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Abstract

In the chapter, a method for measuring the transient temperature of the flowing fluid based on temperature changes of the thermometer is described. In the presented method, the thermometer is considered as an inertial system of first and second order. To reduce the influence of random errors in the temperature measurement, the local polynomial approximation based on nine points is used. As a result, the first and second derivatives of a temperature, which indicate how the temperature of the thermometer varies over time, are determined very accurately. Next, the time constant is defined as a function of fluid velocity for sheathed thermocouples with different diameters. The applicability of the presented method is demonstrated on real data in the experiment. The air temperature is estimated from measurements carried out by the three thermocouples having different outer diameters when the air velocity varied in time. A comparison of the computed temperatures of air gives confidence to the accuracy of the presented method. The method presented in this chapter for measuring the transient temperature of the fluid can be used for the online monitoring of fluid temperature change with time.

Keywords: temperature measurement, transient conditions, first-order model, second-order model, time constant

1. Introduction

Under steady-state conditions, when the fluid temperature is constant, temperature measurement can be performed with the high degree of accuracy owing to the absence of damping and time lag. However, when fluid temperature varies rapidly, for example, during start-up, there is a significant difference between the actual and the measured temperature of the fluid. These
differences occur because it takes time for heat to transfer through the heavy housing to the temperature sensor.

Most of the scientific publications concerning the measurement of temperature mainly discuss the problem of temperature measurement at steady state [1–9]. Only the step response of thermometers is studied to estimate the dynamic error of the temperature measurement. Few studies refer to the measurement of the transient fluid temperature, despite the high practical significance of the problem [10–13].

An example is the measurement of transient temperature steam or flue gases in power plants, which is very difficult. Measured temperature differs significantly from the real temperature of the fluid, which is caused by massive thermowells of the thermometers and their low heat transfer coefficients. Some thermometers may have a time constant of 3 min or more, which makes the implementation of a single temperature measurement requiring about 15 min [13]. On the other side, some designs of thermometers need more than one time constant to describe the unsteady response of temperature sensor inserted into the thermowell. Measuring the fluid temperature in a controlled process may require the knowledge of two or three time constants, which describe the transient response of the thermometer [14].

The problem of dynamic errors in temperature measurements becomes particularly important in superheated steam temperature control systems, which use injection coolers (spray attemperators). Due to the large inertia of the thermometer, the measurements of transient fluid temperature can be inaccurate, which causes the automatic control system of the superheated steam temperature not to work properly.

A similar problem occurs in measuring the exhaust gas temperature as the time constants of the thermometer and the time delay are large.

2. Mathematical models of the thermometers

Generally, thermometers are considered as elements with lumped thermal capacity. It is assumed that the temperature of the thermometer is only a function of time and temperature differences inside it are neglected.

Based on these assumptions, the mathematical model of the thermometer is the differential equation describing the inertial system of the first order [14]:

\[ \tau \frac{dT(t)}{dt} + T(t) = T_f(t) \]  

(1)

where \( \tau \) — time constant of the thermometer in the first-order model in s, \( T(t) \) — measured temperature in °C and \( T_f(t) \) — fluid temperature in °C.

The time constant is expressed by formula:
\[ \tau = \frac{m_c}{h_A} \tag{2} \]

where \( m_c \) — thermocouple mass in kg, \( c_t \) — average-specific heat of the thermocouple in J/(kg·K), \( h_t \) — heat transfer coefficient on the outer surface of the thermocouple in W/(m²·K) and \( A_t \) — outer surface area of the thermocouple in m².

The ordinary differential Eq. (1) was solved for the initial condition:

\[ T(0) = T_0 = 0 \tag{3} \]

where \( T_0 \) signifies initial thermometer temperature in °C.

The initial problem, (Eqs. (1) and (3)), was solved using the Laplace transformation. The operator transmittance \( G(s) \) then assumes the following form:

\[ G(s) = \frac{T(s)}{T_f(s)} = \frac{1}{s + \frac{1}{\tau}} \tag{4} \]

where \( T(s) \) — Laplace transform of the thermometer temperature, \( T_f(s) \) — Laplace transform of the fluid temperature and \( s \) — complex variable.

For the step increase of the fluid temperature from \( T_0 = 0°C \) to the constant value \( T_s \), the Laplace transform of the fluid temperature assumes the form \( T_f(s) = \frac{T_s}{s} \) and the transmittance formula can be simplified to

\[ \frac{T(s)}{T_s} = \frac{1}{\frac{s}{s + \frac{1}{\tau}}} \tag{5} \]

After writing Eq. (5) in the form:

\[ \frac{T(s)}{T_s} = \frac{1}{s - \frac{s}{s + \frac{1}{\tau}}} \tag{6} \]

it is easy to find the inverse Laplace transformation and determine the thermometer temperature as a function of time:

\[ u(t) = \frac{T(t) - T_0}{T_s - T_0} = 1 - \exp\left( -\frac{t}{\tau} \right) \tag{7} \]
where \( u(t) \) denotes unit-step response of the thermometer.

For structurally complex thermometers that measure the temperature of fluid under high pressure, the accuracy of the first-order model (1) is inadequate. The cross section of the temperature sensor placed in the massive housing is shown in Figure 1. This example will be analysed to show that the thermometers can be modelled as second-order inertial systems [14].

Air gap can occur between the outer thermowell and the temperature sensor (Figure 1). The discussion assumes that the thermal capacity \( c_\rho \) is neglected because of its small value.

Introducing the overall heat transfer coefficient \( u_w \), referenced to the inner surface of the housing

\[
\frac{1}{u_w} = \frac{1}{h_m} + \frac{1 + D_w/d}{4k_w} \frac{(D_w - d)}{d} + \frac{D_w}{d} \frac{1}{h_T}
\]

and accounting for the radiation heat transfer from the housing to the inner sensor, the heat balance equation for the sensor located within the housing assumes the form:
\[ A \rho c \frac{dT}{dt} = P_m u_m (T_w - T) + C \left( T_w^4 - T^4 \right) \]  \tag{9}

where the symbol \( C \) denotes:

\[ C = \frac{\pi d \sigma}{1 + \frac{d}{D_m} \left( \frac{1}{\varepsilon_w} - 1 \right)} \]  \tag{10}

In Eqs. (8) and (9), the symbols \( h_m \) and \( h_T \) represent heat transfer coefficient on the inner surface of the housing and the outer surface of the thermocouple in W/(m\(^2\)K), respectively, \( D_m \) inner diameter of the housing in m, \( d \) outer diameter of the thermocouple in m, \( k \) housing thermal conductivity in W/(m\cdot K), \( A \) surface area of the thermocouple cross section in m\(^2\), \( \rho \) average density of the thermocouple in kg/m\(^3\), \( c \) average-specific heat of the thermocouple in J/(kg\cdot K), \( P \) perimeter of the internal surface of the housing in m, \( T_w \) housing temperature in °C, \( \sigma = 5.67 \times 10^{-8} \) W/(m\(^2\)\cdot K\(^4\)) Stefan-Boltzmann constant and \( \varepsilon_w \) and \( \varepsilon_T \) emissivity of the inner housing and the thermocouple surface, respectively.

The convection and conduction of heat transfer between the fluid and the thermometer housing are characterised by the overall heat transfer coefficient \( u_{\text{out}} \) referenced to the outer housing surface:

\[ \frac{1}{u_{\text{out}}} = \frac{1}{h_{\text{out}}} + \frac{1 + D_{\text{out}}/D_m}{2} \frac{\delta_{w}}{k_w} \]  \tag{11}

where \( h_{\text{out}} \) is the heat transfer coefficient on the outer surface of the housing in W/(m\(^2\)K), \( D_{\text{out}} \) outer diameter of the housing in m, \( \delta_{w} \) housing thickness in m and \( k_w \) housing thermal conductivity in W/(m\cdot K).

The formulas (8) and (11) for the overall heat transfer coefficients were derived using the basic principles of heat transfer [2, 4, 8]. The heat transfer equation for the housing (thermowell) can be written in the following form:

\[ A_w \rho_w c_w \frac{dT_w}{dt} = P_{\text{out}} u_{\text{out}} (T_f - T_w) - P_m u_m (T_w - T) - C \left( T_w^4 - T^4 \right) \]  \tag{12}

where \( A_w \) is the surface area of the housing cross section in m\(^2\), \( \rho_w \) average density of the housing in kg/m\(^3\), \( c_w \) average-specific heat of the housing, J/(kg\cdot K) and \( P_{\text{out}} \) perimeter of the external surface of the housing in m.
Further analysis of the heat exchange between the housing of the thermometer and the temperature sensor omits the heat transfer by radiation \[14\]. This is possible when the gap between the thermowell and the temperature sensor is filled with a non-transparent substance or if one of the two emissivities \(\varepsilon_w\) and \(\varepsilon_T\) is near to zero.

Transforming Eq. (9), we get:

\[
T_w = \frac{A \rho c}{P_{in} u_{in}} \frac{dT}{dt} + T
\]  

(13)

Substituting Eq. (13) into Eq. (12) yields:

\[
\frac{(A \rho c_w)(A \rho c)}{(P_{in} u_{in})(P_{out} u_{out})} \frac{d^2T}{dt^2} + \frac{A \rho c_w}{P_{out} u_{out}} \left[ 1 + \frac{(P_{in} u_{in})(A \rho c)}{(P_{out} u_{out})(A \rho c_w)} \right] \frac{dT}{dt} + T = T_f
\]

(14)

Introducing the following coefficients:

\[
a_2 = \frac{(A \rho c_w)(A \rho c)}{(P_{in} u_{in})(P_{out} u_{out})}, \quad a_1 = \frac{A \rho c_w}{P_{out} u_{out}} \left[ 1 + \frac{(P_{in} u_{in})(A \rho c)}{(P_{out} u_{out})(A \rho c_w)} \right]
\]

(15)

the ordinary differential equation of the second order (14) can be written in the form:

\[
a_2 \frac{d^2T}{dt^2} + a_1 \frac{dT}{dt} + T = T_f
\]

(16)

The initial conditions are:

\[
T(0) = T_0, \quad \frac{dT(t)}{dt} \bigg|_{t=0} = v_T = 0
\]

(17)

Equations (16) and (17) are solved using the Laplace transformation. The operator transmittance \(G(s)\) is as follows:
\[
G(s) = \frac{T(s)}{T_f(s)} = \frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad (18)
\]

The time constants \(\tau_1\) and \(\tau_2\) in Eq. (18) are:

\[
\tau_{1,2} = \frac{2a_2}{a_1 \pm \sqrt{a_1^2 - 4a_2}} \quad (19)
\]

The differential Eq. (16) can be written in the following form:

\[
\tau_1 \tau_2 \frac{d^2T}{dt^2} + (\tau_1 + \tau_2) \frac{dT}{dt} + T = T_f \quad (20)
\]

Equation (20) is solved for a step change in fluid temperature from \(T_0 = 0°C\) to the constant value \(T_f\). Laplace transform of the constant fluid temperature \(T_f\) is \(T_f(s) = T_f/s\), and the operator transmittance assumes the following form:

\[
\frac{T(s)}{T_f} = \frac{1}{s(\tau_1 s + 1)(\tau_2 s + 1)} \quad (21)
\]

Eq. (21) can be written in another form:

\[
\frac{T(s)}{T_f} = \frac{1}{s} + \frac{\tau_1}{\tau_2 - \tau_1} \left( \frac{1}{s + \frac{1}{\tau_1}} - \frac{\tau_2}{\tau_2 - \tau_1} \left( \frac{1}{s + \frac{1}{\tau_2}} \right) \right) \quad (22)
\]

Making inverse Laplace transformation of Eq. (22), the thermometer temperature as a function of time is obtained:

\[
u(t) = T(t) - T_0 = 1 + \frac{\tau_1}{\tau_2 - \tau_1} \exp\left( -\frac{t}{\tau_1} \right) - \frac{\tau_2}{\tau_2 - \tau_1} \exp\left( -\frac{t}{\tau_2} \right) \quad (23)
\]

If we assume in Eq. (23) with \(\tau_2 = 0\), then we obtain Eq. (7) with \(\tau = \tau_1\).
A time delay in the time response of the first order inertial system does not appear. Measuring the temperature of the fluid at high pressure requires the use of thermometers with the massive housings. In such cases, there is a time delay between the temperature indicated by the sensor and the changing temperature of the fluid. The inertial system of second order is suitable to describe the response behaviour with a time delay [13].

3. Identification of time constants

The time constant $\tau$ in Eq. (7) or time constants $\tau_1$ and $\tau_2$ in Eq. (23) can be determined on the basis of experimental data. For this purpose, the method of least squares is proposed to use [13, 15]. Finding the minimum of the function $S$

$$S = \sum_{i=1}^{N} [u_m(t_i) - u(t_i)]^2 = \min$$

(24)

allows to determine values of the time constants. In Eq. (24), $u(t_i)$ denotes the approximating function given by Eq. (7), and $N$ is the number of conducted measurements ($t_i, u_m(t_i)$). The sum of the squares of the differences of the measured values $u_m(t_i)$ and the fitted values $u(t_i)$ is minimised. When the time constant $\tau$ or time constants $\tau_1$ and $\tau_2$ are determined, their values can be substituted into Eq. (24) to calculate $S_{\text{min}}$.

The uncertainties of the calculated time constant $\tau$ or time constants $\tau_1$ and $\tau_2$ are calculated using the mean square error [13, 16–18]:

$$S_{\mu} = \sqrt{\frac{S_{\text{min}}}{N - m}}$$

(25)

where $m$ denotes the amount of time constants (i.e. $m = 1$ for Eq. (7) and $m = 2$ for Eq. (7) and $m = 2$ for Eq. (23)). Based on the determined mean square error $S_{\mu}$, which is an approximation of the standard deviation, the uncertainties in the determined time constants can be calculated using the formulas given in the TableCurve 2D software [18].

4. Determination of transient fluid temperature

The fluid temperature can be determined on the basis of measured histories of the thermometer temperature $T(t)$ and known time constant $\tau$, using Eq. (1), or time constants $\tau_1$ and $\tau_2$, using Eq. (20). Temperature of the thermometer $T(t)$ and its first- and second-order time derivatives can be smoothed using the formulas [3, 13]:
Symbol \( f(t) \) in Eqs. (26)–(28) denotes the temperature measured by the thermometer, and \( \Delta t \) is a time step. Application of nine-point digital filter allows eliminating the influence of random errors of the measured temperature \( T(t) \) on the determined temperature of the fluid \( T_f(t) \). If the measured temperature is not disturbed by significant errors, the first and second derivatives of the temperature can be estimated using the central difference method [15]:

\[
T'(t) = \frac{dT(t)}{dt} = \frac{1}{1188\Delta t} \left[ 86f(t - 4\cdot\Delta t) - 142f(t - 3\cdot\Delta t) - 193f(t - 2\cdot\Delta t) - 126f(t - \Delta t) + 126f(t + \Delta t) + 193f(t + 2\cdot\Delta t) + 142f(t + 3\cdot\Delta t) - 86f(t + 4\cdot\Delta t) \right]
\]

\[
T''(t) = \frac{d^2T(t)}{dt^2} = \frac{1}{462(\Delta t)^2} \left[ 28f(t - 4\cdot\Delta t) + 7f(t - 3\cdot\Delta t) - 8f(t - 2\cdot\Delta t) - 17f(t - \Delta t) - 20f(t) - 17f(t + \Delta t) - 8f(t + 2\cdot\Delta t) + 7f(t + 3\cdot\Delta t) + 28f(t + 4\cdot\Delta t) \right]
\]

5. Examples of application of the method for a step change in temperature of the fluid

The first example demonstrates the use of the described method of transient fluid temperature measurement in the case of thin sheathed thermocouple application. During the experiment, the thermocouple with outer diameter 1.5 mm at the ambient temperature was suddenly immersed into hot water at saturation temperature. The results are presented in Figure 2. The time step \( \Delta t \) of the measured temperature was 1.162 s. The measured history of the temperature was approximated by Eq. (7) in order to determine the time constant \( \tau \) of the thermocouple. In the calculations, TableCurve 2D code [18] was used. The calculations of the time constant and the uncertainty at the confidence level of 95% gave the following results: \( \tau = 1.54 \pm 0.09 \) s.
Next, the transient fluid temperature $T_f$ was determined using Eq. (1) together with Eqs. (26) and (27). The first-order time derivative $dT/dt$ in Eq. (1) was also calculated using the central difference quotient of Eq. (29). The results obtained show that the use of a first-order model for thin thermometers is sufficient (Figure 2). The results also indicate that the central difference approximation of the time derivative in Eq. (1) leads to less accurate results, since it is more sensitive to random errors in the experimental data.

Another example shows the application of the method for measuring transient fluid temperature with an industrial thermometer with massive thermowell and its complex construction (Figure 3). As in the previous example, the thermometer at the ambient temperature was suddenly immersed into hot water at a temperature of about 100°C. To compare the two methods of determining the unknown fluid temperature (using the first- and second-order model) for this thermometer, Eqs. (7) and (23) were used as the functions approximating the transient response of the thermometer. The following values with the 95% confidence uncertainty were obtained: $\tau = 14.07 \pm 0.39$ s, $\tau_1 = 3.0 \pm 0.165$ s and $\tau_2 = 10.90 \pm 0.2$ s.

Next, the fluid temperature changes were determined from Eq. (1) for the first-order model and from Eq. (20) for the second-order model (Figure 4).

The analysis of the results presented in Figure 4 indicates that the second-order model delivers more accurate results than the first-order model.
Figure 3. Industrial thermometer and its dimensions: $D = 18$ mm, $l = 65$ mm and $L = 140$ mm.

Figure 4. Fluid and industrial thermometer temperature changes determined from the first-order equation (1) and the second-order equation (20).
6. Experimental determination of time constant as a function of fluid velocity

The method presented in this chapter can also be used to determine the temperature of the flowing fluid. In this case, the time constant of the thermometer $\tau$ should be determined as a function of fluid velocity $w$. After specifying function $\tau(w)$, it should be substituted into Eq. (1).

Research has shown that for thin sheathed thermocouples with a diameter ranging from 0.5 to 6 mm, the equation of time constant as a function of fluid (air) velocity is as follows [15]:

$$\tau = \frac{1}{a + bw}$$

where $\tau$ is the time constant of the thermometer in s, $a$ constant in $1/s$, $b$ constant in $(m\cdot s)^{-1/2}$ and $w$ fluid velocity in m/s.

As an example, the constants $a$ and $b$ were determined for K-type sheathed thermocouples with grounded hot junction with the outer diameter of 0.5, 1.0, 1.5 and 3.0 mm.

Figure 5. Wind tunnel used for determining thermocouple time constant—overall view [19]: (1) test chamber with an opening for the thermometer, (2) differential pressure measurement, (3) data acquisition system and (4) opening for the thermometer insertion.

The thermocouple time constant $\tau$ for various air velocities $w$ was determined in an open benchtop wind tunnel (Figure 5). The WT4401-S benchtop wind tunnel is designed to give uniform flow rate over a 100 mm × 100 mm cross section [19].

The calculated time constants $\tau$ of the sheathed thermocouples with the outer diameter of 0.5, 1.0, 1.5 and 3.0 mm for different velocities of the air are shown in Figure 6. The experimental data collected for all thermocouples, as presented in Figure 6, were approximated by the least
The best estimates for the constants $a$ and $b$, with uncertainty at the 95% confidence level, for the tested thermocouples are [15]:

- $d_i = 0.5$ mm
  
  \[
  a = 0.004337 \pm 0.000622 \text{ } 1/\text{s} \text{ and } b = 0.022239 \pm 0.001103 \text{ (m/s)}^{-1/2}
  \]

- $d_i = 1.0$ mm
  
  \[
  a = 0.020974 \pm 0.006372 \text{ } 1/\text{s} \text{ and } b = 0.103870 \pm 0.011240 \text{ (m/s)}^{-1/2}
  \]

- $d_i = 1.5$ mm
  
  \[
  a = 0.040425 \pm 0.003301 \text{ } 1/\text{s} \text{ and } b = 0.056850 \pm 0.004479 \text{ (m/s)}^{-1/2}
  \]

- $d_i = 3.0$ mm
  
  \[
  a = 0.128220 \pm 0.035716 \text{ } 1/\text{s} \text{ and } b = 0.220641 \pm 0.051122 \text{ (m/s)}^{-1/2}
  \]

The time constant of the thermocouple $\tau$ strongly depends on the heat transfer coefficient $h_i$ on the outer thermometer surface, which results from Eq. (2). The heat transfer coefficient is a function of Nusselt number, and this, in turn, is a function of fluid (air) velocity [20].

Figure 6. Time constants $\tau$ of sheathed thermocouple with outer diameters of 0.5, 1.0, 1.5 and 3.0 mm as a function of air velocity $w$ with 95% confidence intervals.
When the velocity of the fluid and its temperature varies over time, time constant as a function of the velocity formulated by Eq. (31) can be used in Eq. (1) to determine the transient fluid temperature based on measurements made using a sheathed thermocouple.

To show how the described method can improve the temperature readings, the experimental measurements were presented [21]. The temperature of the flowing air in an open wind tunnel (Figure 7) was measured by K-type sheathed thermocouples with outer diameters of 0.5, 1.0 and 1.5 mm. During measurements, the temperature and velocity of air flowing through the tunnel were changed. The thermocouples were placed in the tunnel behind the heat exchanger and very close to each other (i.e. they measured the same temperature, but did not influence each other). The air velocity was measured by the vane anemometer FV A915 S220. Both the temperature and velocity data were collected using the Ahlborn ALMEMO 5990-0 data acquisition system.

![Diagram of an open wind tunnel](image)

**Figure 7.** Diagram of an open wind tunnel [22]: (A) heat exchanger, (B) fan, (C) chamber, (D) air channel, (E) water outlet pipe and (F) hot water feeding pipe.

The comparison of the computed temperatures with the measured temperatures, when the time constants of the thermocouples are known, shows that the above method provides decent results (Figures 8 and 9).

The time histories of temperature obtained from calculations are very similar, especially for thermocouples with the outer diameter of 0.5 and 1.0 mm. In the small degree, the temperature calculated using the measurements with the thermocouple with the outer diameter of 1.5 mm deviates from them. This difference is due to the large time constant of the sheathed thermocouple with the outer diameter of 1.5 mm.
Figure 8. Temperature of the air measured by the thermocouples with outer diameters of 0.5, 1.0 and 1.5 mm and temperature calculated by Eq. (1) when the velocity of the air was constant.

Figure 9. Temperature of the air measured by the thermocouples with outer diameters of 0.5, 1.0 and 1.5 mm and temperature calculated by Eq. (1) when the velocity of the air was changed.
7. Conclusions

Both methods of transient fluid temperature measurement presented in the chapter can be used online.

The first method, in which a mathematical model of the thermometer is first-order differential equation, is the most suitable for thermometers with very small time constants. In such cases, the delay of thermometer indication is small compared to changes in fluid temperature. In turn, the industrial thermometers, which are designed to measure the temperature of the fluid of high pressure, are characterised by a considerable delay of indications in reference to the actual changes of fluid temperature. For such thermometers, the second-order thermometer model, allowing for modelling the signal delay, is more adequate [14].

The method described in this chapter is the most suitable for measuring the transient temperature of gases, such as air or exhaust gases. This is due to the fact that the time constant depends on the heat transfer coefficient on the outer surface of the thermometer and in turn on the Reynolds and Prandtl numbers. For gases such as air or exhaust gas, Prandtl number varies slightly in a wide range of temperatures. However, during measurement of the transient steam temperature, the value of Prandtl number varies significantly depending on the pressure and temperature [23]. In this case, the inverse marching method described in [24] is more appropriate for correction of the dynamic error.

Author details

Magdalena Jaremkiewicz

Address all correspondence to: mjaremkiewicz@pk.edu.pl

Cracow University of Technology, Institute of Thermal Power Engineering, Cracow, Poland

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