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Optimal Design of a New Wheeled Mobile Robot
by Kinetic Analysis for the Stair-Climbing States

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1. Introduction

Many mobile robots have been developed in the various application fields, such as building inspection and security, military reconnaissance, space and underwater exploration, and warehouse services (Muir & Neuman, 1987). Mobile robots are designed with the specific locomotive mechanisms according to the environment of the application field. The various locomotive mechanisms used in mobile robots can be classified into three types: wheeled, tracked, and legged type (Kim, 1999) (Lee et al, 2000). Each locomotion type has its inherent advantages and disadvantages as described below. The wheeled mobile robots (WMRs) weigh less than robots of the other locomotive types and have other inherent advantages, such as high energy efficiency, low noise level, etc (Muir & Neuman, 1987). In comparison with legged mobile robots, the WMRs have a simpler driving part and a plain control strategy, but they have the poor adaptability to the terrain.

Tracked mobile robots have the merit of easy off-road travelling. However, they usually have a heavier driving part and need more power for turning motions, in comparison with mobile robots with other locomotive types. Additionally, tracked mobile robots are usually too noisy to be utilized for in-door applications. Legged mobile robots can easily adapt to the unstructured environments, such as off-road environments, but require more actuators to stabilize themselves than mobile robots in the other two categories. As the locomotion mechanisms are complex and need more complicated control algorithms, legged mobile robots have poor mobility on the plane surfaces.

Various types of mobile robots that are capable of climbing up stairs have been developed but, until now, most of the mobile robots developed have tracked-type locomotive mechanisms (Kohler et al., 1976) (Maeda et al., 1985) (Yoneda et al., 1997) (Iwamoto & Yamamoto, 1985) (Iwamoto & Yamamoto, 1990). For the purpose of developing a robot capable of traversing the stairs, Estier (Estier et al., 2000) proposed a WMR with three 4-bar linkage mechanisms, by applying the concept of the instantaneous centre of rotation. To explore Mars, Volpe (Volpe et al., 1997) developed a WMR named Rockey 7, which is capable of climbing up steps about 1.5 times as large as the wheel diameter.
This paper proposes a new type of locomotive mechanism for WMRs that is capable of travelling up stairs based upon two design concepts: ‘adaptability’ and ‘passivity’. The proposed WMR has a passive linkage-type locomotive mechanism that offers extensive adaptability to rough terrain, especially stair. To fully analyze the behaviours of the proposed passive linkage mechanism during stair climbing, several states analysis was accomplished using the analytical method and the multi-body dynamic analysis software ADAMS\textsuperscript{TM}. From the results of the states analysis, the optimization of the proposed WMR was performed using the multi-objective optimization method.

2. The Proposed Locomotive Mechanism

For the purpose of developing a mobile robot which has a simple structure, light weight, and good energy efficiency, we have elaborately analyzed the features of the three types of locomotive mechanism – wheeled, tracked, and legged. The tracked mobile robots have high off-road capability but usually have heavy weight; the tracked mobile robots have low energy efficiency in turning motions; and the legged mobile robots have extensive adaptability to rough terrain but usually have complex locomotive mechanisms that need complicated control algorithms. Additionally, the legged mobile robots have poor mobility on the plane surfaces. On the other hand, the wheeled mobile robots have simple structure, good mobility on the plain surfaces, and good energy efficiency in turning, but have poor adaptability to the rough terrain. Therefore, considering the indoor applications, we opted to develop a wheeled mobile robot. Our wheeled mobile robot, however, has a locomotive mechanism which enables it to adapt to rough terrain, such as the stair like the legged mobile robot.

Therefore, to develop a wheeled mobile robot that not only has adaptability to stairs but also maintains good energy efficiency, we proposed two design concepts. One is adaptability and the other is passivity (Woo et al., 2001). Based on these design concepts, we developed the first prototype of the WMR, named ROBHAZ-6W, and accomplished stair climbing experiments on several types of stairs (Woo et al., 2002). To improve the WMR’s adaptability to rough terrain and its ability to climb stairs, this paper presents the modified passive linkage-type locomotive mechanism.

2.1 Adaptability

WMRs usually have been utilized in the indoor environment due to their advantages on the indoor applications. To extend the WMR’s application area to the outdoor environment, the WMR must have good adaptability to the environment. In order to improve this adaptability, we proposed a simple linkage-type locomotive mechanism that makes it possible for the driving wheels to move relative to the robot body and for the wheelbases to change among the driving wheels, according to the shape of terrain.

The proposed linkage-type locomotive mechanism is a simple 4-bar linkage mechanism; the side view of the WMR with the proposed mechanism is shown in Fig. 1. As shown in Fig. 1, driving wheels 1 and 2 are interconnected with link 1, which is able to rotate about pin joint P relative to the robot body. Driving wheel 3 is attached to link 2, which is connected with link 1 by pin joint Q. To stabilize the robot body, link 3 is used to connect the robot body to link 2 by pin joints R and S. Therefore, links 1, 2, 3, and the robot body form a 4-bar linkage mechanism which has one degree-of-freedom (DOF). By using the linkage mechanism, the
wheelbases among the driving wheels of the WMR and the relative positions of the wheel axes to the center of gravity of the robot body can be changed, according to the configuration of the proposed linkage-type locomotive mechanism. The WMR has a symmetric structure with the proposed linkage-type locomotive mechanism on the right and the left.

![Side view of the WMR with the proposed linkage-type locomotive mechanism](image1)

Fig. 1. Side view of the WMR with the proposed linkage-type locomotive mechanism

Fig. 2 shows the adaptability of the WMR with the proposed locomotive mechanism according to the two different types of terrain. As shown in Fig. 2, the 6-WMR with the proposed linkage-type locomotive mechanism easily adapts to the two different types of terrain profile; the concave and the convex terrain. The WMR with the proposed linkage locomotive mechanism has more extensive adaptability to the environment than conventional WMRs. The detailed locomotive method of stair-climbing will be described in the next section.

![Extensive adaptability of the proposed WMR](image2)

Fig. 2. Extensive adaptability of the proposed WMR

### 2.2 Passivity

As described in the above section, the proposed linkage-type locomotive mechanism offers extensive adaptability to the terrain. Additionally, as shown in Fig. 1, the proposed mechanism does not have any active mechanical elements, such as motors, that need active control techniques. Therefore, the proposed WMR can passively adapt to the environment, according to the linkage-type locomotive mechanism.

However, while the proposed WMR goes up the stairs, the WMR may be led into states which it cannot climb up the stairs and halts where it is. We called these unexpected states ‘sticking conditions’. Some of the sticking conditions are shown in Fig. 3.

![Sticking conditions](image3)

As shown in Fig. 3 (a), one of the sticking conditions may occur when driving wheels 1 and 2 of the WMR simultaneously come into contact with the wall of the stair. This sticking condition can be evaded by setting the proper wheelbase between driving wheels 1 and 2. Another sticking condition occurs when driving wheels 2 and 3 simultaneously come into contact with the wall of the stair, as shown in Fig. 3 (b). This sticking condition commonly
occurs due to excessive rotation of link 1 relative to the robot body. Not only to avoid this sticking condition and but also to maintain the design concept of ‘passivity’, we suggested a limited pin joint at point P that restricts the excessive rotation of link 1 relative to the robot body, as described in Fig. 1. The maximum allowable angle of link 1 relative to the robot body will be determined by the kinetic analysis in the next section.

Fig. 3. Sticking Conditions

3. Kinetic Analysis

In this section, we introduce the detailed analysis of the WMR’s states while the WMR with the proposed passive linkage-type locomotive mechanism climbs up the stair. The states are classified in the position of the point and the status of contact between the driving wheels and the stair. The kinetics and the dynamics of the proposed locomotive mechanism at each state are also different from each other due to the posture of the WMR and the contact forces on the driving wheels at the points of contact. The reasons for classifying the climbing motion of the WMR into the several states are to describe the contact forces acting on the driving wheels as the analytic functions and analyze the kinetics of the proposed WMR. For the whole states, the contact forces can not be expressed in the analytic function, due to the absence of contact on certain driving wheels. For each state, however, the normal forces and the corresponding friction forces acting on the driving wheels can be described in the analytic functions.

From the kinetic analysis of each state, the geometric constraints to prevent the WMR from falling into the sticking conditions are suggested and the object functions to improve the WMR’s capability to climb up stairs are derived. The design variables of the proposed WMR are shown in Fig. 4.

Fig. 4. Design variables of the WMR with the proposed mechanism
The schematic design in Fig. 4 shows the left side of the WMR having a symmetric structure. In Fig. 4, max indicates the maximum allowable counter-clockwise angle of the link 1 at the pin joint relative to the robot body in order to prevent the WMR from falling into the sticking condition.

Fig. 5 shows the suggested 11 states divided by considering the status of the points of contact, while the mobile robot climbs up the stair. In Fig. 5, the small dot attached around the outer circle of the driving wheels indicates the point of contact between the driving wheels and the stair. If the WMR can pass through the whole states, the WMR is able to climb the stair.

As shown in Fig. 5, the suggested 11 states can be classified into 4 groups. The first group is composed of the states which are kinetically dominant among the whole states, such as states 1, 3, 7, and 10. The capability for the WMR to climb the stair is determined by the states in this group. Therefore, to improve the WMR’s ability to climb up the stair, the object functions being optimized will be obtained from the states in this group.

The second group consists of the states that are kinetically analogous to the states in the first group, such as state 4, 8, and 9. State 4 is similar to state 1 in terms of the points of contact between the driving wheels and the stair. States 8 and 9 are analogous to state 2. Therefore, if the object functions obtained from the kinetic analysis of the states in the first group are optimized, it is supposed that the WMR will automatically or easily pass through the states in this group.

The third group comprises the kinematically surmountable states, such as states 5 and 6. In these states, the WMR moves easily to the next state due to the kinematic characteristics of the proposed mechanism.
Finally, the fourth group is formed by the states in which the WMR can be automatically surmountable, such as states 2 and 11. In these states, due to the absence of forces preventing the WMR going forward, the WMR automatically passes through these states.

In the next subsection, we analyze the kinetics of the WMR for the states in the first group by the analytical method. Additionally, using the multi-body dynamic analysis software ADAMS\textsuperscript{TM}, we will verify the validity of the kinetic analysis of the WMR. From the results of the kinetic analysis the objective functions will be formulated for the purpose of optimizing the design variables of the WMR.

### 3.1 For State 1

As shown in Fig. 5 (a) and Fig. 6, the driving wheel 1 of the WMR comes into contact with the wall of the stair and driving wheels 2 and 3 keep in contact with the floor, because the center of rotation of the proposed linkage-type mechanism is located below the wheel axis.

![Fig. 6. Forces acting on the proposed WMR for the state 1](image)

To find the normal reaction forces and the corresponding friction forces, we supposed that the WMR was in quasi-static equilibrium and the masses of the links composing the proposed mechanism were negligible. The dynamic friction coefficient of the coulomb friction was applied at the points of contact between the driving wheels and the stair.

If link 1 is in the quasi-static equilibrium state, the resultant forces in the x- and y-directions of the Cartesian coordinates must be zero as described in equation (1) and (2), respectively. The resultant z-direction moment of link 1 about point P also should be zero as described in the equation (3).

\[
\sum F_x = 0; \quad \Rightarrow F_{W2} - N_{W1} + P_x + Q_y = 0 \tag{1}
\]

\[
\sum F_y = 0; \quad \Rightarrow F_{W1} + N_{W2} + P_y + Q_y = 2M_{W1g} \tag{2}
\]

\[
\sum (M_z) = 0; \quad \Rightarrow F_{W1} (L_y \cos \theta_1 - L_5 \sin \theta_1 + R) + N_{W1} (L_y \sin \theta_1 + L_5 \cos \theta_1)
+ F_{W2} [(L_3 - L_5) \sin \theta_1 - L_5 \cos \theta_1 + R] - N_{W2} [(L_3 - L_5) \cos \theta_1 + L_5 \sin \theta_1] - Q_y [(L_3 + L_5 - R) \cos \theta_1 + (L_4 + L_6 - L_{14}) \sin \theta_1]
+ Q_y [-(L_3 + L_5 - R) \sin \theta_1 + (L_4 + L_6 - L_{14}) \cos \theta_1]
= M_{W1g} [2L_y - L_{12}] \cos \theta_1 - 2L_y \sin \theta_1 \tag{3}
\]
In equation (3), the forces $P_x$, $P_y$, $Q_x$, and $Q_y$ are x- and y-direction joint forces on the point P and Q, respectively. And $\theta_1$ is the counter-clockwise angle of link 1 relative to the x-axis of the coordinates fixed in the ground, as shown in Fig. 6.

For link 2, the x- and y-direction resultant forces are described in equations (4) and (5), respectively. The resultant moment about the point C is expressed in equation (6).

\[
\sum F_x = 0; \quad \Rightarrow F_{W2x} - Q_x + R_x = 0 \tag{4}
\]
\[
\sum F_y = 0; \quad \Rightarrow N_{W2y} - Q_y + R_y = M_{W2z} \tag{5}
\]
\[
\sum (M_{z2}) = 0; \quad \Rightarrow F_{W2x}R + Q_x \left[ L_4 \sin \theta_2 + (L_3 - R) \cos \theta_2 \right] - Q_y \left[ L_4 \cos \theta_2 - (L_3 - R) \sin \theta_2 \right] - R_x \left[ (L_2 - R) \cos \theta_2 + L_4 \sin \theta_2 \right] - R_y \left[ (L_2 - R) \sin \theta_2 - L_4 \cos \theta_2 \right] = 0 \tag{6}
\]

$R_x$ and $R_y$ are x- and y-direction joint forces on the point R, respectively. $\theta_2$ is the counter-clockwise angle of link 2 relative to the x-axis of the coordinates fixed in the ground.

For link 3, the x- and y-direction resultant forces are described in equations (7) and (8), respectively. The resultant moment about point S is expressed in equation (9).

\[
\sum F_x = 0; \quad \Rightarrow -R_x + S_x = 0 \tag{7}
\]
\[
\sum F_y = 0; \quad \Rightarrow -R_y + S_y = 0 \tag{8}
\]
\[
\sum (M_{z3}) = 0; \quad \Rightarrow R_x \cos \theta_3 + R_y \sin \theta_3 = 0 \tag{9}
\]

$R_x$ and $R_y$ are x- and y-direction joint forces on the point R, respectively. $\theta_3$ is the counter-clockwise angle of link 3 relative to the y-axis of the coordinates fixed in the ground as shown in Fig. 6. $L$ is the length of link 3 as described in equation (10).

\[
L = \left[ (L_7 + L_9 - R)^2 + (L_4 - L_{10})^2 \right]^{1/2} \tag{10}
\]

Finally, for the robot body, the x- and y-direction resultant forces are described in equations (11) and (12), respectively. The resultant moment about point S is expressed in equation (13).

\[
\sum F_x = 0; \quad \Rightarrow -P_x - S_x = 0 \tag{11}
\]
\[
\sum F_y = 0; \quad \Rightarrow -P_y - S_y = M_S / 2 \tag{12}
\]
\[
\sum (M_{z3}) = 0; \quad \Rightarrow P_x \left[ (L_{11} - L_9 - L_{10}) \sin \theta_3 + (L_4 - L_5) \cos \theta_3 \right] - P_y \left[ (L_{11} - L_9 - L_{10}) \cos \theta_3 - (L_4 - L_5) \sin \theta_3 \right] = \frac{1}{2} M_S \left[ (L_2 - L_{10}) \cos \theta_3 - (L_1 + L_9 - R) \sin \theta_3 \right] \tag{13}
\]

$\theta_3$ is the counter-clockwise angle of the robot body relative to the x-axis of the coordinates fixed in the ground.
The friction force $F_{W1}$ can not be determined by the coulomb friction due to the kinematics of the proposed passive linkage-type locomotive mechanism, but $F_{W2}$ and $F_{W3}$ are determined by the coulomb friction, as in equation (14). These relationships between the normal forces and the friction forces will be shown in the simulation results as described in Fig. 7 where $\mu$ represents the dynamic friction coefficient of the coulomb friction.

$$F_{W1} \neq \mu N_{W1}, \quad F_{W2} = \mu N_{W2}, \quad F_{W3} = \mu N_{W3}$$ (14)

![Fig. 7. Normal and friction forces on the driving wheels for the state 1](a) $F_{W1}$  
(b) $N_{W1}$  
(c) $N_{W2}$  
(d) $N_{W3}$

From equations (1) ~ (13), we formulate the 12x12 matrix equation as shown in equation (15) to determine the unknown contact forces $F_{W1}$, $N_{W1}$, $N_{W2}$ and $N_{W3}$.

$$
\begin{bmatrix}
0 & -1 & \mu & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 & -C_{16} & C_{18} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 & C_{47} & -C_{48} & -C_{49} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{10} & C_{11} & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & C_{25} & -C_{12b} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
F_{W1} \\
N_{W1} \\
N_{W2} \\
N_{W3} \\
P_{y} \\
P_{y} \\
M_{y3} \\
M_{y3} \\
2M_{y3} \\
M_{x3} \\
M_{x3} \\
M_{x3} \\
M_{x3} \\
M_{x3} \\
M_{x3} \\
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
2M_{y3} \\
M_{y3}C_{13} \\
0 \\
0 \\
M_{y3}G \\
0 \\
0 \\
0 \\
M_{y3}/2 \\
M_{x3}C_{12b}/2 \\
\end{bmatrix}
$$ (15)

The substituted parameters are described.
From the 12x12 matrix equation (15), we determined the unknown contact forces as in equations (16) ~ (19).

From the 12x12 matrix equation (15), we determined the unknown contact forces as in equations (16) ~ (19).

\[
C_{31} = -L_{x} \sin \theta_{1} + L_{x} \cos \theta_{1} + R \\
C_{32} = -L_{x} (\mu \cos \theta_{1} + \sin \theta_{1}) + (L_{12} - L_{x}) (\mu \sin \theta_{1} - \cos \theta_{1}) + \mu R \\
C_{33} = (L_{x} + L_{3} - R) \cos \theta_{1} + (L_{x} + L_{6} - L_{12}) \sin \theta_{1} \\
C_{34} = -(L_{x} + L_{3} - R) \sin \theta_{1} + (L_{x} + L_{6} - L_{12}) \cos \theta_{1} \\
C_{35} = 2L_{x} \sin \theta_{1} + (2L_{x} - L_{12}) \cos \theta_{1} \\
C_{36} = L \cos \theta_{1} \\
C_{37} = (L_{5} - R) \cos \theta_{2} + L_{4} \sin \theta_{2} \\
C_{38} = (L_{5} - R) \sin \theta_{2} - L_{4} \cos \theta_{2} \\
C_{39} = L \cos \theta_{1} \\
C_{40} = (L_{5} - L_{10}) \sin \theta_{1} + (L_{8} - L_{4}) \cos \theta_{1} \\
C_{41} = (L_{5} - L_{10}) \cos \theta_{1} - (L_{8} - L_{4}) \sin \theta_{1} \\
\]

\[
F_{W1} = \left[ \frac{(4M_{W1} + M_{f})(A_{11} + C_{31}) - 2M_{W1}C_{33} - 2M_{W2}A_{11} + C_{31} - C_{30}}{2A_{11}} \right] \mu \gamma /10^3 \tag{16}
\]

\[
N_{W1} = \left[ \frac{(4M_{W1} + M_{f})C_{31} - 2M_{W1}C_{33} - 2M_{W2}(-C_{31} - C_{30})}{2A_{11}} \right] \mu \gamma /10^3 \tag{17}
\]

\[
F_{W2} = \left[ \frac{(4M_{W1} + M_{f})C_{33} - 2M_{W1}C_{33} - 2M_{W2}(C_{33} - C_{30})}{2A_{11}} \right] \mu \gamma /10^3 \tag{18}
\]

\[
N_{W2} = \left[ \frac{(4M_{W1} + M_{f})C_{33} - 2M_{W1}C_{33} - 2M_{W2}(C_{33} - C_{30})}{2A_{11}} \right] \mu \gamma /10^3 \tag{19}
\]

where,
Here, $\theta_1$, $\theta_2$, $\theta_3$ and $\theta_b$ are determined by the kinematics of the proposed mechanism. For this state, $\theta_2$ is a function of $\theta_1$ as described by equation (20) and $\theta_3$ and $\theta_b$ are functions of $\theta_1$ and $\theta_2$ as in equations (21) and (22), respectively.

$$
\theta_2 = \tan^{-1} \left[ \frac{L_0 K_3 - (L_3 - R) \left( L_0^2 + (L_3 - R)^2 - K_1 \right)^{1/2}}{(L_3 - R) K_3 + L_0 \left( L_0^2 + (L_3 - R)^2 - K_1^2 \right)^{1/2}} \right]
$$

(20)

$$
\theta_3 = \tan^{-1} \left[ \frac{-2(L_11 - L_4 - L_{41}) M_x M_y + (L_4 - L_5) B_y^2 + B_x M_y}{-2(L_5 - L_3) M_x M_y + (L_{11} - L_5 - L_{41}) B_y^2 + B_x M_y} \right]
$$

$$
\theta_b = \tan^{-1} \left[ \frac{B_x^2 - B_y^2 B_x^2 - B_y^2 + B_x M_y \left( M_x (L_{41} - L_4 - L_{40}) + M_y (L_4 - L_5) \right)}{B_x M_y \left( L_4 - L_5 \right) - M_y \left( L_{11} - L_5 - L_{41} \right)} \right]
$$

(21)

where,

$$
K_1(\theta_i) = (L_3 - R) \cos \theta_i + (L_4 - L_{11} + L_{12}) \sin \theta_i
$$

$$
M_x(\theta_1, \theta_2) = (L_4 + L_5 - R) \sin \theta_1 - (L_4 + L_5 - L_{11}) \cos \theta_1 + (L_4 - L_5) \cos \theta_2 + (L_5 - L_4) \sin \theta_2
$$

$$
M_y(\theta_1, \theta_2) = -(L_3 + L_5 - R) \cos \theta_1 - (L_3 + L_5 - L_{11}) \sin \theta_1 + (L_3 - L_5) \sin \theta_2 - (L_5 - L_3) \cos \theta_2
$$

$$
B_1 = \left( (L_4 - L_5)^2 + (L_{11} - L_4 - L_{40})^2 \right)^{1/2}, \quad B_2 = \left( M_x^2 + M_y^2 \right)^{1/2}
$$

$$
B_3 = -(-M_x^2 - M_y^2 + L_x^2 + B_x^2 + L_7^2) + 4L_7^2 B_y^2
$$

$$
B_4 = \left[ M_x (L_{11} - L_4 - L_{40}) + M_y (L_4 - L_5) \right] \left[ -B_x^2 - B_y^2 + L_7^2 \right]
+ \left[ M_x (L_4 - L_5) - M_y (L_{11} - L_5 - L_{41}) \right] B_x^2
$$

$$
B_5 = M_x^2 - M_y^2 - B_x^2 + L_7^2, \quad B_6 = -M_x^2 + M_y^2 - B_y^2 + L_7^2
$$

For this state, the contact forces acting on the driving wheels are described in Fig. 7. The dotted bold lines result from the kinetic analysis as expressed in equations (16) ~ (19) and the solid lines represent the simulation results computed by the multi-body dynamic analysis software ADAMSTM.

As shown in Fig. 7, it is allowable to assume that the WMR are in a quasi-static equilibrium. In Fig. 7, the steep changes in the simulation results are caused by the instantaneous collision between driving wheel 1 and the wall of the stair. From Fig. 7 (a) and (b), the
normal force $N_{W1}$ at the point of contact $C1$ increases as the angle $\theta_1$ of link 1 increases, while the friction force $F_{W1}$ on $C1$ decreases. Therefore, as shown in equation (14), the coulomb friction does not work between the normal force and the friction force at the point of contact $C1$. This is due to the kinematics of the proposed linkage-type locomotive mechanism. The other friction forces $F_{W2}$ and $F_{W3}$ on the points of contact $C2$ and $C3$ can be determined by the coulomb friction.

For the WMR to be in the equilibrium state, the force $F_{W1}$ cannot exceed the friction force produced by the coulomb friction as expressed in equation (22).

$$F_{W1} \leq \mu N_{W1} \tag{22}$$

Fig. 8 shows the force difference between $N_{W1}$ and $F_{W1}$.

As shown in Fig. 8, the force difference between $N_{W1}$ and $F_{W1}$ increases as the driving wheel 1 climbs up the stair, that is, as angle $\theta_1$ increases. If the force difference has a negative value, the force $F_{W1}$ must be higher than the coulomb friction $N_{W1}$ that is needed for the WMR to be in the equilibrium state. As shown in equation (22), that situation cannot happen absolutely. Therefore, whether the WMR can pass through the state 1 or not is determined at $\theta_1=0$. Consequently, to improve the ability for the WMR to climb up stairs, the force difference at $\theta_1=0$ will be selected as the first object function to be optimized.

The relative angle of link 1 to robot body is limited to avoid sticking conditions described in the previous section. The maximum allowable counter-clockwise angle of link 1 relative to the robot body is expressed in equation (23).

$$\theta_{b \text{-} \max} = \theta_1 \text{-} \max - \theta_b \left( \theta_1 \text{-} \max, \theta_2 \text{-} \max \right), \text{ where } \theta_1 \text{-} \max = \sin^{-1} \left( \frac{H_{S \text{-} \max} - R}{L_{12}} \right) \tag{23}$$

Here, $H_{S \text{-} \max}$ is the maximum height of the stair for the WMR to climb and $\theta_b$ is determined by equations (20) and (21) at $\theta_1=\theta_1 \text{-} \max$. 
3.2 For state 3
In this state, as shown in Fig. 5 (c), driving wheels 1 and 3 of the WMR contact with the floor of the stair and driving wheel 2 comes in contact with the wall of the stair. According to the characteristics of the points of contact, state 3 is divided into two sub-states as shown in Fig. 9. In Fig. 9 (a), the coulomb friction does not work on point of contact C3, while in Fig. 9 (b) the coulomb friction does not function on point of contact C2. This characteristic is due to the kinematic characteristics of the proposed passive linkage-type locomotive mechanism.

3.2.1 For state 3-1
In state 3-1, as shown in Fig. 9 (a), the relative angle of link 1 to the robot body is \( \theta_1 \) expressed in equation (23). As mentioned above, the relationships between the normal forces and the friction forces are expressed in equation (24).

\[
\begin{align*}
N_{W1} &= \frac{\left(4M_{W1} + M_b\right)A_{21} - 2M_{W1}D_{131}(D_{44} + D_{82} - \mu D_{68})}{2\left(1 + \mu^2\right)(A_{21} + D_{22}D_{56}) - (D_{44} + D_{82})A_{22} + D_{68}A_{23}} \times 10^3 \\
N_{W2} &= \frac{\left(4M_{W2} + M_b\right)\mu(A_{21} + D_{22}D_{56}) - (D_{44} + D_{82})A_{22} + D_{68}A_{23}}{2\left(1 + \mu^2\right)(A_{21} + D_{22}D_{56}) - (D_{44} + D_{82})A_{22} + D_{68}A_{23}} \times 10^3 \\
N_{W3} &= \frac{\left(4M_{W3} + M_b\right)A_{21} - 2M_{W3}D_{131}(D_{44} + D_{82} - \mu D_{68})}{2\left(1 + \mu^2\right)(A_{21} + D_{22}D_{56}) - (D_{44} + D_{82})A_{22} + D_{68}A_{23}} \times 10^3
\end{align*}
\]
Optimal Design of a New Wheeled Mobile Robot by Kinetic Analysis for the Stair-Climbing States

\[ F_{W3} = \frac{(4M_{W1} + M_d)D_{bb}A_2 - 2M_{W1} \left( 1 + \mu^2 \right)D_{bb}A_3}{2\left( 1 + \mu^2 \right) (A_2 + D_1) - (A_4 + D_2) A_2 + D_{aa}A_2} \times 10^3 \]

Here, the substituted parameters are represented below.

\[ D_{31} = \mu(R - L_b \sin \theta_1 - L_3 \cos \theta_1) + (L_4 \cos \theta_1 - L_5 \sin \theta_1) \]
\[ D_{32} = -(L_{12} - L_3) [\mu \cos \theta_1 + \sin \theta_1] - L_6 (\mu \sin \theta_1 - \cos \theta_1) + \mu R \]
\[ D_{33} = -(L_5 + L_4 - R) \cos \theta_1 - (L_5 + L_4 - L_1) \sin \theta_1 \]
\[ D_{34} = -(L_5 + L_4 - R) \sin \theta_1 + (L_4 + L_4 - L_1) \cos \theta_1 \]
\[ D_{41} = (2L_4 - L_{12}) \cos \theta_1 - 2L_4 \sin \theta_1 \]
\[ D_{44} = R \]
\[ D_{42} = L_4 \sin \theta_1 + (L_5 - R) \cos \theta_2 \]
\[ D_{43} = -L_4 \cos \theta_2 \]
\[ D_{44} = -L_4 \cos \theta_2 + (L_5 - R) \sin \theta_2 \]
\[ D_{45} = \sin \theta_1 \]
\[ D_{51} = (L_{12} - L_4 - L_{10}) \sin \theta_1 + (L_4 - L_5) \cos \theta_1 \]
\[ D_{52} = -(L_4 - L_4 - L_{10}) \cos \theta_1 + (L_4 - L_4) \sin \theta_1 \]
\[ A_{21} = D_{aa}D_{bb} + D_{ba}D_{ba} - D_{21}D_{bb} - D_{bb}D_{ba} \]
\[ A_{22} = \mu D_{32} + D_{33} \]
\[ A_{23} = \mu D_{32} - D_{33} \]
\[ A_{24} = D_{pp}D_{pp} - D_{pp}D_{bb} \]
\[ A_{25} = D_{pp}D_{bb} - D_{pp}D_{pp} \]

In this state, the angle of link 2 relative to the x-axis of the coordinates fixed in the ground is expressed in equation (29) due to the kinematics of the proposed mechanism.

\[ \theta_3 = \tan^{-1} \left( \frac{L_4K_2 - (L_5 - R) \left[ L_4^2 + (L_5 - R)^2 - K_2^2 \right]^{1/2}}{L_4 - R} \right) \]

where, \( K_2(\theta_1) = (L_5 - R) \cos \theta_1 + (L_4 - L_1) \sin \theta_1 + H \)

\( \theta_1 \) and \( \theta_3 \) are determined by equations (20) and (21), respectively. \( \theta_5 \), \( \theta_6 \), and \( \theta_6 \) are defined by the same manner described in section 3.1.
For this state, the contact forces acting on the driving wheels are described in Fig. 10; the dotted bold lines show the results of the kinetic analysis as expressed in equations (25) ~ (28) and the solid lines represent the simulation results obtained by ADAMS®TM.

As shown in Fig. 10, it is also allowable to assume that the WMR are in quasi-static equilibrium. In Fig. 10, the steep changes in the simulation results are also caused by the instantaneous collision between the driving wheel 2 and the wall of stair. As shown in Fig. 10 (c) and (d), the coulomb friction does not work between the normal force and the friction force at point of contact C3 as expressed in equation (24). This is also due to the kinematics of the proposed mechanism. The other friction forces $F_{W1}$ and $F_{W2}$ on points of contact C1 and C2 can be determined by the coulomb friction as shown in equation (24).

$$\mu_{11} = \frac{F_{W1}}{N_{W1}} \quad \mu_{22} = \frac{F_{W2}}{N_{W2}} \quad \mu_{33} = \frac{F_{W3}}{N_{W3}}$$  \hspace{1cm} (30)

In this state, the normal forces and the friction forces can be determined in the same manner as in section 3.1 and as described in equations (31) ~ (34).
Here, the substituted parameters are represented below.

\[
N_{W1} = \left[\frac{(4M_{W1} + M_b)E_{38}}{2A_{32}} - \frac{M_{W1}E_{311}}{A_{31}} - \frac{M_{W2}(E_{38} - E_{33})}{A_{33}} - \frac{M_{W2}E_{38}A_{34}}{2A_{34}A_{33}}\right] / 10^3 \tag{31}
\]

\[
N_{W2} = \left[\frac{(4M_{W1} + M_b)\mu E_{38}}{2A_{32}} - \frac{M_{W1}\mu E_{311}}{A_{31}} - \frac{M_{W2}\mu E_{38}A_{34}}{2A_{34}A_{33}}\right] / 10^3 \tag{32}
\]

\[
N_{W3} = \left[\frac{M_{W2}E_{38}}{A_{32}} - \frac{M_b(E_{38} + E_{121})A_{33}}{2A_{32}A_{33}}\right] / 10^3 \tag{33}
\]

\[
N_{W4} = \left[\frac{M_{W2}E_{38}}{A_{32}} - \frac{M_b(E_{38} + E_{121})A_{33}}{2A_{32}A_{33}}\right] / 10^3 \tag{34}
\]

Here, the substituted parameters are represented.

\[
E_{33} = \left[\mu (\xi + L_s \sin \theta_1 - L_s \cos \theta_1) + (L_s \cos \theta_1 - L_s \sin \theta_1)\right] \quad E_{35} = -\left[(L_s - L_s \sin \theta_1 - L_s \cos \theta_1)\right]
\]

\[
E_{36} = -\left[(L_s + L_s \cos \theta_1 + L_s \sin \theta_1) - (L_s - L_s \cos \theta_1 - L_s \sin \theta_1)\right] \quad E_{37} = \left[(2L_s - L_s \cos \theta_1 + L_s \sin \theta_1)\right]
\]

\[
E_{44} = \mu \xi \quad E_{47} = \left[L_s \sin \theta_2 + (L_s - R) \cos \theta_2\right] \quad E_{48} = -\left[(L_s - R) \cos \theta_2 + L_s \sin \theta_2\right]
\]

\[
E_{49} = \left[(L_s + L_s \cos \theta_2 - L_s \sin \theta_2)\right] \quad E_{38} = \left[L_s \cos \theta_2 + L_s \sin \theta_2\right] \quad E_{39} = \left[L_s \cos \theta_2\right]
\]

\[
E_{41} = \left[L_s \sin \theta_3 - L_s \cos \theta_3\right] \quad E_{11} = \left[(L_s - R) \cos \theta_3 + L_s \sin \theta_3\right] \quad E_{12} = \left[(L_s - L_s \cos \theta_3 + L_s \sin \theta_3) - (L_s - L_s \cos \theta_3 - L_s \sin \theta_3)\right]
\]

\[
A_{33} = \mu \xi \quad A_{34} = \mu \xi \quad A_{35} = \mu \xi \quad A_{36} = \mu \xi \quad A_{37} = \mu \xi \quad A_{38} = \mu \xi
\]

\[
\theta_1, \theta_2, \theta_3, and \theta_b are defined in the same manner described in section 3.1. \theta_1, \theta_2, and \theta_b are determined from equations (29), (20) and (21), respectively.
For this state, the contact forces acting on the driving wheels are described in Fig. 11. The dotted bold lines show the results from the kinetic analysis as expressed in equations (31) ~ (34) and the solid lines represent the simulation results by ADAMS™.

![Fig. 11. Normal and friction forces on the driving wheels for state 3-2](image)

As shown in Fig. 11, it is also allowable to assume that the WMR are in a quasi-static equilibrium. In Fig. 11, the rapid changes in the simulation results are caused by the impact between driving wheel 2 and the wall of the stair. From Fig. 11 (b) and (c), the normal force \( N_{W2} \) at point of contact C2 decreases as the angle \( \theta_1 \) of link 1 decreases, while the friction force \( F_{W2} \) on C2 increases. Therefore, as shown in the equation (30), the coulomb friction does not work between the normal force and the friction force at point of contact C2. This is also due to the kinematics of the proposed linkage-type locomotive mechanism. The other friction forces \( F_{W1} \) and \( F_{W3} \) on points of contact C1 and C3 can be determined by the coulomb friction.

For state 3-1, for the WMR to be in the equilibrium state, the force \( F_{W3} \) can not exceed the friction force produced by the coulomb friction as expressed in equation (35).

\[
F_{W3} \leq \mu N_{W3}
\]  
(35)

Similarly, for state 3-2, the force \( F_{W2} \) can not exceed the friction force produced by the coulomb friction as expressed in equation (36).

\[
F_{W2} \leq \mu N_{W2}
\]  
(36)

Fig. 12 shows the force differences for state 3-1 and the state 3-2. As shown in Fig. 12 (a) and (b), the force difference between \( N_{W3} \) and \( F_{W3} \) and the force difference between \( N_{W2} \) and \( F_{W2} \) decrease as driving wheel 2 climbs up the wall of the stair, that is, \( \theta_1 \) decreases. If equation (36) is satisfied by the design variables of the WMR, equation (35) is satisfied sufficiently for the range of \( \theta_1 \). Therefore, whether the WMR is able to pass through state 3 is determined when the driving wheel 2 comes in contact with the top-edge of the wall of the step, that is, \( \theta_1 = \theta_{1,3} \) as expressed in equation (37). Consequently, to improve the ability of the WMR to climb up the stair, the force difference between \( N_{W2} \) and \( F_{W2} \) at \( \theta_1 = \theta_{1,3} \) will be considered as the second object function.

\[
\theta_{1,3} = \sin^{-1} \left( \frac{R}{L_{12}} \right)
\]  
(37)
In Fig. 12 (b), the dash-dotted line represents the distance between the outer circle of driving wheel 1 and the wall of the stair. We call this value the first ‘Anti-Sticking Constraint (ASC)’. To prevent the WMR from falling into the sticking condition, the ASC \(_{1}\) as expressed in equation (38) must be greater than a certain offset value at \(\theta_1 = \theta_{1_3}\). The offset value (ASC\(_{1}\_off\)) is a fully bounded value as described in equation (39). If the ASC\(_{1}\_off\) increases, the possibility of the sticking condition occurring for this state decreases, even though the WMR climbs the stair with the smaller length than the length of the step \(L_S\).

\[
\text{ASC}_1 = L_3 - L_{12} \cos(\theta_1) \tag{38}
\]

\[
\text{ASC}_1 \geq \text{ASC}_{1\_off} \quad \text{where,} \quad 0 \leq \text{ASC}_{1\_off} \leq R
\tag{39}
\]

### 3.2 For State 7

In this state, as shown in the Fig. 5 (g) and Fig. 13, driving wheel 1 comes into contact with the floor of the stair and wheel 3 comes into contact with the wall of the stair. For this state, we assumed that the proposed linkage mechanism has zero degrees-of-freedom. From this assumption, with the exception of the driving wheels, the WMR will move as a rigid body. In this state, the relationships between the normal forces and the friction forces become as found in equation (40).

\[
F_{W1} = \mu N_{W1}, \quad F_{W3} = \mu N_{W3} \tag{40}
\]
If the WMR is in a quasi-static equilibrium state, the contact forces can be determined by Newton’s 2nd law of motion. The normal forces on points of contact C1 and C3 are determined as described in equations (41) ~ (42).

\[
N_{W1} = \left(\frac{4M_{W1} + 2M_{W2} + M_b}{2(1 + \mu^2)}\right)g / 10^3 \quad (41)
\]

\[
N_{W3} = \left(\frac{4M_{W3} + 2M_{W2} + M_b}{2(1 + \mu^2)}\right)g / 10^3 \quad (42)
\]

For this state, the contact forces acting on the driving wheels are described in Fig. 14. The dotted bold lines show the results of the kinetic analysis as expressed in equations (41) ~ (42) and the solid lines represent the results by ADAMS\textsuperscript{TM}.

![Fig. 14. Normal and friction forces on the driving wheel 1 and 3 for the state 7](image)

In the simulation results, shown in Fig. 14, the oscillation of the forces is caused by the non-rigid body motion of the proposed linkage mechanism. However, it is allowable to assume that the WMR is in a quasi-static equilibrium and moves as a rigid body. For the WMR to overcome this state, the z-axis moment acting on the robot about the point of contact C1 must be a negative value (Fig. 13). The z-axis moment about point of contact C1 is expressed in equation (43).

\[
(M_z)_{C1} = \left[-N_{W3}(D_1 + \mu D_2) + M_{W3}gD_3 + M_{W1}gD_4 + M_bgD_5\right] / 10^3
\]

where,

\[
D_1 = -(L_5 - R)\cos\theta_1 - (L_4 - L_{11})\sin\theta_1 + (L_3 - L)\cos\theta_2 + L_4\sin\theta_2 - R
\]

\[
D_2 = (L_3 - R)\sin\theta_1 - (L_4 - L_{11})\cos\theta_1 - (L_3 - L)\sin\theta_2 + L_4\cos\theta_2 - R
\]

\[
D_3 = (L_3 - R)\sin\theta_1 - (L_4 - L_{11})\cos\theta_1 - (L_3 - R)\sin\theta_2 + L_4\cos\theta_2 - R
\]

\[
D_4 = L_2\cos\theta_3
\]

\[
D_5 = -L_4\sin\theta_1 + L_3\cos\theta_1 + (L_1 + L_6 - R)\sin\theta_2 + (L_{11} - L_2 - L_5)\cos\theta_2
\]

Here, \(\theta_1, \theta_2, \theta_3\), and \(\theta_b\) are described in the equation (44).
Optimal Design of a New Wheeled Mobile Robot by Kinetic Analysis for the Stair-Climbing States

Here, \( \theta_1, \theta_2, \theta_3, \) and \( \theta_b \) are described in equation (45) and \( D_1 \) is computed from equation (43) when \( \theta_1 = \theta_1' \) and \( \theta_2 = \theta_2' \). \( L_R \) is the axle length between driving wheels 1 and 3, when the relative angle of link 1 to the robot body is the maximum allowable angle \( \theta_{1 \text{max}} \) in equation (23), and is computed by equation (46).

\[
\begin{align*}
\theta_1' &= \theta_1' - \theta_1 \\
\theta_2' &= \theta_2' + \theta_{\text{incr}} \\
\theta_3' &= \theta_3' + \theta_{\text{incr}} \\
\theta_b' &= \theta_b' + \theta_{\text{incr}} \\
\text{where,} \quad \theta_{\text{max}} &= \sin^{-1}\left[ \frac{D_1 + R}{L_R} \right] \\
\theta_{\text{max}} &= \sin^{-1}\left[ \frac{H_S + R}{L_R} \right] \\
L_R &= \left\{ \left[ (L_3 - R)\sin(\theta_2') + L_4 \cos(\theta_2') \right] - \left( L_4 - L_{\text{max}} \right) \cos(\theta_{\text{max}}) + (L_3 - R)\sin(\theta_{\text{max}}) \right\}^{1/2}
\end{align*}
\]

where \( \theta_{\text{max}} \) is the maximum allowable angle for the stair-climbing states.

For this state, the z-axis moment is described in Eq. 15, according to the change of \( \theta_2 \). As shown in Fig. 15, from the analytic and simulation results, the z-axis moment increases as driving wheel 3 climbs the wall of the stair, that is, \( \theta_2 \) decreases. So, for the WMR to climb up the stair, the value of the z-axis moment must be sufficiently less than zero at the moment that driving wheel 3 comes in contact with the top-edge of the wall of the stair.

Fig. 15. Z-axis moment about the point of contact C1 for the state 7

In Fig. 15, the dash-dotted line represents the distance between the outer circle of driving wheel 2 and the wall of the stair. We call the value the second ‘Anti-Sticking Constraint’. To prevent the WMR from falling into the sticking condition, the ASC as expressed in equation (47) must be greater than a certain offset value (ASC\(_{2,\text{off}}\)) for all of the range of \( \theta_2 \). The
ASC\textsubscript{2-off} is a fully bounded value as described in equation (48). If the ASC\textsubscript{2-off} increases, the possibility of the sticking condition occurring for this state decreases, even though the WMR climbs the stair with the smaller length than L\textsubscript{S}.

\begin{equation}
AS\textsubscript{C2} = L_5 - L_4 \cos(\theta_1) + (L_3 - R) \sin(\theta_1) + (L_{12} + L_4 - L_{11}) \cos(\theta_1) - (L_3 - R) \sin(\theta_1)
\end{equation}

\begin{equation}
ASC\textsubscript{2} \geq ASC\textsubscript{2-off} \quad \text{where,} \quad 0 \leq ASC\textsubscript{2-off} \leq R
\end{equation}

Consequently, to optimize the design variables of the proposed WMR, we designate the negative z-axis moment about point of contact C\textsubscript{1} at \(\theta_2 = \theta_3\) as expressed in equation (49) as the third object functions.

\begin{equation}
\theta_2 = \theta_2' - \theta_4 - \theta_5\tag{49}
\end{equation}

### 4. Optimization

In the previous section, we analyzed the kinetics of the WMR with the proposed passive linkage-type locomotive mechanism for several states. From the results of the kinetics, we determined the three object functions to improve the ability of the WMR to climb the stair. Additionally, to prevent the WMR from falling into sticking conditions as described in section 2, two anti-sticking constraints (ASCs) were described.

In this section, we optimized the design variables of the proposed WMR by using three object functions. The first object function results from the kinetics for state 1 and is described in equation (50). In equation (50), \(N_{W1}\) and \(F_{W1}\) are computed by equation (16) and (17), respectively. \(\theta_2, \theta_3\) and \(\theta_6\) are determined from equations (20) and (21), respectively.

\begin{equation}
OF_1 = \mu N_{W1} - F_{W1}, \quad \text{when} \quad \theta_1 = 0 \quad \text{for state 1}\tag{50}
\end{equation}

The second object function results from the kinetics for state 3-2 and represents the force difference between \(N_{W2}\) and \(F_{W2}\) on point of contact C\textsubscript{1} at the moment that driving wheel 2 comes in contact with the top-edge of the wall of the step. This object function is expressed in equation (51). The \(N_{W2}\) and \(F_{W2}\) are computed from equations (32) and (33), respectively. \(\theta_2, \theta_3\) and \(\theta_6\) are determined from equations (29) and (21), respectively.

\begin{equation}
OF_2 = \mu N_{W2} - F_{W2}, \quad \text{when} \quad \theta_1 = \theta_3 - \frac{1}{10} \quad \text{for state 3-2}\tag{51}
\end{equation}

The third object function results from the kinetics for state 7 and represents the negative z-axis moment on the WMR about the point of contact C\textsubscript{1} at the moment that the driving wheel 3 comes in contact with the top-edge of the wall of the step. This object function is described by equation (52) where the normal force \(N_{W3}\) and the substituted parameters \((D_1, D_2, D_3, D_4, D_5)\) are computed from equations (42) and (43), respectively.

\begin{equation}
OF_3 = [N_{W3}(D_1 + \mu D_2) - M_{W22}D_3 - M_{W12}D_4 - M_{W3}D_5]/10^3 \quad \text{for state 7}
\end{equation}

\begin{align}
when \quad &\theta_1 = \theta_1' - \theta_{a1} + \theta_{b2}, \quad \theta_2 = \theta_2' - \theta_{a1} + \theta_{b2}, \quad \theta_3 = \theta_3' - \theta_{a1} + \theta_{b2}, \quad \theta_4 = \theta_4' - \theta_{a1} + \theta_{b2} \tag{42}
\end{align}
From the three object functions, we accomplished the optimization of the design variables with the two ASCs as described in equations (38) and (47), respectively, by using the multi-objective optimization method (Gembicki, 1974). The parameters used in the optimization method are listed in Table 1.

<table>
<thead>
<tr>
<th>Object Functions &amp; Parameters</th>
<th>Values</th>
<th>Object Functions &amp; Parameters</th>
<th>Values</th>
</tr>
</thead>
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Table 1. Object functions and parameters used in the optimization.

The convergence of the three object functions and the optimized design variables, after the optimization process, are shown in Fig. 16 and Table 2, respectively.

![Figure 16: Convergence of OF1, OF2, and OF3 by the multi-objective optimization method](image)

<table>
<thead>
<tr>
<th>Design Parameter</th>
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<th>Optimal Value</th>
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<th>Initial Value</th>
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Table 2. The optimal design variables of the proposed WMR
5. Fabrication

Based on the results of optimization, we fabricated the prototype of proposed mobile robot. It is composed mainly of three parts: the driving wheel assembly, the proposed passive linkage mechanism, and the robot body. A motor, a digital encoder, and a harmonic gear are assembled inside the wheel to afford a compact driving unit. The wheel-in motor is shown in Fig. 17.

50Watt Maxon EC flat motors, 500pulse USDigital optical encoders, and 160:1 HarmonicDrive® harmonic gears are used. The passive mechanism is composed of a four bar linkage and a limited pin joint. The pin joint confines the working range of the four bar linkage mechanism to avoid overturning. An electrical subsystem comprised of a single board computer (SBC), a controller area network (CAN) module, a wireless LAN, a motor controller, and Li-ion batteries are placed in the robot body. The prototype of ROBHAZ-6W has eight DOFs: six DOFs for the robot body and two DOFs for the right and left sides of the passive linkage mechanism.

![Wheel-in motor](image)

Fig. 17. Wheel-in motor

The configurations of the right and left sides of the proposed linkage mechanisms are independently changed according to the environment. And the axle distance between the first and the second driving wheels is determined by considering the length of the stair and the axle distance between the 1st and the 3rd driving wheels is designed according to the height of the stairs that the mobile robot will ascend. The prototype of proposed mobile robot is shown in Fig. 18, and the design parameters are described in Table 3. It can navigate stairs and hazardous terrain areas by using the proposed four bar linkage mechanism.

<table>
<thead>
<tr>
<th>Item</th>
<th>Parameters</th>
<th>Value</th>
<th>Parameters</th>
<th>Values</th>
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<td>L₈</td>
<td>-48.0</td>
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<td></td>
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<td>L₉</td>
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<td></td>
<td>L₄</td>
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<td>L₅</td>
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<td>L₇</td>
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<td>Wheels &amp; Linkages</td>
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<tr>
<td></td>
<td>Batteries</td>
<td>3</td>
<td>Total</td>
<td>31</td>
</tr>
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</table>

Table 3. Design parameters of ROBHAZ-6Wheel
Fig. 19 shows a block diagram of the control system for the robot. This control system is composed of the following: a command PC and a joystick at a remote control site, a single board computer (SBC), six motors and controller, wireless LAN equipment, and a controller. The command PC at the remote site communicates with an SBC in the ROBHAZ-6W. The command PC allows an operator to control the mobile robot movement by joystick and receives angular velocity of wheels by CAN Bus. The motor controllers each have a microprocessor and the six wheel-in motors described in the previous section can be independently controlled. These controllers are connected to the SBC by a CAN bus. This kind of control structure enables the SBC to control the six motors in real time.

All the programs are coded by MS Visual C++ on an MS-Windows 2000 Operating system. The programs are composed of three major parts: a TCP/IP server program, a robot program, and a control program. The TCP/IP server program connects the mobile robot to the remote control PC. The mobile robot program calculates the forward/inverse kinematics of the mobile robot, gives motor driving commands via a CAN bus.

Fig. 18. Prototype of the proposed stair-climbing mobile robot

Fig. 19. The full system of proposed mobile robot
6. Experiment

Experimental investigation is performed to prove kinetic analysis and to figure out the characteristics of climbing states. We accomplished the experiments with the fabricated stairs of which length and height is 150 mm and 260 mm and the indoor stair in KAIST of which length and height is 165 mm and 280 mm, respectively. The climbing motions of the fabricated and real stairs are shown in Fig. 20 and Fig. 21, respectively.

![Fig. 20. Experiments for climbing the fabricated stairs](image)

Though the mobile robot is optimized to the stair of which length and height is 165 mm and 280 mm, the proposed mobile robot can easily climb and adapt to the stairs if axle distance is adjusted not to fall in sticking condition.

Table 4 shows the result of adjustment of axle distance to climb the stairs. As \( L_4 \) and \( L_{12} \) become small, WMR overcome the state1 easily with low friction coefficient. Though WMR has a good ability to overcome state1 with small \( L_4 \) and \( L_{12} \), they should not be smaller than specific values which are determined in state 5 and state 7. In state 5 and state 7, there are two important issues which confine minimum length of \( L_4 \) and \( L_{12} \). At first, wheel 1 should be in contact with floor of second stair when wheel 3 start climbing up the wall of first stair not to slip on the wall of stair. Second, not to overturn in state 5, \( L_{12} \) should be long enough for the contact point of wheel 2 and floor to stay on the right side of WMR’s centre of mass.
7. Conclusion

In order to be utilized in building inspection, building security, and military reconnaissance, a new type of WMR was designed with a passive linkage-type locomotive mechanism for improved adaptability to rough terrain and stair-climbing without the active control techniques. Two designed concepts, ‘adaptability’ and ‘passivity’, were considered for the design of the linkage-type locomotive mechanism of the WMR. The proposed mechanism, composed of a simple 4-bar linkage mechanism and a limited pin joint, allows the WMR to adapt passively to rough terrain and to climb stairs.

A state analysis was carried out to determine the states that primarily influence the WMR’s ability to climb the stair. For the several dominant states suggested from the state analysis, a kinetic analysis was accomplished in order to improve the WMR’s ability to climb the stair. The validation of the kinetic analysis was done for the states using ADAMS™. The object functions were formulated from the kinetics of the WMR and we optimized passive link mechanism using the multi-objective optimization method. The proposed WMR with the optimal design values could climb a stair with a height about three times the wheel radius.
8. References


With the advancement of technology, new exciting approaches enable us to render mobile robotic systems more versatile, robust and cost-efficient. Some researchers combine climbing and walking techniques with a modular approach, a reconfigurable approach, or a swarm approach to realize novel prototypes as flexible mobile robotic platforms featuring all necessary locomotion capabilities. The purpose of this book is to provide an overview of the latest wide-range achievements in climbing and walking robotic technology to researchers, scientists, and engineers throughout the world. Different aspects including control simulation, locomotion realization, methodology, and system integration are presented from the scientific and from the technical point of view. This book consists of two main parts, one dealing with walking robots, the second with climbing robots. The content is also grouped by theoretical research and applicative realization. Every chapter offers a considerable amount of interesting and useful information.

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