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Electronic Structure and Piezoelectric Properties of SbSI Crystals

Algirdas Audzijonis, Leonardas Žigas, Raimundas Sereika and Raimundas Žaltauskas

Additional information is available at the end of the chapter
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Abstract

The SbSI crystals are investigated in the paraelectric and ferroelectric phases. The calculations have been performed to find the symmetry and normal coordinates of vibrational modes. We have observed that potential energy with double well create the soft mode of $B_1^u$ symmetry in the microwave range and semisoft modes in the IR range. The $A_u$ and $B_g$ symmetry, top electronic levels of the highest valence band, are degenerate in the paraelectric phase. It is shown that the Jahn-Teller effect is caused by $A_u$ symmetry normal mode interacting with the degenerate $A_u$ symmetry electronic states in the valence band top. The pseudo-Jahn-Teller effect is induced due to the same mode interacting with $A_u$ symmetry electronic states in the valence band and $B_g$ symmetry states in the conduction band bottom. Concerning these two effects, the normal mode force constant $K$ decreases and vibrational constants undergo changes during the phase transition. The theoretical deformation along the crystallographic—$x(a)$, $y(b)$, and $z(c)$—axes were studied for Sb atoms. The major change of piezoelectric modulus takes place in the ferroelectric phase near the phase transition temperature. At lower temperatures piezoelectric modulus changes become slow. The value as well as anomalous temperature dependence of piezoelectric modulus and $\Delta\varepsilon(T)$ is influenced by the change of mean potential energy $\bar{V}_p(z)$ of Sb atoms in soft mode.

Keywords: SbSI crystals, potential energy, pseudo-Jahn-Teller effect, normal vibrational modes, piezoelectric modulus
1. Potential energy of normal vibrational modes

1.1. Introduction

SbSI has a classical ferroelectric phase (FEF) transition at the temperature \( T_c = 295 \) K \([1, 2]\). This crystal has large interest to the fundamental physical properties as well as for the promising application possibilities. It is known that it has large temperature and electric field dependence of the optical characteristics \([3]\), electronic structure \([4, 5]\), and other features. Petzelt \([6]\) observed in the IR reflectivity spectra a strong temperature dependence of the lowest-frequency dielectric constant. Sugawara et al. \([7]\) has determined strong temperature dependence of reflectivity for \( E \parallel c \) in the range of 0–70 cm\(^{-1}\). According to the author \([7, 8]\), the frequencies of the soft mode are 4.0 and 6.5 cm\(^{-1}\) at temperatures 298 and 318 K in the IR range of SbSI spectrum. Although some authors \([9]\) demonstrate result \( n (\approx 90\%) \) of the frequency \( \approx 9 \) cm\(^{-1}\) to the static dielectric constant \( \varepsilon(0) \) at the room temperature, others claim the soft mode with frequency 10 cm\(^{-1}\) to be insignificant \([10]\). The soft mode consisting of two components has been found in the SbSI \([11]\), SbSI-SbSeI \([12]\), SbSI-BiSI \([13]\), and SbSI-SbSBr systems \([14]\) in the microwave range. The first component is a soft microwave mode bringing the major contribution in \( \varepsilon(0) \), and the second is a semisoft mode in the IR range and contributes to \( \varepsilon(0) \), which is less than 10\% \([15]\).

2. Symmetrical and normal coordinates

In \([16]\) we have described the symmetrical and normal coordinates which were used to calculate electronic potential. As known, in the adiabatic approximation the vibration energy \( E = T + V \) of a system of atoms can be expressed via normal coordinates \( Q \) as follows:

\[
T = \frac{1}{2} \sum_k \left( \frac{dQ_k}{dt} \right)^2, \quad V = \frac{1}{2} \sum_k \lambda_k Q_k^2, \quad (1)
\]

The force constants \( \lambda_k \) are solution of the characteristic equation

\[
|\lambda a_q - b_q| = 0 \quad (2)
\]

where \( a_q \) and \( b_q \) are factors of the energy expression in orthogonal symmetry coordinates \( F \):

\[
T = \frac{1}{2} \sum_q a_q \frac{dF_q}{dt}, \quad V = \frac{1}{2} \sum_q b_q F_q F_q. \quad (3)
\]

Symmetry coordinates are defined as particular combinations of Cartesian components \( x_\nu, y_\nu, \) and \( z_\nu \) of displacements \([17]\). Similarly to the vibrations of \( \text{Sb}_2\text{S}_3 \), the ones of SbSI belong to the same irreducible representations \( \Gamma_\nu \) since the space groups of both crystals are \( D_{2d}^{16} \) in the
The ferroelectric phase and \( C^9_{2v} \) in the paraelectric phase. Symmetry operations of the space group \( D^{16}_{2h} \) are S1 to S8 [18, 19]. Some of them, such as \( S_1, S_2, S_4, \) and \( S_8 \), are connected with the space group \( C^9_{2v} \). The effect of these symmetry operations is the same in the unit cell that consists of four molecules (SbSI), as in the SbSI unit cell [20] (Table 1).

<table>
<thead>
<tr>
<th>Atoms Number</th>
<th>( D^{16}_{2h} )</th>
<th>( C^9_{2v} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rao [19]</td>
<td>Balkanski [18]</td>
<td>S1</td>
</tr>
<tr>
<td>SbSI</td>
<td>SbSI</td>
<td>S1</td>
</tr>
</tbody>
</table>

Table 1. Symmetry operations of the space groups \( D^{16}_{2h} \) and \( C^9_{2v} \) in SbSI crystals.

The operator of projection is

\[
\mathbf{S}^{a}_{\mathbf{Y}} = \frac{1}{g} \sum_{k} \Gamma_{a} (S_{k})_{Y} S_{k}.
\]

The Cartesian components \( K_{a} = x_{a} y_{a} z_{a} \) of atom displacements for all their linear combinations have certain symmetry properties with respect to operations of groups \( D^{16}_{2h} \) and \( C^9_{2v} \):

\[
K^{a}_{\mathbf{Y}} (\Gamma_{a}) = \sum_{n} a_{nm} K_{m}.
\]

Here \( A = \text{Sb, S, I} \); \( n = 1, 2, 3, 4 \); \( a_{nm} = \pm 1 \). These combinations of \( K_{a} \) are basis functions of \( \Gamma_{a} \).
Group $D^{16}_{2h}$ has irreducible representations $A_{ig}$, $B_{ig}$, $A_{u}$, and $B_{u}$ of the space group $C^{9}_{2h}$. Table 2 [21]. Subscripts denote the coordinates symmetric ($g$), or antisymmetric ($u$) with respect to inversion $I$. The obtained $K_{Am}$ combinations are also the basic functions of irreducible representations $A_{i}$, $B_{i}$ of the space group $C^{9}_{2h}$.

<table>
<thead>
<tr>
<th>$D^{16}_{2h}$</th>
<th>$C^{9}_{2h}$</th>
<th>$C^{y}_{2}$</th>
<th>$C^{z}_{2}$</th>
<th>$C^{x}_{2}$</th>
<th>$I$</th>
<th>$\sigma_{y}$</th>
<th>$\sigma_{z}$</th>
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<tr>
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<td>r1 $A_{ig}$</td>
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<td>1</td>
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<td>1</td>
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<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>r8 $B_{ig}$</td>
<td>r3 $B_{ig}$</td>
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<td>1</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2. Group characters of irreducible representations of space groups $D^{16}_{2h}$ and $C^{9}_{2h}$ [21].

Orthogonal symmetry coordinates of the entire unit cell can be formed of $K_{Am}(\Gamma_{\alpha})$ by solving equations of orthonormality and are uniform with respect to $A$. Their expressions are as follows:

$$F_{\alpha}(K_{Am}) = N_{\alpha} \sum_{m} C_{n} K_{Am}^{n}(\Gamma_{\alpha})$$

Normalization factors $N_{\alpha}$ and coefficients $C_{n}$ are presented in $z$ direction and $x$, $y$ directions in Table 3 [22]. The group theoretical analysis reveals that they are the unit cell of $D^{16}_{2h}$ point group with 12 atoms per cell and contains 33 optical modes and 3 acoustic modes. Among the optical modes, there are 18 Raman-active modes with the symmetries $6A_{1} + 6B_{1} + 3B_{2} + 3B_{3}$. 3 silent modes belong to the $A_{u}$ irreducible representation, as well as 12 infrared active modes that have symmetries $2B_{1u} + 5B_{2u} + 5B_{3u}$. In $z$ direction, only $2B_{1u}$ modes are infrared active in the paraelectric phase (Table 3), whereas in the same direction, the modes $2A_{1} + 3A_{2} + 3B_{1} + 3B_{2}$ are Raman active in the ferroelectric phase. Note that $2A_{1} + 3B_{1} + 3B_{2}$ are also infrared active among these modes.
Taking into account the fact that SbSI crystal is made of two double chains with a weak interaction between them at $T = 0$ K (20–30 times weaker than within a chain; see Figure 1), the formation of dynamical matrix 3 becomes easier.

<table>
<thead>
<tr>
<th>$\Gamma_s$</th>
<th>$S$</th>
<th>$N_x$</th>
<th>Sb$_1$</th>
<th>Sb$_2$</th>
<th>Sb$_3$</th>
<th>Sb$_4$</th>
<th>S$_1$</th>
<th>S$_2$</th>
<th>I$_1$</th>
<th>I$_2$</th>
<th>I$_3$</th>
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<td>−1</td>
<td>−1</td>
<td>+1</td>
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<td>−1</td>
<td>+1</td>
<td>+1</td>
<td>−1</td>
<td>+2</td>
</tr>
</tbody>
</table>

Table 3. Symmetry coordinates of normal modes of SbSI crystals in $z(c)$ direction.

In our research, force constants for a long chain of strongly bound atoms in the direction of $z(c)$-axis were defined by composing five simplified unit cells (30 atoms). The GAMESS program in the basis sets of atomic functions 3G, 3G + d, 21G, 21G + d were used to calculate force constants.

When atoms are vibrating, interaction between double chains strengthens. Therefore, following Furman et al. [23], we calculated normal models employing the Born-von Karman model and the elementary cell of four pseudomolecules SbSI (12 atoms). The binding constants and ionic charges, 0.19 (Sb), −0.01 (S), −0.18 (I), were also calculated. It appeared that covalent interactions prevailed in the SbSI crystal.
3. The lattice anharmonism on the vibrational spectrum along the \( z(c) \) direction

In [24] we have proposed the method for calculating the total potential energy. The potential energy (PE) is defined as follows:

\[
V(r) = \frac{4\pi}{\Omega} \sum_\alpha |f_\alpha(s)|^2 \exp[-i(\mathbf{R}_\alpha + \mathbf{Q}_\alpha) \cdot \mathbf{s}] \exp[-M_\alpha(s)],
\]

(7)

where \( \Omega \) stands for the volume of the unit cell, \( Q \) represents the normal coordinate, \( R_\alpha \) denotes the radius-vector of the atom position in the unit cell, \( s \) means the reciprocal lattice vector, \( \alpha \) represents the number and kind of the atom in the unit cell, and \( \exp[-M_\alpha(s)] \) represents the Debye-Valler factor that is determined by the mean square amplitudes of atomic displacements (i.e., by the crystal temperature).

In fact, Eq. (7) at a certain point in the unit cell is the PE in terms of normal coordinates (or symmetrized plane waves). The atomic scattering form factor is
where \( nlm \) represents a set of quantum numbers for the atom \( \alpha \). It is noteworthy that the functions \( <nlm> \) of all electronic states of an atom was used for calculating the form factors \( f_\alpha (s) \) by Eq. (8). About 5000 vectors \( s \) were used for the sum in Eq. (7).

For SbSI, we studied the dependence of potential energy \( V(z) \) in the paraelectric phase at \( T = 420 \) K on the normal coordinates of all \( D_{20}^{16} \) symmetry normal modes that formed by displacements of the atoms along the \( z \) direction (along the polar axis \( c \); Figure 2). This dependence was calculated by Eqs. (7) and (8). We determined that the curves \( V(z) \) can be described by the equation

\[
V(z) = V_0 + az + bz^2 + cz^3 + dz^4,
\]

Figure 2. Dependence of the potential energy \( V(z) \) \((T = 415 \) K\) upon the normal coordinates in the \( z(c) \) direction for \( D_{20}^{16} \) symmetry \( B_{1u}(2), B_{1u}(3), B_{2g}(4, 5), B_{3g}(6), B_{3g}(7), B_{1g}(8), B_{1g}(9), A_{1u}(10), A_{1u}(11), \) and \( A_{1u}(12) \) normal modes.

where the coefficients \( a = d = 0 \) in the paraelectric phase. The displacements have been chosen in regard to the equilibrium positions of atoms. The curves \( V(z) \) of symmetry \( B_{1u}(3), B_{2g}(4, 5), B_{3g}(7), \) and \( A_{1u}(10) \) are correspondent to coefficients \( b < 0 \) and \( c > 0 \). These curves \( V(z) \) have two side minima at distances of 0.3 a.u. from the equilibrium position. A potential barrier \( \Delta V = b^2/4c \) separates the minima. Conversely, the curves \( V(z) \) of symmetry \( B_{1u}(2), B_{2g}(6), B_{1g}(8), B_{1g}(9), A_{1u}(11), \) and \( A_{1u}(12) \) have a single minima and the coefficients \( b > 0 \). The coefficient is \( b = Ma^2, \)
where $M$ represents the reduced mass of the normal mode atoms, and $\omega$ denotes the vibration frequency of the normal mode. The frequency of a normal mode with a double-well $V(z)$ is dependent on the height of the potential barrier $\Delta V$. Temperature, electric field, pressure, and fluctuations of atoms in the $x$-$y$ plane influence this potential barrier [15]. In all the three phases, $V(x)$ and $V(y)$ appear to be single-well with the coefficients $b > 0$ in Eq. (9). In the paraelectric phase, $a = d = 0$, whereas in the antiferro- and ferroelectric phases, $a \neq 0$ and $d \neq 0$. Single-well PE turns out to be weakly anharmonic and the frequencies of the $R(k)$ peak should only slightly depend on the temperature.

The experimental results of IR reflectivity $R(k)$ spectrum of SbSI crystals in the paraelectric ($T = 415$ K) and ferroelectric ($T = 273$ K) phases for $E \parallel c$ are presented in Figure 3. The reflectivity measurements of SbSI crystals have been repeated by Dr. Markus Goeppert in Germany in Karlsruhe University using the Bruker Fourier spectrometer.

Figure 3. The reflectivity spectra of SbSI for $E \parallel c$ in the paraelectric phase ($T = 415$ K) and in the ferroelectric phase ($T = 273$ K). Inset: the reflectivity spectra in range 50-95 cm$^{-1}$.

In the reflectivity spectrum for $E \parallel c$ in the range of $k = 10-100$ cm$^{-1}$, the number of peaks $R(k)$ is equal for the paraelectric and ferroelectric phases. However, in the range of $k = 100-200$ cm$^{-1}$, the number of peaks $R(k)$ differs in both phases. Anharmonic modes with double-well $V(z)$ are highly sensitive to dislocations, impurities, and fluctuations of chains in the $x$-$y$ plane.

Figure 2 demonstrates that modes $B_{1g}(3), B_{2g}(4), B_{2g}(5)$, and $B_{2g}(7)$ with a double-well $V(z)$ are strongly anharmonic, whereas modes $B_{1u}(2), B_{2g}(6), B_{3g}(8)$, and $B_{3g}(9)$ with a single-well $V(z)$ appear to be weakly anharmonic. The vibration frequencies of the former modes are lower
compared to the latter ones. Therefore, the $R(k)$ peaks are created by strongly anharmonic modes $B_{1s}(3) \rightarrow A_{u}$, $B_{2g}(4) \rightarrow B_{g}$, $B_{2g}(5) \rightarrow B_{g}$, and $B_{3g}(7) \rightarrow B_{z}$ in the range of $k = 10–100 \text{ cm}^{-1}$ of the IR spectrum. In the range of $k = 100–400 \text{ cm}^{-1}$, the $R(k)$ peaks are created by weakly anharmonic modes. Following the optical selection rules, one $R(k)$ peak in the paraelectric phase is created by the mode $B_{1u}(2)$, while the peaks in the ferroelectric phase are created by modes $B_{1s}(2) \rightarrow A_{u}$, $B_{2g}(6) \rightarrow B_{g}$, and $B_{3g}(9) \rightarrow B_{z}$. In the paraelectric phase, there are three silent modes of $A_{u}$ symmetry: $A_{u}(10)$, $A_{u}(11)$, and $A_{u}(12)$ (Table 3). Two of them, $A_{u}(11)$ and $A_{u}(12)$, comply with the out-of-phase motion of the infrared-active modes. It is expected that the anharmonic mode $A_{u}(10)$ has a low frequency. This mode can be optically active and may cause a weak peak in the $k < 10 \text{ cm}^{-1}$ IR range.

4. The piezoelectricity phenomena

4.1. Introduction

SbSI is a piezoelectric crystal that has a high-volume piezoelectric modulus $d_v = 10 \times 10^{-10} \text{ C/N}$ and electromechanical coupling coefficient $k_3 = 0.87$ [25]. The relation among polarization $P$, piezoelectric modulus $e$, deformation $r = \Delta l/l$, and elastic compliance coefficient is as follows:

$$\Delta P = er, \Delta P = q\Delta z,$$

(10)

$$d = es.$$

(11)

where $q$ is the atom’s ionic charge, $s$ is the elastic compliance coefficient, $\Delta z = z - z_0$, $z_0$ is the atomic equilibrium position, when crystal is not deformed ($r = 0$), and $z$ is the atomic equilibrium position, when crystal is deformed ($r \neq 0$). It is handy to calculate the piezoelectric modulus $d$ along $z$-axis for needle-like crystal deforming it along the crystallographic axes $x(a)$, $y(b)$, and $z(c)$ when $r = \Delta a/a = \Delta b/b = \Delta c/c$… Having $e$ it is possible to calculate $d$ according to Eq. (11). The dependence of elastic compliance coefficients on temperature in SbSI crystal is shown in [26]. From work [27], piezoelectric module $d_{33}$ along $z$-axis and $d_{3i}$ (the so-called “hydrostatic piezoelectric modulus”) were known. As $d_{3i}$ has been found to be of the same magnitude as $d_{3j}$, it has been concluded that $d_{3i}$ is nearly equal to $d_{3j}$ and opposite in sign. Both quantities $d_{3j}$ and $d_{3i}$ are smaller than $d_{33}$. In [28] it is found that $d_{31}$ is of the same order of magnitude as $d_{33}$. The $d_{32}$ coefficient is of nearly the same value as $d_{33}$ but opposite in sign. The high values of $d_0 = d_{33} + d_{32} + d_{31}$ indicate that $d_{33}$ and $d_{32}$ are relatively small or of opposite in sign.
5. Potential energy of Sb atoms in anharmonic soft mode

In [29] we described the method for calculating the potential energy at point \( r \) of the unit cell. The potential energy at the point \( r \) is given in Eq. (7). The average value of the potential energy of Sb atoms of unit cell in the normal mode of SbSI crystal may be written as follows:

\[
V_p = \frac{V_p(R_{Sb1}) + V_p(R_{Sb2}) + V_p(R_{Sb3}) + V_p(R_{Sb4})}{4},
\]

where \( R_{Sb1} = R_{0,Sb1} + Q_{Sb1}; R_{Sb2} = R_{0,Sb2} + Q_{Sb2}; R_{Sb3} = R_{0,Sb3} + Q_{Sb3}; \) and \( R_{Sb4} = R_{0,Sb4} + Q_{Sb4}. \)

Also, the coordinates of all S and I atoms changes according to the equations:

\[
R_\alpha = R_{0,\alpha} + Q_\alpha,
\]

where \( \alpha \) = S\(_1\); S\(_2\); S\(_3\); S\(_4\); I\(_1\); I\(_2\); I\(_3\); I\(_4\). For numerical evaluation of Eq. (12), we need to vary all \( Q_\alpha = z_\alpha \) by small steps from \(-Q_\alpha(max)\) to \(+Q_\alpha(max)\). The atomic form factor is

\[
f_\alpha(s) = \sum_{nlm} \exp[i (r \cdot s)] \times a_{nlm},
\]

where \( nlm \) is a set of quantum numbers for the atom \( \alpha \).

6. Antimony atoms’ equilibrium positions

For theoretical investigations, we used the most sensitive to temperature and deformations soft mode’s \( \bar{V}_p(z) \) dependence on normal coordinates [29]. It is handy to expand \( \bar{V}_p(z) \) using polynomial:

\[
\bar{V}_p(z) = V_0 + a^* z^2 + b^* z^4 + c^* z^6,
\]

where \( a^*, b^*, d^*, \) and \( c^* \) are polynomial expansion coefficients.

The \( \bar{V}_p(z) \) dependence on normal coordinates along \( z(\epsilon) \)-axis in paraelectric phase (PEF) at \( T = 300 \) K is shown in Figure 4 while deforming along \( x(\alpha), y(\beta), \) and \( z(\epsilon) \) axes. The \( \bar{V}_p(z) \) in PEF has a shape of double-well \((b^* < 0, c^*> 0)\). The barrier height when crystal is not deformed is \( \Delta U_C = k T_C \) where \( k \) is the Boltzmann constant. As seen from Figure 4, in paraelectric phase \( \Delta z \) are equal to zero, because \( \bar{V}_p(z) \) of Sb atomic soft mode is symmetrical and double-well. In paraelectric phase, Sb atoms vibrate between two minima of double-well potential \( \bar{V}_p(z) \), provided deformation \( r \neq 0 \) and \( r = 0 \).
Figure 4. The SbSI mean potential energy dependence on soft mode normal coordinates at the Sb atoms sites in paraelectic phase ($T = 300$ K), when deformation $r = 0.03$ along $x(a)$, $y(b)$, and $z(c)$ axes and $r = 0$, $\Delta z = 1$ a.u. = 0.53 Å.

Due to the decrease in the temperature in ferroelectric phase, double-well $\overline{V}_p(z)$ gradually changes its form (see Figure 2) from double-well to single-well ($a^* \neq 0$, $b^* < 0$, $d^* \neq 0$, $c^* > 0$).

We calculated SbSI crystal’s Sb atom potential energy’s dependence on soft mode’s normal coordinates in FEF at $T = 289$ K, when crystal is deformed along $x(a)$, $y(b)$, and $z(c)$ axes, and when crystal is not deformed ($r = 0$; Figure 5). $z_0$ is the equilibrium position of Sb atom when $r = 0$ and $z$ is the Sb atom’s equilibrium position when $r \neq 0$. Calculated temperature dependence of $z$, $\Delta z = z - z_0$ and $\overline{V}_p(z)$ scanning coefficients $a^*$, $b^*$, $d^*$, and $c^*$ are shown in Tables 4 and 5. Temperature dependence of equilibrium position $z_0$ of Sb atoms in ferroelectric phase (when $r = 0$) is calculated using Eq. (8):

$$z_0 = 0.24988 + 0.00221 T - 3.13691 E - 5 T^2 + 1.42721 E - 7 T^3 - 2.37137 E - 10 T^4 (\text{Å}),$$

Figure 5. The SbSI mean potential energy dependence on soft mode normal coordinates at the Sb atoms sites in ferroelectric phase ($T = 289$ K), when deformation $r = 0.03$ along $x(a)$, $y(b)$, and $z(c)$ axis and $r = 0$, $\Delta z = 1$ a.u. = 0.53 Å.
Table 4. Scanning coefficients $a^*$, $b^*$, $c^*$, and $d^*$ at different temperatures in SbSI crystal when crystal is not deformed ($r = 0$) and deformed along $x(a)$, $y(b)$, and $z(c)$ axes, when $r = \Delta a/a = \Delta b/b = \Delta c/c = 0.03$.

<table>
<thead>
<tr>
<th>$T$, K</th>
<th>$a^*$</th>
<th>$b^*$</th>
<th>$c^*$</th>
<th>$d^*$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>0</td>
<td>-0.058422</td>
<td>0.237293</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-0.078223</td>
<td>0.260437</td>
<td>0</td>
<td>$\Delta c/c$</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-0.021201</td>
<td>0.188036</td>
<td>0</td>
<td>$\Delta b/b$</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-0.068322</td>
<td>0.248865</td>
<td>0</td>
<td>$\Delta a/a$</td>
<td></td>
</tr>
<tr>
<td>292</td>
<td>-0.015845</td>
<td>-0.049218</td>
<td>0.221618</td>
<td>0.034324</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>-0.016505</td>
<td>-0.069437</td>
<td>0.245634</td>
<td>0.035488</td>
<td>$\Delta a/a$</td>
</tr>
<tr>
<td></td>
<td>-0.014301</td>
<td>0.013858</td>
<td>0.179530</td>
<td>0.030567</td>
<td>$\Delta b/b$</td>
</tr>
<tr>
<td></td>
<td>-0.030185</td>
<td>0.051400</td>
<td>0.226095</td>
<td>0.031457</td>
<td>$\Delta c/c$</td>
</tr>
<tr>
<td>289</td>
<td>-0.044289</td>
<td>-0.030932</td>
<td>0.207956</td>
<td>0.094969</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>-0.046523</td>
<td>-0.049664</td>
<td>0.228457</td>
<td>0.099234</td>
<td>$\Delta a/a$</td>
</tr>
<tr>
<td></td>
<td>-0.039959</td>
<td>0.004717</td>
<td>0.164141</td>
<td>0.084533</td>
<td>$\Delta b/b$</td>
</tr>
<tr>
<td></td>
<td>-0.05896</td>
<td>-0.033813</td>
<td>0.210681</td>
<td>0.093388</td>
<td>$\Delta c/c$</td>
</tr>
</tbody>
</table>

Table 5. Temperature dependence of Sb atom’s equilibrium positions displacements’ $\Delta z_{31}$, $\Delta z_{32}$, and $\Delta z_{33}$ along $x(a)$, $y(b)$, and $z(c)$ axes when deformation is $r = \Delta a/a = \Delta b/b = \Delta c/c = 0.03$, respectively. $\Delta z = 1$ a.u. = 0.53 Å.

<table>
<thead>
<tr>
<th>$T$, K</th>
<th>$\Delta z_{31}$ a.u.</th>
<th>$\Delta z_{32}$ a.u.</th>
<th>$\Delta z_{33}$ a.u.</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>292</td>
<td>0.0759</td>
<td>-0.068310</td>
<td>0.097</td>
</tr>
<tr>
<td>289</td>
<td>0.0487</td>
<td>-0.04383</td>
<td>0.0694</td>
</tr>
<tr>
<td>274</td>
<td>0.0329</td>
<td>-0.02961</td>
<td>0.0384</td>
</tr>
<tr>
<td>250</td>
<td>0.0228</td>
<td>-0.02052</td>
<td>0.0284</td>
</tr>
<tr>
<td>225</td>
<td>0.0173</td>
<td>-0.01587</td>
<td>0.0219</td>
</tr>
<tr>
<td>195</td>
<td>0.0142</td>
<td>-0.01278</td>
<td>0.019</td>
</tr>
<tr>
<td>170</td>
<td>0.0141</td>
<td>-0.01269</td>
<td>0.0158</td>
</tr>
<tr>
<td>150</td>
<td>0.0124</td>
<td>-0.01116</td>
<td>0.0152</td>
</tr>
<tr>
<td>130</td>
<td>0.0107</td>
<td>-0.00963</td>
<td>0.0137</td>
</tr>
</tbody>
</table>

where $T < T_C$ is the temperature of the SbSI crystal in ferroelectric phase.

Since $e = \frac{\Delta P}{r}$ and $r = \text{const.}$, then $e = \Delta P = \Delta z$. Consequently, using the data from Table 5 we present $\Delta z$ dependence on temperature, when crystal is being deformed equally along three axes: $r = \Delta a/a = \Delta b/b = \Delta c/c$ (Figure 6).
Figure 6. Temperature dependence of Sb atom’s equilibrium position’s displacements’ $\Delta z_{31}$, $\Delta z_{32}$, and $\Delta z_{33}$ along $x(a)$, $y(b)$, and $z(c)$ axes when deformation of SbSI crystal is $r = \Delta a/a = \Delta b/b = \Delta c/c = 0.03$, respectively, $\Delta z = 1$ a.u.

Figures 4 and 6 show that close to $T_C$ potential energy of Sb atoms $\bar{V}_{j}(z)$ and Sb atom’s equilibrium position’s displacements $\Delta z_{31}$, $\Delta z_{32}$, and $\Delta z_{33}$ are very sensitive to the temperature changes and deformation of SbSI crystal. In temperature range $T < 250 \text{ K}$, the sensitivity of $\bar{V}_{j}(z)$ and $\Delta z_{31} - e_{31}$, $\Delta z_{32} - e_{32}$, $\Delta z_{33} - e_{33}$ to their temperature changes and deformation directly decreased.

In Figure 7, we compared theoretical results of Sb atoms $\Delta z_{33} - e_{33}$ from Table 5 with experimental piezoelectric modulus $e_{33} = d_{33}/s$ from [2, 11] dependence on temperature when crystal is deformed in direction of $z(c)$-axis.

Figure 7. Temperature dependence of Sb atom’s equilibrium position’s displacement $\Delta z_{33}$ along $z(c)$-axis and experimental [2, 11] piezoelectric modulus $e_{33} = d_{33}/s$. $\Delta z = 1$ a.u.

As seen from Figure 7, the temperature dependences of Sb atom’s equilibrium position’s displacements $\Delta z_{33} - e_{33}$ coincide well with experimental piezoelectric modulus $e_{33} = d_{33}/s$ when crystal is deformed in direction of $z(c)$-axis because $\Delta z_{33}$ is created by anharmonic Sb atom’s potential energy $\bar{V}_{j}(z)$.
Existence of the piezoelectric effect experimentally detected up to a temperature several degrees above the \( T_c \) (Figure 4). The effect above \( T_c \) (Figure 7) may be attributed to the compositional inhomogeneity or the existence of the internal mechanical stresses.

7. Electron-phonon interaction and Jahn-Teller effect (JTE)

7.1. Introduction

Examining the state of instability, the authors [30] have determined that in this respect the normal coordinate of the vibrational mode is \( K = K_0 - K_v < 0 \), where \( K_0 = 1/2 M V_0^2 \) represents the bare force constant, while \( K_v \) stands for a vibrational coupling term, which is dependent on the vibrational coupling constant \( F \) and the energy gap \( \Delta E \). \( C_2^h \) operations are \( S_1 = E/\{0,0,0\} \), \( S_2 = C_2(z)/(0,0,1/2) \), \( S_3 = I/\{0,0,0\} \), and \( S_4 = \sigma_V/(0,0,1/2) \) for the PEP space group, whereas only \( S_1 \) and \( S_2 \) are for the FEP space group \( C_2^h \). The movement of atoms within the chain is marked by energy states of independent normal vibrations, associated with vibrational symmetry coordinates that are composed of the projections of atomic displacements \( K_i = X_i, Y_i, Z_i \) on the axes of local coordinate systems [31, 32]:

\[
\chi_i^{(v)} = N^{(v)} \sum c_{i\mu} K_i
\]

Table 6. Characters of irreducible representations of symmetry space group \( C_2^2 \) and \( C_2^h \).

<table>
<thead>
<tr>
<th>( r_i (C_2^2) )</th>
<th>( r_i (C_2^h) )</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>( S_3 )</th>
<th>( S_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>( A_z )</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>( B )</td>
<td>( B_z )</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td>( A )</td>
<td>( A_y )</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>( B )</td>
<td>( B_y )</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
</tr>
</tbody>
</table>

The coefficients \( c_{i\mu} \) are developed from the properties of irreducible representations \( \Gamma \), obtained when coordinates \( \chi_i^{(v)} \) with symmetry operations \( S_\mu \) are transformed (see Table 6 for the characters of the representations). Symmetry coordinates that are composed of projections \( Z_i \) of atomic displacements usually have symmetry \( A_x \) and \( B_y \) whereas the coordinates that are composed of projections \( X_i \) and \( Y_i \) possess symmetry \( A_z \) and \( B_z \). Consequently, by solving the system of vibrational equations, normal coordinates \( Q_i^{(a)} \) are obtained, in which \( X_i \) and \( Y_i \) are interlinked and \( Z_i \) participates separately. Thus, the normal vibrations in the SbSI chain along the \( z \)-axis can be characterized by \( A_x \) and \( B_z \) symmetry coordinates, whereas the vibrations in the direction of \( x \)- and \( y \)-axes are related to \( B_x \) and \( A_z \) symmetry coordinates. The modes
$X_7(B_u)$ and $X_{13}(B_u)$ comply with acoustic vibrations, whereas the remaining ones correspond to the optical vibrations of the SbSI chain (Table 7).

<table>
<thead>
<tr>
<th>$r_\alpha(c_2^2)$</th>
<th>$r_\gamma(c_{2h}^2)$</th>
<th>$N^{\text{opt}}$</th>
<th>Sb3</th>
<th>Sb4</th>
<th>S3</th>
<th>S4</th>
<th>I1</th>
<th>I2</th>
<th>K1</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>$\frac{1}{\sqrt{6}}$</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>z(1)</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>$\frac{1}{\sqrt{12}}$</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>-2</td>
<td>-2</td>
<td>z(2)</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>$\frac{1}{\sqrt{2}}$</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>z(3)</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>$\frac{1}{\sqrt{6}}$</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>z(4)</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>$\frac{1}{\sqrt{12}}$</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>-2</td>
<td>+2</td>
<td>z(5)</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>$\frac{1}{\sqrt{2}}$</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>z(6)</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>$\frac{1}{\sqrt{6}}$</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>x,y(7, 13)</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>$\frac{1}{\sqrt{12}}$</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>-2</td>
<td>-2</td>
<td>x,y(8, 14)</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>$\frac{1}{\sqrt{2}}$</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>x,y(9, 15)</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>$\frac{1}{\sqrt{6}}$</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>x,y(10, 16)</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>$\frac{1}{\sqrt{12}}$</td>
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<td>+1</td>
<td>-1</td>
<td>-2</td>
<td>+2</td>
<td>x,y(11, 17)</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>$\frac{1}{\sqrt{2}}$</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>x,y(12, 18)</td>
</tr>
</tbody>
</table>

Table 7. Vibrational symmetry coordinates in the simplified unit cell of the SbSI chain.

8. Electron-phonon interaction in one atomic chain

Figure 2 provides the dependence of one-electron energies in the valence band of a long SbSI chain on the number of atoms $N$, which is calculated by the unrestricted Hartree-Fock method (UHF) [33] in the basis set of $H_w$ functions using the pseudopotential. As it is seen from Figure 2, the highest energy levels in the valence band degenerate when $N$ turns to 40 or more. As it is shown in Figure 3, the highest $A_u$ and $A_g$ one-electron energy levels are degenerated in the PEP close to the band gap. The lowest levels of the conduction band are set apart from the highest levels of the valence band by a band gap of $E_g = 5.712$ eV.
Table 8 has been composed with reference to Table 7, where symmetry types of one-electron levels are presented. In Table 8, the energies of the degenerate $A_u$ and $B_g$ symmetry electronic levels in the valence band are highlighted by boldface, where the energies of level 177 $B_g$ and level 178 $A_u$ are $-0.3344 \text{H}$, and the energies of level 179 $A_u$ and level 180 $B_g$ are $-0.3257 \text{H}$ (assuming that $1 \text{H} = 27.21 \text{ eV}$).

<table>
<thead>
<tr>
<th>Valence band</th>
<th>Energy (H)</th>
<th>Conduction band</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetry</td>
<td>Energy (H)</td>
<td>Symmetry</td>
</tr>
<tr>
<td>$176 \ B_g$</td>
<td>$-0.3363$</td>
<td>$181 \ A_u$</td>
</tr>
<tr>
<td>$177 \ B_g$</td>
<td>$-0.3344$</td>
<td>$182 \ B_g$</td>
</tr>
<tr>
<td>$178 \ A_u$</td>
<td>$-0.3344$</td>
<td>$183 \ A_u$</td>
</tr>
<tr>
<td>$179 \ A_u$</td>
<td>$-0.3257$</td>
<td>$184 \ B_g$</td>
</tr>
<tr>
<td>$180 \ B_g$</td>
<td>$-0.3257$</td>
<td>$185 \ A_u$</td>
</tr>
</tbody>
</table>

Note: The degenerate electronic states in the valence band are denoted by boldface.

Table 8. One-electron energies in the conduction band bottom and valence and top for an SbSI chain containing $N = 60$ atoms.

In the phase transition, i.e., when equilibrium positions of the Sb and S atoms alter along the $z(c)$-axis, the degeneracy is removed, and the band gap $E_g$ is narrowed. The decrease of the interatomic force factor $R$ determines the decrease of $E_g$ (Figure 8a). However, the ionic charges $q$ has no influence on the decrease of $E_g$, as they remain almost constant throughout the phase transition (Figure 8b).

Figure 8. The variation of the interatomic bond strengths $R$ and atomic charges $q$ in the SbSI chain containing 60 atoms during the phase transition. Here $z_0$ denotes the displacement of Sb atoms in the direction of $z$-axis.
The normal vibrational mode of symmetry $A_u$ (further for clarity denoted here as $u$) interacts with the electronic states of symmetry $A_u$ and $B_g$ if $A_u \times B_g = u$. The electron-phonon interaction involving the highest degenerate $A_u$ and $B_g$ symmetry levels of the valence band is called the Jahn-Teller effect. The pseudo-Jahn-Teller effect (PJTE) occurs when the normal $u$ mode interacts with the $A_u$ electronic state at the top of the valence band and with the $B_g$ electronic state at the bottom of the conduction band, as well as with the band gap of $E_g$ between them. Because of the PJTE, the dependence of the harmonic term $\varepsilon_{\alpha\beta}$ on normal coordinates $Q$ is split into two terms:

$$\varepsilon_{\alpha\beta}(Q) = \frac{1}{2} \left( K_0 - \frac{2F^2}{E_g} \right) Q^2$$  \(16\)

where $K_0$ represents nonvibronic part of the force constant (the bare force constant), $K_V$ stands for the vibronic coupling term. $K = K_0 - \left( \frac{2F^2}{E_g} \right) = K_0 - K_v$ denotes the force constant considering the PJTE. $F$ which is found from the following equation represents the vibronic coupling constant:

$$F^2 = \frac{\Delta E E_g}{2Q_0^2}.$$  \(17\)

Here $Q_0 = z_0$ is the atomic displacement, and $\Delta E$ is the variation of the band gap during the phase transition. From Figure 9 and Eq. (17), it follows that for $z_0 = 0.2$ Å, $\Delta E = 2.23$ eV, and $E_g = 5.7$ eV, we would get $F = 13$ eV/Å. From the experimental results of Ref. [34], we derived $F = 4$ eV/Å. Therefore, the magnitude of the term $2F^2 = E_g$ may attain as much as $2H/\text{Å}$.

Figure 9. The variation of the one-electron energy spectrum of the SbSI chain containing 60 atoms during the phase transition. Here $z_0$ denotes the displacement of Sb atoms in the direction of $z$-axis.
In the case of the normal mode $\omega$, the PJTE and JTE vibronic mixing leads to the force constant

$$K = K_0 - \left( \frac{2F^2}{E_\tau} + G \right) = K_0 - K_{\tau},$$

where $G$ is a quadratic vibronic constant from JTE, and $K_{\tau}$ is the vibronic coupling term of PJTE and JTE. The phase transition occurs when the low-frequency mode is unstable, i.e., when $K < 0$, see Table 9. The symmetry coordinates of this mode are Sb(+1), S(+1), and I(−2). In pursuance of finding $K$, the total energy $E_\tau$ of the vibrational mode has to compute. Separate energy contributions constitute the total energy $E_\tau$:

$$E_\tau = E_k + E_{\text{es}} + E_{\text{me}} + E_{\text{in}},$$

(18)

where the kinetic energy of electrons is

<table>
<thead>
<tr>
<th>State</th>
<th>AFEP ($T = 300$ K)</th>
<th>FEP ($T = 215$ K)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_{\text{max}}$ (eV)</td>
<td>$E_{\text{min}}$ (eV)</td>
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</tr>
<tr>
<td>I 4d</td>
<td>51.35</td>
<td>50.35</td>
</tr>
</tbody>
</table>
the interelectron interaction energy is

\[ E_{\text{ee}} = \frac{1}{2} \sum_{i<j}^n \frac{e^2}{|\vec{r}_i - \vec{r}_j|} \]  

(20)

the electron-nuclear interaction energy is

\[ E_{\text{en}} = -\sum_{i=1}^{n,N} \frac{e Z_i}{|\vec{R}_i - \vec{r}_i|} \]  

(21)

and the internuclear interaction energy is

\[ E_{\text{nn}} = -\sum_{i \neq j=1}^{N} \frac{e Z_i Z_j}{|\vec{R}_i - \vec{R}_j|} \]  

(22)

Figure 10. The dependence of the total energy \( E_T \) of the \( u \) mode upon normal coordinates (relative displacements \( Z_\alpha \) of atoms (\( \alpha = \text{Sb, S, I} \)) from equilibrium position), where \( z \) denotes the displacement of Sb atoms in the \( \tilde{Z} \) mode from their equilibrium position. The temperatures of the chain are 400 K in PEP and 284 and 248 K in FEP. In all cases \( E_{T0} = 0 \).
The aforesaid energies are dependent on nuclear and electronic coordinates \( \vec{R}, \vec{r}_e \). Their evaluation has been conducted with the unrestricted Hartree-Fock method using the computer program GAMESS described in [33]. In pursuance of finding \( K \), the dependence of the total energy \( E_T \) of the \( u \) mode has been computed on the displacements \( z_\alpha \) of atoms \( (\alpha = \text{Sb, S, I}) \) from their equilibrium positions (Figure 10). Figure 10 demonstrates that \( z \) measures the displacements of Sb atoms in the \( u \) mode from their equilibrium positions \( z_0\alpha \). Vibrational displacements \( z_\alpha \) of atoms \( (\alpha = \text{Sb, S, I}) \) from their equilibrium positions \( z_{0\alpha} \) are identified employing the equation

\[
z_\alpha = z_0 + u_\alpha \frac{1}{\sqrt{m_\alpha}} \frac{i}{20},
\]

(23)

where \( i = 0–19, u_\alpha m_\alpha^{-1/2} \) is the vibrational amplitude of atoms \( (u_\alpha \) is the normal coordinates of the \( u \) vibrational mode). Normal coordinates of the \( u \) vibrational mode are found by diagonalization of the dynamical matrix [23].

By expressing \( E_T = f(z) \) as a fourth-order polynomial

\[
E_T = E_{T0} + K(z)^2 + c(z)^4.
\]

(24)

For the \( u \) mode in PEP (300 K), \( K = 2.05 \text{ H/Å}^2 \) and \( c = 2.3 \text{ H/Å}^4 \). Since the obtained \( K \) value is positive, it is supposed that the electron-phonon interaction will only decrease \( K \). The interaction itself will not induce a phase transition because under the influence of the electron-phonon interaction the \( u \) mode becomes only slightly anharmonic. However, due to phonon-phonon interaction in the SbSI chain, the \( u \) mode is strongly anharmonic, as its \( K < 0 \) and \( c > 0 \) [14]. Simultaneous action of electron-phonon and phonon-phonon interactions increase even more the anharmonicity of the \( u \) mode as compared to its anharmonicity determined by the phonon-phonon interaction alone. Therefore, the phase transition in the SbSI crystals is due to both electron-phonon and phonon-phonon interactions.

9. Physical parameters in the phase transition region

The vibration frequencies of the normal mode \( A_u \) are temperature dependent [10]. Therefore, the influence of the electron-phonon interaction and variation of unit cell parameters in the phase transition region on temperature dependence of frequency should be assessed properly.
By putting vibrational displacements $z_\alpha$ of atoms ($\alpha = \text{Sb, S, I}$) from their equilibrium positions equal to $z_\alpha = 0$, Eq. (24) is transformed into

$$E_{10} = E_{k0} + E_{\text{cel}} + E_{\text{an0}} + E_{\text{at0}}.$$  

It follows from Eqs. (18) and (22) that separate terms in Eq. (25) are functions of atomic coordinates $\vec{R}$ and distances between atoms ($\vec{R}_A - \vec{R}_B$). During the phase transition, due to variation of the volume of the simplified unit cell $V_0 = (a/2)bc$ [30]. Just this causes anomalies in the temperature dependences of $E_{10}$ and its separate components in the phase transition region Figures 9 and 11. Figure 11 demonstrates temperature dependences of $E_{10}$ and of the unit cell volume $V_0$.

**Figure 11.** Temperature dependences of the unit cell volume $V_0 = 2 \text{ V}$ (where V is the volume of the simplified unit cell) (1) and of the total energy $E_{10}$ (2) in the phase transition region. For curve 2, $V_0$ is assumed to depend upon temperature, and for curve 3 it is taken as constant: $V_0 = 354.25 \text{ Å}^3$.

**Figure 12.** Temperature dependences of the total kinetic $E_{\text{TK0}}$ and potential energy $E_{\text{TP0}}$ in the phase transition region.
In PEP (or in the antiferroelectric phase, according to Ref. [14]), the anomalies of $E_{\tau 0}$ and $V$ correlate in the temperature range of 295–400 K. It means that $E_{\tau 0}$ decreases due to the growth of $V_0$ and vice versa. **Figures 12 and 13** show that the temperature dependence of $E_{\tau 0}$ is mainly determined by the components $E_{\tau 0} = E_{\text{ee0}} + E_{\text{ne0}} + E_{\text{nn0}}$.

**Figure 13.** Temperature dependences of the total potential energy $E_{\tau 0}$ components $E_{\text{ee0}}$, $E_{\text{ne0}}$, and $E_{\text{nn0}}$ in the phase transition region.

In the FEP in the temperature range 280–295 K, a sharp increase of the unit cell volume $V_0$ leads to a decrease of $E_{\tau 0}$, curves 1 and 2 in **Figure 11**. However, in FEP at the temperatures of 220–280 K, $E_{\tau 0}$ grows if the temperature is decreased, while the unit cell volume $V_0$ changes only slightly. Curve 3 in **Figure 11** shows the calculated temperature dependence of $E_{\tau 0}$ provided that $V_0 = \text{constant}$. In this temperature range, when $V_0 = \text{constant}$, the growth of $E_{\tau 0}$ is caused by the variable shift of the equilibrium positions of Sb and S atoms. Thus, curve 2 in **Figure 11** demonstrating the growth of $E_{\tau 0}$ can also be considered as caused solely by the variation of Sb and S equilibrium positions.

**Figure 14.** Temperature dependences of the harmonic constant $K = (K_0 - K_V^c)$ Å and of the anharmonicity factor $\epsilon \, (B)$ for the $\bar{A}_u$ normal mode in the phase transition region. Dashed lines show the temperature dependences of $K$ and $\epsilon$ assuming that $V_0 = 354.25$ Å$^3$. 
The three temperature intervals, i.e., 295–400 K in PEP, 280–295 K in FEP, and 220–280 K in FEP, clearly reveal the anomalies in the temperature dependences of the coefficients $K$ and $c$ of the polynomial (10) (Figures 14a and b). The temperature dependences of $K$ and $c$ are determined by the variation of the unit cell volume $V_0$ in the PEP interval at 295–400 K, whereas the behavior of $K$ and $c$ is determined by $V_0$ and the variable shift of the equilibrium positions of Sb and S atoms in the FEP interval at 280–295 K.

Pursuing to separate the influence of the volume $V_0$ and of the shift of Sb and S equilibrium positions on the anomalies of $K$ and $c$, the calculated temperature dependences of $K$ and $c$ at a constant $V_0$ are demonstrated in Figures 14(a) and (b) (dashed lines). In this temperature range, $K$ and $c$ rapidly change due to the variation of the equilibrium position of Sb and S atoms if $V_0$ = constant. Consequently, the rapid change of $K$ and $c$ marked by the solid lines in Figures 14(a) and (b) is also caused merely by the shift of Sb and S atoms. The rapid variation of $K$ in this temperature range occurs due to the decrease of $K V^*$ that is determined by the variation of degeneracy (see Figure 9). Since $K = m \tau^2$, where $m$ is the reduced mass of the $\omega$ mode and $\omega$ is the frequency of this mode, then the dependence of $K$ upon temperature (Figure 14a) is similar in its shape to the temperature dependence of the soft mode $s^2$ [14]. Therefore, the temperature dependence of the soft mode frequency $s^2$ is essentially affected by the variation of the unit cell volume $V$ and of the polynomial coefficient $K$ in the phase transition region: the total temperature dependence of $s^2$ can be obtained provided the anharmonicity caused by phonon-phonon interaction is considered.

10. The electronic structure of SbSI cluster at the phase transition region

10.1. Introduction

As it is known [25, 34, 35], a ferroelectric phase transition of the first kind occurs in SbSI, though it is close to the phase transition of the second kind. What is more, it has been theoretically proved in Ref. [36] that the phase transition in SbSI takes an intermediate position between order-disorder and displacement types. In general, it is known that ferroelectric phase is at $T < T_c = 295$ K, antiferroelectric phase exists at $T_c < T < 410$ K, and paraelectric phase at $T > 410$ K. Lukaszewicz et al. [17] determined the crystal structure of SbSI in the temperature region at 170–465 K. They found that there are three phases in the phase diagram of SbSI: ferroelectric phase (FEP) III (space group Pna2$_1$) below $T_{c1} = 295$ K, antiferroelectric phase (AFEP) II (P 2$_1$2$_1$2$_1$) in the temperature region ($T_{c2} = 295–410$ K), and high temperature paraelectric phase (PEP) I above 410 K. AFEP is characterized by the double polar chains [((SbSI)$_2$)$_2$]$_2$, which are ordered antiferroelectrically: these cause a disorder in the crystal lattice. Experimental investigation of X-ray absorption, fluorescence, and electron emission [37, 38] adjusts the energy band structure of SbS$x$Se$_{1−x}$I ($x = 0.25; 0.5; 1$), SbSBr. Its explanation necessitates for a more precise calculation of the contribution of separate atoms to the total density of states. The
authors [39] had measured X-ray photoelectron emission spectra (XPS) of the SbSI crystals of valence bands (VB) and core levels (CL). XPS results showed the splitting of the CL. However, they have not explained theoretically the XPS spectra of the VB.

11. *Ab initio* method description

The UHF method in the MOLCAO approximation serves as the basis of the method of calculation. The molecular orbital (MO) \( \phi_i \) can be expanded in the atomic orbital (AO) \( \chi_\mu (r) \) basis set

\[
\phi_i (r) = \sum_{\mu} C_{i\mu} \chi_\mu (r). \tag{26}
\]

Here \( \mu \) denotes the number of AO. Matrix \( C \) is obtained by solving the Hartree-Fock matrix equation

\[
FC = ESC. \tag{27}
\]

The roots of this secular equation lie in the molecular energies \( E_i \) of electrons, where \( F \) stands for the Fockian matrix and \( S \) for the matrix of overlap integrals:

\[
S_{\mu\nu} = \int \chi_\mu (r) \chi_\nu (r) dr. \tag{28}
\]

The coefficients \( C_{i\mu} \) allowed us to find the density matrix of the electron distribution:

\[
P_{\mu\nu} = 2 \sum_{i=1}^{N/2} C_{i\mu} C_{i\nu}^*, \tag{29}
\]

where \( N \) is the number of electrons, and the sum is over all occupied MO. We can find the bond order between the A and B atoms,

\[
P_{AB} = \sum_{\mu\nu} \sum_{i,j} n_{ij} \tag{30}
\]

where \( n_{ij} \) is the number of electrons.
and the charge of atoms,

\[ \rho_A = Z_A - \sum_{\mu \in A} P_{\mu \nu} \]  

(31)

The calculations were performed by the GAMESS program [33]. Atomic coordinates have been used from the work of Lukaszewicz et al. [17].

12. Investigation of SbSI crystal valence band

The paraelectric structure \((T > T_{C2} = 410 \text{ K})\) of SbSI is made of atom chains that belong to the paraelectric space group Pnam, which form square-pyramidal SbI3 groups with the Sb ion at the center of the pyramid base. All atoms lie on mirror planes normal to the \(c\)-axis. (Figure 15).

Figure 15. Crystal structure of SbSI projected on the \(a-b\) plane (a), and one double chain of molecular cluster in projection on the \(a-c\) plane (b).

On passage into the AFEP \((T_{C2} > T > T_{C1} = 295 \text{ K})\) and FEP \((T < T_{C1})\), the position parameters normal to the \(c\)-axis are substantially unchanged. In AFEP \((T = 300 \text{ K})\) the displacements of equilibrium position occur for all Sb3 atoms along the \(c\)-axis \(z_0 = 0.02 \text{ Å}\) and for all Sb4 atoms.
\( z_0 = -0.02 \text{ Å} \) \cite{37}. In FEP \((T < T_{c1})\) all Sb and S atoms move along the \( c \)-axis toward these I sites, which leads to the removal of the mirror plane symmetry. In FEP \((T = 215 \text{ K})\), the displacements of equilibrium position occur for all Sb atoms along the \( c \)-axis \( z_0 = 0.2 \text{ Å} \).

The molecular cluster model of one SbSI crystal chain was used to perform the theoretical \textit{ab initio} calculation of energy levels (Figure 15). As seen from Figure 16, energy levels only slightly change with the increase of the cluster. Figure 17 shows that the energy of some levels increases and of others decreases.

**Figure 16.** Dependence of energy levels of VB on the number \( N \) of SbSI molecules in a molecular cluster in AFEP (300 K).

**Figure 17.** Dependence of energy levels of VB on equilibrium positions \( z_0 \) of Sb atoms in SbSI molecular cluster. The displacements of equilibrium position \( z_0 \) of Sb atoms: for all Sb3 atoms in AFEP (300 K), \( z_0 = 0, 0.2 \text{ Å} \), and for all Sb4 atoms \( z_0 = -0.02, 0.2 \text{ Å} \), and for all Sb3 and Sb4 atoms in FEP (215 K).
We have calculated the bond orders between the atoms Sb3-S4, Sb4-S3, Sb3-I1, and Sb4-I2 along a cluster in AFEP and FEP (Figures 18 and 19). When the phase transition takes place, bond orders change. As a consequence, VB experiences the broadening of some energy levels and the narrowing of others.

![Figure 18. Bond orders between the atoms Sb3–S4 (a) and Sb4–S3 (b) along the SbSI molecular cluster in AFEP and FEP.](image1)

![Figure 19. Bond orders between the atoms Sb3–I1 (a) and Sb4–I2 (b) along the SbSI molecular cluster in AFEP and FEP.](image2)

While the temperature decreases further in the FEP, Sb and S atoms change their positions relative to I atoms, and VB becomes broader. In Grigas et al. [39], the authors present the X-ray photoelectron spectra (XPS) of the VB and CL of the SbSI single crystals. The XPS are
measured in the energy range 0–1400 eV and the temperature range 130–330 K. They compared experimentally obtained energies of CL with the results of theoretical \textit{ab initio} calculations. However, the experimentally obtained energies of the VB were not compared with the results of theoretical \textit{ab initio} calculations.

We have calculated the density of states of VB for Sb, S, and I atoms and the total molecular cluster in AFEP ($T = 300$ K) and FEP ($T = 215$ K). The density of states is

$$g = \frac{\Delta N}{\Delta E},$$

where $\Delta N$ is the number of states in the energy interval $\Delta E$ (eV). The dimension of $g$ is (eV$^{-1}$).

In Figure 20, the experimental results of XPS VB spectra of SbSI crystals are compared with the theoretically calculated total density of states of a molecular cluster that consists of 20 SbSI molecules in AFEP ($T = 300$ K) and FEP ($T = 215$ K) is the number of states in the energy interval $\Delta E$ (eV). The dimension of $g$ is (eV$^{-1}$). In Figure 19, the experimental results of XPS VB spectra of SbSI crystals are compared with the theoretically calculated total density of states of a molecular cluster that consists of 20 SbSI molecules in AFEP ($T = 300$ K) and FEP ($T = 215$ K). The average total density of states is as follows:

$$< g > = \frac{1}{n} \sum_{i}^{n} g_i,$$

where $n$ is the number of normal modes ($n = 15$).
As seen from Figures 20 and 21, the experimental and theoretical results in AFEP and FEP are in good agreement in the energy range of 6–17 eV. In the range of 17–22 eV, there is a great difference between experimental and theoretical results.

The peaks of Figures 21(a) and (b) have no distinct shape in the XPS (at $T = 300$ K). This difference is explicable because the theoretical calculation does not take into account vibrational displacements of atoms. Average densities of states shown in Figure 21(c) are more similar to the experimental XPS spectrum than those from Figure 21(b). Comparison of Figures 20 and 21 shows the contribution of atomic states to the total density of states.

13. Core-level splitting in the antiferroelectric and ferroelectric phases

The mechanisms of the XPS splitting in SbSI crystals were discussed by Grigas et al. [39]. After the breaking of the crystals under high vacuum conditions some bonds of atoms (see Figure 15b) at the surface become open.

Employing the UHF method [40], the energy of CL and ionic charges of atoms Sb and S were calculated using the SbSI molecular cluster model. Figure 22 demonstrates the calculated ionic charges of atoms Sb and S along a molecular cluster of 20 SbSI molecules.
As it is seen from Figures 22(a) and (b), the ionic charges at the edges of the SbSI cluster in AFEP considerably differ from those in the bulk of the sample. Besides, the differences of charges $\Delta q_j$ between Sb3 and Sb4 and between S3 and S4 have increased at the cluster edges. The differences of Sb3 and Sb4 are equal at both edges in AFEP and unequal in FEP.

The difference of charge $\Delta q_j$ at the cluster edges forms the binding energy difference ($\Delta E_{\text{clust}} = E_{\text{max}} - E_{\text{min}}$) between atoms Sb3 and Sb4, S3 and S4, as well as I1 and I2 (Table 9). Moreover, the differences ($\Delta E_{\text{clust}} = E_{\text{max}} - E_{\text{min}}$) at both cluster edges are equal in AFEP and unequal in FEP.

<table>
<thead>
<tr>
<th>State</th>
<th>Energy (eV)</th>
<th>State</th>
<th>Energy (eV)</th>
<th>State</th>
<th>Energy (eV)</th>
</tr>
</thead>
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<td>S 2p</td>
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<td>1d</td>
<td>657.86</td>
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<td>S+2 2p</td>
<td>169.58</td>
<td>1+3 3d</td>
<td>649.00</td>
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<tr>
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<tr>
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<td>593.13</td>
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Table 10. The energies of CL of isolated neutral Sb, S, and I atoms and isolated Sb+1, Sb+2, S+1, S+2, and I+1 ions.

Pursuing to explain the splitting of the CL, the eigenvalues of isolated neutral Sb, S, and I atoms and isolated Sb+1, Sb+2, S+1, S+2, and I+1 ions have been calculated by the Hartree-Fock method using the N21 orbital basis set (Table 10).
As shown in Table 10, the eigenvalues of isolated neutral atoms Sb and S and isolated ions Sb\(^{+1}\) and S\(^{-1}\) differ for all states. Besides, as demonstrated in Table 10, 3d state energies of an isolated Sb atom and Sb\(^{+1}\) ion vary by \(\Delta E = 9.6\) eV. The difference for 2p states of S and S\(^{-1}\) makes \(\Delta E = 11\) eV. Consequently, \(\Delta E\) is proportional to ion charges \(\Delta q\). The ion charges at the edge of the SbSI cluster vary between two Sb\(^4\) (and between two Sb\(^3\)) by \(\Delta q = 0.23\) a.u. and by \(\Delta q = 0.18\) a.u. between two S\(^4\) (and between two S\(^3\); Figure 22). Hence, the energy difference \(\Delta E^*_{\text{clus}}\) of the 3d state between two Sb atoms and of the 2p state between two S atoms at the edges of an SbSI cluster can be calculated as follows:

\[
\Delta E^*_{\text{clus}} = \frac{\Delta q}{q_i} \Delta E
\]

where \(\Delta E\) is the difference between energies and \(q_i\) is the difference between ionic charges of isolated atoms (Table 10).

The proportion between \(\Delta E^*_{\text{clus}}\) and \(\Delta q\), according to Eq. (33) allows us to explain the splitting, \(\Delta E_{\text{exp}}\), of the CL in XPS. So in AFEP XPS along the c-axis, \(\Delta E^*_{\text{clus}}\) and \(\Delta q\) are equal at both edges of the cluster (Figures 22a and b). However, in FEP their values are different. At phase transition, \(\Delta E^*_{\text{clus}}\) changes differently at the left and right edges of the cluster (Figures 22c and d). Thus, at the left edge \(\Delta E^*_{\text{clus}}\) (AFEP) > \(\Delta E^*_{\text{clus}}\) (FEP) and at the right edge \(\Delta E^*_{\text{clus}}\) (AFEP) < \(\Delta E^*_{\text{clus}}\) (FEP). On the other hand, \(\Delta E^*_{\text{clus}}\) is in good agreement with the difference \(\Delta E_{\text{clus}} = E_{\text{max}} - E_{\text{min}}\) of the same CL energies of atoms at the edges of the cluster. For example, from Table 4 we get \(\Delta E_{\text{clus}} = 1.93\) eV between 3d of Sb\(^3\) and Sb\(^4\), 2.77 eV between 3d of I\(^1\) and I\(^2\), and 1.88 eV between 3d of S\(^3\) and S\(^4\) in AFEP (Table 11).

<table>
<thead>
<tr>
<th>State</th>
<th>(\Delta E) (eV) of isolated neutral atoms and ions with (q_i = 1) (a.u.)</th>
<th>(\Delta q) (a.u.) of surface atoms of SbSI cluster</th>
<th>(\Delta E^*_{\text{clus}}) (eV)</th>
<th>(\Delta E_{\text{clus}} = E_{\text{max}} - E_{\text{min}}) (from Table 1) (eV)</th>
<th>(\Delta E_{\text{exp}}) (eV)</th>
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<td>2.50</td>
</tr>
<tr>
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<td>11.51</td>
<td>0.18</td>
<td>2.07</td>
<td>1.88</td>
<td>2.20</td>
</tr>
<tr>
<td>I 3d</td>
<td>8.85</td>
<td>0.25</td>
<td>2.21</td>
<td>2.77</td>
<td>2.70</td>
</tr>
</tbody>
</table>

Table 11. The calculated splitting of the deep CL \(\Delta E^*_{\text{clus}}\) and \(\Delta E_{\text{clus}}\) in SbSI cluster and experimentally observed \(\Delta E_{\text{exp}}\) of XPS levels in SbSI crystal.

As it is shown in Table 11, the experimentally observed \(\Delta E_{\text{exp}}\) of the XPS levels in an SbSI crystal is in good agreement with the calculated splitting of the deep CL \(\Delta E^*_{\text{clus}}\) and \(\Delta E_{\text{clus}}\) in an SbSI cluster. Thus, it can be concluded that both \(\Delta q\) and \(\Delta E_{\text{clus}}\) and the splitting of CL in an SbSI crystal are sensitive to the displacements of equilibrium position \(z_0\) (depending on the temperature change).
14. The energy gap of the SbSI crystals

14.1. Introduction

Investigation of the total density of states of SbSI crystals [41] has shown that the absolute valence band top is formed in both phases of 3p orbitals of S, while the absolute conduction band bottom of 5p orbitals of Sb.

We have undertaken an attempt of a more detailed calculation of the electronic structure and some properties of SbSI from the first principles using the empirical pseudopotential method [42]. The method for calculating the band structure of SbSI was employed in Refs. [43–45]. In Ref. [43], purely ionic and partially covalent models were applied, and an indirect energy gap of 2.28 eV at point S was obtained. Nevertheless, the accuracy turned out to be 0.2 eV. In Ref. [44], the pseudopotentials were corrected applying the data on direct gaps. The absorption band edge in the paraelectric phase was determined at 1.82 eV for $E \parallel c \left( S_{5\rightarrow 6}^{7\rightarrow 8} \right) \rightarrow S_{7\rightarrow 8}^{7\rightarrow 8}$ and 1.91 eV for $E \perp c \left( S_{1\rightarrow 2}^{7\rightarrow 8} \right) \rightarrow S_{7\rightarrow 8}^{7\rightarrow 8}$. The valence band top corresponded to $S_{5\rightarrow 6}^{7\rightarrow 8}$, whereas the conduction band edge complied with $Z_{1}^{(C)}$ with the energy of indirect gap of 1.41 eV. The energy gap sides matched to points Z and R in the ferroelectric phase. Analyzing the light reflection spectra, the indirect band gap of 1.82 eV between $\Gamma_{6}^{(V)}$ and $S_{1}^{(V)}$, as well as the minimum direct gap of 2.08 eV were obtained in reference [45].

In Ref. [45], a purely ionic model was assumed. The form factors of the pseudopotential were adjusted by fitting the calculated band gap values to the ones obtained both experimentally and theoretically by other authors. Form factors for Cl were used instead of I.

15. Investigation by using the pseudopotential method

The infinite crystal approximation, which states that the crystal properties obtained for the primary cell are extended then to the entire crystal employing periodical boundary conditions, has been employed. Hence, within the primary cell, the position of the $j$th atom of kind $\alpha (\alpha = \text{Sb, S, or I})$ is

$$R_{j}^{(\alpha)} = R_{\text{cell}} + \tau_{j}^{(\alpha)},$$

where $R_{\text{cell}}$ is the translation vector of the orthorhombic system. The vectors $\tau_{j}^{(\alpha)}$ describe onprimitive translations that may be written as

$$\tau_{j}^{(\alpha)} = \lambda_{j}^{(\alpha)} t_{x} + \mu_{j}^{(\alpha)} t_{y} + \nu_{j}^{(\alpha)} t_{z},$$
where \( t_x, t_y, \) and \( t_z \) are the lattice parameters along the three coordinate axes. The pseudopotential of the crystal has been selected as a sum of atomic pseudopotentials:

\[
V(r) = R_{\text{cell}} \sum_{\alpha} \sum_{R_{\text{cell}}} \sum_{i} \nu_{\alpha} \left( r - R_{\text{cell}} - t_{i}^{(\alpha)} \right).
\]  

(35)

Here, \( \nu_{\alpha}(r) \) is the atomic pseudopotential of atom \( \alpha \). It is assumed to be localized and energy independent. Weakness of the pseudopotential \( V_{\text{ps}}(r) \) provides good convergence of the pseudo-wave functions expanded in terms of plane waves:

\[
\Psi_{k_i}^{\text{ps}}(r) = \sum_{G} C_{k_i}(G) e^{i(k \cdot G)r},
\]  

(36)

where \( G \) is the reciprocal lattice vector, \( C_{k_i}(G) \) are Fourier coefficients. The Schrödinger equation in the pseudopotential method has the form

\[
(H + V_{\text{ps}}) \Psi_{k_i}^{\text{ps}} = E_{k_i} \Psi_{k_i}^{\text{ps}},
\]  

(37)

The pseudo-wave functions of the valence electrons \( \Psi_{k_i}^{\text{ps}}(r) \) approach the true ones outside the core region, though they show no oscillations within the core. The Fourier component of this potential (or the matrix element in the plane-wave basis set) is expressed

\[
V_{k_i}^{\text{ps}}(r) = \sum_{\alpha} S_{\alpha}(G) \nu_{\alpha}(G),
\]  

(38)

where \( \Omega_{\alpha} \) and \( \Omega_{c} \) are the volume of the corresponding atom and of the primary cell as a whole. The atomic structural factor \( S_{\alpha}(G) \) and atomic form factor \( \nu_{\alpha}(G) \) are defined as

\[
S_{\alpha}(G) = \sum_{\alpha} e^{-iG \cdot \alpha},
\]  

(39)

\[
\nu_{\alpha}(G) = \frac{1}{\Omega_{\alpha}} \int \nu_{\alpha}(r) e^{-iG \cdot r} d^{3}r.
\]  

(40)

The integrals are taken over the whole volume \( \Omega_{\alpha} \). Provided the atomic coordinates within the cell are known, the atomic structural factor can be readily evaluated. The pseudopotential forms factors for Sb, S, and I, so the following equation [39, 43] was preliminary determined to be used:
\[ v_G(G) = \frac{4\pi}{\omega} \sum_{nlm} |nlm| \exp \left[ -i (Gr) \right] nlm. \] (41)

where \( nlm \) denotes the set of electron quantum numbers. Neutral functions for Sb, S, and I were involved by the numerical evaluation of the form factors [43, 45, 46]. It is significant that the agreement between the theoretical and experimental data shall be obtained solely employing neutral functions. It is noteworthy that some other authors [47, 48], who theoretically studied the band structure of the SbSI crystal, employed an ionic model of chemical bonding (Sb\(^{3+}\) S\(^{-2}\) I\(^{-1}\)) instead.

Figure 23. The scheme and notation of 27 special points in the irreducible part of the Brillouin zone.

<table>
<thead>
<tr>
<th>No</th>
<th>Point</th>
<th>Coordinates</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td>( k_x )</td>
</tr>
<tr>
<td>1</td>
<td>( \Gamma )</td>
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</tr>
<tr>
<td>2</td>
<td>( \Delta )</td>
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</tr>
<tr>
<td>3</td>
<td>( Y )</td>
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</tr>
<tr>
<td>4</td>
<td>( \Lambda )</td>
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</tr>
<tr>
<td>5</td>
<td>ZTY ( \Gamma )</td>
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</tr>
<tr>
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</tr>
<tr>
<td>7</td>
<td>( Z )</td>
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</tr>
<tr>
<td>8</td>
<td>( B )</td>
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<td>9</td>
<td>( T )</td>
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</tr>
<tr>
<td>10</td>
<td>( \Sigma )</td>
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</tr>
<tr>
<td>11</td>
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</tr>
<tr>
<td>12</td>
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</tr>
<tr>
<td>13</td>
<td>( \Gamma_X U_Z )</td>
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</tr>
<tr>
<td>14</td>
<td>( O )</td>
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</tr>
<tr>
<td>15</td>
<td>( \Gamma_Y S_R )</td>
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</tr>
</tbody>
</table>
Table 12. Coordinates of the special points in the irreducible part of the Brillouin zone of SbSI (in relative units), see also Figure 23.

<table>
<thead>
<tr>
<th>No</th>
<th>Point</th>
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<th>( k_y )</th>
<th>( k_z )</th>
</tr>
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<td>0</td>
</tr>
<tr>
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<td>0.5</td>
<td>0</td>
</tr>
<tr>
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<td>0.25</td>
</tr>
<tr>
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<td>0</td>
<td>0.5</td>
</tr>
<tr>
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<td>0.5</td>
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<td>R</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Figure 24. A diagram of the forbidden gap of SbSI in antiferroelectric phase at 308 K and its change after transition to ferroelectric phase (at 278 K). \( E_g^{\text{antiferro}} \) is the indirect transition energy (1.42 eV) in antiferroelectric phase between points U and Z of the irreducible part of the Brillouin zone. \( E_g^{\text{ferro}} \) is the indirect transition energy (1.36 eV) in ferroelectric phase between points R and Z of the irreducible part of the Brillouin zone. The direct gap takes its minimum value at point U for both phases (a) and (b) correspond to different directions in the crystal.
The chemical bond in SbSI is of mixed kind, with contributions of both ionic and covalent components. As we have demonstrated in Ref. [43], it may be described by an approximate model formula $\text{Sb}^{+0.3}\text{S}^{-0.2}\text{I}^{-0.1}$. The band structure for both phases was estimated in 27 points of the irreducible part of the eightfold Brillouin zone, which is the total of 216 points over the Brillouin zone. The points are schematically described in Figure 23, as well as their coordinates are provided in Table 12. A total of 600 plane waves were included in the basis set for the calculation. The experimental energy gap values were estimated from the exponential light absorption tail at $\ln K = 6$, where $K$ represented the absorption coefficient [2]. As it is seen from Figure 24, the most significant changes in the valence band at the phase transition occur at points $Q$ and $C$ (energy variation was 0.92 and 0.86 eV, respectively).

Moreover, the changes at points $R$ (0.37 eV), $H$ (0.55 eV), and $E$ (0.42 eV) should be noted. As far as the conduction band is concerned, similar significant changes occur at points $H$ (0.53 eV) and $E$ (0.51 eV). At all the remaining points of the Brillouin zone, the band gap profile has changed insignificantly. All these changes have only a slight effect on the main characteristics of the band structure (except point $R$, obviously). As it is seen from Figure 24, the SbSI crystal has an indirect forbidden gap both in antiferroelectric phase and in ferroelectric phase [42]. The conduction band bottom in both phases is located at point $Z_1^{(C)}$, the valence band top in antiferroelectric phase is at the point $U_{5-6}^{(V)}$, and in ferroelectric phase at $R_{3-4}^{(V)}$. Our experimental and theoretical results of the SbSI crystal electronic structure sometimes differ from those obtained by other authors.

16. Band structure at the first-order phase transition

Direct (Brillouin zone point $U$) and indirect (jump $U \rightarrow Z$) dependence of the forbidden band on the temperature that has been calculated by us is demonstrated in Figure 25. As shown in Figures 25 and 26, the width of the indirect forbidden band is 1.42 eV in antiferroelectric phase and 1.36 eV in ferroelectric phase. The following results match well to the results that have been obtained earlier [49]: 1.42 eV in antiferroelectric and 1.36 eV in ferroelectric phase. The indirect width of the forbidden band is 1.36 eV in that band structure, which was obtained by moving to the point of phase transition from ferroelectric phase point B1; and it is 1.41 eV when moving from antiferroelectric phase point A1. Hence, the indirect forbidden band moving from antiferroelectric to ferroelectric phase alters by 0.05 eV, according to our calculations. This number complies with the change 0.06 eV of the width of the indirect band, which has been experimentally set [48, 49]. Figures 3 and 4 reveal that the direct forbidden band is narrowest in point $U$, 1.83 eV. The calculations presented in Ref. [49] demonstrate the value of 1.82 eV. The minimal direct forbidden band in ferroelectric phase occurs in point $G$, 1.94 eV, whereas this gap is 1.98 eV in point $U$. The narrowest forbidden band is in point $U$ of Brillouin zone, approaching from both antiferroelectric and ferroelectric sides. This has been estimated in band structure in the area of phase transition (Figures 26b and c). In the first case, it turned out to be 1.83 eV, point $A2$ (Figure 25), and in the second one it is 1.87 eV, point $B2$ [43] (Figure 25). For this reason, the observed jump of the direct forbidden band is 0.04 eV during
the first-order phase transition. The change of the grating parameter along axis \( e(y) \), which affects the band structure of SbSI crystal, causes this jump. It is possible to find the width of the forbidden direct band in Brillouin zone point U in Ref. [49, 50], which is experimentally measured. Unfortunately, the SbSI exponential edge of absorption has not been taken into account. The exponential edge of absorption is explored in greater detail in antiferroelectric and ferroelectric phases and in the area of the temperatures in the phase transition in Ref. [34]. The edge of the absorption in antiferroelectric phase complies with the Urbach’s rule:

\[
K = K_0 \exp \left[ \frac{\sigma(E_{\text{ex}} - E_{\text{th}})}{kT} \right].
\]  

(42)

Figure 25. We got the direct (2) and the indirect (1) forbidden band width’s dependence on temperature. The brake of the curve in points \( A_1 \) and \( A_2 \) is got when moving to phase transition from antiferroelectric phase, and in points \( B_1 \) and \( B_2 \) when moving to phase transition from ferroelectric phase.

here

\[
\sigma = \sigma_0 \frac{2kT}{\omega_0^2} \text{th}^{-1} \frac{\omega_0}{2kT}.
\]  

(43)
where $\sigma$ stand for Urbach’s parameter, which denotes the outspread of the absorption edge. The $\sigma_0$ is a constant, which describes the intensity of interaction between electrons and phonons, $\hbar\omega_0$ denotes effective phonons energy, $K_0$ denotes “oscillator’s strength” or the maximal absorption coefficient, $E_0$ represents characteristic “gap” of the forbidden band, and $E_K$ denotes light quantum energy for a particular absorption coefficient $K$. During the phase transition, the characteristic “gap” differs:

$$\Delta E_0 = E_{0F} - E_{0AF},$$

(44)

where $E_{0F}$ and $E_{0AF}$ stand for the values of the energy “gap” in ferroelectric (F) and antiferroelectric (AF) phases, respectively. As seen from Eq. (43), temperature dependences $E_K(T)$ and $\sigma/kT$ should be measured when $K = $ const. and $K_0(T)$ in both phases in the area of the phase transition in order to determine $E_{0F}$ and $E_{0AF}$. $K_0(T)$ is not temperature dependent in antiferroelectric phase and it is experimentally defined. Different temperatures are matched finding the point of crossing of the curve $\ln K(E)$. In ferroelectric phase, $K_{0F}$ is determined as follows:

$$\ln K_{0F} = \ln K_0 - \gamma \left( \frac{\sigma}{kT} \right) \ln P_S,$$

(45)

where $\ln K_0$ and $(\sigma/kT)_{AF}$ are the parameters in the antiferroelectric phase, and $\gamma$ represents the coefficient of proportionality (polarization potential). It is possible to find the values of $\gamma$, $K_{0F}$, $P_S(T)$ and Eq. (42) in Ref. [34]. Figure 27 demonstrates temperature dependencies of $E_K(T)$ and $\sigma/kT(T)$, which have been measured experimentally employing the dynamic method with a
continuously variable temperature [51]. Using experimental results provided in Figure 27, when temperature is 295 and 278 K, as well as employing the data indicated in Ref. [34], we estimate that according to Eqs. (43) and (46), $\Delta E_0 = 0.12 \pm 0.02$ eV. The experimental $\Delta E_0$ value coincides with the theoretic variation of the width of the forbidden band in margins of error, which is 0.11 eV in Brillouin zone point U. It appears due to the variation of the grating parameters along the axis $c(y)$, which affects the band structure of crystal (Figures 26c and d).

Figure 27. The experimental iso absorption curve energy $E_K (eV)$ dependence on temperature when $K = 65$ cm$^{-1}$ (1) and the dependence $\sigma/KT (T)$ (2) are measured in SbSI at phase transition.

17. Conclusions

The potential energy of $A_{u}(10)$, $B_{1u}(3)$, $B_{2g}(4)$, $B_{2g}(5)$, and $B_{3g}(7)$ normal modes in the paraelectric phase are anharmonic with a double-well $V(z)$, while $B_{2u}(2)$, $B_{3g}(6)$, $B_{3g}(8)$, and $B_{3g}(9)$ modes possess only one minimum. The semisoft modes $B_{1u}(3) \rightarrow A_{u}$, $B_{2g}(4) \rightarrow B_{vi}$, $B_{2g}(5) \rightarrow B_{vi}$, and $B_{3g}(7) \rightarrow B_{3}$ evoke experimental reflection $R(k)$ peaks for $E \parallel c$ in the range of $k = 10$–100 cm$^{-1}$ for both paraelectric and ferroelectric phases. The $R(k)$ peak for $E \parallel c$ in the paraelectric phase
is created by mode $B_{1u}(2)$, but in the ferroelectric phase the $R(k)$ peaks are caused by modes $B_{1u}(2) \rightarrow A_{1g}$, $B_{2g}(6) \rightarrow B_{1g}$, $B_{3g}(8) \rightarrow B_{2g}$ and $B_{1u}(9) \rightarrow B_{2u}$. It has been determined that strong lattice anharmonisity, as well as interaction between chains, can split the mode $B_{1u}(3)$ into two components, from which one is soft in the microwave range and the other $B_{1u}(3)$ is semisoft in the IR range. The semisoft modes $B_{1u}(3)$, $B_{2g}(4)$, $B_{3g}(5)$, and $B_{3g}(7)$ increase the large (absorption) peak in the range $k = 5–100$ cm$^{-1}$ and the dielectric contribution of $\varepsilon =5000$. The reflection spectra also show large peaks due to its strong temperature dependence. The strongest temperature dependence of reflection is observed in the ferroelectric phase. While in the paraelectric phase it becomes relatively weak.

In the long SbSI chains, the highest levels of one-electron energies in the valence band top are degenerate. Therefore, Jahn-Teller effect appears as an important factor. The ferroelectric phase transition in SbSI crystals is caused by electron-phonon and phonon-phonon interactions. The electron-phonon interaction reduce harmonic coefficient $K$, whereas phonon-phonon interaction reverses its sign, i.e., $K < 0$; $c > 0$. In the phase transition region, anomalous behavior is demonstrated by the coefficients of the polynomial expansion of the total energy $E_T$ dependence upon the $\nu$ normal mode coordinate. The $\nu$ mode temperature dependence of the coefficient $K = m^2_\nu$ is similar to the temperature dependence of the soft mode frequency, $\omega^2_S$.

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References


