We are IntechOpen, the world’s leading publisher of Open Access books
Built by scientists, for scientists

4,100
Open access books available

116,000
International authors and editors

125M
Downloads

154
Countries delivered to

TOP 1%
Our authors are among the most cited scientists

12.2%
Contributors from top 500 universities

WEB OF SCIENCE™
Selection of our books indexed in the Book Citation Index in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com
A PSO Approach in Optimal FACTS Selection with Harmonic Distortion Considerations

H.C. Leung and Dylan D.C. Lu

Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/54555

1. Introduction

Static Var Compensator (SVC) has been commonly used to provide reactive power compensation in distribution systems [1]. The SVC placement problem is a well-researched topic. Earlier approaches differ in problem formulation and the solution methods. In some approaches, the objective function is considered as an unconstrained maximization of savings due to energy loss reduction and peak power loss reduction against the SVC cost. Others formulated the problem with some variations of the above objective function. Some have also formulated the problem as constrained optimization and included voltage constraints into consideration.

In today’s power system, there is trend to use nonlinear loads such as energy-efficient fluorescent lamps and solid-state devices. The SVCs sizing and allocation [2-4] should be properly considered, if else they can amplify harmonic currents and voltages due to possible resonance at one or several harmonic frequencies and switching actions of the power electronics converters connected. This condition could lead to potentially dangerous magnitudes of harmonic signals, additional stress on equipment insulation, increased SVC failure and interference with communication system.

SVC values are often assumed as continuous variables whose costs are considered as proportional to SVC size in past researches. Moreover, the cost of SVC is not linearly proportional to the size (MVAr). Hence, if the continuous variable approach is used to choose integral SVC size, the method may not result in an optimum solution and may even lead to undesirable harmonic resonance conditions.

Current harmonics are inevitable during the operation of thyristor controlled rectifiers, thus it is essential to have filters in a SVC system to eliminate the harmonics. The filter banks can not only absorb the risk harmonics, but also produce the capacitive reactive power. The SVC
uses close loop control system to regulate busbar voltage, reactive power exchange, power factor and three phase voltage balance.

This chapter describes a method based on Particle Swarm Optimisation (PSO) [5] to solve the optimal SVC allocation successfully. Particle Swarm Optimisation (PSO) method is a powerful optimization technique analogous to the natural genetic process in biology. Theoretically, this technique is a stochastic approach and it converges to the global optimum solution, provided that certain conditions are satisfied. This chapter considers a distribution system with 9 possible locations for SVCs and 27 different sizes of SVCs. A critical discussion using the example with result is discussed in this chapter.

2. Problem formulation

2.1. Operation principal of SVC

The Static Var Compensator (SVC) are composed of the capacitor banks/filter banks and air-core reactors connected in parallel. The air-core reactors are series connected to thyristors. The current of air-core reactors can be controlled by adjusting the fire angle of thyristors. The SVC can be considered as a dynamic reactive power source. It can supply capacitive reactive power to the grid or consume the spare inductive reactive power from the grid. Normally, the system can receive the reactive power from a capacitor bank, and the spare part can be consumed by an air-core shunt reactor. As mentioned, the current in the air-core reactor is controlled by a thyristor valve. The valve controls the fundamental current by changing the fire angle, ensuring the voltage can be limited to an acceptable range at the injected node(for power system var compensation), or the sum of reactive power at the injected node is zero which means the power factor is equal to 1 (for load var compensation).

2.2. Assumptions

The optimal SVC placement problem [6] has many variables including the SVC size, SVC cost, locations and voltage constraints on the system. There are switchable SVCs and fixed-type SVCs in practice. However, considering all variables in a nonlinear fashion will make the placement problem very complicated. In order to simplify the analysis, the assumptions are as follows: 1) balanced conditions, 2) negligible line capacitance, 3) time-invariant loads and 4) harmonic generation is solely from the substation voltage supply.

2.3. Radial distribution system

Figure 1 clearly illustrates an m-bus radial distribution system where a general bus \( i \) contains a load and a shunt SVC. The harmonic currents introduced by the nonlinear loads are injected at each bus.

At the power frequency, the bus voltages are found by solving the following mismatch equations:
A PSO Approach in Optimal FACTS Selection with Harmonic Distortion Considerations

\[ P_i = |V_i|^2 G_{ii} + \sum_{j=1}^{m} |V_i|^1 |V_j|^1 \cos(\theta_{ij} - \delta_{ij}) \quad i = 1,2,3...m \]  
\[ Q_i = -|V_i|^2 B_{ii} + \sum_{j=1}^{m} |V_i|^1 |V_j|^1 \sin(\theta_{ij} + \delta_{ij}) \quad i = 1,2,3...m \]

where
\[ P_i = P_{ii} + P_{aii} \]  
\[ Q_i = Q_{ii} + Q_{aii} \]  
\[ Y_{ij} = Y_{ii} - Y_{ij} \begin{cases} -y_{ij} & \text{if } i \neq j \\ y_{i-1,j} + y_{i+1,j} + y_{i1} & \text{if } i = j \end{cases} \]
\[ Y_{ii} = G_{ii} + jB_{ii} \]

2.4. Real power losses

At fundamental frequency, the real power losses in the transmission line between buses \( i \) and \( i+1 \) is:
\[ P_{\text{loss}(i,i+1)} = R_{i,i+1} |V_{i+1} - V_i|^2 \]
So, the total real losses is:
\[ P_{\text{loss}} = \sum_{n=1}^{N} \left( \sum_{i=0}^{m-1} P_{\text{loss}(i,i+1)}^n \right) \]

2.5. Objective function and constraints

The objective function of SVC placement is to reduce the power loss and keep bus voltages and total harmonic distortion (HDF) within prescribed limits with minimum cost. The constraints are voltage limits and maximum harmonic distortion factor, with the harmonics...
taken into account. Following the above notation, the total annual cost function due to SVC placement and power loss is written as:

Minimize

\[
f = K_i K_p P_{\text{loss}} + \sum_{j=1}^{m} Q_{c_j} K_j
\]  

(9)

where \( j = 1,2,\ldots,m \) represents the SVC sizes

\[
Q_{c_j} = j^* K_s
\]

(10)

The objective function (1) is minimized subject to

\[
V_{\text{min}} \leq |V_i| \leq V_{\text{max}} \quad i = 1,2,3\ldots m
\]

(11)

and

\[
\text{HDF}_i \leq \text{HDF}_{\text{max}} \quad i = 1,2,3\ldots m
\]

(12)

According to IEEE Standard 519 [7] utility distribution buses should provide a voltage harmonic distortion level of less than 5% provided customers on the distribution feeder limit their load harmonic current injections to a prescribed level.

3. Proposed algorithm

3.1. Harmonic power flow [8]

At the higher frequencies, the entire power system is modelled as the combination of harmonic current sources and passive elements. Since the admittance of system components will vary with the harmonic order, the admittance matrix is modified for each harmonic order studied. If the skin effect is ignored, the resulting n-th harmonic frequency load admittance, shunt SVC admittance and feeder admittance are respectively given by:

\[
Y_{ii}^n = \frac{P_{li}}{|V_i|^2} - j Q_{li} \frac{|V_i|^2}{n|V_i|^2}
\]

(13)

\[
Y_{ij}^n = n Y_{ij}^1
\]

(14)

\[
Y_{li,j+1}^n = \frac{1}{R_{li,j+1} + j n X_{li,j+1}}
\]

(15)

The linear loads are composed of a resistance in parallel with a reactance [9]. The nonlinear loads are treated as harmonic current sources, so the injection harmonic current source introduced by the nonlinear load at bus \( i \) is derived as follows:
In this study, $C(n)$ is obtained by field test and Fourier analysis for all the customers along the distribution feeder. The harmonic voltages are found by solving the load flow equation (18), which is derived from the node equations.

$$Y^n V^n = I^n$$  \hspace{1cm} (18)

At any bus $i$, the r.m.s. value of voltage is defined by

$$|V_i| = \sqrt{\frac{1}{N} \sum_{n=1}^{N} |V_{i,n}|^2}$$  \hspace{1cm} (19)

where $N$ is an upper limit of the harmonic orders being considered and is required to be within an acceptable range. After solving the load flow for different harmonic orders, the harmonic distortion factor (HDF) [8] that is used to describe harmonic pollution is calculated as follows:

$$HDF_i(\%) = \left( \frac{\sqrt{\sum_{n=1}^{N} |V_{i,n}|^2}}{|V_i|} \right) \times 100\%$$  \hspace{1cm} (20)

It is also required to be lower than the accepted maximum value.

### 3.2. Selection of optimal SVC location

The general case of optimal SVC locations can be selected for starting iteration. PSO calculates the optimal SVC sizes according to the optimal SVC locations. After the first time iteration, the solution of SVC locations and sizes will be recorded as old solution and add more locations to consideration. PSO is used to calculate a new solution. If the new solution is better than the old solution, the old solution will be replaced by the new solution. If else, the old solution is the best solution. Therefore, this project will continue to consider more locations until no more optimal solution, which is better than the previous solution. In this chapter, the selection of optimal SVC location is based on the following criteria: voltage, real power loss, load reactive power and harmonic distortion factor with equal weighting.

### 3.3. Solution algorithm

PSO is a search algorithm based on the mechanism of natural selection and genetics. It consists of a population of bit strings transformed by three genetic operations: 1) Selection
or reproduction, 2) Crossover, and 3) mutation. Each string is called chromosome and represents a possible solution. The algorithm starts from an initial population generated randomly. Using the genetic operations considering the fitness of a solution, which corresponds, to the objective function for the problem generates a new generation. The string’s fitness is usually the reciprocal of the string’s objective function in minimization problem. The fitness of solutions is improved through iterations of generations. For each chromosome population in the given generation, a Newton-Raphson load flow calculation is performed. When the algorithm converges, a group of solutions with better fitness is generated, and the optimal solution is obtained. The scheme of genetic operations, the structure of genetic string, its encode/decode technique and the fitness function are designed. The implementation of PSO components and the neighborhood searching are explained as follows.

4. Implementation of PSO

This section provides a brief introductory concept of PSO. If \( X_i = (x_{i1}, x_{i2}, ..., x_{id}) \) and \( V_i = (v_{i1}, v_{i2}, ..., v_{id}) \) are the position vector and the velocity vector respectively in \( d \) dimensions search space, then according to a fitness function, where \( P_i = (p_{i1}, p_{i2}, ..., p_{id}) \) is the pbest vector and \( P_g = (p_{g1}, p_{g2}, ..., p_{gd}) \) is the gbest vector, i.e. the fittest particle of \( P_i \), updating new positions and velocities for the next generation can be determined.

```
For each particle
  Initialize particle
END

Do
  For each particle
    Calculate fitness value
    If the fitness value is better than the best fitness value (pbest), set current value as the new pbest
  End

  Choose the particle with the best fitness value of all the particles as the gbest
  For each particle
    Calculate particle velocity from equation (21)
    Update particle position from equation (22)
  End

  Perform mutation operation with \( p_m \)
  While maximum iteration is reached or minimum error condition is satisfied
```

Figure 2. The pseudo code of the PSO method
A PSO Approach in Optimal FACTS Selection with Harmonic Distortion Considerations

\[ V_{id}(t) = \omega V_{id}(t-1) + C_1 \times \text{rand1} \times (P_{id} - X_{id}(t-1)) + C_2 \times \text{rand2} \times (P_{gd} - X_{id}(t-1)) \]  

(21)

\[ X_{id}(t) = X_{id}(t-1) + V_{id}(t) \]  

(22)

where \( V_{id} \) is the particle velocity, \( X_{id} \) is the current particle (solution). \( P_{id} \) and \( P_{gd} \) are defined as above. \( \text{rand1} \) and \( \text{rand2} \) are random numbers which is uniformly distributed between [0,1]. \( C_1 \) and \( C_2 \) are constant values which is usually set to \( C_1 = C_2 = 2.0 \). These constants represent the weighting of the stochastic acceleration which pulls each particle towards the \( p_{best} \) and \( g_{best} \) position. \( \omega \) is the inertia weight and it can be expressed as follows:

\[ \omega = (\omega_i - \omega_f) \cdot \frac{\text{iter}_{\text{max}} - \text{iter}}{\text{iter}_{\text{max}}} + \omega_f \]  

(23)

where \( \omega_i \) and \( \omega_f \) are the initial and final values of the inertia weight respectively. \( \text{iter} \) and \( \text{iter}_{\text{max}} \) are the current iterations number and maximum allowed iterations number respectively.

The velocities of particles on each dimension are limited to a maximum velocity \( V_{\text{max}} \). If the sum of accelerations causes the velocity on that dimension to exceed the user-specified \( V_{\text{max}} \), the velocity on that dimension is limited to \( V_{\text{max}} \).

In this chapter, the parameters used for PSO are as follows:

- Number of particles in the swarm, \( N = 30 \) (the typical range is 20 – 40)
- Inertia weight, \( \omega_i = 0.9, \omega_f = 0.4 \)
- Acceleration factor, \( C_1 \) and \( C_2 = 2.0 \)
- Maximum allowed generation, \( \text{iter}_{\text{max}} = 100 \)
- The maximum velocity of particles, \( V_{\text{max}} = 10\% \) of search space

There are two stopping criteria in this chapter. Firstly, \( \text{i(t) is the number of iterations since the last change of the best solution is greater than a preset number. The PSO is terminated while maximum iteration is reached.} \)

For the PSO, the constriction and inertia weight factors are introduced and (21) is improved as follows.

\[ V_{id}(t) = k \left[ \omega V_{id}(t-1) + \phi_1 \times \text{rand1} \times (P_{id} - X_{id}(t-1)) + \phi_2 \times \text{rand2} \times (P_{gd} - X_{id}(t-1)) \right] \]  

(24)

\[ k = \frac{2}{2 - (\phi_1 + \phi_2) - \sqrt{(\phi_1 + \phi_2)^2 - 4(\phi_1 \phi_2)}} \]  

(25)

where \( k \) is a constriction factor from the stability analysis which can ensure the convergence (i.e. avoid premature convergence) where \( \phi_1 + \phi_2 > 4 \) and \( k_{\text{max}} < 1 \) and \( \omega \) is dynamically set as follows:
\[ \omega = \omega_{\text{max}} - \frac{\omega_{\text{max}} - \omega_{\text{min}}}{t_{\text{Total}}} \times t \]  

(26)

where \( t \) and \( t_{\text{Total}} \) is the current iteration and total number of iteration respectively and \( \omega_{\text{min}} \) and \( \omega_{\text{max}} \) is the upper and lower limit which are set 1.3 and 0.1 respectively.

The advantage of the integration of mutation from GAs is to prevent stagnation as the mutation operation choose the particles in the swarm randomly and the particles can move to difference position. The particles will update the velocities and positions after mutation.

\[
\text{mutation}(x_{id}) = x_{id} - \omega_r (-1 < r < 0) \quad (27)
\]

\[
\text{mutation}(x_{id}) = x_{id} + \omega_r (1 > r \geq 0) \quad (28)
\]

where \( x_{id} \) is a randomly chosen element of the particle from the swarm, \( \omega \) is randomly generated within the range \([0, \frac{1}{10} \times (\text{particle max} - \text{particle min})]\) (\( \text{particle max} \) and \( \text{particle min} \) are the upper and lower boundaries of each particle element respectively) and \( r \) is the random number in between 1 and -1.

Implementation of an optimization problem is realized within the evolutionary process of a fitness function. The fitness function adopted is derived as equation (9). The objective function is to minimize \( f \). It is composed of two parts; 1) the cost of the power loss in the transmission branch and 2) the cost of reactive power supply. Since PSO is applied to maximization problem, minimization of the problem take the normalized relative fitness value of the population and the fitness function is defined as:

\[
f_i = \frac{f_{\text{max}} - f_i}{f_{\text{max}}} \quad (29)
\]

where \( f_i = K_i K_p P_{\text{loss}} + \sum_{j=1}^{m} Q_{ij} K_{ij} \)

5. Software design

Figure 3 depicts the main steps in the process of this chapter. The predefined processes of optimal SVC location and Particle Swarm Optimisation calculation are illustrated in Figure 4 and Figure 5.
A PSO Approach in Optimal FACTS Selection with Harmonic Distortion Considerations

Start

- Input setting, system file and capacitor file.
- Setup a Y-matrix
- Setup the possible choice of the SVC sizes and costs.

Optimal SVC location calculation subprogram *

PSO calculation subprogram **

Any feasible solution found?

- Yes
  - Record the best result and set as old solution
  - Optimal SVC location calculation subprogram *
  - PSO calculation subprogram **
  - Record the best result and set as new solution
  - Is new solution better than old solution?
    - Yes
      - Output the new setting result
    - No
      - Output the old setting result

- No
  - Old solution replaced by new solution

- Is all location considered?
  - Yes
    - Output the new setting result
  - No
    - Any feasible solution found?

End

* refer to Figure 4 and ** refer to Figure 5

Figure 3. Flow chart of main operation
Figure 4. Flow chart of ‘Optimal SVC location calculation subprogram’ in Figure 2
A PSO Approach in Optimal FACTS Selection with Harmonic Distortion Considerations 71

Figure 5. Flow chart of ‘Particle Swarm Optimisation calculation subprogram’ in Figure 3
Enter

Set first harmonic order

Adjust Y-matrix

Calculate Harmonic current source

Solve V*Y=I

Is the highest harmonic order considered?

Yes

Calculate the harmonic distortion factor

No

Next harmonic order

Exit

Figure 6. Flow chart of ‘Harmonic distortion calculation subprogram’ in Figure 4 and Figure 5

6. Numerical example and results

In this section, a radial distribution feeder [10] is used as an example to show the effectiveness of this algorithm. The testing distribution system is shown in Figure 7. This feeder has nine load buses with rated voltage 23kV. Table 1 and Table 2 show the loads and feeder line constants. The harmonic current sources are shown in Table 3, which are generated by each customer.

Figure 7. Testing distribution system with 9 buses
A PSO Approach in Optimal FACTS Selection with Harmonic Distortion Considerations

<table>
<thead>
<tr>
<th>Bus</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(kW)</td>
<td>1840</td>
<td>980</td>
<td>1790</td>
<td>1598</td>
<td>1610</td>
<td>780</td>
<td>1150</td>
<td>980</td>
<td>1640</td>
</tr>
<tr>
<td>Q(MVAr)</td>
<td>460</td>
<td>340</td>
<td>446</td>
<td>1840</td>
<td>600</td>
<td>110</td>
<td>60</td>
<td>130</td>
<td>200</td>
</tr>
<tr>
<td>Non-linear (%)</td>
<td>0</td>
<td>55.7</td>
<td>18.9</td>
<td>92.1</td>
<td>4.7</td>
<td>1.9</td>
<td>38.2</td>
<td>4.5</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Table 1. Load data of the test system

<table>
<thead>
<tr>
<th>From Bus i</th>
<th>From Bus j</th>
<th>R_{i,i+1}(\Omega)</th>
<th>X_{i,i+1}(\Omega)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.1233</td>
<td>0.4127</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0.0140</td>
<td>0.6051</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.7463</td>
<td>1.2050</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>0.6984</td>
<td>0.6084</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>1.9831</td>
<td>1.7276</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>0.9053</td>
<td>0.7886</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>2.0552</td>
<td>1.1640</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>4.7953</td>
<td>2.7160</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>5.3434</td>
<td>3.0264</td>
</tr>
</tbody>
</table>

Table 2. Feeder data of the test system

<table>
<thead>
<tr>
<th>Bus</th>
<th>Harmonic current sources(%) in harmonic order</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>9.1</td>
</tr>
<tr>
<td>3</td>
<td>3.1</td>
</tr>
<tr>
<td>4</td>
<td>6.2</td>
</tr>
<tr>
<td>5</td>
<td>17.7</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0.3</td>
</tr>
<tr>
<td>8</td>
<td>0.8</td>
</tr>
<tr>
<td>9</td>
<td>15.1</td>
</tr>
</tbody>
</table>

Table 3. The harmonic current sources

K_p is selected to be US $168/MW in equation (9). The minimum and maximum voltages are 0.9 p.u. and 1.0 p.u. respectively. All voltage and power quantities are per-unit values. The base value of voltage and power is 23kV and 100MW respectively. Commercially available SVC sizes are analyzed. Table 4 shows an example of such data provided by a supplier for 23kV distribution feeders. For reactive power compensation, the maximum SVC size $Q_{c(max)}$ should not exceed the reactive load, i.e. 4186 MVAr. SVC sizes and costs are shown in Table 5 by assuming a life expectancy of ten years (the placement, maintenance, and running costs are assumed to be grouped as total cost.)
Table 4. Available 3-phase SVC sizes and costs

<table>
<thead>
<tr>
<th>j</th>
<th>$Q_j$ (MVAr)</th>
<th>$K_j$ ($/MVAr$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150</td>
<td>0.500</td>
</tr>
<tr>
<td>2</td>
<td>300</td>
<td>0.350</td>
</tr>
<tr>
<td>3</td>
<td>450</td>
<td>0.235</td>
</tr>
<tr>
<td>4</td>
<td>600</td>
<td>0.220</td>
</tr>
<tr>
<td>5</td>
<td>750</td>
<td>0.183</td>
</tr>
<tr>
<td>6</td>
<td>900</td>
<td>0.170</td>
</tr>
<tr>
<td>7</td>
<td>1050</td>
<td>0.228</td>
</tr>
<tr>
<td>8</td>
<td>1200</td>
<td>0.170</td>
</tr>
<tr>
<td>9</td>
<td>1350</td>
<td>0.207</td>
</tr>
</tbody>
</table>

Table 5. Possible choice of SVC sizes and costs

<table>
<thead>
<tr>
<th>j</th>
<th>$Q_j$ (MVAr)</th>
<th>$K_j$ ($/MVAr$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1500</td>
<td>0.201</td>
</tr>
<tr>
<td>11</td>
<td>1650</td>
<td>0.193</td>
</tr>
<tr>
<td>12</td>
<td>1800</td>
<td>0.187</td>
</tr>
<tr>
<td>13</td>
<td>1950</td>
<td>0.211</td>
</tr>
<tr>
<td>14</td>
<td>2100</td>
<td>0.176</td>
</tr>
<tr>
<td>15</td>
<td>2250</td>
<td>0.197</td>
</tr>
<tr>
<td>16</td>
<td>2400</td>
<td>0.170</td>
</tr>
<tr>
<td>17</td>
<td>2550</td>
<td>0.189</td>
</tr>
<tr>
<td>18</td>
<td>2700</td>
<td>0.187</td>
</tr>
</tbody>
</table>

The effectiveness of the method is illustrated by a comparative study of the following three cases. Case 1 is without SVC installation and neglected the harmonic. Both Case 2 and 3 use PSO approach for optimizing the size and the placement of the SVC in the radial distribution system. However, Case 2 does not take harmonic into consideration and Case 3 takes harmonic into consideration. The optimal locations of SVCs are selected at bus 4, bus 5 and bus 9.

Before optimization (Case 1), the voltages of bus 7, 8, 9 are violated. The cost function and the maximum HDF are $132138 and 6.15% respectively. The harmonic distortion level on all buses is higher than 5%.

After optimization (Case 2 and 3), the power losses become 0.007065 p.u. in Case 2 and 0.007036 p.u. in Case 3. Therefore, the power savings will be 0.000747 p.u. in Case 2 and 0.000776 p.u. in Case 3. It can also be seen that Case 3 has more power saving than Case 2.

The voltage profile of Case 2 and 3 are shown in Table 6 and Table 7 respectively. In both cases, all bus voltages are within the limit. The cost savings of Case 2 and Case 3 are $2,744 (2.091%) and $1,904 (1.451%) respectively with respect to Case 1. Since harmonic distortion is considered in Case 3, the sizes of SVCs are larger than Case 2 so that the total cost of Case 3 is higher than Case 2.

The maximum HDF of Case 2 of Case 3 are 1.35% and 1.2% respectively. The HDF improvement of Case 3 with respects to Case 1 is

\[
HDF \; improvement \; % = \frac{6.15 - 1.20}{6.15} \times 100 = 80.49\% 
\]

The HDF improvement of Case 3 with respects to Case 2 is

\[
HDF \; improvement \; % = \frac{1.40 - 1.20}{1.40} = 14.29\% 
\]
The improvement of the harmonic distortion is quite attractive and it is clearly shown in Figure 7. The reductions in HDF are 80.49% and 14.29% with respect to Case 1 and Case 2.

The optimal cost and the corresponding SVC sizes, power loss, minimum / maximum voltages, the average CPU time and harmonic distortion factor are also shown in Figure 8.

Figure 8. Effect of harmonic distortion on each bus

<table>
<thead>
<tr>
<th>Bus</th>
<th>V1</th>
<th>V5</th>
<th>V7</th>
<th>V11</th>
<th>V13</th>
<th>V17</th>
<th>V19</th>
<th>V23</th>
<th>V25</th>
<th>V_min</th>
<th>HDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.993</td>
<td>4.41</td>
<td>2.96</td>
<td>1.57</td>
<td>1.25</td>
<td>9.60</td>
<td>8.12</td>
<td>7.47</td>
<td>4.72</td>
<td>0.992</td>
<td>5.78</td>
</tr>
<tr>
<td>2</td>
<td>0.987</td>
<td>4.43</td>
<td>2.98</td>
<td>1.58</td>
<td>1.26</td>
<td>9.69</td>
<td>8.19</td>
<td>7.53</td>
<td>4.76</td>
<td>0.987</td>
<td>5.85</td>
</tr>
<tr>
<td>3</td>
<td>0.963</td>
<td>4.45</td>
<td>2.98</td>
<td>1.58</td>
<td>1.26</td>
<td>9.70</td>
<td>8.18</td>
<td>7.54</td>
<td>4.74</td>
<td>0.963</td>
<td>6.02</td>
</tr>
<tr>
<td>4</td>
<td>0.948</td>
<td>4.47</td>
<td>3.00</td>
<td>1.59</td>
<td>1.27</td>
<td>9.76</td>
<td>8.21</td>
<td>7.59</td>
<td>4.75</td>
<td>0.947</td>
<td>6.15</td>
</tr>
<tr>
<td>5</td>
<td>0.917</td>
<td>4.23</td>
<td>2.78</td>
<td>1.46</td>
<td>1.18</td>
<td>9.02</td>
<td>7.49</td>
<td>6.98</td>
<td>4.24</td>
<td>0.916</td>
<td>5.95</td>
</tr>
<tr>
<td>6</td>
<td>0.907</td>
<td>4.14</td>
<td>2.71</td>
<td>1.41</td>
<td>1.14</td>
<td>8.61</td>
<td>7.14</td>
<td>6.65</td>
<td>4.05</td>
<td>0.907</td>
<td>5.86</td>
</tr>
<tr>
<td>7</td>
<td>0.889</td>
<td>4.02</td>
<td>2.61</td>
<td>1.34</td>
<td>1.08</td>
<td>8.11</td>
<td>6.72</td>
<td>6.22</td>
<td>3.79</td>
<td>0.888</td>
<td>5.78</td>
</tr>
<tr>
<td>8</td>
<td>0.859</td>
<td>3.80</td>
<td>2.43</td>
<td>1.23</td>
<td>0.98</td>
<td>7.31</td>
<td>6.05</td>
<td>5.57</td>
<td>3.40</td>
<td>0.858</td>
<td>5.60</td>
</tr>
<tr>
<td>9</td>
<td>0.838</td>
<td>3.66</td>
<td>2.32</td>
<td>1.15</td>
<td>0.91</td>
<td>6.79</td>
<td>5.61</td>
<td>5.13</td>
<td>3.15</td>
<td>0.837</td>
<td>5.49</td>
</tr>
</tbody>
</table>

Table 6. The voltage profile of Case 1
### Voltages in harmonic order

<table>
<thead>
<tr>
<th>Bus</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>V&lt;sub&gt;ms&lt;/sub&gt;</th>
<th>HDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>x10&lt;sup&gt;-2&lt;/sup&gt;</td>
<td>x10&lt;sup&gt;-3&lt;/sup&gt;</td>
<td>x10&lt;sup&gt;-4&lt;/sup&gt;</td>
<td>x10&lt;sup&gt;-5&lt;/sup&gt;</td>
<td>x10&lt;sup&gt;-6&lt;/sup&gt;</td>
<td>x10&lt;sup&gt;-7&lt;/sup&gt;</td>
<td>x10&lt;sup&gt;-8&lt;/sup&gt;</td>
<td>x10&lt;sup&gt;-9&lt;/sup&gt;</td>
<td>x10&lt;sup&gt;-10&lt;/sup&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.997</td>
<td>1.190</td>
<td>5.86</td>
<td>1.93</td>
<td>1.22</td>
<td>7.45</td>
<td>6.27</td>
<td>4.49</td>
<td>3.33</td>
<td>0.999</td>
<td>1.40</td>
</tr>
<tr>
<td>2</td>
<td>0.999</td>
<td>1.190</td>
<td>5.90</td>
<td>1.94</td>
<td>1.23</td>
<td>7.51</td>
<td>6.32</td>
<td>4.53</td>
<td>3.56</td>
<td>0.988</td>
<td>1.40</td>
</tr>
<tr>
<td>3</td>
<td>0.988</td>
<td>1.130</td>
<td>5.34</td>
<td>1.62</td>
<td>0.99</td>
<td>5.51</td>
<td>4.37</td>
<td>2.94</td>
<td>2.05</td>
<td>0.980</td>
<td>1.32</td>
</tr>
<tr>
<td>4</td>
<td>0.980</td>
<td>1.100</td>
<td>5.02</td>
<td>1.44</td>
<td>0.85</td>
<td>4.36</td>
<td>3.23</td>
<td>2.05</td>
<td>1.29</td>
<td>0.980</td>
<td>1.26</td>
</tr>
<tr>
<td>5</td>
<td>0.962</td>
<td>0.887</td>
<td>3.42</td>
<td>0.81</td>
<td>0.52</td>
<td>2.29</td>
<td>1.33</td>
<td>0.96</td>
<td>0.29</td>
<td>0.962</td>
<td>1.02</td>
</tr>
<tr>
<td>6</td>
<td>0.954</td>
<td>0.861</td>
<td>3.28</td>
<td>0.79</td>
<td>0.51</td>
<td>2.12</td>
<td>1.24</td>
<td>1.12</td>
<td>0.49</td>
<td>0.954</td>
<td>0.99</td>
</tr>
<tr>
<td>7</td>
<td>0.939</td>
<td>0.827</td>
<td>3.10</td>
<td>0.73</td>
<td>0.46</td>
<td>1.90</td>
<td>1.12</td>
<td>0.97</td>
<td>0.44</td>
<td>0.939</td>
<td>0.95</td>
</tr>
<tr>
<td>8</td>
<td>0.915</td>
<td>0.751</td>
<td>2.72</td>
<td>0.60</td>
<td>0.36</td>
<td>1.45</td>
<td>0.89</td>
<td>0.68</td>
<td>0.34</td>
<td>0.915</td>
<td>0.89</td>
</tr>
<tr>
<td>9</td>
<td>0.900</td>
<td>0.682</td>
<td>2.37</td>
<td>0.47</td>
<td>0.25</td>
<td>1.04</td>
<td>0.69</td>
<td>0.39</td>
<td>0.25</td>
<td>0.901</td>
<td>0.82</td>
</tr>
</tbody>
</table>

**Table 7.** The voltage profile of Case 2

### Voltages in harmonic order

<table>
<thead>
<tr>
<th>Bus</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>V&lt;sub&gt;ms&lt;/sub&gt;</th>
<th>HDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>x10&lt;sup&gt;-2&lt;/sup&gt;</td>
<td>x10&lt;sup&gt;-3&lt;/sup&gt;</td>
<td>x10&lt;sup&gt;-4&lt;/sup&gt;</td>
<td>x10&lt;sup&gt;-5&lt;/sup&gt;</td>
<td>x10&lt;sup&gt;-6&lt;/sup&gt;</td>
<td>x10&lt;sup&gt;-7&lt;/sup&gt;</td>
<td>x10&lt;sup&gt;-8&lt;/sup&gt;</td>
<td>x10&lt;sup&gt;-9&lt;/sup&gt;</td>
<td>x10&lt;sup&gt;-10&lt;/sup&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.998</td>
<td>1.05</td>
<td>5.08</td>
<td>1.64</td>
<td>1.03</td>
<td>6.41</td>
<td>5.45</td>
<td>3.95</td>
<td>2.98</td>
<td>0.998</td>
<td>1.20</td>
</tr>
<tr>
<td>2</td>
<td>1.000</td>
<td>1.06</td>
<td>5.11</td>
<td>1.65</td>
<td>1.04</td>
<td>6.46</td>
<td>5.50</td>
<td>3.98</td>
<td>3.00</td>
<td>1.000</td>
<td>1.19</td>
</tr>
<tr>
<td>3</td>
<td>0.991</td>
<td>0.99</td>
<td>4.54</td>
<td>1.33</td>
<td>0.80</td>
<td>4.42</td>
<td>3.53</td>
<td>2.38</td>
<td>1.69</td>
<td>0.991</td>
<td>1.11</td>
</tr>
<tr>
<td>4</td>
<td>0.983</td>
<td>0.95</td>
<td>4.20</td>
<td>1.14</td>
<td>0.66</td>
<td>3.25</td>
<td>2.36</td>
<td>1.47</td>
<td>0.91</td>
<td>0.983</td>
<td>1.07</td>
</tr>
<tr>
<td>5</td>
<td>0.963</td>
<td>0.81</td>
<td>3.08</td>
<td>0.75</td>
<td>0.52</td>
<td>2.35</td>
<td>1.36</td>
<td>1.05</td>
<td>0.28</td>
<td>0.963</td>
<td>0.90</td>
</tr>
<tr>
<td>6</td>
<td>0.955</td>
<td>0.79</td>
<td>2.96</td>
<td>0.74</td>
<td>0.50</td>
<td>2.18</td>
<td>1.26</td>
<td>1.20</td>
<td>0.49</td>
<td>0.955</td>
<td>0.89</td>
</tr>
<tr>
<td>7</td>
<td>0.944</td>
<td>0.76</td>
<td>2.81</td>
<td>0.68</td>
<td>0.45</td>
<td>1.95</td>
<td>1.14</td>
<td>1.04</td>
<td>0.44</td>
<td>0.940</td>
<td>0.86</td>
</tr>
<tr>
<td>8</td>
<td>0.917</td>
<td>0.69</td>
<td>2.48</td>
<td>0.57</td>
<td>0.35</td>
<td>1.49</td>
<td>0.90</td>
<td>0.73</td>
<td>0.34</td>
<td>0.917</td>
<td>0.80</td>
</tr>
<tr>
<td>9</td>
<td>0.902</td>
<td>0.63</td>
<td>2.18</td>
<td>0.45</td>
<td>0.25</td>
<td>1.05</td>
<td>0.69</td>
<td>0.40</td>
<td>0.25</td>
<td>0.902</td>
<td>0.74</td>
</tr>
</tbody>
</table>

**Table 8.** The voltage profile of Case 3

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum voltage (p.u.)</td>
<td>0.999</td>
<td>0.999</td>
<td>1.000</td>
</tr>
<tr>
<td>Minimum voltage (p.u.)</td>
<td>0.837</td>
<td>0.901</td>
<td>0.902</td>
</tr>
<tr>
<td>Total power loss (p.u.)</td>
<td>0.007812</td>
<td>0.007065</td>
<td>0.007036</td>
</tr>
<tr>
<td>Q&lt;sub&gt;(4)&lt;/sub&gt; (p.u.)</td>
<td>0.024</td>
<td>0.036</td>
<td></td>
</tr>
<tr>
<td>Q&lt;sub&gt;(5)&lt;/sub&gt; (p.u.)</td>
<td>0.024</td>
<td>0.018</td>
<td></td>
</tr>
<tr>
<td>Q&lt;sub&gt;(9)&lt;/sub&gt; (p.u.)</td>
<td>0.009</td>
<td>0.009</td>
<td></td>
</tr>
<tr>
<td>Cost ($ / year)</td>
<td>131238</td>
<td>128494</td>
<td>129334</td>
</tr>
<tr>
<td>Average CPU Time (sec.)</td>
<td>0.8</td>
<td>1.20</td>
<td>3.39</td>
</tr>
<tr>
<td>Maximum HDF (%)</td>
<td>6.15</td>
<td>1.40</td>
<td>1.20</td>
</tr>
</tbody>
</table>

**Table 9.** Summary results of the approach
7. Conclusion

This chapter presents a Particle Swarm Optimisation (PSO) approach to searching for optimal shunt SVC location and size with harmonic consideration. The cost or fitness function is constrained by voltage and Harmonic Distortion Factor (HDF). Since PSO is a stochastic approach, performances should be evaluated using statistical value. The performance will be affected by initial condition but PSO can give the optimal solution by increasing the population size. PSO offers robustness by searching for the best solution from a population point of view and avoiding derivatives and using payoff information (objective function). The result shows that PSO method is suitable for discrete value optimization problem such as SVC allocation and the consideration of harmonic distortion limit may be included with an integrated approach in the PSO.

Nomenclature

\[ f_{\text{max}} \]  the maximum fitness of each generation in the population
\[ N \]  the number of harmonic order is being considered
\[ Q_i \]  the size of SVC (MVAr)
\[ K_c \]  the equivalent SVC cost ($/MVAr)
\[ K_l \]  the duration of the load period
\[ K_p \]  the equivalent annual cost per unit of power losses ($/kW)
\[ K_s \]  the SVC bank size (MVAr)
\[ y_{ci} \]  frequency admittance of the SVC at bus \( i \) (pu)
\[ V_i \]  voltage magnitude at bus \( i \) (pu)
\[ P_i, Q_i \]  active and reactive powers injected into network at bus \( i \) (pu)
\[ P_{li}, Q_{li} \]  linear active and reactive load at bus \( i \) (pu)
\[ P_{ni}, Q_{ni} \]  nonlinear active and reactive load at bus \( i \) (pu)
\[ \theta_{ij} \]  voltage angle different between bus \( i \) and bus \( j \) (rad)
\[ G_{ii}, B_{ii} \]  self conductance and susceptance of bus \( i \) (pu)
\[ G_{ij}, B_{ij} \]  mutual conductance and susceptance between bus \( i \) and bus \( j \) (pu)

Superscript

\( I \)  corresponds to the fundamental frequency value
\( n \)  corresponds to the \( n \)th harmonic order value

Author details

H.C. Leung and Dylan D.C. Lu
Department of Electrical and Information Engineering,
The University of Sydney, NSW 2006,
Australia
8. References


