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Abstract

Molecular magnetic materials have become flourishing fields for research and technological developments due to their novel behavior compared to classical magnetic materials. Molecular magnetism modeling has reached a certain degree of maturity, although several experimental findings are still open problems. This chapter is aimed at providing a general introduction to physical modeling in molecular materials with a special emphasis placed on spin crossover compounds. This presentation includes Ising-type models and their generalizations, such as Wajnflasz and Pick, Bousseksou et al., Zimmermann and König, Sorai and Seki, and Nasser et al., along with their applications to the characterization of phase transition, hysteresis behavior, and thermal relaxations in spin crossover compounds. Recent experimental findings are explained in this context and the relevance of theoretical results for technological applications is also discussed.

Keywords: molecular magnetism, spin crossover compounds, hysteresis, phase transitions, thermal relaxations, modeling and simulations

1. Introduction

Molecular magnetic materials have been in the research spotlight since the current silicon based devices and conventional magnetic storage devices approached their technological and physical limits [1, 2]. In this context, spin crossover (SCO) materials present a special interest due to their unique properties, high versatility, and a wide variety of potential technical applications [3, 4].
The most widely studied SCO materials involve transition metal centers (3d^4–3d^7) in an octahedral $O_h$ symmetry of ligands. These materials present molecular bistability that can be triggered by various external stimuli such as temperature, pressure, light, magnetic and electric field, and gas absorption [5–16]. The molecular bistability results from the balance between the spin pairing energy and the crystal field that leads to the possibility to populate the 3d orbitals in two different ways characterized by different values of the total spin $S$. These two situations are known in the literature as high-spin (HS) and low-spin (LS) states, respectively. Schematic representation of 3d electron configurations in an octahedral ligand field for Fe (II) complexes in the low-spin ($S = 0$) and the high-spin ($S = 4$) states are presented in Figure 1.

The two stable states may feature different magnetic, optical, electrical, and vibrational/structural properties. In the solid state, the structural changes associated to the spin transition propagate in a cooperative manner, which can lead to a hysteretic behavior. From an applicative point of view, SCO materials have been proposed for various technological applications such as recording media, sensors (thermal, pressure, or gas) [17–21], displays [4], molecular switches [22, 23], memristor devices [24–28], or actuators [29, 30].

Molecular magnetism modeling has reached a certain degree of maturity, although several experimental findings are still open problems from theoretical point of view. Here, we provide a review of physical modeling in molecular materials with a special emphasis placed on spin crossover compounds, including Ising-type models and their generalizations, such as Slichter and Drickamer [9], Bousseksou et al. [31], Sorai and Seki [32], and Nasser et al. [33, 34], along with the novel approaches developed in our research group and applied to the characterization of phase transition and hysteresis behavior in molecular magnets [35–41]. Recent experimental findings are explained in this context and the relevance of theoretical results for technological applications is also discussed.

![Figure 1. 3d electron configurations in an octahedral ligand field for Fe (II) complexes in the low-spin ($S = 0$) and the high-spin ($S = 4$) states, respectively.](image)
2. Modeling of spin crossover compounds

The theoretical analysis of SCO materials has been developed in two main directions: the molecular approach [31–34, 42–64] and macroscopic approach [9, 65–70], respectively. In the case of the “molecular” models, the interactions between molecules disturb the molecular system and, thus, the statistical thermodynamics can be used to correlate the microscopic parameters of the system with the experiments. The “macroscopic” models use the laws of thermodynamics (phenomenological equations and/or mixture theory) and do not reflect the intrinsic structure of the molecular system.

The first model based on the concept of intermolecular interaction, which was able to qualitatively simulate the spin transition in SCO materials, was proposed by Wajnflasz and Pick in the early 1970s [42, 43]. They assumed that each metal center is a four-level system (two ion beams, each one of them having two spin states) and introduced a fictitious charge to characterize each spin state. The interaction between ionic centers was described by an Ising-like coupling term and the corresponding Hamiltonian was resolved in the mean-field approximation. This model was able to predict both discontinuous and continuous $HS \leftrightarrow LS$ transitions, but its main limitation is related to the nature of the interaction term that is not entirely justified.

The main macroscopic model accounting intermolecular interactions was proposed during the same period by Slichter and Drickamer [9]. This model successfully described the effect of an external applied pressure on the spin transition phenomena and represented the starting point for many macroscopic models attempting to describe the origin of the interactions (atom-phonon coupling, elastic energy, etc.). It is based on the theory of regular solutions assuming a random distribution of molecules in the material. This model was also able to reproduce hysteresis phenomena accompanying discontinuous spin transitions in SCO materials.

In 1974, Sorai and Seki [32] proposed an alternative thermodynamic model by considering that the molecules of the same spin state are organized in spin-like domains, assumed to be independent from each other. The model is similar to the theory of heterophase fluctuations developed by Frenkel and is widely used when the calorimetric data for SCO transition are available. However, their model is rather limited since it is not able to reproduce a hysteretic transition.

In 1977, Zimmermann and König [45] developed a novel model that takes explicitly into account the lattice vibrations. Intramolecular interactions were introduced by using an Ising-like Hamiltonian that was resolved in the Bragg-Williams approximation. The vibration modes were assumed in the Debye approximation, regardless of their intra- or intermolecular origin. The authors also showed a formal equivalence between the two-level models approached in the mean-field approximation and the macroscopic thermodynamic models based on the theory of regular solutions.

The next step was made by Kambara [46] in 1979. This model, based on the theory of the ligand field, assumes that the $HS \leftrightarrow LS$ transition is induced by the Jahn-Teller coupling between the 3d electrons and the local distortion of the base complex Fe.
The link between the thermodynamic model and the experimental results was made by Gutlich et al. [70], changing the input parameters of the Sorai and Seki model.

The first real physical approach of the cooperativity origin was proposed by Onishi and Sugano [65] and Spiering et al. [66], respectively. The authors consider the active molecules, which are susceptible of undergoing a spin transition, as hard spheres inserted into an elastic medium. Indeed, it has been observed that the spin transition was accompanied by a volume change of the SCO molecules. This change will induce a stress field that will propagate in the whole crystallographic lattice due to its elasticity. These spheres were treated as point defects.

A new approach for two-level models was proposed by Bousseksou et al. in the 1990s [31, 71]. They consider that the degeneracy of the two-spin states is much higher than the spin degeneracy ratio. Such degeneracy results from the entropy variation during the spin transition. This new approach allows fitting of the experimental data using input parameters from calorimetric measurements.

Ising-like models can be easily applied, but the nature of the interactions, introduced to describe the cooperativity among the molecules, is not specified. In this regard, in recent years, another series of elastic-like models was proposed. Here, we can distinguish two different approaches. The first approach was proposed by Nishino et al. [58] and Enachescu et al. [60], where they consider that the interactions are elastic in their nature, i.e., the molecules are interacting via the change of the molecular volume. The second approach is proposed by Nasser et al. [33, 34], which is known in the literature as “atom-phonon coupling” (APC) model. The basic idea of the APC model is that the elastic constants that characterize the coupling between neighboring molecules depend on their spin state. This model has proven its complexity, being able to reproduce a wide range of characteristic behaviors to spin transition compounds [35–37, 39, 41, 72, 73].

2.1. Slichter and Drickamer model

The model proposed by Slichter and Drickamer assumes a mixture of both HS and LS constituents in the solid. If we denote by \( n_{LS} \) and \( n_{HS} \) the associated mole fractions of the LS and HS states, respectively, the free enthalpy of the interacting ions centers can be then expressed as

\[
\Gamma = n_{LS}G_{LS} + n_{HS}G_{HS} - TS_{\text{mix}} + \Gamma n_{HS}n_{LS},
\]

where \( \Gamma \) is the term of intermolecular interaction and \( S_{\text{mix}} \) is the mixed entropy of an ideal mixing solution of LS and HS molecules, given by \( S_{\text{mix}} = -R(n_{LS} \ln n_{LS} + n_{HS} \ln n_{HS}) \) with \( R \) being the gas constant (\( = 8.3144598(48) \) J K\(^{-1}\) mol\(^{-1}\)). If \( G_{LS} \) is considered as origin level for energies, then \( G_{LS} = 0 \), while \( G_{HS} = \Delta G = \Delta H - T\Delta S \), with \( \Delta H \) and \( \Delta S \) being the entropy and the enthalpy variations, respectively, during the spin transition. As a consequence, the free enthalpy can be written as follows:
The equilibrium condition of the system, \( \left( \frac{\partial G}{\partial n_{HS}} \right)_T = 0 \), leads to the implicit expression of \( n_{HS} \) as a function on \( T \):

\[
G = n_{HS} \Delta H - \Gamma n_{HS}(1 - n_{HS}) + RT \left[ (1 - n_{HS}) \ln(1 - n_{HS}) + n_{HS} \ln n_{HS} + n_{HS} \frac{\Delta S}{R} \right].
\]

The solution of Eq. (3) for HS molar fraction as a function of temperature is plotted in Figure 2, for several values of intermolecular interaction constants considering variations of enthalpy and entropy specific to Fe(II) SCO compounds.

Figure 2. Thermal variations of the HS molar fraction for three selected values of the intermolecular interaction constant \( \Gamma = 0.2, 0.6, \) and 1 kJ. The following values have been used for the enthalpy and entropy variations: \( \Delta H = 7 \text{ kJ mol}^{-1} \) and \( \Delta S = 50 \text{ JK}^{-1} \text{mol}^{-1} \).

2.2. Sorai and Seki model

The model proposed by Sorai and Seki considers that LS and HS molecules are not randomly distributed but organized in spin-like domains, i.e., regions of molecules having the same spin state. Thus, the SCO system consists of several noninteractive domains having the same
number of molecules \( (n) \) and transition temperature around a critical temperature \( T_c(=\Delta H/\Delta S) \).

The equilibrium condition leads to the following explicit expression of \( n_{HS} \) as a function on \( T \):

\[
n_{HS} = \frac{1}{1+\exp\left(\frac{n\Delta H}{\Delta S}\frac{1}{T} - 1\right)}.
\]  

(4)

For single-molecule domain, the solution is the same with the one found from Eq. (3) in the case of noninteractive particles. The introduction of spin-like domains in the model leads to a better control of the spin transition path, as reflected by Figure 3(a), where the thermal variation of HS molar fraction is plotted for various numbers of molecules included in a spin-like domain. The Sorai and Saki model is widely used in the case when the calorimetric data of the SCO compounds are available. From calorimetric measurements, such as molar heat capacity at constant pressure as a function of temperature, \( C_p = f(T) \), the number of molecules per domain can be calculated by using the following formula:

\[
n = \frac{4RT_c^2}{(\Delta H)^2} \left( C_p(T_c) - \frac{1}{2} (C_{LS}(T_c) + C_{HS}(T_c)) \right),
\]  

(5)

where \( C_{LS} \) and \( C_{HS} \) are defined as “normal” heat capacities of LS and HS states as described in Figure 3(b). Although \( n \) is considered a relevant measure of the spin cooperativity during phase transition, the lack of interaction between domains leads to an important limitation of Sorai and Seki model, since it is not able to reproduce hysteretic transition.

Figure 3. (a) Thermal variations of the HS molar fraction for three selected values of the number of molecules per domain \( n = 1, 5, \) and 100. The following values have been used for the enthalpy variations and critical temperature: \( \Delta H = 7 \text{ kJ mol}^{-1} \) and \( T_c = 150 \text{ K} \) and (b) typical heat capacity variation as a function of temperature and definitions of \( C_{LS} \) and \( C_{HS} \).
2.3. Ising-like model

The model proposed by Varret research group [31] has taken into account the degenerate energy levels and it was successfully used to describe both static and dynamic properties of SCO compounds under different external stimuli such as temperature, pressure, magnetic, and electric fields [74, 75]. The model Ising-like Hamiltonian can be written as follows:

\[
H = \frac{1}{2} \sum_i (\Delta - k_B T \ln g) \sigma_i^z - \sum_{\langle i,j \rangle} J_{ij} \sigma_i \sigma_j - L(\bar{\sigma}) \sum_i \sigma_i^z ,
\]  

(6)

where \(\sigma_i^z\) is the fictitious spin operator associated to molecule \(i\) having the eigenvalues +1 (when the molecule is in the HS state) and −1 (when the molecule is in the LS state), \(\Delta\) is the internal energy gap between the HS and LS states, \(k_B\) is the Boltzmann constant (= 1.38064852(79) × 10^{-23} \text{ J K}^{-1}\), \(g\) is the degeneracy ratio between HS and LS energy levels (>1), \(J_{ij}\) stands for the short-range interaction parameter representing the cooperative interaction that only exists between the nearest-neighboring pairs (\(J_{ij}\) includes the nearest-neighbors number), \(i, j\) denotes the fact that the sum is only made over the nearest-neighbors pairs, and \(L\) stands for long-range interaction parameter that is expressed here in a mean-field approach with \(\langle \bar{\sigma} \rangle\) being the average fictitious spin.

Figure 4. Pressure effect on the configuration diagram.

Hamiltonian (6) takes into account both short- and long-range interactions, while spin and energy degeneracies are considered as a temperature-dependent effective field \((\Delta - k_B T \ln g)/2\) acting on each spin. In noninteractive system, the critical temperature \(T_{\text{cr}}\) at which
\( n_{\text{HS}} = n_{\text{LS}} = 0.5 \), corresponds to a zero effective field and, consequently, it is related to the energy gap and degeneracy ratio as follows: \( T_{\text{c}} = \Delta/k_b \ln g \). Let us now denote by \( T_{\text{c}} \) the critical temperature at which phase transition happens in purely ferromagnetic Ising systems formed by the second term in Hamiltonian (6) with \( J \) positive. If the critical temperature in purely Ising model is smaller than the critical temperature in noninteractive model (\( T_{\text{c}} < T_{\text{c}} \)), then a gradual spin transition from LS to HS takes place by increasing the temperature. Otherwise, the spin transition is discontinuous and is associated to a first-order phase transition. The addition of long-range interactions, the last term in Hamiltonian (6), can significantly change the system behavior.

For example, purely one-dimensional Ising systems generate no phase transition, and, consequently, the addition of degeneracy term does not contribute to the appearance of phase transition (since \( T_{\text{c}} \) is larger than \( T_{\text{c}} = 0 \)). However, when long-range interactions are introduced, various types of spin transition are possible, including multiple-step transitions and hysteretic transitions [76–79].

Next, we will discuss how this model can be adapted and applied to analyze the effects of external pressure on SCO behavior. It was experimentally observed that variations of the external pressure can give rise to transitions between HS state and LS state. This process, governed by entropy, can be better understood by assuming two potential wells, each representing a possible spin state as a function of a configuration coordinate (metal-ligand distance), as represented in Figure 4. The application of an external pressure decreases intermolecular distances and, by consequence, the metal-ligand distances. Since the ligand field is inversely proportional to the metal-ligand distance (to the sixth power), the application of external pressure has a similar effect as the substitution of ligands to the central metal, so the potential wells can move vertically and horizontally by changing the metastable state [9, 31, 43, 74, 80].

The energy gap dependence on the pressure can be expressed as:

\[
\Delta(p) = \Delta(p = 0) + p \delta V, \tag{7}
\]

where \( \delta V \) is the volume variation of the SCO molecules during the spin transition phenomenon and \( p \) is the externally applied pressure. By considering the mean-field approach, the behavior of the system described by Eq. (6) coupled with Eq. (7) can be approximated by behavior of a noninteracting system of spins with one-site Hamiltonian \( H_{\text{mf}} \) written as

\[
H_{\text{mf}} = \frac{-\Delta(p) - k_b T \ln g}{2} \hat{\sigma} - (qJ + L) \hat{\sigma} \cdot \hat{\sigma} < \hat{\sigma} >, \tag{8}
\]

where \( q \) is the number of nearest neighbors. The average fictitious spin \( \langle \hat{\sigma} \rangle \) is also known as “fictitious magnetization” and will be further denoted by \( m \). The resulting mean-field equation \( m \) has the following form:
For the sake of simplicity, in the following calculations we use several normalizations, namely, the energy gap $\Delta = \Delta(p)/k_B$, the interaction parameter $\Gamma = (qJ + L)/k_B$ (which includes both short- and long-range interaction), and the volume variation $\alpha = \delta V \cdot 10^{-5}/k_B$ (where the factor $10^{-5}$ stands for the conversion of the pressure units from pascal to bar). Sample results for hysteretic transitions driven by temperature and pressure variations are presented in Figure 5, for $\Delta = 1000K$, $\ln g = 7$, $\Gamma = 400K$, $\alpha = 4K/bar$, where the HS fraction $n_{HS}$ is simply related to the fictitious magnetization $m$ by $n_{HS} = (1 + m)/2$.

Figure 5. (a) Thermal hysteresis loop computed for pressure $p = 1$ bar and (b) pressure hysteric loop for $T = 200K$. The normalized parameters used in these calculations are $\Delta = 1000K$, $\ln g = 7$, $\Gamma = 400K$, $\alpha = 4K/bar$.

By increasing the applied pressure, the switching temperatures will shift to higher values, thus making it possible to control the system switching temperature by external inputs. However, the width of hysteretic loop is reduced by this action and, at some critical value of the applied pressure, the SCO system will lose its cooperativity, i.e., the hysteretic loop disappears leading to a continuous spin transition. The 3D phase diagrams’ $(T, p, n_{HS})$ coordinates are presented in Figure 6, for a system with noninteractive molecule (left) and for a cooperative system with interaction constant $\Gamma = 400K$ (right). The other parameters are the same as in the previous discussion $\Delta = 1000K$, $\ln g = 7$, $\alpha = 4K/bar$. In order to separate the hysteretic part from nonhysteretic part of the phase diagram, let us observe that the transition temperatures (pressures) can be obtained as maxima and minima of the temperature (pressure) as a function of $n_{HS}$ or, equivalently, as a function of $m$. The expression of temperature in terms of $m$ can be obtained by inverting Eq. (9) as follows:

$$m = \tanh \left( \frac{(qJ + L)m}{k_B T} + \frac{k_B T \ln g - \Delta(p)}{2k_B T} \right).$$

(9)
\[
T = \frac{\Delta(0) + \alpha p - 2\Gamma m}{\ln g - \ln \left(\frac{1+m}{1-m}\right)}.
\]

In order to find the extrema of \(T\) as a function of \(m\), the derivative \(dT/dm\) is set to be zero, leading to the following algebraic equation:

\[
\frac{1}{2}(m^2 - 1)\left(\ln \left(\frac{1+m}{1-m}\right) - \ln g\right) + 2m = \frac{1}{\ln g - \ln \left(\frac{1+m}{1-m}\right)}(\Delta(0) + \alpha p).
\]

By varying pressure, the solution of this equation describes in \((T, p, n_{HS})\) space, a critical curve, also known as spinodal curve, which delimits the hysteretic part of the phase diagram from the nonhysteretic one, as presented from Figure 6.

2.4. Atom-phonon coupling model

Atom-phonon coupling model has been proposed by Nasser et al. [33, 34], with the aim to overcome some deficiencies of Ising-like models concerning the origin of the interaction parameter. Moreover, APC model is taking into account the results of Sorai and Seki, regarding the dependence of the Debye temperature as a function of the HS fraction [32]. In Sorai and Seki model, the importance of the contribution of phonons in the transition phenomenon is emphasized. A similar approach has been also discussed by other authors in Refs. [81, 82]. Although the APC model was initially proposed for the characterization of the thermal behavior of the SCO compounds, its applicability has been extended afterwards in order to describe the process of photoexcitation observed at low temperature light-induced excited spin
state trapping (LIESST) [37, 39, 41], the relaxation process of the photo-excited metastable states and light-induced thermal hysteresis (LITH) [35], as well as the effect of externally applied pressure [36, 37].

In the APC model, the spin crossover molecules are modeled as atoms linked by springs, i.e., the intramolecular vibrations are not taken into account. As in the Ising-like model, a fictitious spin \( \hat{\sigma}_i \) is associated to each molecule that can take two eigenvalues: \(-1\), corresponding to the fundamental level LS, and \(+1\), corresponding to the HS level. According to the spin state of the neighbor atoms, three values of the elastic constants taken into account are denoted by \( \lambda \) when both molecules are in the LS state, \( \nu \) when both molecules are in the HS state, and \( \mu \) when one atom is in the LS state and the other is in the HS state. From Brillouin and Mossbauer spectroscopy [83, 84] data, it was shown that the elastic constants are smaller in the HS state than in the LS state. Thus, at a given temperature, phonons promote the HS state while the HS-LS energy gap promotes the LS state, leading to an interesting competition between two mechanisms. Consequently, in the APC model, it is considered as \( \nu < \mu < \lambda \).

The APC Hamiltonian is given by the sum of two contributions: electronic and vibrational:

\[
H = H_{\text{spin}} + H_{\text{ph}} = \left( \frac{N}{2} \Delta \hat{\sigma}_i \right) + \left( E_c + E_p \right) ,
\]  

where \( N \) is the number of spins in the system, \( \Delta \) is the energy gap between these LS and HS states, and \( E_c \) and \( E_p \) are the kinetic energy and the potential energy of the system, respectively. The kinetic energy is simply written as:

\[
E_c = \sum_{i=1}^{N} \frac{p_i^2}{2m_a} ,
\]

where \( p_i \) is the impulse of atom and \( m_a \) is the mass of the SCO atom. For a chain of \( N \) atoms, the potential energy can be written as:

\[
E_p = \sum_{i=1}^{N-1} \frac{1}{2} e_{i,i+1}(u_{i+1} - u_i)^2 ,
\]

where \( u_i \) is the longitudinal displacement of the \( i \)th atom from its equilibrium position that is assumed to be independent of the electronic state of the atom \( i \) and its neighbors and \( e_{i,i+1} \) is the elastic constant between the \( i \)th atom and the \((i+1)\)th atom. By taking into account the previous discussion related to the state-dependent elastic constants, a concise formula can be obtained for \( e_{i,i+1} \) in terms of \( \lambda, \mu, \nu \) and the fictitious neighboring spins \( \hat{\sigma}_i \) and \( \hat{\sigma}_{i+1} \):

\[
e_{i,i+1} = \frac{\lambda + 2\mu + \nu}{4} + \frac{\nu - \lambda}{4} (\hat{\sigma}_i + \hat{\sigma}_{i+1}) + \frac{\lambda - 2\mu + \nu}{4} \hat{\sigma}_i \hat{\sigma}_{i+1} .
\]
The periodic boundary conditions are considered, as apparent from Figure 7, so $\hat{\sigma}_{N+1}$ is actually $\hat{\sigma}_1$. By plugging Eq. (15) in Eq. (14), the potential energy can be rewritten as the sum of three terms $V_0$, $V_1$, and $V_2$ where:

$$V_0 = \sum_{i=1}^{N} \frac{\lambda + 2\mu + \nu}{8} (u_{i+1} - u_i)^2,$$

$$V_1 = \sum_{i=1}^{N} \frac{\nu - \lambda}{8} \left[ (u_{i} - u_{i+1})^2 + (u_{i+1} - u_i)^2 \right] \sigma_i,$$

$$V_2 = \sum_{i=1}^{N} \frac{\lambda - 2\mu + \nu}{8} (u_{i+1} - u_i)^2 \sigma_i \sigma_{i+1}.$$  

Figure 7. Chain of atoms bound together by springs with periodic boundary conditions.

It is apparent from Eq. (17) that $V_1$ has the expression of Zeeman-like energy, each spin $\sigma_i$ being subject to an effective field $h_i$ given by:

$$h_i = \frac{\nu - \lambda}{8} \left[ (u_{i} - u_{i+1})^2 + (u_{i+1} - u_i)^2 \right].$$

where $\nu < \lambda$ in this field promotes the HS state (eigenvalue +1). It is apparent from Eq. (18) that $V_2$ has the expression of exchange-like energy, so an effective exchange parameter $J_{i,i+1}$ can be defined as follows:

$$J_{i,i+1} = \frac{\lambda - 2\mu + \nu}{8} (u_{i+1} - u_i)^2.$$

The sign of exchange constant depends on the relation between $\mu$ and $(\nu + \lambda)/2$, and the exchange term disappears when the elastic constant for HS-LS case is the average of the elastic constants for LS-LS and HS-HS states, i.e., $\mu = (\nu + \lambda)/2$.

In the mean-field approximation, the elastic constant $e_{i,i+1}$ can be replaced by an effective elastic constant $K$ that does not depends on site $i$. Thus, the effective field $h_i$ generated by the phonons is uniform throughout the chain and the effective exchange parameter $J_{i,i+1}$ is the same for each pair of neighboring spins. The system can be studied by using transfer-matrix method [85], and the resulting mean value for the phonon Hamiltonian can be expressed as follows:
\[
\langle H_{\text{phon}}(K) \rangle_T = \frac{\hbar \omega_M(\lambda)}{2} \sqrt{\frac{K}{\lambda}} \sum_{i=1}^{N} \sin \frac{\pi i}{N} \coth \left( \frac{\hbar \omega_M(\lambda)}{2 k_{B} T} \sqrt{\frac{K}{\lambda}} \sin \frac{\pi i}{N} \right),
\]  

(21)

where $\hbar$ is the reduced Planck constant ($= 1.054571800(13) \times 10^{-34}$ J s) and $\omega_M(\lambda)$ is the maximum phonon angular frequency for a periodic chain with an elastic constant $\lambda$ ($\omega_M(\lambda) = 2 \sqrt{\lambda/m_a}$). The effective elastic constant $K$ can be obtained by applying the variational method \[86\]:

\[
K = \frac{2\mu + \lambda + \nu}{4} + 4 \hbar c \cdot m + 2 J_{c} \cdot s,
\]

(22)

where $h_0 = \frac{\nu - \lambda}{4}$, $J_0 = \frac{\lambda - 2\mu + \nu}{4}$, and $m = \hat{\sigma} \cdot \hat{\chi}$, $s = \hat{\sigma} \cdot \hat{\chi} \cdot \hat{\sigma} \cdot \hat{\chi}$ are the two-order parameters known as the fictitious magnetization and correlation parameter, respectively. From the transfer matrix method \[85\], the following system of two nonlinear equations is obtained for the two unknown $m$ and $s$:

\[
\begin{align*}
m &= \frac{\exp(J/k_{B}T) \sinh(h_{z}/k_{B}T) - \exp(-2J/k_{B}T)}{\sqrt{\exp(2J/k_{B}T) \sinh^{2}(h_{z}/k_{B}T) + \exp(-2J/k_{B}T)}} \\
s &= 1 - \frac{2m^{2} \exp(-4J/k_{B}T)}{\sinh(h_{z}/k_{B}T) \left( m \cosh(h_{z}/k_{B}T) + \sinh(h_{z}/k_{B}T) \right)},
\end{align*}
\]

(23)

where the implicit dependencies of the right-hand side terms on $m$ and $s$ are presented in $h_z$ and $J$ that depend on $K$ (see also Eqs. (21) and (22)) as follows:

\[
\begin{align*}
h_{z} &= \frac{\Delta}{2} - \frac{2k_{B} \langle H_{\text{phon}}(K) \rangle_{T}}{N} + k_{B}T \ln \frac{g}{2} \\
J &= \frac{J_{c} \langle H_{\text{phon}}(K) \rangle_{T}}{N},
\end{align*}
\]

(24)

where $g$ is the degeneracy factor equal to the ratio between the degeneracy of the HS energy level and the degeneracy of the LS energy level. In conclusion, once the SCO parameters are given, namely, $\Delta$, $g$, $N$, $h_0$, $J_0$, $\lambda$, $m_c$, the formula (23) contains two self-consistent equations for $m$ and $s$ that can be solved for each value of the temperature $T$.

Since the HS fraction $n_{HS}$ is simply related to the fictitious magnetization $m$ by the formula $n_{HS} = (1 + m)/2$, the dependence of $n_{HS}$ on temperature is thus obtained. For the sake of simplicity in the following calculations, we use several normalizations, namely:
• the reduced temperature \( t_r = \frac{k_B T}{\hbar \omega_M(\lambda)} \);

• the dimensionless electronic excitation energy \( \delta = \frac{\Delta \hbar \omega_M(\lambda)}{\nu} \); and

• the dimensionless parameter \( y \) defined by \( \mu = \frac{\lambda + \nu}{2} + \frac{\lambda - \nu}{2} y \).

The dimensionless parameter \( x \) is a measure of the interaction intensity in the system, a small value of \( x \) implying strong interactions in the system. The dimensionless parameter \( y \) shows how close is \( \mu \) to \( \lambda \) as compared to \( \nu \), a positive value of \( y \) implying that \( \mu \) is closer to \( \lambda \) while a negative value of \( y \) implying that \( \mu \) is closer to \( \nu \). It is apparent that for \( y = 0 \), \( \mu \) is the mean value of \( \lambda \) and \( \nu \). By using this normalization and the property of the above algebraic system that uses only ratios of elastic constants to \( \lambda \), the 8-parameter problem is now reduced to 6-parameter problem: 4 normalized parameters presented above along with the number of atoms \( N \) and degeneracy ratio \( g \). An example of hysteretic transition for a SCO chain driven by temperature, computed in the APC model, is plotted in Figure 8.

**Figure 8.** Thermal hysteresis loop for a SCO chain computed in the APC model. The reduced parameters used in these calculations are \( x = 0.2, y = 0, \delta = 0.606 \), while \( \ln g = 5, N = 2000 \).

The size effect plays a very important role in applications such as information storage technology. The problem is to estimate the minimum size for which the system still maintains its hysteretic properties. An important advantage of the APC model is that the system size appears naturally in the Hamiltonian, allowing a direct analysis of the size effect, i.e., the dependence
of the hysteresis loop width on the number of atoms $N$. Since the intermolecular interactions also play a crucial role in the width of hysteresis loop, we present in Figure 9, the variation of hysteresis loop width with the system size for two values of the elastic constant ratio $x$. As expected, the hysteretic loop is wider when stronger interactions are present in the system (i.e., $x$ is smaller), but it is interesting to observe that critical system size for hysteretic behavior is also decreasing with the increase in the interaction strength. A saturation phenomenon in hysteretic loop width is also observed with the increase in system size. A more detailed analysis of this important topic can be found in Refs. [36, 80].

![Figure 9](image)

**Figure 9.** Size dependence of the SCO hysteresis width computed in the APC model for two different values of the elastic constant ratio $x$. The other reduced parameters used in these calculations are $y = 0$, $\delta = 0.606$, while $\ln g = 5$.

![Figure 10](image)

**Figure 10.** (Left) Thermally driven transitions computed for selected values of pressure indicated in the figure. (Right) Pressure-driven transitions computed for selected values of the reduced temperature indicated in the figure. The parameters used in these calculations are $x = 0.4$, $y = 0.8$, $\delta_0 = 0.525$, $\alpha = 10^{-4}$ “bar$^{-1}$”, $\ln g = 5$, and $N = 2000$. 
Next, we will focus on the pressure effect on the spin transition driven by temperature as well as on the temperature effect on the spin transition driven by external pressure. Based on several results obtained by Raman spectroscopy, the vibrational properties of SCO systems seem to remain stable when external pressure is applied [84]. Thus, we can assume that the ratio of the elastic constants does not depend on pressure and the pressure effect will be accounted only in the energy gap: \( \Delta_p = \Delta_0 + p\delta V \), as it was also considered in the Ising-like model. By substituting the new expression for the energy gap in Eq. (28), the solution for the fictitious magnetization will be dependent on the external pressure. The dimensionless electronic excitation energy will be now defined as \( \delta_0 = \Delta_0/\hbar\omega_{\text{M}}(\lambda) \) and a normalized volume variation is introduced as \( \alpha = \delta V \cdot 10^{-5}/\hbar\omega_{\text{M}}(\lambda) \) (where the factor \( 10^{-5} \) stands for the conversion of the pressure units from pascal to bar). Sample results for spin transitions driven by temperature and pressure variations are presented in Figure 10, for \( x = 0.4, y = 0.8, \delta_0 = 0.525, \alpha = 10^{-4}\text{bar}^{-1}, \ln g = 5, \) and \( N = 2000. \)

It should be noted that the pressure hysteresis loops computed in the APC model present an elongation at high pressure, in good agreements with the experimental observations [87]. However, in some coordination compounds, the electronic transitions do not always arise from a simple configuration. The external pressure can also induce a crystallographic phase transition, giving rise to an unusual behavior, such as the increase of the hysteretic loop width with the increase in the applied pressure [16].

2.4.1. Relaxation process of the photo-excited HS metastable states at low temperature

The bistable nature of spin crossover materials coupled to their sensitivity to light irradiation is of particular interest for the development of data-recording media. At low temperatures (typically below 40 K), the system can be switched from the ground low-spin state to the metastable high-spin state using appropriate wavelengths known as light-induced excited spin state trapping effect [6]. The relaxation from high-spin state to low-spin state (HS → LS) appears if, after photoexcitation, the light is switched off.

![Figure 11. Configuration diagram of a spin crossover molecule showing the energy barrier variation.](image-url)
At low temperature (<30 K), the relaxation process occurs by tunneling, while at higher temperature (>40 K), this process is thermally activated and consequently it is very fast at room temperature, where the life time is in the order of $10^{-8}$ to $10^{-6}$ s [88]. The relaxation process from high-spin state to low-spin state was studied in detail by Andreas Hauser and the Mainz group [88–90]. This analysis was based on the energy barrier properties of the state (HS) and led to the understanding of the main properties of the SCO cooperative relaxation. As illustrated in Figure 11, the energy gap $\Delta$ is increasing and the activation energy $E_a$ is decreasing with the decrease of the molar high-spin fraction $n_{HS}$. Consequently, the relaxation (HS $\rightarrow$ LS) becomes faster with the decrease of $n_{HS}$ leading to a self-accelerating process.

The expression of the relaxation rate, $K_{HL}$, was introduced by Hauser, on experimental and theoretical bases, by considering a linear dependence of the barrier energy with $n_{HS}$. In APC model, a more realistic formula for the energy barrier was derived and used to describe the relaxation process. Thus, the activation energy can be expressed as follows:

$$E_a = E_0 - E_{HS} = E_0 - \frac{\Delta}{2} - \frac{2k_0}{K} \left( \frac{H_{phon}}{N} \right)_T,$$

(25)

where $E_0$ is the level of the energetic barrier. In the thermal activation regime, the relaxation rate is given by the standard relation:

$$K_{HL}(T,n_{HS}) = k_c \exp \left( -\frac{E_a}{k_BT} \right),$$

(26)

where $k_c$ is the high-temperature relaxation rate (associated to vibrational frequencies). The variation in time of the high-spin fraction is given by the master equation:

$$\frac{dn_{HS}}{dt} = -n_{HS}K_{HL}(T,n_{HS}),$$

(27)

where $K_{HL}(T,n)$ is given by Eq. (26) in which $E_a$ is given by Eq. (25).

Sample results for the SCO relaxation curves calculated by using the above formulas derived in the APC model are presented in Figure 12, for several values of the interaction parameter $x$, and in Figure 13, for several values of the temperature at which relaxation process occurs. The values for the reduced parameters of the system (introduced above) used in these calculations are given in the figures’ captions. In addition, a new normalized parameter is introduced and denoted by $A$, which is defined by $A = E_0 / \hbar \omega (\lambda)$.

As can be seen from Figure 12, the relaxation process is getting slower when the interaction between the molecules is getting stronger (i.e., smaller values of $x$). It is important to observe the sigmoidal shape of the relaxation curve which is in good agreement with the experimental data. Moreover, APC model also resolves one of the weaknesses of Hauser model, which was
not able to reproduce the so-called “tail-effect,” i.e., the slow-relaxation effect observed for small values of molar high-spin fraction. Since these results were derived in the framework of classical Arrhenius-type activation, the relaxation process of the photo-excited metastable states can also be analyzed by using the Arrhenius representation $\ln(K_{HS}) = f(n_{HS})$, as shown in Figure 14. Here, the shape of the resulting Arrhenius diagram shows two slopes, in good agreement with the experimental data, unlike the Hauser model, where the Arrhenius diagram is a straight line [91].

Figure 12. Relaxation curves recorded in dark for selected values of interaction parameter $x$. The parameters used in the calculations are $y = 0$, $\delta_0 = 0.6$, $t_r = 0.03$, $\ln g = 5$, $N = 2000$, $A = 0.8$, and $k_\infty = 100$.

Figure 13. Relaxation curves recorded in dark for selected values of the reduced temperature $t_r$ at which relaxation process occurs. Parameters used in the calculations are $y = 0$, $\delta_0 = 0.6$, $x = 0.2$, $\ln g = 5$, $N = 2000$, $A = 0.8$, $k_\infty = 100$. 
Since we focused on thermally activated relaxations, it is apparent that the relaxation process speeds up with the increase of temperature, as shown in Figure 13, as well. In practice, the switch from HS state to LS state can be achieved either by direct nonradiative relaxation or by the reverse LIESST effect when the samples are irradiating in the HS state absorption band [8].

2.4.2. Light-induced thermal hysteresis

Experiments under permanent light irradiation illustrate the competition between the noncooperative photoexcitation (generating LS → HS transitions) and the cooperative relaxations (HS → LS) that are thermally activated. This competition leads to hysteretic transition known as light-induced thermal hysteresis. Light-induced thermal hysteresis was discovered almost simultaneously by the groups of F. Varret from Versailles [88] and O. Kahn from Bordeaux [92], respectively.

The LITH explanation given by Varret’s group goes as follows: at low temperatures, the metastable HS states are populated by LIESST effect; by increasing the temperature, while maintaining the light irradiation, thermally activated relaxations (HS → LS transitions) get stronger leading the system into a metastable state where thermal relaxations balance the LIESST effect. Quantitatively, this situation can be expressed by introducing in relaxation Eq. (27) an additional term accounting for the effect of continuous light irradiation:

$$\frac{dn_{\text{HS}}}{dt} = -n_{\text{HS}}(t)K_{\text{HS}}(t, n_{\text{HS}}(t)) + I_0 \sigma_a (1 - n_{\text{HS}}(t)) ,$$

where $I_0 \sigma_a$ counts as the LS → HS switching probability rate ($I_0$ is the light beam intensity and $\sigma_a$ is the light absorption cross section). Depending on the temperature change rate, two cases can be identified.
• **Static LITH**: when the temperature change rate is small as compared to the rate of the other processes involved, so it can be neglected. The metastable states can be identified as fixed points of Eq. (28), which is equivalent to solving, for each value of temperature \( t_r \), the algebraic equation:

\[
K_{IH}(t_r, n_{HS}) = I_0 \sigma_r \left( \frac{1}{n_{HS}} - 1 \right)
\]  

(29)

Sample results for the metastable states calculations are plotted versus (reduced) temperature in **Figure 15**, for two values of the interaction parameter \( x \). The static LITH phenomenon is apparent in these two examples. As expected, the intermolecular interaction has an important role in LITH, due to the cooperative nature of the relaxation process.

• **Dynamic LITH**: where the temperature change interferes with the other processes and, consequently, the temperature change rate cannot be neglected. Typical hysteretic loops obtained by using Eq. (28) and APC model for \( K_{HL} \) are plotted in **Figure 16**, for several values of temperature change rate \( v_r \). The APC model can successfully reproduce the main features of the experimental data reported in LITH properties [93]. By increasing the cooperativity of a SCO system, the LITH width increases accordingly. A similar effect can be obtained "artificially" by increasing the temperature change rate; thus, by increasing the temperature change rate, the system does not have enough time to react at the light effect, so the hysteresis loop is widened.

**Figure 15.** Static LITH for two values of the interaction parameter \( x \). Parameters used in these calculations are \( y = 0, \delta_0 = 0.6, x = 0.1, \ln g = 5, N = 2000, A = 0.8, k_\infty = 100, I_0 \sigma_a = 5 \times 10^{-4} \text{s}^{-1} \).

Although the APC model is able to reproduce a wide range of phenomena, experimentally observed in molecular SCO compounds, it also has some limitations that need to be addressed.
by further studies. The first physical limitation of the APC model consists of the difficulty of measuring the elastic constants and consequently, the impossibility of measuring the reduced parameter $y$. This constraint leads to the application of the trial-and-error method for identifying a proper value of this parameter needed in a quantitative study instead of a physical identification that would make the model more focused on technical applications. Another important limitation of this model concerns the analysis of the spin-like domains spreading in spin crossover single crystals. Indeed, the spin crossover molecules are considered in APC model as fixed atoms that limit the analysis of the local distortions. These distortions are not only affecting the nearest neighbors but also causing long-range interactions between the system molecules. It is important to notice that some limitations were overcome by newly developed models, such as mechanoelastic model designed by Enachescu et al. [60], where the molecular volume change during the spin transition is taken into account.

Figure 16. Dynamics LITH for three values of the temperature change rate $v_r$. Parameters used in the calculations are $x = 0.5, y = 0, \delta_0 = 0.6, x = 0.1, \ln g = 5, N = 2000, A = 0.8, k_\infty = 100, I_0 = 5 \times 10^{-4} \text{ s}^{-1}$.

3. Conclusions

This chapter provided a general introduction of physical modeling in molecular materials with a special emphasis placed on spin crossover compounds. It covered multiple types of spin crossover compounds and analyzed intermolecular interactions, size effects, thermal behavior, pressure effects, the photoexcitation processes (such as LIESST), relaxation process, and light-induced thermal hysteresis. Numerous illustrations were provided to help the reader in grasping highly abstract concepts, explaining molecular materials behavior, and designing various applications. In this way, we try to support students and young researchers get acquainted to this research area and to facilitate the communication between the mathematical
and physical communities, chemical experimentalists, and engineering practitioners in this field.

Although the models presented here are able to reproduce a wide range of phenomena, experimentally observed in molecular SCO compounds, they also have some limitations that emphasize the need for further research in this area. However, an attentive analysis of the bibliography is needed before deciding to work on a specific problem, since several generalizations have not been explicitly addressed in this short introduction to this novel and dynamic area of research.

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