pores showed that the heterogeneity of the mixture was mixed more evenly.

In our previous work, using the Betti numbers for gas-liquid-solid three-phase mixing effects of molten salt system based on the reaction of CH\(_4\) + ZnO were characterized. Nitrogen was used to imitate the gas phase (CH\(_4\)) and mainly mixing effect in the sink. The zeroth Betti numbers were used to measure the number of pieces in the patterns, bring about beneficial
parameter to describe the mixture homogeneity, which was the number of masters in the micro-structure occupied by one of the two phases. The first Betti numbers were introduced to describe the mixing heterogeneity of mixture. Because we only quantified the solid-liquid mixed flow pattern, the mixture of nitrogen bubble will disappear after image binarization.

1.6. Heat transfer performance

It must be pointed out that Gulawani et al. [20] studied and founded that the turbulent flow pattern in a gas-liquid interface heat transfer coefficient and the immersed surface has a significant impact. Under Gulawani et al. [20, 21] inspiration and guidance, our work is mainly described the flow pattern characteristics of bubbles under the effect of heat transfer in the DCHE. Dahikar et al. [22] and Tayler et al. [23] used the Betti numbers to represent the heterogeneous and pseudo-homogeneous of bubbles. In addition, the relationship between the Betti numbers and the heat transfer coefficient has been obtained in a DCHE.

1.7. Mixing time.

Both mixing speed and phase transition time in the direct contact boiling heat transfer process are fast. An accurate mixing time is critical to appropriately evaluate computational fluid dynamics models and then enhances equipment understanding and develops scale down models for process characterization and design space definition during late stage process development. In the past few years, many researchers have studied the mixing time and many methods were proposed to measure mixing time. But at present, there is no generally accepted method of measuring mixing time, mainly because of each method is not universal, that is each method has its own limitations, such as conductivity [24], pH [25], the dual indicator system method [26], tracer concentration [27–30], electrical resistance tomography [9], coloration decolouration methods [31], the box counting with erosions method [15] and Betti numbers with image analysis [32]. The limitation of each method has been described in details [31]. In all of the above-mentioned technologies, the Betti numbers are one of the most worthy methods to measure the mixing time and get further information of the mixing process. The Betti numbers can be effectively quantitative mixing time, the development process of mixing and degree of homogeneity. It has been used to characterize the evolution of the bubble group in direct contact with the boiling heat transfer process. But, we found that the Betti number method to be used for mixing time and the different evaluation indexes for mixing time have a similar trend, such as the slope \( p \) [15], pH, tracer concentration \( c_t \), the percentage of mixed pixels, \( M(\%) \) [31], and the standard deviation (\( \sigma_G \)) [26]. These indicators change at the beginning of the mixing and quickly tend to be stable after fluctuations. The mixture of non-uniformity caused these fluctuations. This is the inevitable process of pseudo-homogeneous critical point to determine the influence accurate estimates of mixing time, which has often been overlooked. A literature survey shows that it is mainly used for determining the critical point of mixing time, including the mean value of Betti number (mean method) [32], slope \( p \) (slope method) [31], and standard deviation (SD) and the selected threshold [33]. Accurate estimation about the mixing time of work was published less than others, especially the critical point determination impact. The idea of a three-sigma method is inspired and motivated by statistical process control (SPC). According to Woodall [34], SPC can commonly be divided into two
phases. The data of phase I are clean gathered under stable operating conditions, whereas the major of phase II is to detect any changes. The $3\sigma$ principle is that if the sample data come from a normal distribution $N(\mu, \sigma^2)$, most of the data (99.73%) will lie within the range $[\mu-3\sigma, \mu+3\sigma]$. It is imposed to detect outliers in the quality control of samples. If the result is normal, the process of the product specification will lie within the scope $\mu \pm 3\sigma$ of the standard value. Otherwise, the production process is considered to be abnormal. Homoplastically, a changed three-sigma edit test has been successfully used in distributed self-fault diagnosis algorithm for large-scale wireless sensor networks [35]. Our research confirmed that the critical point of the response time ignored may result in significant error in mixing time estimation.

1.8. $L_2$-star discrepancy

The mixing process in DCHE has been studied by many experiments. Similarly, at present, there is no generally accepted way to measure mixing homogeneity, mainly because each method has its own deficiencies, such as thermal method, conduct metric method, pH method, decolourization methods, Schlieren method, Betti numbers method [36], etc. In all of the above-mentioned technologies, the Betti numbers have been used to characterize the evolution of the heterogeneous and pseudo-homogeneous bubbling regimes. But, with the Betti numbers for characterization of mixing uniformity have a space-time limitation; it may lead to significant errors in the evaluation of mixing uniformity. The key question is how to measure the random bubble swarm of minimum difference of space-time consistency bubble swarm of domain. Fang and Wang put forward the concept of UD (uniform design) that dispersed experimental points uniformly scattered on the domain. One should choose a set of given all possible designs with amount of minimum difference of laboratories under the design of all possible factors and experimental runs. The above is the basic idea of UD [37]. UD has been widely used since 1980 [38]. Inspired and motivated by Fang [37–39], our main research objects are the study of characteristics of time-space features and analyse the mixing process of numerical simulation and experimental analysis. Uniform design theory and image analysis have been applied to quantitative uniformity of time and space in a DCHE.

1.9. Modified $L_2$-star discrepancy

Recently, we were vitalized and motivated by Xu et al. [39], by the literature that introduces the relatively not complicated and accurately uniformity coefficient (UC) technology, which is based on image processing technology and the theory of uniform design to determine the mixing time and uniformity in a DCHE. The space-time characteristics can be quantified by means of the uniformity coefficient method, which based on $L_2$-star discrepancy (UC-LD) and provides a method of direct measurement about the macro-mixing evolution. With the same Betti number just is aimed to separate the local and global uniform [40]. Whereas, the nature of the UC such as rotation invariance has not been explored and it has a lot dependence of calculating the initial conditions of UC, namely UC-LD. Clearly, the $L_2$-star discrepancy is much easier to calculate numerically according to Heinrich [41]. Unfortunately, the $L_2$-star discrepancy shows some shortcomings, as pointed out by Hickernell [42]. For instance, it is influenced by all the sub-dimensions of the projection uniformity. In order to overcome these shortcomings, Hickernell studied uniformity of some new measure methods, which are also associated
with the $L_2$ norm, namely, the centred discrepancy (CD) and the wrap-around discrepancy (WD). The centred discrepancy (CD) and the wrap-around discrepancy (WD) satisfy a Koksma-Hlawka type inequality according to Xu et al. [39]. According to the theory of UC-LD, if the image shows a low CD or WD that can be called homogeneous mixing of a set or bubble swarm pattern, and the use of UC-CD and UC-WD provides at least likely to get a good approximation of mixing bubble swarm of spatial distribution. Additionally, UC-CD and UC-WD exhibit some advantages including rotation invariance, reflection invariance and projection uniformity [39].

1.10. Chapter structure

The chapter is organized as follows. In the next section, experiments and methodology are presented; the results and discussion are presented subsequently; the conclusion is briefly summarized in this section finally. Then the acknowledgements and references are presented in the end.

2. Measurement of mixing uniformity in DCHE

2.1. Experimental

2.1.1. Apparatus

The schematic of the experiment employed in the present research is sketched in Figure 1 [32]. There are two circulation loops in the test device for this experiment. The first loop, which consists of the DCHE (1), electric heater (2), heat transfer fluid (HTF), pump (7) and connecting inlet and outlet pipelines, is a continuous-phase circulation loop for fluid flow, and the other, which consists of the DCHE (1), centrifugal pump (4), plate condenser (5), centrifugal pump (6) and connecting inlet and outlet pipelines, is a dispersed-phase circulation loop for working medium flow. The temperature control device, gear oil pump (3), regulates the initial temperature difference arising from heat exchange. The frequency control cabinet, gas mass flow-meter (8), regulates the rates of flow of the HTF and working medium. The patterns were imaged by a high-speed shutter video camera, which was placed at the second viewing window. In the bubble evaporation process, we could observe the most active stage of the bubbling regime. HTF and the refrigerant R-245fa (1, 1, 1, 3, 3 pentafluoropropane) were used as the continuous phase and the dispersed phase in all runs, respectively.

2.1.2. Experimental design

The settings of the experimental plan affecting the heat transfer capacity of the tested DCHE are determined through the orthogonal array (OA) experimental design method. As listed in Table 1, design parameters with four factors and three levels were selected to investigate the influence of heat transfer capacity. The $L_9(3^4)$ orthogonal array table was chosen for designing the experiment. The interaction between the design parameters was neglected in the present study. $H$ is the height of HTF in the DCHE, $\Delta T$ is the initial heat transfer temperature difference, $U_g$ is the refrigerant flow rate, and $U_o$ is the flow rate of the HTF.
As shown in Table 2, the numbers E₁–E₉ denote different experimental levels according to the orthogonal array table.

### 2.2. Pattern acquisition and processing

A high-speech video camera was employed to obtain the patterns, and the brand used was PRAKTICA from Germany, with resolution 4 million pixels with no LED light. The images,

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Design parameter</th>
<th>Unit</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>H</td>
<td>mm</td>
<td>460</td>
</tr>
<tr>
<td>B</td>
<td>ΔT</td>
<td>K</td>
<td>80</td>
</tr>
<tr>
<td>C</td>
<td>$U_\text{g}$</td>
<td>m³/s</td>
<td>$1 \times 10^{-4}$</td>
</tr>
<tr>
<td>D</td>
<td>$U_0$</td>
<td>kg/s</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1. Design parameters and levels.

As shown in Table 2, the numbers E₁–E₉ denote different experimental levels according to the orthogonal array table.

<table>
<thead>
<tr>
<th>Experimental</th>
<th>E₁</th>
<th>E₂</th>
<th>E₃</th>
<th>E₄</th>
<th>E₅</th>
<th>E₆</th>
<th>E₇</th>
<th>E₈</th>
<th>E₉</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$(mm)</td>
<td>460</td>
<td>460</td>
<td>460</td>
<td>530</td>
<td>530</td>
<td>530</td>
<td>600</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>ΔT(K)</td>
<td>80</td>
<td>100</td>
<td>120</td>
<td>80</td>
<td>100</td>
<td>120</td>
<td>80</td>
<td>100</td>
<td>120</td>
</tr>
<tr>
<td>$U_\text{g}$ ($10^4$m³/s)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$U_0$ (kg/s)</td>
<td>0</td>
<td>0.15</td>
<td>0.3</td>
<td>0.3</td>
<td>0</td>
<td>0.15</td>
<td>0.15</td>
<td>0.3</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2. Design experiments according to four factors and three levels orthogonal table.
which were blurred in photographing, can be improved using some image processing tech-
niques. It takes 8 minutes to shoot in each occasion of the orthogonal experiment. Because of
difficulties in storing and calculating these images, we choose equal interval sampling from
6000 images, in total, 12,000 images are collected.

**Figure 2** is randomly obtained in the present image-processing process. In order to sup-
press the background of the original image, eliminate noise and enhance the image, gray-
scale transformation, top-hat transform is used here. The binarization operation was used
to calculate the Betti numbers. With a dilation process, an erosion process named as an
opening was executed. This process, aiming at removing tiny or isolate points at the finer
locations, and smoothing the boundaries of larger points, could not change the size of the
image significantly. In contrast, with a dilation erosion process, a dilation process named
as an opening was executed. This operation, aiming at filling up tiny pores within the
points, connecting nearby points, and smoothing the borders, could not alter the size of
the image significantly. The opening is used here to remove small holes representing sile
bubbles or small bubble swarms of the binarization images. Since the behavioural char-
acteristics of bubble swarms could not be accurately portrayed by binarized images with
noise, an opening operation must be executed to eliminate image noise by the appropriate
thresholds selected.

Thus, the white area indicates the bubble swarm, and the black area refers to the continu-
ous phase. As the experimental conditions, the captured image is relatively fuzzy; however,
its quality can be improved by using the digital image processing techniques. The resultant
image that could be used for the following analysis was identifiable.

![Figure 2. Treatment for one piece of bubble swarm patterns.](image-url)
2.3. Methodology

2.3.1. Performance evaluation of the DCHE model

Owing to the complexity of the DCHE multiphase structure, heat exchange performance has often been expressed in terms of the volumetric heat transfer coefficient, \( h_v \), which is given by [43]:

\[
h_v = \frac{Q}{V \times LM\text{T}D} \tag{1}
\]

where \( V \) is the volume of the continuous phase in the DCHE, and \( Q \) is the rate of heat transfer from the continuous phase to the dispersed phase, given by:

\[
Q = m(h_d - h_i) \tag{2}
\]

where \( m \) is mass flow-rate of the dispersed phase steam, and \( h \) is the enthalpy of the dispersed phase. The \( LM\text{T}D \) in Eq. (1) is the logarithmic mean temperature difference, which is defined as:

\[
LM\text{T}D = \frac{(T_{ci} - T_{do}) - (T_{co} - T_{di})}{\ln \left( \frac{T_{ci} - T_{do}}{T_{co} - T_{di}} \right)} \tag{3}
\]

where \( T \) is temperature. In all the equations, the subscript \( c \) refers to the continuous phase, \( d \) refers to the dispersed phase, \( i \) refers to the inlet, and \( o \) refers to the outlet.

2.3.2. Computational homology (Betti numbers)

Box-counting with erosions method, which was developed by Le Coënt et al. [15], can be applied to quantify the mixture homogeneity; however, it is not available for quantifying the mixture non-homogeneity. As shown in the experiment, some agglomerates still exist in the vessel after stirring for quite a long time. With computational homology, an original analysis method aiming at getting the quantification of the mixture homogeneity and non-homogeneity was proposed.

As we all know that the zeroth Betti number and the first Betti number have the following information [18, 44]: \( \beta_0 \) equals the number of connected components that make up the space, and \( \beta_1 \) provides a measure of the number of tunnels in the structure. In a two-dimensional domain, tunnels are reduced to loops. Since an image is three-dimensional, it has three Betti numbers: \( \beta_0, \beta_1, \) and \( \beta_2 \). \( \beta_2 \) measures the number of completely enclosed cavities, such as the interior of a sphere. \( \beta_0 \) indicates the number of pieces, and \( \beta_1 \) represents the number of the holes. In other words, the mixing effect will vary with the number of pieces in the glass vessel. So \( \beta_0 \) and \( \beta_1 \) are used to get such a characterization of the mixture homogeneity and the mixture non-homogeneity, respectively.

\( \beta_0, \beta_1 \), and their averages \( \bar{\beta}_0, \bar{\beta}_1 \), of the binary images of the patterns can be obtained at different submerged lengths of the lance and flow rates of the gas. Also, we may obtain the value of
time $T$ (time unit: seconds) at which $\beta_0$ of the black/white image is equal to $\bar{\beta}_0$. The time $T$ can be employed to obtain the minimum mixing time.

Set

$$
\chi^+ := \{ t \mid \beta_0(t) > \bar{\beta}_0, t > T \}
$$

$$
\chi^- := \{ t \mid \beta_0(t) < \bar{\beta}_0, t > T \}
$$

$$
A := \frac{1}{2} \left[ m \sum_{t \in \chi^+} \beta_0(t) - n \sum_{t \in \chi^-} \beta_0(t) \right] \tag{4}
$$

where $\beta_0(t)$ denotes the zeroth Betti number of the binary image of the pattern, which is captured at the time $t$, and $m$, $n$ are the numbers of elements in $\chi^+$, $\chi^-$. $A$ is used to estimate the deviation amplitude of $\beta_0(t)$ from their average $\bar{\beta}_0$.

In two-dimensional cases, $\bar{\beta}_0$ is the number of connected components, such as black regions. The number of these holes, which is completely enclosed by cubes/pixels, is measured by $\beta_1$, and $\beta_1$ represents the number of the holes in the domain. One can easily count these white regions. As shown in Figure 2, $\bar{\beta}_0$ represents the number of continuous phases, whereas $\beta_1$ represents the number of bubble swarms.

The calculation of Betti number is difficult, and the methods are only in their early stages [44]. The free software package CHomP was used to calculate Betti numbers [44, 45]. We could compute $\beta_0$ and $\beta_1$ of the open operation images of the patterns at different experimental levels using the CHomP software package [45].

Subsequently, we obtained the value of time $t$ (seconds) that can be used to estimate the pseudo-homogeneous time with $\bar{\beta}_0$ representing the average of $\beta_0$ of the open operation image after the pseudo-homogeneous process. The bubble sizes were found to be almost the same by inspecting many test images. With the pseudo-homogeneous time, the entire visible area was covered by the bubbles [36]. Just as we all known, combining the evolution of Betti numbers, we can distinguish whether the distribution is uniform or not. In the beginning, the Betti number increases and then rapidly stabilizes after fluctuations.

As Figure 3 shows, a conversion operation of open operation images was performed. The results showed that a black-and-white conversion directly leads to a switch between the corresponding objects of the zeroth and first Betti numbers [46]. To illustrate, $\beta_0$ and $\beta_1$ represent the number of the continuous phase in Figure 3a, and the opposite in Figure 3b, respectively. Since it is the white pores that most directly reflect the flow patterns of the bubble swarms, $\beta_1$ is still used to characterize the number of bubble swarms.

### 2.3.3. Three-sigma method

Let $X$ be a normally $N(\mu, \sigma^2)$ distributed random variable. For any $k > 0$, $P(|X- \mu| < k\sigma) = 2\Phi[k] - 1$, where $\Phi[\cdot]$ is the distribution function of the standard normal law; whence, in particular, for $k = 3$ it follows that $P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973$. The latter equation means that $X$ can differ from its expectation by a quantity exceeding $3\sigma$ on the average in not more than 3 times in a thousand trials [47]. This circumstance is sometimes used by an experimenter in certain problems, by assuming that $|X- \mu| > 3\sigma$ is practically impossible, and consequently, $|X- \mu| < 3\sigma$ is practically certain. The probability of exceeding the range of “$\mu \pm 3\sigma$” occurring
twice is $7.29 \times 10^6$. Indeed the experimental time series of Betti numbers approximate normal distribution, as shown in Figure 4 [36].

![Figure 3. Influence of the boundary on the Betti numbers.](image)

![Figure 4. Betti number histogram with a normal distribution fit.](image)

Two consecutive points beyond the limits are viewed as exception criteria.

Step 1: Giving a time point $t_o > t$, and $t$ is the mixing time, a homogeneous trend was presented during the evolution of Betti number time series of bubble swarm.

Step 2: Calculating the mean $\mu$ and standard deviation $\sigma$ of Betti numbers time series after the time $t_o$. 

Step 3: Determining whether an event exceeds the range of \( \mu \pm 3\sigma \) occurring twice, as \( t_0 \) in reverse order. If so, then the moment is defined as mixing time, \( t \). In order to quantify the macro-mixing efficiency using the Betti numbers, the data of the Betti numbers satisfying approximately normal distribution are collected from mixing homogeneity process, then let \( \mu \) represents the estimated mean and \( \sigma \) represents the estimated standard deviation, and the mixing time is the time when the critical point exceeds the range of \( \mu - 3\sigma \) in reverse order twice.

The technique by itself is not limited to transparent tanks. It can be used in conjunction with electrical resistance tomography (ERT), position emission tomography (PET) and magnetic resonance imaging (MRI) [36].

2.3.4. Measures of uniformity

A popular figure of merit is the star discrepancy [48] and its generalization the \( L_p \)-star discrepancy. Let \( F_u(x) = x_1 x_2 \ldots x_s \) be the uniform distribution function on \( C_s \), where \( x = (x_1, x_2, \ldots, x_s) \). Let \( F_p(x) \) be the empirical distribution function of \( P = \{x_1, x_2, \ldots, x_n\} \):

\[
F_p(x) = \frac{1}{n} \sum_{i=1}^{n} 1_{[x_i, \infty)}(x) \tag{5}
\]

where \( 1_{(\cdot)}(x) \) is the indicator function. Then the \( L_\infty \)-star discrepancy can be defined as the \( L_\infty \)-norm of difference between uniform and empirical distribution function, and then the \( L_p \) discrepancy can be defined as:

\[
D_p^*(P) = \left( \int_{C_s} |F_u(x) - F_p(x)|^p \, dx \right)^{1/p} \tag{6}
\]

By taking \( p = \infty \), \( L_\infty \) discrepancy, which defined as the maximum deviation between these two distributions, is called the star discrepancy [48]. It is probably the most commonly used and can be expressed in another way as follows:

\[
D_\infty^*(P) = \sup_{x \in C_s} |F_u(x) - F_p(x)| \tag{7}
\]

With the discrepancy criterion in mind, we next discuss how to construct a uniformity coefficient. \( x=(x_1, x_2, \ldots, x_s) \in C_s \), \( 0, x \ast [0, x_1] \times [0, x_2] \times \ldots \times [0, x_s] \) is the rectangle determined by the origin \( O \) and \( x \) decided on \( C \). \( \text{Vol}([0, x]) \) denotes the volume of the rectangular solid \([0, x]\), where \( \text{Vol}([0, x])=x_1 x_2 \ldots x_s=F_u(x) \). Let \( \| \) be the gained number of points in a group. The function of \( |P \cap [0, x]|/n \) represents an empirical distribution, as shown below:

\[
F_p(x) = \frac{1}{n} \sum_{i=1}^{n} 1_{[0, x]}(x_i) = \frac{|P \cap [0, x]|}{n} \tag{8}
\]

Definition 2.1. The local discrepancy function is

\[
dis_c^*(x) = F_x(x) - F_p(x) = \text{Vol}([0, x]) - \frac{|P \cap [0, x]|}{n} \tag{9}
\]

The difference between theory and empirical distribution can be used to measure the local discrepancy function with a rectangle \([0, x]\). It can be expressed in another way as follows:
\[ \text{dis} \{c(s, t) = \frac{\text{vol}(s, t)}{\text{vol}(c, t)} - \text{hol}(s, t) \} \]

where \( i \) denotes the four corners of an image, \( i = 1, 2, 3, 4 \), \( \text{vol} \) denotes the volume of the rectangular solid, \( \text{hol} \) is the number of bubbles.

Definition 2.2. The mean absolute discrepancy is often defined as follows:

\[ \text{MAD}(s, t) = \frac{1}{4} \sum_{i=1}^{4} |\text{dis} \{c(s, t)\} | \]

In Figure 5, the influence of iteration steps on the measurement is not pronounced. The MAD (mean absolute discrepancy) is conducted by the four corners of an image.

Definition 2.3. Uniformity coefficient (UC) at time \( t \) is often defined by

\[ \text{UC}(t) = 1 - \text{Median}\{\text{MAD}(s, t)\} \]

\[ \text{UC}(t) = 1 - \sqrt{\frac{1}{S} \sum_{s=1}^{S} \text{MAD}(s, t)} \]

In every case, the degree of mixing uniform could be detected successfully by the uniformity coefficient method (Figure 6). After certain processing, the value range of UC is usually \([0, 1]\). We also denote that the measurement is not pronouncedly affected by the iterative steps.

In Figure 7, when the pixels sizes are reduced from 16:9 to 4:3, the influence of homogenization curve by the uniformity coefficient method is not reduced [40]. However, the trend of homogenization curve by Betti numbers method becomes unclear.
The evolution of the UC and Betti numbers of binary images at different image sizes was clearly shown in Figure 7.

Quasi-Monte Carlo method is the most commonly used measure of uniformity in the literature, especially when \( p = \infty \) and 2. When \( p = 2 \), Warnock [49] gave an analytic and simple formula for calculating \( L_2 \)-star discrepancy as follows:

\[
D^*_2(P) = \left\{ \left( \frac{1}{3} \right)^s - \frac{1}{n} \frac{1}{2} \prod_{j=1}^{s} \left( 1 - x_{ij}^2 \right) + \frac{1}{n} \sum_{i,j=1}^{s} \left( 1 - \max \left( x_{ij}, x_{lj} \right) \right) \right\}^{\frac{1}{2}}
\]

(14)

where \( x_k = (x_{k1}, x_{k2}, \ldots, x_{ks}) \). Unfortunately, the \( L_2 \)-star discrepancy exhibits some limitations, as pointed out by Heinrich and Hickernell [41, 42]. To overcome these disadvantages, other discrepancies were proposed. From the definition of discrepancy, its formula is calculated as follows if objective function takes the uniform distribution function on \( X \):
\[ D(P, K) = \left\{ \int_{\mathbb{R}^2} K(x, y) dF_x(x) dF_y(y) - \frac{2}{\pi} \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} K(x, y) dF_x(x) dF_y(y) + \frac{1}{n} \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} K(x_i, x_j) \right\}^{\frac{1}{2}} \]  

(15)

According to Fang et al. [50], the reproducing kernel functions are taken, respectively, as follows,

\[ K(z, t) = 2^n \prod_{j=1}^{n} \left( 2 + \left| z_j - \frac{1}{2} \right| + \left| t_j - \frac{1}{2} \right| - \left| z_j - t_j \right| \right) \]  

(16)

\[ K^{(x)}(z, t) = \prod_{j=1}^{n} \left( \frac{3}{2} - \left| z_j - t_j \right| + \left| z_j - t_j \right| \right) \]  

(17)

hence, the analytical expressions for centred discrepancy and wrap-around discrepancy are as follows:

\[ \text{CD}(t) = \left\{ \left( \frac{13}{12} \right)^{\frac{1}{2}} - \frac{3}{2n} \sum_{j=1}^{n} \left( 1 + \frac{1}{2} \left| x_j - 0.5 \right| - \frac{1}{2} \right) \right\}^{\frac{1}{2}} \]  

(18)

\[ \text{WD}(t) = \left\{ -\left( \frac{3}{4} \right)^{\frac{1}{2}} + \frac{3}{2} \left( \frac{3}{2} \right)^{\frac{1}{2}} + \frac{3}{2n} \sum_{j=1}^{n} \sum_{k=1}^{n} \left( \frac{3}{2} - \left| x_j - x_k \right| + \left| y_j - y_k \right| \right) \right\}^{\frac{1}{2}} \]  

(19)

UC-CD and UC-WD related to time \( t \) are defined and denoted by

\[ \text{UC} - \text{CD}(t) = 1 - \text{CD}(t), \quad \text{UC} - \text{WD}(t) = 1 - \text{WD}(t) \]  

(20)

where \( \text{CD}(t) \) and \( \text{WD}(t) \) denote the modified discrepancy.

Many bubble patterns are related to time \( t \) and one piece of patterns corresponds to one \( t \). For the calculation of UC-CD and UC-WD, using the \( x \)-axis and \( y \)-axis values of a Cartesian coordinate system to determine the position of bubble swarm is necessary.

Step 1: Transform image matrix to 0–1 matrix.

Step 2: Search the coordinates of one bubble located in top-left and bottom-right corners.

Step 3: Calculate the mean values of rows and columns of the two above coordinates.

In our work, \( M = 1280 \) and \( N = 720 \), then readjusting every piece of bubble patterns to unified pixel size of \( M \times N \). The image pixel matrix is transformed into coordinate within \([0,1]\)

\[ x = \frac{x_0 - 1}{M - 1}, \quad y = \frac{N - y_0}{N - 1} \]  

(21)

and where \( x_0 \) and \( y_0 \) denote the column and row of one image matrix, \( x \) and \( y \) denote the coordinates of horizontal axis and vertical axis in Cartesian coordinate system. Furthermore, in this work, \( s = 2 \), so \((x, y)\) of one experimental point is equal to \((x_{ij}, y_{ij})\) of Eqs. (7) and (8).

The origin of coordinates lies in the bottom-left (BL) corner of one piece of pattern. Certainly, other three groups of transformation ways are used to make origin of coordinates locate in bottom-right (BR), top-right (TR) and top-left (TL) coiners’ of one piece of patterns, respectively. Detailed formulas as follows,
More interesting, these transform methods are different but corresponding to the coordinates rotate operation for the rectangular plane coordinate system. Hence, we will talk about the rotational invariance and neglect the different transform methods in next section.

2.4. Mixing quantification by Betti numbers

2.4.1. Multiphase mixing quantification

Now this new method is used to study the influence of the flow rate and the submerged length on the degree of the mixing homogeneity and non-homogeneity of solid and liquid. The acquisition system was shown in Figure 8. The patterns were gained at the speed of 30 frames per second by a camera taking 10,000 images in each experiment.

Figure 9 shows that an initial image was subtracted from each image.

2.4.2. Characterization of heat transfer process

Figure 10 shows the evolution of $\beta_1$ in the open operation images produced in representative experimental cases. $\beta_1$ for each pattern is shown as a solid line, and the horizontal dotted line corresponds to the average of $\beta_1$ [32]. The vertical dotted line corresponds to the pseudo-homogeneous time $t$.

Experiments indicate that, in Figure 11, volumetric heat transfer coefficient shows good correlation with average Betti number and pseudo-average-time value [32]. An interesting tendency is found in the better cases of $L_6$ and $L_{2'}$, in which the larger first Betti numbers averages and
Figure 9. Binarization for one piece of images.

Figure 10. The evolution of Betti numbers at $E_c$.

Figure 11. Fitting of $\beta_t$ and $h \sim V$. 
shorter pseudo-homogeneous times correspond to a higher volumetric heat transfer coefficient \( h_V \), while \( L_4 \) and \( L_7 \) are the worse cases. As shown in Table 3, \( t \) and \( \bar{h}_V \) are nearly opposite in all cases except \( L_5, L_8 \) and \( L_9 \). Therefore, the parameters \( t \) and \( \bar{h}_V \) which can be used to characterize a bubbling flow pattern, are both related to \( \bar{h}_V \).

\[
\beta_t = \bar{\beta}_1 \cdot t^{1.26} - 0.86 \quad \text{and} \quad \bar{h}_V = 2.75 \times \beta_t + 1.11
\]

Let \( \beta_t = \bar{\beta}_1 \cdot t^{1.26} \), the tendency of \( \beta_t \) is consistent with that of \( \bar{h}_V \). According to our analysis, a linear relation between \( \beta_t \) and \( \bar{h}_V \) seems to be the outcome. Let \( \bar{h}_V = a \times \beta_t + b \). The least-squares fitting method was used to obtain the parameters \( a \) and \( b \) between \( \beta_t \) and \( \bar{h}_V \). In this work, \( a = 0.4241, b = 0.4547 \). The linear relationship is illustrated in Figure 11. The correlation coefficient is 0.95. In the end, we constructed a model on the parameters \( \beta_t \) and \( \bar{h}_V \), which points out the relationship between the flow pattern of a bubble swarm and the heat transfer performance of a DCHE.

### 2.4.3. Accurate estimation of mixing time

(1) Mixing time estimations by different methods

Based on the above, \( \beta_t \) has been defined by the Betti number average as well as the mixing time, which is synergistic with \( \bar{h}_V \). Correlation degree and correlation coefficient are used to investigate about the bubble swarm patterns and heat transfer performance for the mixing time evaluation effectiveness. According to reference [36], the computing results show that the synergy by our 3σ method between \( \beta_t \) and \( \bar{h}_V \) is much better than the other methods. In Figure 12, the plots show the evolution and determination of mixing time measured by different methods [36].

It is found that the correlation coefficient between \( t_{3\sigma} \) and mixing time estimated by these methods is \(-0.2304 \) (mean method), 0.9494 (slope method), 0.9265 (SD method) and 0.9731 (3σ method).

In Table 4, \( \mu \) is the mean of the Betti number time series after the time \( t_{3\sigma} \) is the mixing time by mean method. \( t_{slope} \) is the mixing time by slope method. \( t_{SD} \) is the mixing time by SD method. \( t_{3\sigma} \) is the mixing time defined by 3σ method. \( T_{3\sigma} \) is the inhomogeneous time by 3σ method. \( \delta_t \) is the difference between \( t_{3\sigma} \) and \( T_{3\sigma} \). \( \beta_{mean} \) is calculated by mean method, and the others are defined similarly [40]. It can be quantified by time intervals \( \delta_t \) between inhomogeneous time and mixing time. From the view point of the time interval, the transitional state appears the following forms: sudden change case (\( \delta_t = 0 \)); interval case (\( \delta_t > 0 \)); overlapping case (\( \delta_t < 0 \)).

(2) Simulation experiments

By real data analysis of the Betti number data, we have compared the proposed method with mean method, slope method and SD method. In order to assess the effectiveness of the new

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( E_1 )</th>
<th>( E_2 )</th>
<th>( E_3 )</th>
<th>( E_4 )</th>
<th>( E_5 )</th>
<th>( E_6 )</th>
<th>( E_7 )</th>
<th>( E_8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>159</td>
<td>93</td>
<td>120</td>
<td>226</td>
<td>126</td>
<td>92</td>
<td>264</td>
<td>105</td>
</tr>
<tr>
<td>( \bar{\beta}_1 )</td>
<td>197</td>
<td>186</td>
<td>208</td>
<td>177</td>
<td>197</td>
<td>189</td>
<td>187</td>
<td>176</td>
</tr>
<tr>
<td>( \beta_t )</td>
<td>1.26</td>
<td>2</td>
<td>1.23</td>
<td>0.79</td>
<td>1.61</td>
<td>2.25</td>
<td>0.71</td>
<td>1.50</td>
</tr>
<tr>
<td>( \bar{h}_V )</td>
<td>0.96</td>
<td>1.21</td>
<td>0.86</td>
<td>0.83</td>
<td>1.20</td>
<td>1.44</td>
<td>0.75</td>
<td>1.11</td>
</tr>
</tbody>
</table>

Table 3. The data of the \( t, \beta_t, \bar{h}_V \) and \( h_V \) for the entire orthogonal array table.
method and provide more evidences of good performance of this method, the mean absolute error (MAE) and the mean square error (MSE) are often used.

\[
MAE = \frac{1}{n} \sum_{i=1}^{n} |t_i - \hat{t}_i| \quad MSE = \frac{1}{n} \sum_{i=1}^{n} (t_i - \hat{t}_i)^2
\]

where \(t_i\) is the real mixing time, and \(\hat{t}_i\) is the estimate of \(t_i\).

**Table 4. Computing results of mixing performance by four methods.**

<table>
<thead>
<tr>
<th>Index</th>
<th>(E_1)</th>
<th>(E_2)</th>
<th>(E_3)</th>
<th>(E_4)</th>
<th>(E_5)</th>
<th>(E_6)</th>
<th>(E_7)</th>
<th>(E_8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M)</td>
<td>197</td>
<td>185</td>
<td>206</td>
<td>177</td>
<td>197</td>
<td>190</td>
<td>186</td>
<td>197</td>
</tr>
<tr>
<td>(t_{\text{mean}})</td>
<td>159</td>
<td>93</td>
<td>120</td>
<td>226</td>
<td>126</td>
<td>92</td>
<td>264</td>
<td>105</td>
</tr>
<tr>
<td>(t_{\text{slope}})</td>
<td>153</td>
<td>80</td>
<td>112</td>
<td>216</td>
<td>123</td>
<td>83</td>
<td>249</td>
<td>97</td>
</tr>
<tr>
<td>(t_{\text{sd}})</td>
<td>135</td>
<td>71</td>
<td>158</td>
<td>177</td>
<td>134</td>
<td>101</td>
<td>232</td>
<td>99</td>
</tr>
<tr>
<td>(t_{3\sigma})</td>
<td>159</td>
<td>93</td>
<td>150</td>
<td>225</td>
<td>124</td>
<td>87</td>
<td>262</td>
<td>103</td>
</tr>
<tr>
<td>(t_{\delta})</td>
<td>159</td>
<td>99</td>
<td>165</td>
<td>215</td>
<td>115</td>
<td>79</td>
<td>252</td>
<td>85</td>
</tr>
<tr>
<td>(\delta_{\text{mean}})</td>
<td>0</td>
<td>-6</td>
<td>-15</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>(\beta_{\text{mean}})</td>
<td>1.24</td>
<td>1.99</td>
<td>1.72</td>
<td>0.78</td>
<td>1.56</td>
<td>2.07</td>
<td>0.70</td>
<td>1.88</td>
</tr>
<tr>
<td>(\beta_{\text{slope}})</td>
<td>1.29</td>
<td>2.31</td>
<td>1.84</td>
<td>0.82</td>
<td>1.60</td>
<td>2.29</td>
<td>0.75</td>
<td>2.03</td>
</tr>
<tr>
<td>(\beta_{\text{sd}})</td>
<td>1.46</td>
<td>2.61</td>
<td>1.30</td>
<td>1.00</td>
<td>1.47</td>
<td>1.88</td>
<td>0.80</td>
<td>1.99</td>
</tr>
<tr>
<td>(\beta_{3\sigma})</td>
<td>1.24</td>
<td>1.99</td>
<td>1.37</td>
<td>0.79</td>
<td>1.59</td>
<td>2.18</td>
<td>0.71</td>
<td>1.91</td>
</tr>
</tbody>
</table>
From Table 5, we can see that proposed method has a distinct advantage [36]. Figure 13 shows an example of 1000 simulation results.

<table>
<thead>
<tr>
<th>Index</th>
<th>Mean method</th>
<th>Slope method</th>
<th>SD method</th>
<th>3σ method</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAE</td>
<td>6.22</td>
<td>6.66</td>
<td>8.07</td>
<td>3.38</td>
</tr>
<tr>
<td>MSE</td>
<td>41.78</td>
<td>44.85</td>
<td>101.85</td>
<td>12.11</td>
</tr>
</tbody>
</table>

Table 5. Comparison of computer simulation results by 1000 times.

2.5. Measuring bubbles uniformity by discrepancy

2.5.1. Quantifying mixing efficiency using $L_2$-star discrepancy

(1) Quantification of mixing efficiency

The variation of the uniformity coefficient with frames can be an effective method to determine the critical mixing time and mixing uniform.

In Figure 14, quantitative comparisons of the homogenization curve and mixing time predicted by the uniformity coefficient method are conducted with reported experimental data and other predictions by the Betti numbers method.

The comparisons show that good agreements of the mixing time obtained by Betti numbers method and uniformity coefficient method have also been achieved as given in Table 6 [40].

(2) Recognition of local and global uniformity

**Numerical simulations.** Generated small sets were used with the same Betti number randomly to assess the performance of the UC implementation for approximating the discrepancy of a given set of points. By checking a large number of experimental images, we found the sizes
of the bubbles are nearly the same. Two hundred and thirty-four bubbles have the same small blank area of the radius of the circle. MATLAB software randomly selects the centre. Although 234 bubbles have the same size, they are spread in different places.

These plots of Figure 15 are got by simulation with 720 lines and 1280 rows. Among them, the simulation 1 (Figure 15a) is the corresponding local uniform description, simulation 2 (Figure 15b) corresponds to the portrayal of global uniform. The lattice points generate the example of lattice uniform used in the demonstration of the algorithm. Under the guidance of the lattice points method, the simulation 3 (Figure 15c) performs for a lattice points set, which has 234 bubbles. It is can be seen that the lattice uniform is most accurate uniformly. One can take set of points or objects, which are generated by these experiments and simulations to check the algorithm.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$E_1$</th>
<th>$E_2$</th>
<th>$E_3$</th>
<th>$E_4$</th>
<th>$E_5$</th>
<th>$E_6$</th>
<th>$E_7$</th>
<th>$E_8$</th>
<th>$E_9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>159</td>
<td>93</td>
<td>120</td>
<td>226</td>
<td>126</td>
<td>92</td>
<td>264</td>
<td>105</td>
<td>145</td>
</tr>
<tr>
<td>$t'$</td>
<td>151</td>
<td>97</td>
<td>172</td>
<td>224</td>
<td>120</td>
<td>84</td>
<td>259</td>
<td>118</td>
<td>127</td>
</tr>
</tbody>
</table>

Table 6. Computing results of mixing time by the Betti numbers method ($t$) and uniformity coefficient method ($t'$) at the whole orthogonal arrays table.
Experimental examples. The plots in Figure 16 are obtained by experiments E₃ and E₆. The experiments are response to the worse and better cases. Figure 16a–c shows the difference of mixing uniformity in case 1 of E₆ with \( \beta_1 = 160 \), whereas Figure 16e, d, and f shows the difference of mixing uniformity in case 2 of E₃ with \( \beta_1 = 194 \). Comparison results show that the different experimental cases with the same Betti numbers can be identified by the MAD evolutions. The results in Figure 16 indicate that the uniformity coefficient method might be enough to obtain a good estimation and quantification of multiphase mixing effects.

Figure 16. MAD evolutions of different experimental cases with the same Betti numbers.
2.5.2. Modified $L_p$-star discrepancy for measuring mixing uniformity

(1) Video-frequency image sequence of experimental cases

In Figure 17, quantitative comparisons of the homogenization curve utilizing uniformity coefficient with modified discrepancy methods are conducted with reported experimental data and the other method is UC-LD. The variation of the UC-LD and UC-WD versus $i$ can be effectively used to estimate the critical mixing time and mixing uniform because of the similar evolutionary trend and regularity of most experimental cases. Especially here there is a clearly and distinctively different at the early phase of mixing process of experimental case $E_2$ (see Figure 17a), and obvious differences in numerical performance have a deep significant role to assess the mixing efficiency. For another experimental case $E_7$, UC-CD and UC-WD successfully measured the mixing process from no uniformity to uniformity clearly, just like UC-LD (see Figure 17b).

(2) Verification of properties

Suppose coordinates of bubble swarms in Figure 2 can be denoted as follows,

$$X_p = \begin{pmatrix} x_{11} & x_{21} \\ x_{12} & x_{22} \\ \vdots & \vdots \\ x_{1n} & x_{2n} \end{pmatrix}$$

where $n$ is the number of bubble swarms or experimental points noted earlier. With one important note as mentioned above, the first rank elements $x_{11}, x_{21}, \ldots, x_{n1}$ of $X_p$ correspond to the $x$ values identified in Eq. (21) the second rank elements $x_{12}, x_{22}, \ldots, x_{n2}$ of $X_p$ correspond to the $y$ values identified in Eq. (21). UC-CD and UC-WD are formulated as the two methods of measures of uniformity. In addition, they are dimensionless, which vary from 0 to 1. And it is hard for it to reach the certain endpoint values.

Invariance to permutation. UC-CD and UC-WD are permanent for disrupted order of the experimental factors or the experimental points. Table 7 shows verification data about invariance to permutation. Bubbles disordered in Table 7 means that the operator randomly shuffles

![Figure 17. Uniformity coefficients of $E_2$ and $E_7$.](http://dx.doi.org/10.5772/66630)
rows in Eq. (24). Coordinates disordered in Table 7 means that the operator randomly shuffles columns in Eq. (24). It is quite clear found that the two parameters identically equal to 0.9751 and 0.9723 individually even though the disorder of rows or ranks happened.

<table>
<thead>
<tr>
<th>Modified UC</th>
<th>In proper order</th>
<th>Disordered Coordinates</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>UC-CD</td>
<td>0.9289</td>
<td>0.9289</td>
<td>0.9289</td>
</tr>
<tr>
<td>UC-WD</td>
<td>0.9311</td>
<td>0.9311</td>
<td>0.9311</td>
</tr>
</tbody>
</table>

Table 7. The data of verification of invariance to permutation.

**Invariance under reflection.** In theory, UC-CD(P) and UC-WD(P) are invariant if \( x_{i1} \) and \( x_{i2} \) are replaced individually by \( 1-x_{i1} \) and \( 1-x_{i2} \). The data of verification of invariance under reflection are depicted in Table 8. More in detail, \( x = 1/2 \) in Table 8 means that the operator rotates the origin of coordinates system from bottom-left to bottom-right in a piece of patterns, \( y = 1/2 \) in Table 8 means that the operator rotates the origin of coordinates system from bottom-left to top-left in a piece of patterns, \( x = 1/2 \) and \( y = 1/2 \) (both) in Table 8 means that the manipulator spins the origin of coordinates system from bottom-left to top-left. It is obviously found that the two parameters individually equal to 0.9751 and 0.9723 even though the origin of coordinates system is rotated.

**Projection uniformity.** The projection uniformity over all sub-dimensions can be considered, and UC-CD(P) and UC-WD(P) are also invariant to it in theory. The data of verification of projection uniformity are depicted in Table 9. Projected to \( x \)-axis in Table 9 refers to set \( x_{i2} = 0 \) in Eq. (24), projected to \( y \)-axis refers to \( x_{i1} = 0 \) and projected to origin refers to \( x_{i1} = 0 \) and \( x_{i2} = 0 \). It is quite clear that the projection uniformity over all sub-dimensions can be obtained. All this to say the consideration is not insignificant for high-dimensional cases.

<table>
<thead>
<tr>
<th>Modified UC</th>
<th>No reflected</th>
<th>( x = 1/2 )</th>
<th>Reflected</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>UC-CD</td>
<td>0.9289</td>
<td>0.9289</td>
<td>0.9289</td>
<td>0.9289</td>
</tr>
<tr>
<td>UC-WD</td>
<td>0.9311</td>
<td>0.9311</td>
<td>0.9311</td>
<td>0.9311</td>
</tr>
</tbody>
</table>

Table 8. The data of verification of invariance under reflection.

<table>
<thead>
<tr>
<th>Modified UC</th>
<th>No projected</th>
<th>( y = 0 )</th>
<th>( x = 0 )</th>
<th>Projected to origin</th>
</tr>
</thead>
<tbody>
<tr>
<td>UC-CD</td>
<td>0.9289</td>
<td>0.3946</td>
<td>0.3936</td>
<td>0.0554</td>
</tr>
<tr>
<td>UC-WD</td>
<td>0.9311</td>
<td>0.5263</td>
<td>0.5253</td>
<td>0.3128</td>
</tr>
</tbody>
</table>

Table 9. The data of verification of projection uniformity.

(3) **Time complexity.** The time complexity of different methods is shown in Table 10. Through experimental comparison, we may draw the conclusion that UC-CD and UC-WD can replace
UC-LD and Betti numbers to some extent. Determination of the position of bubble swarms spends too much time, which leads to make the upper time complicated. But, other progressive technology can change this disadvantage.

(4) Numerical simulations and experimental examples. In order to assess the performance of UC-CD and UC-WD implementation for approximating the discrepancy of a given set of points, the three sets in Figure 14 were used. Table 11 shows that UC-LD of the three simulated images are affected by initial position, but UC-CD and UC-WD not. Comparing the modified UC of Figure 14b and c, the absolute difference $|0.9751 - 0.9657| = 0.0094$ is less than $|0.9623 - 0.9464| = 0.0159$ since Figure 14b and c seems to have the same degree of mixing uniformity. So it is concluded that UC-CD may outperform UC-WD and perform more sensitive for practical engineering application in some sense. The data in Table 11 also show that the difference of mixing uniformity coefficients including UC-LD, UC-CD and UC-WD with the same Betti numbers in Figure 16a, b, d and e. Meanwhile, it is noticed that different initial positions are response to different UC-LDs, which bring unreasonable and bias measurement of uniformity in practice. In other words, UC-LD may result in multiple values, but UC-CD and UC-WD do not have this problem. Moreover, the absolute difference of UC-CDs is larger than that of UC-WDs. The comparison result shows that UC-CD performs more sensitive than UC-WD in identifying the different patterns with the same Betti numbers. Those are the major of our presented work in this part.

2.6. Conclusions

1. Because a new technique based on algebraic topology was introduced for quantifying the efficiency of multiphase mixing, the mixture homogeneity and the non-homogeneity

| Table 10. The data of time of different methods. |
|---|---|---|---|---|
| Single image | Betti numbers | UC-LD | UC-CD | UC-WD |
| Reckoning | $\beta_1 = 174$ | 0.9632 | 0.9289 | 0.9311 |
| Running time (s) | 3.07 | 2.61 | 18.33 | 18.23 |

| Table 11. The data of numerical simulations and experimental examples. |
|---|---|---|---|---|---|---|
| Plots | UC-LD | UC-CD | UC-WD | TL |
| Figure 15 | 0.9326 | 0.9033 | 0.9189 | 0.8052 | 0.8493 | 0.8954 | 0.9866 |
| (b) | 0.9854 | 0.9955 | 0.9751 | 0.9623 | 0.9925 | 0.9932 | 0.9930 | 0.9925 |
| (c) | 0.9657 | 0.9464 | 0.8699 | 0.9077 | 0.9094 | 0.9033 | 0.9102 |
| Figure 16 | 0.9170 | 0.9826 | 0.9523 | 0.9823 | 0.9869 | 0.9474 | 0.9436 |
| (d) | 0.8686 | 0.9507 | 0.8645 | 0.9056 | 0.8793 | 0.9121 | 0.9121 |
| (e) | 0.9676 | 0.9794 | 0.9504 | 0.9825 | 0.9429 | 0.9306 |
of the mixture can be characterized by the Betti numbers for binary images of the patterns. The zeroth Betti numbers $\beta_0$ are used to estimate the numbers of pieces in the patterns, leading to a useful parameter to characterize the mixture homogeneity. The first Betti numbers $\beta_1$ are introduced to characterize the non-homogeneity of the mixture. This novel method may be applied for studying a variety of multiphase mixing problems in which multiphase components or tracers are visually distinguishable.

2. In a DCHE, Betti number can estimate the number of bubbles assembling in flow patterns and to get the pseudo-homogeneous time. Experimental analysis constructs a simple linear model representing a bubble swarm and the heat transfer performance of a DCHE. In addition, the Betti number average and the pseudo-homogeneous time $t$ define a new index. A better fitting curve about and the volumetric heat transfer coefficient average is received and its correlation coefficient is 0.95. A paradigm is established on the basis of this novel method for the study of flow patterns and heat transfer performance. And the paradigm offers an optional route to study the relationship of flow patterns and heat transfer in other heat transfer processes.

3. A novel method relying on image analysis and statistics was developed to estimate the mixing time accurately in a DCHE. The three sigma method researches the critical point determination of the pseudo-homogeneous process, which satisfies approximately normal distribution and surpasses the range of occurring twice. The mean value method, slope method and standard deviation method make quantitative comparisons of the mixing time. In addition, time intervals between in-homogeneous time and mixing time quantify the quasi-steady state. Neglecting critical point could make substantial errors in mixing time estimation, which is proved.

4. A straightforward method, uniformity coefficient (UC) method based on $L_2$-star discrepancy (UC-LD), presented for assessing the uniformity and mixing time of bubbles behind the viewing windows in a DCHE is effective. An imaging technique processed in the MATLAB software tracks the evolution of bubbles movement. The local discrepancy of a set of bubbles seems to be helpful to judge the difference between theory and empirical distribution. The UC links to a discrepancy, leading to a useful parameter which expresses the mixture homogeneity and mixing time. A comparison was made between the mixing time and uniformity obtained by UC method and the data obtained by Betti numbers method. Discussing the simulation and experiments conducted between local and global uniform (with the same Betti numbers) and examples are given for illustration. UC method calculates the space-time features of the mixing process successfully. The UC curves can study and compare mixing efficiency of different systems with the novel method, which can generate accurate mixing information and has a well reliability.

5. The properties of UC have been explored and there was a great influence of calculating the initial position on the original UC, namely UC-LD. The UC-LD method applies to the modified uniformity coefficient based on modified $L_2$-star discrepancies. A Koksma-Hlawka-type inequality is applicable to uniformity coefficient based on wrap-around discrepancy (UC-WD) as well as uniformity coefficient based on centred discrepancy (UC-CD) theoretically. In addition, they show some advantages includ-
ing rotation invariance (reflection invariance), permutation invariance and the ability to measure projection uniformity. Analysing real experimental cases and simulating to evaluate the performance of the novel method. The experimental results show that UC-CD presents more sensitive performance than UC-WD so the UC-CD is more appropriate for industry.

In summary, we believe that on the basis of a large amount of previously published works, the complexity of the bubble swarm patterns can be reduced and their mechanisms clarified, and the heat transfer performance in a DCHE can be elucidated.

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