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Stateflow® Aided Modelling and Simulation of Friction in a Planar Piezoelectric Actuator

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1. Introduction

Research works have focused on friction over more than 500 years. It is indeed a complex phenomenon arising at the contact of the surfaces that is encountered in a wide variety of engineering disciplines including contact mechanics, system dynamics and controls, aeromechanics, geomechanics, fracture and fatigue, structural dynamics, and many others [1]. Recently, the action of friction generated by a surface under the finger has been exploited in continuous structure tactile (sensorial touch) interfaces.

Indeed, in daily life, various tasks may be more speedily and efficiently completed if kinesthetic feedback is exploited [2]. However, human beings are only using visual and audio feedback when interacting with numerous interfaces such as computers, mobile phones, etc. In such a context, the utilization of haptic devices to get back forces corresponding to feelings from virtual objects manipulation enhances the realism of the global experience. This is even true in the instance of forces and pressure applied on fingers and hand in tele-robotics applications. As a matter of fact, as soon as one has to grasp, touch, and feel objects it becomes necessary to involve haptic devices. Thus, for many researchers, such interfaces represent a new human-machine communication medium, which is a growing topic of interest and prime importance to the research community. Developments in the field have been observed over the last years. Besides tele-operation, haptic feedback can have so many over applications either in engineering, CAD, electronic games, education, learning, etc. Force feedback devices are commercialised. Phantom from Sensable® or Virtuose from Haption®, are examples of haptic feedback pen [3]. Some other solutions have been proposed including « exoskeleton » devices that directly apply forces on the hand or the finger. Typically, these haptic peripherals have an essential place in design, simulation and virtual assembly for instance in car production in the automobile industry. So many additional illustrations in terms of utilisation are
available and helping a user in identifying limits and virtual shape changes may be achieved by such interfaces.

Nevertheless, it is remarkable that force feedback devices are mostly based on electromagnetic technologies. Consequently, numerous mechanical links are often involved in movement transformation. This results in systems integration and dynamic problems, etc. For this reason piezoelectric actuators present a reliable alternative especially since high forces, rapid response and compactness are seen to be added advantages. It is indeed shown that piezoelectric actuators make variable friction phenomena available [4] and exploitable for the purpose of haptic feedback.

The actuator to be discussed follows this trend: it is based on electro mechanical conversion principle specific to piezoelectric systems.

Indeed, exciting a plate in certain conditions of vibratory amplitude and frequency the texture represented in Figure 1 e.g. can be simulated [5][6]. For that it suffices to correlate the vibratory amplitude with the level of friction reduction between an exploring finger and the textured plate. To complete the process, the control of the finger position over the plate enables textures feeling. In fact, during the touching of the excited surface, that is vibrating, the finger perceives smoothness. If the vibrations source is switched off a roughness is perceived. To reform a given texture, we can thus measure the finger displacement and control the vibratory amplitude as a function of the explored feeling (smoothness or roughness) and the finger position. This example is chosen to illustrate that controlled friction is an interesting alternative in terms of tactile feelings generation. The challenge is however in modelling the friction and to deduce appropriate control laws.

![Figure 1. Image of the texture](image_url)

A feature of stick-slip vibration is a saw tooth displacement time evolution with stick and slip phases clearly defined in which the two surfaces in contact stick respectively slip over each other [7]. From the later definition a relative similarity may be perceived between the modelling of stick-slip vibration and the texture in figure 1. In the chapter is presented the approach we propose using Matlab-Simulink-Stateflow® to deal with our structure as a finite state structure.

Thus an overview of piezoelectric materials and the impact they may have on resulting piezoelectric systems, in particular on transducers technology is given. This is followed by a brief discussion on tactile feedback designs using actuation other than piezoelectric and the drawbacks and advantages of piezoelectric actuators versus other types. From now on
we present the design, modelling and implementation of our proposal which is a passive 2Dof (two Degree of freedom) device able to provide different resistant feelings when a user moves it.

2. Piezoelectric materials contribution to transducers technology

In this section, piezoelectric materials such as lead zirconium titanate (PZT) utilised elsewhere in our design are briefly discussed. We focus on the benefits that transducers technology in general can draw from those materials.

Piezoelectric materials produce an electric charge when subjected to mechanical loads (direct effect) and/or vibrations. Those materials deform when subjected to a magnetic field (inverse effect) [8]. The piezoelectric effect is expressed in materials such as single crystals, ceramics, polymers, composites, thin films and relaxor-type ferroelectric materials.

Of the existing piezoelectric materials polymers and ceramics have been widely explored as transducers materials [8]. Ceramic piezoelectric materials comprise barium titanate (BaTiO3), lead zirconate – lead titanate of general formula Pb(Zr-Ti)O3-PZT and lead titanate (PbTiO3, PCT). Those materials have been extensively studied because most of them have in common a perovskite (ABO3) structure [9][8], a material in which the application of an intense electric field aimed at aligning polarization of elementary ferroelectric microcrystal (polarization operation) enables introduction of necessary anisotropy to piezoelectric existence. The dielectric and piezoelectric constants of barium titanate vary with temperature, stoichiometry, microstructure and doping [10] whereas the piezoelectric constants for PZT are not strongly dependant upon temperature but rather on material composition. However, piezoelectric constants for certain stoichiometry compositions of PZT are far more sensitive to temperature dependencies than others [11]. In [12] it can be seen that the characteristics of BaTiO3 single crystal favoured their usage in certain applications such as electromechanical transducers for operation at high frequencies. However, nowadays, PZT have been the material of reference in the field of piezoelectric motors. Regarding, lead titanate material, Samarium-modified PCT was investigated for application in infrared sensors, electro-optic devices and ferroelectric memory devices [13].

Polymer piezoelectric material such as polyvinylidene difluoride (PVDF) or PVF2 demonstrate the piezoelectric effect when they are stretched or formed during fabrication [10]. However, it is among composite piezoelectric materials made of a piezoelectric and a polymer that composites comprised of PZT rods embedded within a polymer matrix are predicted to be one of the most promising structures for transducers and acoustic applications [10].

Relaxor-type ferroelectric materials differ from traditional ferroelectric material because they have a broad phase transition from paraelectric to ferroelectric state, dielectric relaxation and weak remnant polarization. Indeed those materials have the ability to become polarized when subjected to applied electric field. Unlike in traditional ferroelectric materials, this can happen even if there is no permanent electric dipole that exists in the material. The
result of removing the field is polarisation in the material returning to zero [14]. Single crystals of Pb(Mg$_{1/3}$Nb$_{2/3}$)O$_3$ (PMN), Pb(Zn$_{1/3}$Nb$_{2/3}$)O$_3$ (PZN), PMN-PT and PZN-PT are currently under investigation for transducer technology because of their large coupling coefficients, large piezoelectric constants and high strain levels so far higher than other piezoelectric materials [15].

3. Tactile feedback interfaces overview

As we have indicated in the introduction, piezoelectric technology is promising in haptics, restricted in this chapter to tactile feedback, but other alternatives have been exploited. Therefore several methods used for tactile or coetaneous feedback and several forms of technological proposals will be described. We chose to sort those proposals by highlighting related function and technology.

Until now, two methods of tactile feedback have mainly been proposed: shape tactile display and vibrotactile stimuli application. Shape tactile display appears to designers of tactile stimulators to reforming a material state of surface [16]. Conversely, tactile stimulation, as it is perceived actually deals with direct stimulation through vibrators or skin mechanoreceptors stimulation. Each stimulator has its own domain of validity and tactile devices are divided according to discrimination and space covered by stimulation area [17][18]. From a technological point of view, tactile stimulation can be realised from different ways. Therefore, various actuation principles are discussed. So, in shape tactile displays (quasi static) actuators are of: electromagnetic, Shape Memory Alloy, pneumatic technology, etc. In a second group, vibrotactile matrices systems use piezoelectric and electromagnetic actuators. Finally, the third subset is made of vibrating systems with continuous structure and friction reduction generally actuated using piezoelectric technology.

3.1. Shape tactile displays (Quasi-static)

3.1.1. Electromagnetic actuators

Electromagnetic (EM) actuators used in shape stimuli display are generally DC rotary actuators or electromagnets. They are generally bulky because of mechanisms involved and their miniaturisation is challenging. Consequently very few electromagnetic micro actuators are available in the market. Two examples are given with a graphic illustration.

The FEELEX from University of Tsukuba is an interface actuated by rotary motors controlled in position and deforming a plane surface of 3 mm thick thanks to a rods matrix. Each rod is actuated by a DC motor which the movement is transformed.

The FEELEX2 [19] allows a maximum rods displacement and force of respectively 18 mm and 1.1 kgf; such displacements and high forces are the main advantages of EM actuation.
A device similar to the one in this study was proposed by Schneider et al [20] who developed a common computer mouse to move on a steel mat, to which force feedback function is added by including in an electromagnet. In function of cursor position on the computer screen and effort needed a reference voltage is applied inducing therefore a magnetic field and then a continuous friction force depending upon the voltage value. The modified mouse is a Hewlett-Packard 5187-1556 (Figure 3). The maximum friction force obtained is 2.0 N which is still relatively high. However, a notable problem came from the localized magnetic force at the back of the mouse, causing a rotation around the magnet; a rotation which needed to be counterbalanced.
3.1.2. Shape Memory Alloys (SMA)

SMA are alloys of tremendous properties among metal materials wherein the capability to “keep in memory” an initial shape and to get back even after a deformation. Usually SMA follows a plastic deformation at relatively low temperatures (Martensite) and gets back their original shape (Austenite) if they are heated at high temperature.

For teleoperation and virtual reality applications, researchers from Harvard University have developed a tactile prototype [21] made of one line of rods. Reduced space between rods, significant force and roughness developed, and displacements amplitude make up SMA technologically adapted to tactile feedback. However, SMA are not so often used because of their relatively long response time and their integration remains challenging.

3.1.3. Pneumatic technology

Apart from air bladders inserted in the gloves of TeleTact Glove and filled up by a compressor, other devices exist. That is the case for pistons actuated by motors [22] allowing the feeling of a variable roughness membrane, the case as well of devices using a pump to expel [23] or aspire some air by means of binary electromagnet micro valves [24]. Although those devices from pneumatic technology can generate high forces, they are not always comfortable and are relatively heavy.

3.1.4. Electro-Rheologic Fluids (ERF)

A matrix of bladders full of ERF can allow space distribution of normal forces on the finger pulp that pre constrains the device then. Indeed, a local change of fluid viscosity by electric field application yields a variable roughness under the finger pulp. It turns out that some progress in terms of precision is necessary [25].

3.1.5. Other technologies

In this field of application, research groups have investigated other solutions such as MEMS (Micro-Electro Mechanical Systems) [26] and active polymers but developments are still marginal.

3.2. Vibro tactile matrices systems

The chosen approach in this sub section differs from the latter one because this time it is the vibro tactile stimuli application that generates the prospected feeling. Previously the prospected effect was obtained by means of normal indentation of the skin according to shape reconstitution usually related to discrete representation of the state of surface or 3D explored asperity. High amplitude and low frequency of rods displacement feature those structures whereas in vibro tactile stimulators rods have a high frequency (around 200 Hz) and very low amplitude (around 10 μm) displacements. “Reproducing” surface asperities is no longer the aim but conversely, producing a sort of appropriate excitation on differ-
ent skin mechanoreceptor populations. Conceptually, electromagnetic and piezoelectric technologies are resorted to.

In this category the Vital [17] is an example of electromagnetic system.

The devices belonging to the family to be discussed in next section are based on electro mechanical conversion principle specific to piezoelectric systems. That principle will be used in the rest of this work.

### 3.3. Continuous structure based vibro tactile systems

In order to create shape displays or vibro tactile feelings, the technologies described above generally have the particularity to communicate the explored feeling to the user finger through a rods matrix. The advantage of that rods matrix structure is to allow refinement in control (each rod being independently controllable) but the structure is limited in integration. Conversely, some other devices present a continuous structure producing a pulse or a controllable friction under the finger.

Using this principle, the impulse display proposed by Poupyrev et al [27] was able to be incorporated on the sensitive screen of a PDA (Personal Digital Assistant).

### 3.4. Variable friction devices

#### 3.4.1. Variable friction creation principle

The action of friction generated by a surface under the finger is a second alternative exploited in continuous structure tactile interfaces. Watanabee [28] pioneered friction coefficient adjustment. Let’s describe Watanabee experience briefly for the slot it opened in the design of number of structures in this category. Watanabee used a steel beam which one end was attached to a Langevin transducer [29]. The Langevin transducer excited at 77 kHz communicates its maximum 2 μm vibrations to the beam. As a result, the feeling procured to a finger that explored the surface of the non excited beam was different from the feeling obtained with the vibrating beam: in the latter case the surface is very slippery and smooth. Watanabee also observed that as long as the vibratory frequency is greater than 20 kHz it has no influence on the perceived feeling.

In the follow up T. Nara [30] proposed a tapered plate and as in the introduction the idea inspired among others the designs presented in [2] and [3].

From the description of the later devices it can be seen that controlled friction is an interesting alternative in terms of tactile feelings production. Controlled friction is generated by continuous surfaces, limited in size and hence fully inerrable. In addition the control of these devices is rather global since it is the entire structure that is excited not a matrix rods. The accurate knowledge of the finger position is necessary and texture to be explored have to be processed in terms of friction coefficient in order to provide reference inputs to the effector.
4. The system

4.1. Description

Figure 4 shows the proposed structure; a resonant physical device that in this specific case is a piezoelectric transducer converting electrical energy into mechanical energy. Driven in a simple bending mode of vibration, the structure consists of a set of PZT polarised piezo-ceramics of 12x12x1mm glued on the upside of a copper-beryllium substrate whose size is 64x38x3 mm. On the opposite side (Figure 1), four built-in feet support the plate. Considering this polarisation, piezo-ceramic electrodes are conveniently supplied by a sinusoidal voltage of some ten Volts to create a standing wave using the piezoelectricity inverse effect with 40.7 kHz driving frequency (resonant frequency). Earlier some studies have been carried out using a system closed to this structure [31][32]. The main difference is that this plate will only move normally. Indeed, as we will see later, this particular structure is not supposed to move by itself along the tangential direction.

In Figure 4 b), alongside the four feet appears also a measurement ceramic glued on the plate. That flat round ceramic is acting as a vibratory sensor without altering the structure voltage supply that is kept unbroken.

4.2. Working principle

The feet are positioned exactly at the antinodes (Figure 5) of the vibrating plate and they are in contact with a plane steel substrate for example. Therefore, when no voltage is applied to the ceramics, if users move the actuator, they can feel the classical Coulomb friction force ($F_t$ in Figure 7 a) acting at the interface feet-substrate.

When voltage is applied to the ceramic electrodes, a standing wave is generated and the friction between feet and substrate is decreased: this happens according to amplitude vibration. As a matter of fact, from a given wave amplitude, an intermittent contact may occur at the interface. Consequently, at the feet base, transitions between stick or slip conditions are created.
4.3. Modelling

Many researchers studied stick-slip vibrations with switch models which Leine et al. [7] claim the solution they proposed is an improved version. The model proposed by Leine treats the system as three different sets of ordinary differential equations (ODE): one for the slip phase, a second for the stick phase and a third for the transition from stick to slip. Restricted to its normal movement component, our system can be seen as the “stick-slip” defined above with the “same” (contact, separation and transition from contact to separation) three states. The problem is solved using Simulink-Stateflow®, a convenient tool for finite state machine simulation and control: here is all the interest that is to see how it provides with a “switch block” to deal with the transition phase source of some difficulties including dealing with equation for stick to slip transition in the pseudo code used by Leine et al., sensitivity analysis, etc, as developed in [7].

4.3.1. Feet-plan Contact model

The structure in study involves contact and separation sequences between the feet of a body and the floor or substrate: contact and separation are induced by the foot elasticity. The movement so defined on the component normal to the plane of contact is broken down into compression – relaxation – separation sequences. This approach identifies the feet in contact with the floor to a mass-spring system.

The particular surface topology required by the device, that is a substrate highly rigid and a minimal surface roughness ($R_a \leq 0.6 \mu m$), the low clearances of the effector tip in the range of a few micrometers make the approximation a priori acceptable.

The model will enable characterizing the contact intermittence.
4.3.1.1. Foot mass – spring model

From the plate which the kinetic is briefly reminded here, a refined analysis of the bond up effecter can be carried out. For sake of convenience, figure 6 shows the plate kinetic diagram in an Oxz coordinates system. For a plate constrained in pure bending mode as in figure 6, more precisely a plate in vibratory mode (0,6) as in figure 5 with feet located at $\lambda/4$, it can be shown [33] [34][35] that:

$$w\lambda(u,v,t) = w(t) = W(t) \sin(\omega t)$$  \hspace{1cm} (1)

$A$ is the foot end located at intersection of the foot and the plate.

![Figure 6. Kinematic of plate deformation](image)

W(t) is the dynamic vibration amplitude at the actuator centre and $\omega$ the vibratory mode pulsation.

It is recalled in the preceding that only the displacement $w$ is considered. Also, the feet location which the tip is at antinode ($\lambda/4$) is assumed constant because of the pure bending mode assumption. In addition, from the similar feet positioning with respect to the wave, the mechanical study may be restricted to that of one foot subjected to a pre stress $F_n$ equally distributed upon n feet. Moreover, external loads are reported to the plate partial centre of gravity G. Conversely, the plate has four feet, making the problem hyperstatic because of many number of unknown contact variables. Elsewhere, planarity of the contact surface may contribute to invalidate the load equal distribution upon the feet. Nevertheless, considering the global contact approach, the load equal repartition is retained.

That yields the equivalent mechanical diagram used to define the system dynamic relations.

 Each foot can now on be described as a mass-spring system and we represented it in Figure 7 a. In Figure 7 a, $M_{ext}$ depicts the load applied on the top of the device to assume pre-stress. This load lies on an elastic element whose stiffness $k_m$ is low enough to assume that the force
\( F_n \) due to \( M_{\text{ext}} (F_n = 9.81 M_{\text{ext}}) \) is constant. The mass of the vibrating plate is denoted \( m \) and the number of feet, \( n \). The foot mass \( m_f \) is too low to be considered and its normal stiffness is \( k_n \). Finally the displacement \( w_A(t) \) is imposed by the plate vibrations whereas \( R_n \) denotes the normal reaction force at the foot tip.

**Figure 7.** a) equivalent mechanical scheme and forces acting on the foot b) system

### 4.3.1.2. Actuator behaviour along the normal axis

Describing the behaviour of actuator along the normal axis \( O_z \) amounts to writing the foot tip equation of movement characterizing the induced separation and contact periods along that \( O_z \) axis. The separation phase is also called flight. Equation obtained by applying the general dynamic laws to the system assuming the substrate is ideally rigid and therefore only the foot is storage element of potential energy, which yields:

\[
\frac{m}{n} z_A = \frac{m}{n} w_A - \frac{F_n}{n} - \frac{m}{n} g + R_n(t)
\]

with

\[
\frac{m}{n} z_A = \frac{m}{n} w_A - \frac{F_n}{n} - \frac{m}{n} g + R_n(t), \quad \text{during the contact phase}
\]

\[
R_n(t) = 0, \quad \text{during the separation phase}
\]
Where $k_n$ is the elasticity of the foot, $h$ the height of the foot when no pressure is exerted on it and $d_n$ a damping coefficient on the main compression of the foot induced by variation of $z_G$.

Prior to solving equation (2) let us examine the detail of transition conditions from contact to separation phase. At rest, under external mass $M_{ext}$, the plate plane is located from the ground at a distance $z_G$ lesser than $h$, the height of relaxed/released foot: the foot is compressed. Vibrating, the plate imposes a sinusoidal normal displacement of the foot end $z_A$. In the instance of a displacement large enough for the ordinate $z_A$ to be greater than the height $h$, there is separation. It is underlined that for $k_n$ a priori high, eventual longitudinal vibrations of the foot are neglected during the separation. The following transition condition results:

$$z_A > h, \quad \text{foot in separation} \quad (5)$$

$$z_A \leq h, \quad \text{foot in contact} \quad (6)$$

Taking

$$z_A = z_G - w_A, \quad (7)$$

yields

$$(z_G + w_A) \leq h : \quad \text{contact} \quad (8)$$

$$(z_G + w_A) > h : \quad \text{separation} \quad (9)$$

Taking into account equation (7), equation (2) may be rewritten:

$$\frac{m}{n} z_G = -\frac{F_n}{n} - \frac{m}{n} g + R_n(t) \quad (10)$$

or after rewriting equation (2):

$$\frac{m}{n} z_G + d_n z_G + k_n z_A = -\frac{F_n}{n} - \frac{m}{n} g + k_n h \quad \text{If there is contact} \quad (11)$$

$$\frac{m}{n} z_G = -\frac{F_n}{n} - \frac{m}{n} G. \quad \text{If there is separation} \quad (12)$$
The set of these two equations may mathematically describe the system. The solutions of the piecewise ODE are of course function of the initial conditions and the focus is only on the steady state in presence of an intermittent contact; without the intermittence we are in the instance of a spring which one end is fixed while to the other end is attached a mass in sinusoidal motion. It is assumed that the first phase is a contact phase governed by equation (11) and the final conditions of the contact phase \((z_A > h)\) are the initial conditions of the separation phase \((z_A \leq h)\) governed by equation (12) and so on.

The structure requires taking into account vibratory phenomena alternating transient-steady-transient states. For an arbitrary value of vibratory amplitude high enough to induce intermittent contact, that behaviour is graphically illustrated in Figure 8 where \(t_c\) denotes the start up of the \(i^{th}\) phase arising after a transient state of the periodic phenomenon described by Equation 11 of period \(T\). \(t_{c(i+1)}\) represents the beginning of the \((i+1)^{th}\) contact phase.

\[ \text{Figure 8. Contact-separation-contact sequence and post } t_c \text{ detail} \]

Bearing in mind that the aim is to compute the normal reaction \(R_n\), it is worth the while to look at the set of ODE implementation. Figure 9 shows the Simulink implementation of equation (10). The subsystem that is a self defined Simulink system uses the Constant, Sum, Integrator, Outport and Gain templates taken from the linear block library of Simulink. The first summation generates the quantity in the right hand side of equation (10). The later sum is divided by the gain \("1/n"\) and then integrated twice to obtain \(z_m\) from which \(z_G\) is derived. The reason for integrators chain (two integrators) is because we are dealing with a second order equation.

Therefore we integrate \(z\) twice to get \(z\).

Of course, the integrators must be initialized to correspond to initial values; thus in this specific case, the initial value of first integrator is set to 0 according to the system initial conditions (null foot speed). The initial value of the second integrator is set to \(h\) according to the initial conditions.

A second sum is used to get \(z_A\) as in equation (7).
To proceed further, we use the Simulink system shown in Figure 10. The distinctive feature of this system is that it contains an algebraic loop and a Stateflow® chart.

As can be seen in Figure 10, $R_n$ is delivered to the input of FPDnorm broke down above.

On the other hand, this input depends directly on the output function given variables as expressed in equation (10). But, $R_n$ is conditional, which suggests resorting to Stateflow® through “motion state” chart.

Stateflow® is a Simulink toolbox convenient for modelling and simulating finite state machines. A finite state machine being a representation of an event-driven system that is a system making a transition from one state (mode) to another prescribed state, provided that the condition defining the change is true.
In Stateflow® representations states and transitions form the basic building blocks of the system.

Stateflow® block is used within the Simulink model here to dynamically simulate the system state changes. The chart block where the system is modelled is open in Figure 11. The device has two states: contact and separation that are represented by a rectangular block and named accordingly. Transition lines indicate the next state that the system in the current state can transit to. According to our algorithm these lines are from contact to fly and vice versa. Also, a default transition is assigned to a default or very first state, the state machine has to be in when it starts, chosen to be the “contact” state. In this case, the first state will be “contact” when the execution begins.

Actions to be taken when entering each state are defined. In this case, this is fly \((flight = 1)\) or its logic complement, not fly \((flight = 0)\) for contact and flight state respectively. For the Simulink model it is the machine output \((0, 1)\) that is present at the Stateflow® chart output port. Entry command will be executed when entering the state.

Conversely, let us notice the chart block input port set to control the Stateflow® chart that “sees” \((h - z_A)\) component from Simulink model. This is the “flight_cond” standing for flight condition associated to transition lines.

Next, at its input 2, from Stateflow® chart, a switch block compares the machine output \((0, 1)\) to a threshold chosen to have the arbitrary value 0.5 as criterion for the switch to pass either through switch input 1 where \(k_n(h - z_A(t)) - d_n z_G\) is present or through switch input 3 holding the 0 value. One or the other later amount will ultimately output the switch yielding the \(R_n(t)\) (see equation (11) and (12)) of the subsystem output. In every iteration step, Simulink will try to bring the algebraic loop mentioned earlier which involves \(R_n\) “into balance”.

This block enables watching state changes when simulating.

![Figure 11](image-url). Open chart block named “motion state” in figure 10
Regarding the diagrams in Figure 9 and 10 the interested reader may connect outport carrying $R_n$ to corresponding inport, the same for inports $W_a$, replaces outports $d_{zm}, z_c$ and $z_A$ with scopes e.g., set up a sinewave for $W_a$ and use the given values to run the simulation. The solution may be calculated using the ode23 (Bogacki-Shampine) procedure with step size control activated (parameters: Initial Step Size=Auto, Max Step Size=1e-7, Min Step Size=Auto, relative and absolute tolerance=Auto), over the time interval [0, 0.5]. Here is an example of numerical values set: $k_n=9.7$ MN/m, $F_n=10$ N, $m=0.0723$ kg, $d_n=200$ N/ms, $h=0.004$ m. A MATLAB function, similar to the one in Appendix may be used to calculate the quantities of the example above, to be supplied to Simulink interfaces.

In Figure 12 are shown the time evolution of normal reaction $R_n$; for low vibratory amplitude, the normal reaction does not get null but is modulated at the excitation frequency and according to the vibratory amplitude. For higher vibratory amplitudes

![Figure 12. Continuous-time variation of normal reaction $R_n$ a) low b) high vibratory amplitude](image)

Figure 12 shows an annulment of $R_n$ and thus an intermittent contact. The impact of the switch in transitions contact-separation-contact is remarkable. For a purpose, in steady state the exciting frequency is taken equal to 35 kHz in this simulation.

For the purpose of verification this simulation was compared to the one performed in [36]. To that end, attention was given to a parameter capable to learn on the flight duration over a vibratory period. The parameter is named flight ratio and defined as:

$$\beta = \frac{t_s-t_c}{T} \times 100$$

with $t_s$: instant of separation debut

and

MATLAB
Figure 13 shows results for $\beta$ that are compared to those obtained by [37] for feet located at $\lambda/8$. Since our simulation is somehow validated by this result, we show in Figure 9 b) the flight rate variations as a function of vibratory amplitude for our actuator.

![Figure 13](image)

**Figure 13.** Flight ratio as a function of vibratory amplitude a) for $k_n$ equals 10 MN/m; comparison with [LJF00]; feet at $\lambda/8$ b) feet at $\lambda/4$.

In steady state, for different preload values, for a stiffness of 10 MN/m and feet located at $\lambda/8$, results in Figure 13 a) show the expected flight ratio variations that increase with the vibratory amplitude and decrease with the preload.

Experimental tests performed by [36] to measure the flight ratio allowed the verification of the model relevance observing foot contact intermittence with the floor. Also, the phenomenon remains periodic and of same period with the vibratory wave in steady state.

Regarding the simulation results in conditions of feet located at the wave crest, they show the same trend with a general curves shift toward the left that is a flight ratio greater for given vibratory amplitude.

The “similarity” introduced earlier on when the system was restricted to its normal movement component and the “stick-slip” vibrations as defined by Leine et al. [7] has been used up to this point.

### 4.3.1.3. Actuator behaviour along the tangential axis

To calculate the tangential force variation, a classical approach would have consisted of applying at the contact surface Coulomb friction modelling. It turns out that if the device is manipulated by a user, non-zero relative tangential speed always exists between the foot and the substrate. Coulomb law suggests therefore that $R_t = \mu R_n$ during contact phase and $R_t = 0$ otherwise. Since in flight event $R_n = 0$, it is possible to generalize that $R_t = \mu R_n$. 
Calculating $R_t$, average value over a vibratory period, if a constant friction coefficient $\mu$ is considered, $<R_t> = \mu <R_n>$ ($\langle \cdot \rangle$ denotes average value of $f$). But, Equation 10 shows that in steady state $<R_n> = \frac{F_n}{n} + \frac{m}{n} g$. It follows hence that regardless the flight ratio, $<R_t>$ is constant for constant $\mu$. That is not what was experimentally observed, justifying thus the consideration of a time variable friction coefficient.

The approach consists of considering in a more refined way the sliding triggering phenomena occurring over every vibratory period. Indeed it has been seen that feet – substrate contact can be intermittent. In such an instance, at every resuming contact, while the actuator is tangentially moving due to the user action, the feet are first in adhesion on the substrate, then speedily in partial slip and finally in total slip before flying again: the consideration of partial slip phase is source of tangential force average variation. That phase may be characterized by an elastic behaviour of the foot, characterized by its tangential stiffness $k_t$. This corresponds to the definition of a time varying friction coefficient $\mu$ obeying Coulomb – Orowan law \[38\].

Figure 14 depicts friction coefficient as a function of $\delta$ and the plot is divided approximately in two parts. The first part, relative to tangential stiffness of the contact, is linear and describes the partial slip. Then, from a critical displacement ($\delta_{\text{crit}}$) corresponding to total slip, $\mu$ is constant.

As explained above, the feet are positioned exactly at the antinodes (Figure 5) of the vibrating plate and they are in contact with a plane steel substrate for example. Therefore, when no voltage is applied to the ceramics, if users move the actuator, they can feel the classical Coulomb friction force acting at the interface feet/substrate.

$\delta_{\text{crit}}$ is defined from the following relationship after prior identification of tangential stiffness $k_t$:

$$\delta_{\text{crit}} = \mu_0 CR_n = \frac{\mu_0 R_n}{k_t}$$

(14)

where $C$ (m/N) is the compliance and $\mu_0$, the maximum friction coefficient at the interface (static friction).
As a consequence, we are able to obtain $\mu(t)$ during the foot/substrate contact time, limited by $t_c$ and $t_s$ which are respectively the contact and separation instants during one period of our vibrating device. We can then write:

$$\text{if } t \in [t_c, t_s] \text{ and } \delta < \delta_c \text{ then } \mu(t) = \frac{\mu_0}{\delta_c} \delta(t)$$

(15)

$$\text{if } t \in [t_c, t_s] \text{ and } \delta > \delta_c \text{ then } \mu = \mu_0$$

(16)

The displacement $\delta(t)$ is computed from the tangential speed integration.

$$\delta(t) = \int_0^t V_i(t) dt$$

(17)

where $V_i$ is the relative sliding speed between the two surfaces in contact.

The determination of $<R_t>$ is a key point for this study since it is the reactive force sensed by the device user and, equations (11) and (12) show that it is a function of the pre-load, displacement speed and wave amplitude. To this end, we will have to control the wave amplitude, the two other variables being not suitable for control: the tangential speed will be imposed externally by the user, and the normal pre-load is set at a fixed value.

5. Control of the vibration amplitude

The control of the vibratory amplitude may be achieved following different approaches. In [39], the wave amplitude control is done thanks to the phase control of the standing wave according to the voltage signal supply. The advantage of this method is its high robustness against resonance frequency variations. One drawback is a lower dynamic behaviour due to the response time imposed by a phase locked loop (PLL).

Another way to control the wave amplitude is to tune the supply frequency around the resonance value. This approach comes from the characteristic frequency – vibratory amplitude which shows that beyond the resonant frequency, the wave amplitude $W$ decreases quasi-linearly, making possible its control [40]. The method presents the advantage of being easy to implement and the loop dynamic is fast. Conversely, it has the disadvantage that changes in temperature displace the resonant frequency and lead to discrepancies in the control. Rigorously, to avoid that inconvenience, an algorithm to track the resonant frequency should be implemented to anticipate the preload influence on the resonant frequency. Nevertheless we have chosen this approach, also easier to implement.
6. Features of friction forces

From equations introduced in section 5, it was possible to compute the behaviour of the actuator for a given wave amplitude, a given tangential speed and a given normal load. The obtained results are as shown in Figure 15.

![Graph](image)

Figure 15. Friction force for a) a fixed preload as a function of vibratory amplitude and tangential displacement speed, b) a given tangential speed as a function of the wave amplitude and the normal pre-load.
To characterize the friction forces we use, for different vibratory amplitudes, a DC motor Maxon® controlled in speed to which the plate is attached by means of an inextensible cable and a 10 mm diameter pulley. The measured motor current is therefore an image of torque and thus force developed by the motor. That force is in absolute value equal to the explored friction force.

An optical encoder is used to measure the motor rotational speed and the so constituted setup is controlled by a dSPACE DS1104 application. Several simulations based on the contact conditions described all along were performed and the results as compared to the experimental are shown in Figure 15.

The experimental results presented in Figure 15 were obtained in such a way that a load $M_{ext}$ was applied on the top of the device to assume pre-load. This load lied on an elastic element whose stiffness was low enough to consider constant the force $F_n$ due to $M_{ext}$ ($F_n=9.81M_{ext}$). Also, a steel substrate ($\mu = 0.2$) was used for these trials. Finally the time variable displacement $w$ is imposed by the plate vibrations.

These results illustrate the overall behaviour of the structure and show the existence of a critical wave amplitude beyond which friction reduction is noticed.

### 7. Evaluation of the device for touch feedback application

The aim of the evaluation of the device for tactile feedback is to determine whether the device qualifies or not for the dedicated application. In his study, U. Spälter [41] indicates that there is no standard evaluation procedure for haptic devices. In this specific case, the profile in Figure 16 that shows an alternation of “notches” was considered. One aspect of the evaluation was to know if the alternation of apparent friction coefficient ($\mu_1$, $\mu_2$) induced by vibratory amplitude enabled generation of “notches”. In Figure 16, $\mu_1$ and $\mu_2$ correspond respectively to friction coefficient before and beyond the critical wave amplitude identified in Figure 12.

![Figure 16. Profile of alteration of notches](image-url)
The other aspect of the evaluation was to determine if it was possible to discriminate the two profiles in Figure 17.

![Simulated notches (different spatial periods)](image)

Figure 17. Simulated notches (different spatial periods)

A preliminary psychophysical evaluation discussed in [42] showed how to assess the validity of the structure to low force feedback application.

8. Summary

The concept of friction coefficient reduction has been presented to design a 2Dof passive low force feedback device in this chapter. For a design utilizing piezoelectric technology, piezoelectric materials and their effect in transducers technology mainly, together with several existing solutions using technologies other than piezoelectric actuation in the field of touch feedback were briefly discussed. Modelling of the device with an emphasis on the normal reaction force, leading to the expression of the tangential force felt by a user moving the actuator the way he moves a common computer mouse was also presented. In particular, some details were given on a way to use Stateflow® to deal with modelling and simulation aspects of the normal reaction force in the system, regarded as finite state machine when restricted to its normal movement component. We also showed how results from experimental and theoretical investigations agree on the fact that it is possible to control the resulting friction force even if this force highly depends on normal pre-load and tangential speed.

Finally, the proposed device may be a solution to cope with the lack of compactness and simplicity often encountered in haptics interfaces. Complementary experiments are needed to assess its response to touch feedback. Also required is a study of the device behaviour over the time to consider feet wear, or at least variation of contact conditions so that the initial vibratory amplitude control can anticipate such changes. Consequently a direct comparison between the solution proposed in this study and the demonstrated high-performance and practical electromagnetic mouse described in section 3.1.1 e.g. may not be easily sustainable. However, apart from simplicity and compactness characteristics common to both of them, it is an additional advantage in terms of behaviour that this design anticipated issues like the observed rotation of the electromagnetic mouse which resulted from the localized magnetic force.
Appendix

function [m,kn,dn,Fn,h,n] = planeactuatpm(Li,l,h_s,h_p,rho_s,rho_p)

% Function activebrakepm
% Call: [m,kn,dn,Fn,h,n]=planeactuatpm(Li,l,h_s,h_p)
% MATLAB function for parametrizing the Simulink system
% could be called activebrake.mdl
% Input data: Li      plate width [m]
%             l       plate length [m]
%             h_s     thickness of the metal layer of the plate [m]
%             h_p     thickness of the ceramic layer of the plate [m]
%             rho_s   metal layer density [kg/m^3]
%             rho_p   ceramic layer density [kg/m^3]
% with the parameters of the Simulink simulation window not set here
% Output data: the parameters of Equation (11) and (12)
%
%*****************************************************
% declaration of the pre-load
%*****************************************************
% The pre-load is constant
Fn=10; % [N/m]
%
%*****************************************************
% declaration of the foot length and number of feet
%*****************************************************
% The length of the foot and the number of the feet are constant
h=0.0040; % length of the foot [m]
n=3; % chosen number of the feet
%
% Calculation of the mass of the plate
m=rho_s*(h_s*l*Li)+rho_p*(h_p*l*Li);
% gravity is constant
g=9.81; % [N/kg]
% Calculation of the weight 'mg'
mg=g*m;
%
%*****************************************************
% Damping and stiffness coefficient
%*****************************************************
% normal damping coefficient is constant
dn=200; % [kg/m]
%
% The stiffness coefficient function of the pre-load is taken constant
kn=9.7e6; % [N/m]
A call for the parameters of our actuator then yields:

\[ [m,kn,dn,Fn,h,n] = \text{planeactuatpm}(0.038,0.064,0.003,0.00065,8250,7650) \]

\[ m = 0.0723 \]
\[ kn = 9700000 \]
\[ dn = 200 \]
\[ Fn = 10 \]
\[ h = 0.0040 \]
\[ n = 3 \]

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