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Design of Heat Transfer Surfaces in Agitated Vessels

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Abstract

The project on heat transfer surfaces in agitated vessels is based on the determination of the heat exchange area, which is necessary to abide by the process conditions as mixing quality and efficiency of heat transfer. The heat transfer area is determined from the overall heat transfer coefficient ($U$). The coefficient ($U$) represents the operation quality in heat transfers being a function of conduction and convection mechanisms. The determination of $U$ is held from the Nusselt’s number, which is related to the dimensionless Reynolds and Prandtl’s, and from the fluid’s viscosity relation that is being agitated in the bulk temperature and the viscosity in the wall’s temperature of heat exchange. The aim of this chapter is to present a summary for the literature concerning heat transfer in agitated vessels (equipped with jackets, helical coils, spiral coils, and vertical tube baffles) and also the many parameters of Nusselt’s equation for these surfaces. It will present a numerical example for a project in an agitated vessel using vertical tube baffles and a 45° pitched blade turbine. Subsequently, the same procedure is held with a turbine radial impeller, in order to compare the heat transfer efficiencies.

Keywords: vessels, overall heat transfer coefficient, Nusselt, jackets, coils, vertical tube baffles, mechanical impeller, convection

1. Introduction

Vessels with mechanical impellers are largely used by chemical, petrochemical, food, textile, and other industries, operating as extractors, concentrators, flotators, storage vessels, and chemical reactors (producing polymers, paints, fertilizers, and resins, for instance).

The aforementioned processes need heating or cooling, these being provided by jackets, coils, and vertical tube baffles, where the heat transfer area is the target parameter for the project as a function of the overall heat transfer coefficient.
The jackets are surfaces of heat transfer characterized by encompassing the tank, given that the thermal fluid roams the space between the jacket and the tank. On these equipments, the heat transfer efficiency is low due to the heat source being in the wall, which provokes uneven heating of the fluid within the tank, besides the structural problems in big units. Nevertheless, this system presents advantages as easy cleaning and available project data provided in the current literature [1].

The helical coils are made out of tubes between the impeller center line and the tank’s wall. The contact area with the fluid to be heated or cooled is much bigger around the jackets, which end up increasing the heat transfer efficiency; however, the difficulty is in the cleaning of the equipment. Most agitated vessels involving heat transfer operations are projected with helical coils due to the availability of project data [2].

The spiral coils consist of wounded tubes generally placed on the bottom of the tank. They are mostly applied to heating viscous fluids in pumping transport. Spiral coils have as disadvantage the located heat transfer, fostering uneven heating. Another downside is the lack of data in the current literature [3].

The three heating transfer surfaces mentioned above demand baffles alongside the tank’s wall to avoid vortex, which is characterized by the development of a bottleneck formed by the agitated fluid around the mechanical impeller. The vortex is an indicator of inefficiency at mixing and low heat transfer, due to the circular and organized stream lines.

The vertical tube baffles are surfaces that provide significant heating transfer rate and also eradicate the development of a bottleneck, due to its spatial structure, which makes necessary to allocate extra baffles alongside the tank’s wall. The drawback of this technique is the lack of data project, especially on continuous operation [4].

However, each type of heat transfer surface will present parameters for Nusselt’s equation particular to each system in which they were determined, in such a way that it is possible to obtain a general model that covers all surfaces for heat transfer in agitated vessels.

For batch processes the heat transfer area is determined by transient energy balances, while for continuous processes, on the large majority, the area is obtained by energy balances for steady-state operation.

2. Fundamentals

2.1. Steady-state operation

Heat transfer in agitated vessels is carried through heat exchange surfaces, like jackets, helical coils, spiral coils, and vertical tubular baffles [5]. The surfaces of heat exchange are designed as a function of the area necessary to carry the heating or cooling, based on the overall heat transfer coefficient, which is a function of the dimensionless groups Reynolds, Prandtl, and Nusselt. The classic project equation of the heat exchange area for steady-state operation is shown in Eq. (1).
A = \frac{Q}{U L M T D} \quad (1)

in which the overall heat transfer coefficient (U) is a function of local coefficients of internal convection, related to the external surface area or internal and external convection coefficient (h_i and h_o) for the hot and cold fluid, respectively.

Eq. (2) shows the dependence of the overall coefficient U to the convection coefficients (internal and external), the resistance to heat transfer presented by the heat exchange surface (conduction), and the resistances by internal and external fouling in relation to the pipelines.

\[ \frac{1}{U A} = \frac{1}{h_i A_i} + \frac{1}{R_{d_i}} + \frac{R_c}{A_i} + \frac{R_{d_0}}{A_0} + \frac{1}{h_o A_o} \quad (2) \]

The thermal resistance presented by conduction can be disregarded when the thickness of the surface wall is negligible in relation to the internal and external diameter and when it is made out of a material with high thermal conductivity. Therefore, the term \( R_c \) (referring to the thermal resistance presented by heat conduction) in Eq. (2) can be disregarded [6]. The terms regarding the resistance by fouling can also be neglected if the pipelines used are new or with operational time inferior to 5 years. Eq. (2) can be written as shown in Eq. (3), being the overall heat transfer coefficient only dependent of the internal and external convection.

\[ \frac{1}{U A} = \frac{1}{h_i A_i} + \frac{1}{h_o A_o} \quad (3) \]

The internal film coefficient by definition is associated to the fluid's flow in the interior of the heat exchange surface, while the external coefficient, in the case of a vessel's surface, is related to the mixture of the fluids that will be agitated. If the mixture, which will be agitated, is the one that will be heated, the heat transfer happens on the external surface of contact of the hot source with the cold fluid; therefore, the internal convection coefficient must be corrected and placed as a function of this external area, called then as corrected internal coefficient (\( h_{io} \)), calculated from the relation between the internal and external diameter of the pipeline, as shown in Eq. (4).

\[ h_{io} = h_i \frac{D_i}{D_e} \quad (4) \]

Adding Eq. (4) to Eq. (3) and indicating the areas as a function of the diameters, Eq. (5) is obtained.

\[ \frac{1}{U \pi D_e L} = \frac{1}{(D_i/D_e) h_{io} \pi D_e L} + \frac{1}{h_o \pi D_e L} \quad (5) \]

It can be seen in Eq. (5) that the area of heat transfer \( A (\pi D_e L) \) corresponds to the external area of the tube's surface \( A_e \). Grouping the common terms in Eq. (5), the overall coefficient of heat transfer \( U \) can be calculated as a function of the corrected internal convection coefficient and the external convection coefficient, as shown in Eq. (6).

\[ \frac{1}{U} = \frac{1}{h_{io}} + \frac{1}{h_o} \quad (6) \]

The determination of the rectified internal convection coefficient (\( h_{io} \)) implies the determination of the coefficient \( h_o \), which is a function of the fluid's physical properties (viscosity, density,
specific heat, and thermal conductivity), the heat transfer surface geometry (tube diameter), the kind of flow (laminar or turbulent), the speed flow, and the temperature gradient [7].

The analytic determination of the equation, which relates the internal film coefficient to the aforementioned variables, can be done through the application of the hydrodynamic and thermal boundary layer theory on the heat transfer surface exposed to the hot fluid and the cold fluid. After using dimension analysis and some similar equations to those of continuity, momentum, and energy, it can be noticed that the Nusselt’s number is a function of the Reynolds and Prandtl’s number. However, this analytical method can be difficult to be solved due to the nonhomogeneous partial differential equations, which will depend on the specific boundary conditions as a function of the heat transfer surface geometry. To avoid this situation, the Buckingham’s Pi theorem can be used, which relates the variables that may affect the answer variables, in this case the film coefficient, through dimensionless groups [8]. With dimension analysis the internal film coefficient can be related with the many variables aforementioned with a function in which \( h_i \) is the dependent variable, as seen in Eq. (7).

\[
h_i = f(\mu, \rho, c_p, D_i, k, u)
\]  (7)

The function expressed in Eq. (7) can be written powering the dependent variables to an exponent, adding a proportionality constant \( K \) and a dimensional constant \( K_h \), which will be reduced to 1,0 if all the other variable dimensions, when combined, lead to thermal quantities, as illustrated in Eq. (8).

\[
h_i = Ku^a \rho^b c_p^c D_i^d k^e \mu^f K'^g
\]  (8)

Expressing all the terms in Eq. (8) as fundamental quantities and then rearranging it, the semiempiric model known as Nusselt’s equation can be obtained (Eq. (9)).

\[
\frac{h_i D_i}{k} = K \left( \frac{D_i \mu}{\rho} \right)^a \left( \frac{c_p \mu}{k} \right)^b
\]  (9)

The term on the left side of Eq. (9) is the dimensionless Nusselt, and the terms on the right, going from left to right, are the dimensionless Reynolds and Prandtl, respectively; hence, Eq. (9) can be written in the following manner.

\[
Nu = K.Re^a.Pr^b
\]  (10)

Many researches established the exponents and constant values in Eq. (24) for forced flow on the interior of the cylindrical heat transfer surface, that is, on the interior of the pipelines.

Sieder-Tate [9] added a correction factor in Eq. (9), taking into account the viscosity effects on wider variations of temperature, a relation between the fluid’s viscosity on the bulk temperature, and the fluid’s viscosity on the wall’s temperature, as shown in Eq. (11), which is valid for turbulent flows on the interior of cylindrical surfaces with Reynolds above 10,000 and smooth pipes.
\[ Nu = 0.027R_e^{0.8}Pr^{1/3}\left(\mu/\mu_w\right)^{0.14} \]  
\hspace{1cm} (11)

Eq. (11) is a simplified form to determine the internal coefficient film; however, it carries an overall error of 40%, being necessary to implement correction factors on the design of the heat transfer area. Geankoplis [10] presents an equation to determine the internal coefficient film for water with temperatures ranging from 4 to 105°C on smooth pipes’ interior in fully developed turbulent flow, as shown in Eq. (12), which carries an overall error of 25%.

\[ h_i = 1429\left(1 + 0.0146T\right)u^{0.8}/D^{0.2} \]  
\hspace{1cm} (12)

Eq. (11) and Eq. (12) are simple to be used and present satisfactory results for the majority of heat exchanger projects, despite the high error associated with them. Gnielinski [11] presents a correlation to determine the internal convection coefficient, with an error lower than 10%, for fully developed turbulent flows, with Reynolds ranging from 3000 to $5 \times 10^6$, Prandtl between 0.5 and 2000, and for a relation $(L/D_i) > 10$. The correlation is presented in Eq. (13).

\[ Nu = \frac{(f'/8)(Re - 1000)Pr}{1 + 12.7(f'/8)^{1/2}\left(Pr^{2/3} - 1\right)} \]  
\hspace{1cm} (13)

The friction factor ($f'$) can be obtained from Moody’s diagram or by specialized correlations. In the case of Eq. (13), which is for fully developed turbulent flow, the friction factor can be calculated through Eq. (14).

\[ f' = (0.790lnRe - 1.64)^{-2} \]  
\hspace{1cm} (14)

The external convection coefficient ($h_o$) is a function of the same variables of the internal convection coefficient ($h_i$); however, the heat transfer surface’s geometry and the mechanical impeller have large influence in heat transmission. The expression to determine the $h_o$ can be obtained through the Buckingham’s Pi theorem [5], similar to the determination of $h_i$; hence, Eq. (9) can be rewritten by modifying the Nusselt and Reynolds’ numbers for agitation ($Re_a$), presented in Eq. (15), and adding the viscosity correction factor proposed by Sieder-Tate, as shown in Eq. (16).

\[ Re_a = \frac{ND_i^2 \rho}{\mu} \]  
\hspace{1cm} (15)

\[ \frac{h_o D_i}{k} = K \left(\frac{ND_i^2 \rho}{\mu}\right)^a \left(\frac{\rho \mu}{k}\right)^b \left(\frac{\mu}{\mu_w}\right)^c \]  
\hspace{1cm} (16)

Differently from Eq. (9), which is generic for any type of fluid, as long as the heat transfer surface is cylindrical and has a fully developed turbulent flow, Eq. (16) has the following limitations: (i) the position and geometry of the mechanical impeller, (ii) the flow’s regime, (iii) the geometry of the heat transfer surface, and (iv) the tank’s geometry. These limitations do
not allow to obtain the values for the $K$ constant and the exponents generically, for all surfaces of heat transfer in agitated vessels [12].

### 2.2. Unsteady-state operation

The project of the heat transfer surface for agitated vessels will depend on whether the tank is operating in discontinuous mode (batch) or continuous mode.

The batch processes in agitated vessels are common in many industrial processes, specially, in chemical reactors. In batch operations, there is the occurrence of two typical situations: (1) the design parameter is the operation time for heating or cooling; hence, the surface area is unknown; (2) the area for heat exchange is known and the operation time is unknown.

These processes can be carried out on isothermal operations and with phase change (heating with steam or cooling with thermal fluid vaporization) or non-isothermal operations and fluids without phase change.

An important consideration must be highlighted regarding the $U$ coefficient: for heat exchangers in systems operating on countercurrent or cocurrent flow, if the fluid’s physical properties do not change significantly during the process, the $U$ coefficient is almost constant over time. However, when the system has at least one of its fluids with considerable variation of physical properties, like viscosity in the case of oils heating, the $U$ coefficient undergoes variation as function of time, making its determination complex, since it is necessary to know the pattern in which it changes over time. Similarly, this consideration can be applied to tanks’ heating or cooling.

Considering a vessel with a generic heat transfer surface for heating a fluid using a isothermal source, the heat exchange area is determined from the macroscopic energy balance (Eq. (17)) with the following hypothesis:

\[
\sum Q + \dot{w}_c \left( h_s + \frac{v_s^2}{2} + g_x + C_1 \right) = \frac{dE_{nc}}{dt} + \dot{w}_c \left( h_s + \frac{v_s^2}{2} + g_x + C_1 \right) + \sum W
\]

(17)

a. Few physical properties vary in the fluid to be heated; hence, the $U$ coefficient is assumed to be constant over all the heat exchange surface.

b. Hot fluid constant flow (on this isothermal heating, water steam will be considered).

c. Perfect mixing tank (in such a way that the temperature will be the same at any point of the tank).

d. Perfect tank insulation.

e. Workflow in control volume is neglected, being only considered to be significant when the fluid has a large viscosity.
Eq. (18) presents a simplification of Eq. (17) following the aforementioned hypothesis.

\[ \dot{w}(h_e - h_s) = UA(T - t_b) = \frac{Mcp_c}{\theta} \frac{dt_b}{d\theta} \]  

(18)

Integrating Eq. (18) with the specific boundary condition in \( \theta = 0 \) with \( t_b = t_{b1} \) (fluid initial temperature in the beginning of the heating process), Eq. (19) is obtained, which allows to calculate the heat exchange area with a constant \( U \) coefficient over the entire surface.

\[ A = \frac{Mcp_c}{\theta} \ln \frac{T - t_{b1}}{T - t_{b2}} \]  

(19)

In a similar way, Eq. (20) presents the expression to calculate heat transfer area in the case of isothermal cooling.

\[ A = \frac{Mcp_c}{\theta} \ln \frac{t_{b1} - T}{t_{b2} - T} \]  

(20)

The energy balance described in Eq. (18) can be applied to nonisothermal processes, where Eq. (21) presents the heat exchange area calculation for heating and Eq. (22) presents the heat exchange area calculation for cooling, highlighting that both equations are valid for hot and cold fluid constant flows and constant inlet temperature.

\[ A = \frac{Mcp_c}{\theta} \ln \left\{ 1 - \left[ \frac{1}{\ln \frac{T - t_{b1}}{T - t_{b2}}} \right] \frac{Mcp_c}{\dot{w}_c cp_t} \right\} \]  

(21)

\[ A = \frac{Mcp_c}{\theta} \ln \left\{ 1 - \left[ \frac{1}{\ln \frac{t_{b1} - T}{t_{b2} - T}} \right] \frac{Mcp_c}{\dot{w}_c cp_t} \right\} \]  

(22)

On continuous operations, the same hypotheses assumed for the processes in batch mode can be considered, adding only that the flow of both fluids will be constant during the whole operation with constant inlet temperatures.

Eqs. (23)–(25) show the obtainment of the heat transfer area for an isothermal process operating at continuous mode.

\[ \ln \frac{K_1 - K_2 t_{b2}}{K_1 - K_2 t_{b1}} = - \frac{K_2}{Mcp_c} \theta \]  

(23)

\[ K_1 = \dot{w}_c cp_t t_{b1} + UAT \]  

(24)

\[ K_2 = \dot{w}_c cp_t + UA \]  

(25)

Similarly, Eqs. (26)–(28) present the heat exchange area for a nonisothermal process operated in continuous mode.
\[
\frac{K_3 - K_4 b_2}{K_3 - K_4 b_1} = -\frac{K_4}{Mcp_c} \theta 
\]

(26)

\[
K_3 = \dot{w}_c c_p c_{t_1} + \dot{w}_c c_p c_{t_2} \left[ 1 + \frac{1}{\epsilon C_{18}} \right] + T_1 
\]

(27)

\[
K_4 = \dot{w}_c c_p c_{t_1} + w_a c_p c_{t_1} 
\]

(28)

The equations for continuous operation described in Eqs. (26)–(28) can only be used for processes operated in an unsteady state; however, for projects on tanks operating continuously, they are carried as steady state, in such a way that the design equation described by Eq. (26) is reduced to Eq. (1), as shown in item 1.

3. Expressions for Nusselt’s number in several heat transfer surfaces

3.1. Jackets

Jackets are surfaces for heat exchange often used by the majority of processes of heating and cooling. Structurally, they consist of an external coating for the tank like a shirt, where the thermal fluid flows through the space between the jacket and the tank. The jackets are mainly classified as standards, spiral, and half-jackets, in which the difference is only the structure; however, these three types have the same placement inside the tank.

The heat transfer through these thermal surfaces is not efficient due to its location, since the heat transfer occurs on the tank’s walls; therefore, baffles are placed in order to increase the turbulence aiming to implement the heat transfer efficiency [13]. A big disadvantage of using jackets, more than just the low heat transfer, is its dimension, since on tanks with large volumes, like fermentation vats with an average volume of 5000 m$^3$, the construction becomes structurally unfeasible due to the high cost. Industrially, jackets are used in large scale, even with the aforementioned disadvantages, because the exponents and the constant in Eq. (16) are already known for many tank’s dimensions and several kinds of fluids operating in continuous or discontinuous mode.

Chilton et al. [14] carried out a pioneer work concerning the determination of the external convection coefficient, in a tank with 0.3 m of diameter for water and glycerin solution heating with axial mechanical impeller. Eq. (29) presents Nusselt’s equation with constant and exponents determined by Chilton.

\[
Nu = 0.36 Re_a^{0.67} Pr^{0.33} \left( \frac{H}{\mu w} \right)^{0.14} 
\]

(29)

The exponents and the constant in Eq. (29) were determined in a tank without baffles and discontinuous operation, although, as the dimension of the studied tank was quite inferior to industrial dimensions, Eq. (29) has deviations of 40% magnitude from the real values of external convection coefficients obtained experimentally in tanks of other sizes.
Uhl and Gray [15] carried experiments in a tank with 0.6 m of diameter and axial impeller, turbine type, and anchor type, with ultra-viscous fluids and four vertical baffles, being operated on discontinuous mode. However, because of the elevated viscosity, the equations are only valid if Reynolds ranges between 20 and 300. Eq. (30) presents the expression for the Nusselt determination using the axial impeller.

\[
Nu = 0.415Re_a^{0.67}Pr^{0.33} \left( \frac{\mu}{\mu_w} \right)^{0.24}
\]  

(30)

Bourne et al. [16] determined the exponents and the constant for Eq. (16) in a tank with diameter of 0.51 m on the standard conditions for dimensions, described on Rushton et al. [17] pioneer work, conditions that are being used up until the present day. Electrolytic solutions and a turbine radial impeller with six flat blades were used; however, Bourne et al. [16] disregarded the term related to viscosity due to electrolytic solutions having a viscosity close to the one of the water. In Eq. (31), the model obtained on the aforementioned conditions is shown.

\[
Nu = 0.42Re_a^{0.694}Pr^{0.33}
\]  

(31)

Having mathematical software’s backing, the exponents and the constant of Eq. (16) have been expressed in terms of more complex functions, decreasing the error generated by the model. For instance, Karcz and Streck [18] carried out experiments for continuous operations on water heating using steam, varying the type of impeller and its height in relation to the tanks bottom. With the data obtained and arbitrating the exponents values on Prandtl’s number as 0.33 and on viscosity relation as 0.14, like Mohan et al. [19], Karcz and Streck [18] developed a polynomial for the \( K \) constant determination in Eq. (16) for each mechanical impeller used. Eq. (32) presents the model determined on the experiments.

\[
Nu = KRe_a^{0.67}Pr^{0.33} \left( \frac{\mu}{\mu_w} \right)^{0.14}
\]  

(32)

In Eq. (33), the polynomial obtained from Eq. (32) with axial mechanical impeller to determine \( h_o \) is illustrated.

\[
K = 0.3119 + 0.3333 \times 10^{-3} \left( \frac{F - 6}{2} \right) + 1.75 \times 10^{-3} \left( \frac{G - 0.06}{0.015} \right) - 6.8333 \times 10^{-3} \left( \frac{Y - 0.184}{0.0694} \right) \\
+ 2.3333 \times 10^{-3} \left( \frac{X - 0.0128}{0.0638} \right)
\]  

(33)

where \( F, G, Y, \) and \( X \) are dimensional parameters, expressed in International System of Units relating to the tank and mechanical impeller.
Nassar and Mehrotra [20] present the coefficients in Eq. (16) for an isothermal heating equipped with a six-flat-blade turbine impeller, using condensed steam as heating fluid. This is shown in Eq. (34).

\[
Nu = 0.44Re^{0.67}Pr^{0.33}\left(\frac{H}{\mu_w}\right)^{0.24}
\]  
(34)

3.2. Helical coils

The helical coils present several advantages over jackets, because they have a large heat transfer area, are compacts, and, due to their geometry, promote an efficient turbulence on the fluids to be agitated in the tank. Moreover, it is an equipment that has low cost and it is easy to build [21]. The increase on the heat transfer through this kind of surface has two disadvantages: the need for baffles to avoid the formation of vortexes and the difficulty of maintenance and cleaning when submitted to fouling fluids. The helical coils can be single ribbon and double ribbon. The single ribbon coils are more efficient than the double ribbon ones, because as they have only one tube filament, there is a small formation of stationary zones, while for double ribbon coils, there is formation of stationary zones between the two tube filaments, decreasing the turbulence on this region and, as a consequence, the heat transference.

The characteristics of an efficient mixture with helical coils are consequence of developments of secondary flows throughout the pipeline, which is known as Dean’s effect [22]. The secondary flows appear due to the action of centrifugal force on the fluid unit, being the difference of axial speed on the fluid unit for the tubes cross section as a function of a force gradient. The fluid elements in the tube center are projected perpendicularly to the tubes, that is, against the tank’s wall, where they undergo a speed decrease, and when returning to the tube center, they find the main flows, resulting in vortexes throughout the helical coil. These vortexes are called Dean’s cells, which are responsible for the increase of heat transfer efficiency by providing intensification on local turbulence close to the coil [23].

However, this effect is significant when in laminar and transient flows, because the vortexes’ stability is kept, although, in turbulent flows, the vortexes end up being negligible in relation to the turbulence generated on the system.

Similarly with jackets, several researchers provided the exponents and the constant for Eq. (16) for single ribbon helical coils, which can also be used on double ribbon coils just adding safety factors.

Cummings and West [24] determined the exponents of Eq. (16) for organic liquids heating in a 0.76 m diameter tank equipped with a six-flat-blade radial impeller without baffles, on discontinuous operation. The obtained model is expressed in Eq. (35).

\[
Nu = 1.01Re^{0.62}Pr^{0.33}\left(\frac{H}{\mu_w}\right)^{0.14}
\]  
(35)
DeMaerteleire [25] carried out a similar study to the one done by Cummings and West [24]; however, he added four baffles to the tank and varied the turbine impeller diameter. A term was added to the obtained model, which relates the tank’s diameter to the mechanical impeller diameter, thus obtaining a more realistic model that is valid for a Reynolds ranging from 26,000 to 110,000. The obtained expression is illustrated in Eq. (36).

$$Nu = 1.778 Re_a^{0.628} Pr^{0.33} \left( \frac{\mu}{\mu_w} \right)^{0.20} \left( \frac{D_t}{D_a} \right)^{0.382}$$  \hspace{1cm} (36)

The dimensionless group \((D_t/D_a)\) relates the tanks’ diameter to the impeller’s diameter; however, as demonstrated by Karcz and Streck [18], the best mixing conditions occur when the relation \((D_t/D_a) = 3\). For instance, if the tank’s diameter is 1 m, according to the standard condition, the impeller’s diameter should be 0.33 m; replacing these values with the dimensionless group and powering it to 0.382, the value 1.527 is found. This value will be fixed regardless of the chosen tank’s diameter, being the only request to follow the aforementioned standard condition. The influence of an increase or decrease on the tank’s diameter and consequently the mechanical impeller diameter is evident on the flow regime (Reynolds) and on the heat transference (Nusselt).

Havas et al. [26] determined the exponents and the constant in Eq. (16) for water heating in a tank with 0.4 m of diameter and radial impeller with different diameters. The same experiments were carried out in a tank with 0.8 m of diameter. The model obtained by Havas et al. shows a greater coverage compared with the ones presented in Eqs. (35) and (36), because it takes into account a wider range of tank’s diameter. Eq. (37) shows the determined model based on the obtained results.

$$Nu = 0.187 Re_a^{0.688} Pr^{0.36} \left( \frac{\mu}{\mu_w} \right)^{0.11} \left( \frac{D_t}{D_a} \right)^{0.62}$$  \hspace{1cm} (37)

Yet, regarding Eq. (16), Couper et al. [27] recommended the Prandtl’s number powered to a 0.37 exponent and the \(c\) exponent, which relates the fluid viscosities, determined by an empirical expression as function of the viscosity in the average inlet and outlet temperature of the fluid to be heated, as pointed by Eq. (38).

$$c = \frac{0.714}{T^{0.77}}$$  \hspace{1cm} (38)

Dias et al. [28] determined the exponents and the constant in Eq. (16) for water heating equipped with an axial impeller with four flat blades inclined by 45° and a turbine radial impeller with six flat blades, based on Couper’s assumption. The obtained models for axial and radial impellers are expressed by Eqs. (39) and (40), respectively.
\[ Nu = 0.317 \cdot Re_a^{0.588} Pr^{0.37} \left( \frac{\mu}{\mu_w} \right)^{0.79} \] (39)

\[ Nu = 0.195 \cdot Re_a^{0.664} Pr^{0.37} \left( \frac{\mu}{\mu_w} \right)^{0.79} \] (40)

3.3. Spiral coils

Spiral coils are used, primarily, in tanks for storing heavy oil, where flow is not recommended when at ambient temperatures, due to the oil’s high viscosity. As a solution, these coils are placed on the tank’s base, where heating steam transfers energy to the oil, which increases oil’s temperatures, making the viscosity on that spot decrease, allowing pumping transportation.

As these heat transfer surfaces meet in the vessel’s bottom, even with intense turbulence generated by the mechanical impeller, it is evident that the heat transfer will occur predominantly on the tank’s bottom and will spread on the axial axis over time. In view of this problem, Ho and Wijeysundera [29] simulated the heat transfer on the air dehumidification process with spiral coil and numerical simulations; they determined the internal heat transfer coefficient \((h_i)\) and, as a consequence, the heat transference performance throughout the process.

Naphon [30] determined a correlation for the \(h_i\) coefficient in a vessel without mechanical impeller for cooling of hot water by using cold water. The model was based on Prandtl’s number and Dean’s number; however, the expression presented is of a totally particular nature to the experiment conditions used, not being possible, and then to extrapolate it for industrial units. The unit is operated in discontinuous state and natural convection, given that the obtained model is expressed in Eq. (41).

\[ Nu = 27.358 D_0^{0.287} Pr^{-0.949} \] (41)

Rosa et al. [31] conducted a study as to predict the Nusselt’s number when heating organic solutions in tanks equipped with spiral coils for two kinds of mechanical impellers, Rushton turbine (RT) with six flat blades and pitched blade turbine (PBT) with four blades at 45°. The obtained models are based in Eq. (16) and can be applied for Reynolds between 2000 and 500,000 and Prandtl around 3.8 and 140 and tanks containing baffles. The models for RT and PBT impellers are presented in Eqs. (42) and (43).

\[ Nu = 0.10 \cdot Re_a^{0.83} Pr^{0.33} \left( \frac{\mu}{\mu_w} \right)^{0.14} \] (42)

\[ Nu = 0.81 \cdot Re_a^{0.64} Pr^{0.33} \left( \frac{\mu}{\mu_w} \right)^{0.14} \] (43)

3.4. Vertical tube baffles

The vertical tubular baffles consist of tube bank connected between themselves, which are located in the interior side of the tank, in the same way of vertical baffles used in tanks with jackets and coils. The dimensions to construct tube banks follow the standard agitation
configurations determined by Rushton et al. [32], which states that the tube bank’s diameter should have 1/10 of the tank’s diameter in order to reach the best thermal and mixing efficiency.

According to Tatterson [33], the vertical tubular baffles increases 37% the heat transfer efficiency on the turbulent region; this happens due to the large heat transfer area of tubular baffles and the intense turbulence caused by the mechanical impeller, in such a way that tanks with this kind of surface do not need additional baffles for vortex breaking, since the very thermal surface also works as a baffle.

Studies carried out by Rosa et al. [34] with water and sucrose solutions heating in tanks equipped with axial and radial impellers determined that the rotation in which the heat transfer is maximized with the lowest energy usage by mechanical impeller ranges from 200 to 300 rpm.

The external convection coefficient of heat transfer ($h_o$) is a function of several variables, which have already been mentioned on item 1; however, when using vertical tubular baffles, this coefficient is also reliant on the amount of bench tubes used, on their spatial placement in the tank’s interior, and on the kind of operation (continuous or discontinuous) [35].

The exponents and the constant in Eq. (16) were not generically determined for any tank dimension, vertical tubular baffles, and amount of tube banks, due to the complexity presented in the previous paragraph, that is, most part of the models presented on today’s literature are for discontinuous operations.

The first model determined in order to estimate the $h_o$ coefficient using vertical tubular baffles was presented on the pioneer work carried out by Rushton et al. [17] in tanks abiding by standard dimensions. The experiments were ran in a 0.366 m diameter tank where hot water was used to heat oil through four vertical tube bank using a turbine impeller with six flat blades and a radial impeller with four flat blades. The operation was performed as a discontinuous operation.

The models obtained by Rushton et al. were found by plotting directly the experimental values of the $h_o$ coefficient in function of the variables within the Reynolds number modified for agitated tanks, which are the impeller’s diameter, the mechanical impeller’s rotation, the density and the viscosity of the fluid in study.

The models for turbine radial impellers with six flat blades and with four flat blades are represented in Eqs. (44) and (45), respectively.

$$h_o = 0.00285D^2N\rho\mu^{-1}$$  \hspace{1cm} (44)

$$h_o = 0.00235D^{1.1}N^{0.7}\rho^{0.7}\mu^{-0.7}$$  \hspace{1cm} (45)

Dunlap and Rushton [36] continued the study carried by Rushton et al. [17] with vertical tube baffles, incorporating to the obtained model the effect of the impeller’s diameter in relation to the tank’s diameter and the effect of the amount of tube bank ($n_b$) on the heat transfer from hot water to oils. Eq. (46) presents the determined model.
\[ Nu = 0.09Re_a^{0.65}Pr^{0.33} \left( \frac{\mu}{\mu_w} \right)^{0.4} \left( \frac{D_s}{D_t} \right)^{0.33} \left( \frac{2}{\mu} \right)^{0.2} \]  

(46)

Havas et al. [37] carried out experiments in a tank with 0.4 and 0.8 m of diameter for water and fuel oils heating with five-tube bank and radial impeller equipped with six flat blades. The operation was conducted on discontinuous mode. Based on the experimental data, the researchers concluded that the effect caused by the impellers’ diameter and by the amount of tube bank is negligible in relation to the turbulence generated by the mechanical impeller. The obtained model incorporates the effects, which are directly disregarded on the Reynolds number, as presented in Eq. (47).

\[ Nu = 0.208Re_a^{0.65}Pr^{0.33} \left( \frac{\mu}{\mu_w} \right)^{0.4} \]  

(47)

Karcz and Strek [38] defined a model to determine the external convection coefficients using an axial mechanical impeller with three inclined blades in nonstandard conditions for the vertical tubular baffle dimensions. The model is presented in Eq. (48).

\[ Nu = 0.494Re_a^{0.67}Pr^{0.33} \left( \frac{\mu}{\mu_w} \right)^{0.14} \]  

(48)

Lukes [39] expanded the studies carried by Karcz and Strek [38], using an axial impeller with three inclined blades and a standard axial impeller with four blades inclined 45°, obtaining an expression similar to Eq. (48). The obtained function is presented in Eq. (49).

\[ Nu = 0.542Re_a^{0.65}Pr^{0.33} \left( \frac{\mu}{\mu_w} \right)^{0.40} \]  

(49)

Rosa et al. [40] carried experiments in a tank with 0.4 m diameter for heating sucrose solutions at 20% and 32% (concentration?) using hot water, through four-tube bank using an axial impeller with four blades inclined 45°; however, this is the first model presented operating in continuous operation, as illustrated in Eq. (50).

\[ Nu = 17.88Re_a^{0.27}Pr^{0.29} \left( \frac{\mu}{\mu_w} \right)^{0.37} \]  

(50)

Rosa et al. [41] carried out experiments under the same conditions that Rosa et al. [35], however, this time using a turbine radial impeller. The model is shown in Eq. (51).

\[ Nu = 25.03Re_a^{0.38}Pr^{0.11} \left( \frac{\mu}{\mu_w} \right)^{0.20} \]  

(51)
4. Project of a tank with heat exchange by vertical tubular baffles

Specify the heat exchange system model with vertical tube baffles for a tank with useful volume of 3 m$^3$ on continuous operation, as shown in Figure 1. The vertical tube baffles are made of brass composed by vertical pipelines (one baffle/three pipelines). The unit must heat an aqueous solution with 20% in mass of sucrose and a 2.0 m$^3$/h flow ($V_{sol}$) from 20 to 42°C, in steady-state operation. The liquid available for heating is water at 90°C with a 10 m$^3$/h flow.

The impeller, which has four blades, inclined 45°, works at 150 rpm. Data: (a) solutions' specific mass ($\rho_{sol}$) of 1074.2 kg/m$^3$, (b) solutions' specific heat ($C_{psol}$) of 3650 J/kg K, (c) water specific mass ($\rho_{water}$) of 1000 kg/m$^3$, (d) water specific heat ($C_{pwater}$) of 4180 J/kg K, (e) water viscosity ($\mu_{water}$) of 0.001 kg/(m s), (f) solutions’ viscosity ($\mu_{sol}$) of 0.0017 kg/(m s), (g) solutions’ thermal conductivity ($k_{sol}$) of 0.43 W/(m °C), and (h) combined fouling factor ($R_d$) of 0.001 (h ft$^2$ °F)/Btu.

The described system has as project equation for heat transfer area, Eq. (52) (nonisothermal continuous operation) (see Eq. (1)).

![Figure 1. Tank with axial impeller and four blades inclined by 45° and system of heat exchange by vertical tubular baffles. Continuous operation [42].](http://dx.doi.org/10.5772/66729)
The solution of Eq. (52) includes the determination of the overall coefficient of heat transfer. Due to the high thermal conductivity of brass, the conduction will be disregarded in this process; hence, the $U_d$ coefficient will only depend on the convection coefficients (Eq. (53)).

$$\frac{1}{U_d} = \frac{1}{h_{iv}} + \frac{1}{h_v}$$  (53)

The $h_i$ coefficient will be calculated from the expression proposed by Bondy and Lippa [6] (Eq. (12) of item 2)—Eq. (54)—and corrected regarding the pipeline’s external surface (Eq. (55)).

$$h_i = 1429(1 + 0.0146T)^{0.8}/D^{0.2}$$  (54)

$$h_{iv} = \frac{h_i D_i}{D_e}$$  (55)

The average speed $u$ in the interior of the vertical tubular baffles’ pipeline is calculated from the continuity equation ($\dot{V} = uA$); however, the pipeline’s diameter is not known yet. This procedure involves the determination of the vessel’s internal geometry, which will be calculated from the standards recommended by Rushton et al. [32]. With 3 m³ volume and applying the geometric relations, the following is obtained:

$$V = \left(\pi D_i^2 / 4\right)H = 1.56 \text{ m}$$  (56)

$$D_t = H = 1.56 \text{ m}$$  (57)

$$D_t / D_a = 3 \rightarrow D_a = 0.52 \text{ m}$$  (58)

$$E / D_a = 1 \rightarrow E = 0.52 \text{ m}$$  (59)

$$W / D_a = 1/5 \rightarrow W = 0.104 \text{ m}$$  (60)

$$X = W / 0.707 \rightarrow X = 0.147 \text{ m}$$  (61)

$$f / D_i = 0.1 \rightarrow f = 0.156 \text{ m} = 6.14 \text{ in.} = 6 \text{ in.}$$  (62)

Since the baffle is composed by three pipelines, their external ($D_e$) and internal ($D_i$) diameters can be specified:

$$D_e = 6 \text{ in.} / 3 = 2 \text{ in.}$$  (63)

Considering a commercial pipeline with $D_N$ of 1 1/2 in., Sch 40S and $D_i$ of 1.61 in., and thickness (ep) of 0.145 in. therefore
\[ D_e = D_i + 2\varepsilon p = 0.04826\,\text{m} \quad (64) \]
\[ D_i = 0.040894\,\text{m} \quad (65) \]

The average temperature \( T \) is calculated from the arithmetic average between the hot fluid’s inlet and outlet (\( T_1 \) and \( T_2 \)), respectively. However, on non-isothermal conditions, the temperature \( T_2 \) also varies over time. Considering that the vessel is perfectly insulated (isolated?), in such a way that all the heat given by the hot fluid (water) will be transferred to the cold fluid (solution), the temperature \( T_2 \) and the heat flow \( Q \) can be obtained by an energy balance, as shown in Eq. (66).

\[ \dot{w}_h c_p \frac{dT}{d\theta} = \dot{w}_c c_p \frac{dT}{d\theta} \quad (66) \]

Integrating in \( \theta = 0 \) with \( T = T_1 \) and \( t_b = t_{b1} \), the following is obtained:

\[ T_2 = \frac{\dot{w}_h c_p}{\dot{w}_h c_p} (t_{b2} - t_{b1}) + T_1 = 85.9^\circ\text{C} \quad (67) \]
\[ Q = \dot{w}_h c_p \frac{dT}{d\theta} = 48180\,\text{W} \quad (68) \]

Hence, the average temperature \( T \) is given by Eq. (69) and the average speed \( u \) by Eq. (70).

\[ T = 87.95^\circ\text{C} \quad (69) \]
\[ u = \frac{4V}{\pi D_i^2} = 2.11\,\text{m/s} \quad (70) \]

Replacing Eqs. (65), (69), and (70) in Eq. (54), the \( h_l \) coefficient (Eq. (71)) is obtained, and replacing the value of \( h_l \) and of the Eqs. (64) and (65) in Eq. (55), the \( h_{io} \) coefficient is given (Eq. (72)).

\[ h_l = 11238.20\,\text{W/m}^2{^\circ}\text{C} \quad (71) \]
\[ h_{io} = 9522.90\,\text{W/m}^2{^\circ}\text{C} \quad (72) \]

The \( h_o \) coefficient for the current project example will be calculated with the expression given by Rosa et al. [40] (Eq. (73)).

\[ Nu = 17.88Re_{\alpha}^{0.27}Pr_{\alpha}^{0.29} \left( \frac{\mu}{\mu_w} \right)^{0.37} \quad (73) \]

Eqs. (74) and (75) show the calculation for the Reynolds (\( Re_{\alpha} \)) and Prandtl numbers. The relation \( \mu/\mu_w \) will be assumed as 1, considering that fluid temperature in the tank \( t_b \) will be equal to the wall temperature \( t_{w} \).
Replacing Eqs. (74) and (75) in Eq. (73), the following is given:

\[ \text{Nu} = 1247.14 \]  
\[ h_0 = \frac{\text{Nu} \cdot K_{\text{sol}}}{D_t} = 343.76 \text{W/m}^2\text{°C} \]  
\[ U_C = \frac{h_0 \cdot h_0}{h_0 + h_0} = 331.78 \text{W/m}^2\text{°C} \]  
\[ U_D = 313.47 \text{W/m}^2\text{°C} \]

The LMTD will be calculated considering the agitation system operating on countercurrent. Eqs. (80) and (81) present the LMTD calculation.

\[ \text{LMTD} = \frac{\Delta t_q - \Delta t_f}{\ln{(\Delta t_q/\Delta t_f)}} = \frac{\Delta t_f - \Delta t_q}{\ln{(\Delta t_f/\Delta t_q)}} \]  
\[ \text{LMTD} = 56.4 \text{°C} \] 

Finally, replacing Eqs. (68), (79), and (81) in Eq. (52), the necessary heat exchange area for this project is found (Eq. (82)).

\[ A = 2.72 \text{m}^2 \] 

The total pipeline length \( L \), the number of pipelines \( N_t \), and the number of tubes per baffle \( N_b \) are given by Eqs. (83)–(85), respectively.

\[ L = \frac{A}{\pi D_c} = 17.9 \text{m} \]  
\[ N_t = \frac{L}{H} = 11.5 \text{tubes} = 12 \text{tubes} \]  
\[ N_b = \frac{N_t}{4 \text{baffles} \cdot 3 \text{tubes per baffle}} \]

Hence, the vessel described by the given example must have four vertical tubular baffles, and each one must have three tubes. If the tank’s heating were to be carried out with agitation promoted by a radial impeller, the \( h_0 \) coefficient should be calculated by the equation...
proposed by Dunlap and Rushton [36], shown in item 3.4 (Eq. (51)), obtaining a heat transfer area of just 0.91 m$^2$.

5. Conclusions

Comparing the value obtained to the areas for both impellers, the agitation with radial impeller is much more efficient in terms of heat transfer when compared to the axial impeller, due to the large turbulence promoted by the radial impeller and to the fact that the radial impeller sends the fluid directly to the tank’s wall, where the vertical tubular baffles are located. However, the power consumed by the mechanical impeller must also be analyzed in order to find the most economical rotation with the maximum heat exchange.

Therefore the choice of the kind of heat transfer surface suitable for the process to be projected in agitated vessels (jackets, helical coils, spiral coils, and vertical tube baffles) must be done very strictly, specially doing an analysis between the kind of impeller and its interaction with the adopted surface, because, as present on this paper, the difference between the areas obtained by each kind of surface can vary significantly.

Nomenclature

\[
\begin{align*}
a & \quad \text{Exponent of Eq. (16)} \\
A & \quad \text{Heat transfer area (m}^2\text{)} \\
A_i & \quad \text{Heat transfer area (inside a pipe) (m}^2\text{)} \\
A_o & \quad \text{Heat transfer area (outside a pipe) (m}^2\text{)} \\
b & \quad \text{Exponent of Eq. (16)} \\
c & \quad \text{Exponent of Eq. (16)} \\
c_p & \quad \text{Specific heat (kJ/kg °C)} \\
c_{pc} & \quad \text{Specific heat of cold fluid (kJ/kg °C)} \\
c_{ph} & \quad \text{Specific heat of hot fluid (kJ/kg °C)} \\
D_i & \quad \text{Internal diameter of tube (m)} \\
D_e & \quad \text{External diameter of tube (m)} \\
D_v & \quad \text{Vessel diameter (m)} \\
D_a & \quad \text{Impeller diameter (m)} \\
DE & \quad \text{Dean’s number} \\
\epsilon & \quad \text{Wall thickness of tube (m)} \\
E & \quad \text{Distance of impeller from the bottom of the tank (m)} \\
E_{dc} & \quad \text{Global energy in control volume (kJ)} \\
f & \quad \text{Friction factor of Moody}
\end{align*}
\]
\( F \)  
Constant of Eq. (33)

\( G \)  
Constant of Eq. (33)

\( g \)  
Acceleration of gravity (m/s²)

\( h_i \)  
Heat transfer coefficient at internal surface of tube (W/m² °C)

\( h_{io} \)  
Heat transfer coefficient at the internal surface in relation of external surface (W/m² °C)

\( h_e \)  
Heat transfer coefficient at the external surface

\( h_s \)  
Specific enthalpy of flow inlet in control volume (kJ/kg)

\( h_o \)  
Specific enthalpy of flow outlet in control volume (kJ/kg)

\( J \)  
Diameter of bank of tubes (m)

\( k \)  
Thermal conductivity of fluid (W/m °C)

\( K \)  
Constant of Eq. (16)

\( K'' \)  
Constant of Eq. (8)

\( \text{LMTD} \)  
Logarithmic mean temperature difference (°C)

\( L \)  
Length of tube (m)

\( M \)  
Mass of fluid in the vessel (kg)

\( N_b \)  
Number of bank tube of tubular vertical baffles tubes

\( N_t \)  
Total number of tubular vertical baffles tubes

\( N \)  
Rotation of impeller (rpm)

\( Nu \)  
Nusselt’s number

\( Pr \)  
Prandtl’s number

\( Q \)  
Heat transfer rate in control volume (W)

\( R_c \)  
Thermal conduction resistance (m² °C/W)

\( Rd_i \)  
Internal resistance by fouling (m² °C/W)

\( Rd_o \)  
External resistance by fouling (m² °C/W)

\( Re \)  
Reynolds number for inside tube

\( Re_a \)  
Reynolds number for agitation in vessels

\( T_b \)  
Bulk temperature in vessel (°C)

\( T_{b1} \)  
Inlet bulk temperature in vessel (°C)

\( T_{b2} \)  
Outlet bulk temperature in vessel (°C)

\( T \)  
Constant temperature of hot fluid in isothermal process (°C)

\( T' \)  
Constant temperature of cold fluid in isothermal process (°C)

\( T_1 \)  
Inlet temperature of hot fluid in nonisothermal process (°C)

\( T_2 \)  
Outlet temperature of hot fluid in nonisothermal process (°C)

\( T \)  
Mean temperature of hot fluid (°C)

\( u \)  
Velocity of fluid (m/s)

\( U, U_l, U_d \)  
Overall heat transfer coefficient, clean coefficient, and design coefficient (W/m² °C)
Design of Heat Transfer Surfaces in Agitated Vessels

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References


Greek letters

\( \mu \) Dynamic viscosity of agitated liquid at bulk temperature in the vessel (kg/m s)

\( \mu_w \) Dynamic viscosity of agitated liquid at wall temperature of thermal surface (kg/m s)

\( \eta \) Dynamic viscosity of agitated liquid at mean temperature (kg/m s)

\( \rho \) Density of agitated liquid (kg/m³)

\( \Delta t_h \) Hot terminal temperature difference (°C)

\( \Delta t_c \) Cold terminal temperature difference (°C)

Inlet velocity of fluid in control volume (m/s)

Outlet velocity of fluid in control volume (m/s)

Height of blade impeller (m)

Workflow in control volume (W)

Length of blade impeller (m)

Constant of Eq. (33)

Constant of Eq. (33)

Elevation (m)


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