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Chapter 16

Some Remarks on Modelling of Mass Transfer Kinetics During Rehydration of Dried Fruits and Vegetables

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Additional information is available at the end of the chapter

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1. Introduction

Dehydration operations are important steps in the food processing industry. The basic objective in drying food products is the removal of water in the solids up to a certain level, at which microbial spoilage is minimized. The wide variety of dehydrated foods, which today are available to the consumer (dried fruits, dry mixes and soups, etc.) and the interesting concern for meeting quality specifications, emphasize the need for a thorough understanding of the operation [1].

Dehydrated products can be used in many processed or ready-to-eat foods in place of fresh foods due to several advantages such as convenience in transportation, storage, preparation and use. Dehydrated products need to be rehydrated before consumption or further processing [2]. Rehydration is a process of moistening dry material [3]. Rehydration is usually carried out by soaking the dry material in large amounts of water, although, instead of this, some authors have used air with high relative humidity, either statically or in a drying chamber with air circulation [4].

Three main processes take place simultaneously during rehydration: the imbibition of water into the dried material and the swelling and the leaching of solubles [5]. It is a very complex phenomenon that involves different physical mechanisms such as water imbibition, internal diffusion, convection at the surface and within large open pores, and relaxation of the solid...
Capillary imbibition is very important during the early stages, leading to an almost instantaneous uptake of water. Tension effects between the liquid and the solid matrix may also be relevant [6]. In the rehydration process, two main crosscurrent mass fluxes are involved, a water flux from the rehydrating solution to the product, and a flux of solutes (sugars, acids, minerals, vitamins) from the food product to the solution, and the kinetics depends on the immersion medium [2,6,7].

Rehydration is influenced by several factors, grouped as intrinsic factors (product chemical composition, pre-drying treatment, drying techniques and conditions, post-drying procedure, etc.) and extrinsic factors (composition of immersion media, temperature, hydrodynamic conditions) [8]. Some of these factors induce changes in the structure and composition of the plant tissue, which results in the impairment of the reconstitution properties [9]. Therefore equilibrium moisture content at saturation does not reach the moisture content of the raw materials prior to dehydration, indicating that the dehydration procedure is irreversible [1]. Physical and chemical changes that take place during drying affect the quality of the dehydrated product, and by a simple addition of water, the properties of the raw material cannot be restored [10]. Rehydration cannot be simply treated as the reverse process to dehydration [11]. Hence, rehydration can be considered as a measure of the injuries to the material caused by drying and treatments preceding dehydration [12].

Rehydration characteristics are therefore employed as a parameter to determine quality [5]. Optimal reconstitution can be achieved by controlling the drying process and adjustment of the rehydration conditions [13]. The knowledge of the rehydration kinetics of dried products is important to optimise processes from a quality viewpoint since rehydration is a key quality aspect for those dried products that have to be reconstituted before their consumption [14]. The most important aspect of rehydration technology is the mathematical modelling of the rehydration process. Its purpose is to allow design engineers to choose the most suitable operating conditions. The principle of modelling is based on having a set of mathematical equations that can adequately describe the system. The solution of these equations must allow prediction of the process parameters as a function of time. Therefore the use of a simulation model is a valuable tool for prediction of performance of rehydration systems [15].

Many models have been investigated to predict migration of water in foods and, for example, relate moisture content to time. The models are theoretical, empirical, semi-empirical, exponential, and non-exponential ones, and despite the widespread use of computers and their associated softwares, empirical equations are still widely used in view of their simplicity and ease of computations [16]. Theoretical models, however, are based on the general theory of mass and heat transfer laws. They take into account fundamentals of the rehydration process and their parameters have physical meaning. Therefore, theoretical models can give an explanation of the phenomena occurring during rehydration. On the other hand however these models are more difficult in application compared to other mentioned models [15].

The aim of the present chapter was to discuss the suitability of rehydration models and indices for describing mass transfer kinetics during rehydration of dried fruits and vegetables.
2. Mathematical description of rehydration process

2.1. Rehydration indices

There is a large number of research reports in which authors measure the ability of dry material to rehydrate. Results of experiments are expressed in variable ways and quite often the same index is differently named. The most common index used to express rehydration of dry plant tissue is rehydration ratio defined as follows:

\[
\text{Rehydration ratio (RR)} = \frac{\text{mass after rehydration}}{\text{mass before rehydration}}
\]

(1)

Rehydration ratio was used to express the rehydration of the dried products such as carrots [17], mushrooms [18], pears [19], potatoes [20], and coriander leaves [21]. RR is sometimes named rehydration capacity [22,23]. To facilitate a mathematical description of the rehydration phenomenon, the experimental reconstitution data were correlated with time according to a second order polynomial relation [20,24]:

\[
\text{RR} = at^2 + bt + c
\]

(2)

During rehydration together with water acquisition, soluble compounds can be leached. Observed increase in mass is a net result of those processes, and practically gives no information about the amount of absorbed water or the mass of lost solubles. Lewicki in [12] proposed three indices to estimate the rehydration characteristics of dried food. These are the water absorption capacity (WAC), the dry matter holding capacity (DHC), and the rehydration ability (RA).

The water absorption capacity gives information about the ability of the material to absorb water with respect to the water loss during dehydration and varies in the range 0≤WAC≤1. The more the water absorption capacity is lost during dehydration the smaller the index. Water absorption capacity is defined by:

\[
\text{WAC} = \frac{\text{mass of water absorbed during rehydration}}{\text{mass of water removed during drying}}
\]

(3)

and can be calculated from the formula:

\[
\text{WAC} = \frac{m_r(1-s_r) - m_d(1-s_d)}{m_r(1-s_r) - m_d(1-s_d)}
\]

(4)

The dry matter holding capacity is a measurement of the ability of the material to retain soluble solids after rehydration and provides information on the extent of tissue damage and its
permeability to solutes. The more the tissue is damaged the smaller the index. The dry matter holding capacity varies in the range $0 \leq \text{DHC} \leq 1$. The index is calculated by:

$$\text{DHC} = \frac{m_s s_t}{m_p s_d}$$  \hspace{1cm} (5)

The rehydration ability measures the ability of the dried product to rehydrate and shows the total damage of the tissue by drying and soaking processes. The index varies in the range $0 \leq \text{RA} \leq 1$. The more the tissue is damaged the smaller is the index. The rehydration ability is given by:

$$\text{RA} = \text{WAC} \cdot \text{DHC}$$  \hspace{1cm} (6)

The indices proposed by Lewicki in [12] were employed to express the rehydration of the dried products such as apples [25,26], *Boletus edulis* mushrooms [4], chestnuts [5], and *Morchella esculenta* (morel) [14].

### 2.2. Mathematical models

The analysis of the rehydration kinetics can be very useful for optimizing process condition. Many theoretical and empirical approaches have been employed and in some cases empirical models were preferred because of their relative ease of use.

#### 2.2.1. Empirical and semi-empirical models

Among the empirical models, the one proposed by Peleg in [27] is a two parameter, non-exponential equation to describe water transport from the surface to the interior of the solids. The model proposed by Peleg in [27] is as follows:

$$M = M_0 \pm \frac{t}{A_1 + A_2 t}$$  \hspace{1cm} (7)

where $A_1$ is the Peleg rate constant (s) and $A_2$ is the Peleg capacity constant.

In Eq. (7) “±” becomes “+” if the process is absorption or adsorption and “−” if the process is drying or desorption.

The rate of sorption can be obtained from the first derivative of the Peleg equation:

$$\frac{dM}{dt} = \pm \frac{A_1}{(A_1 + A_2 t)^2}$$  \hspace{1cm} (8)
and at the very beginning (t=0):

\[
\frac{\text{d}M}{\text{d}t} \bigg|_{t=0} = \pm \frac{1}{A_1}
\]

(9)

If time of the process is long enough (t→∞), the equilibrium moisture content can be calculated by:

\[
M_e = M_0 \pm \frac{1}{A_2}
\]

(10)

Linearization of Eq. (7) gives:

\[
\frac{t}{M-M_0} = \pm (A_1 + A_2 t)
\]

(11)

allowing for the determination of \(A_1\) and \(A_2\) values by linear regression of experimental data.

Some of the authors correlated \(A_1\) value by means of exponential equation according to an Arrhenius type relationship [4,5]:

\[
A_1 = A_0 \exp(-\frac{E_A}{RT})
\]

(12)

where \(A_0\) is the constant.

The Peleg [27] model has been widely used due to its simplicity, and has been reported to adequately describe the rehydration of various dried products such as apples [28], bambara [29], candied mango [7], carrots [30], chickpea [31], red kidney beans [32], and wheat [33]. Bilbao-Sáinz et al. in [34] applied Peleg model to volume recovery data assuming the following form of equation:

\[
V = V_0 + \frac{t}{A_1 + A_2 t}
\]

(13)

Marques et al. in [35] modified the Peleg model obtaining the following form of equation:

\[
\frac{m(t)}{m_d} = \left[ -\frac{m}{m_d} \right]_{t \to \infty} \left[ -\frac{1}{A_2} \right] + \frac{t}{A_1 + A_2 t}
\]

(14)
and applied it for modelling of dried tropical fruits rehydration.

Pilosof et al. in [36] proposed empirical, two parameter, non-exponential equation to describe kinetics of water uptake to food powders. The Pilosof-Boquet-Batholomai model [36] is as follows:

$$M = M_0 + \frac{A_3 t}{A_4 + t}$$

(15)

where $A_3$ and $A_4$ are constants.

If time of the process is long enough ($t \to \infty$), the equilibrium moisture content can be calculated by:

$$M_e = M_0 + A_4$$

(16)

Linearization of Eq. (15) gives:

$$\frac{t}{M - M_0} = \frac{1}{A_3} t + \frac{A_4}{A_3}$$

(17)

It can be deducted from Eq. (11) (for absorption or adsorption) and (17) that $A_1 = A_4 / A_3$ and $A_2 = 1 / A_3$.

The Pilosof-Boquet-Batholomai model [36] has been used by Sopade et al. in [16] for describing water absorption of wheat starch, whey protein concentrate, and whey protein isolate.

Singh and Kulshrestha in [37] proposed empirical, two parameter, non-exponential equation to describe kinetics of water sorption by soybean and pigeonpea grains. The model developed by Singh and Kulshrestha [37] is as follows:

$$M = M_0 + \frac{A_5 A_6 t}{A_5 t + 1}$$

(18)

where $A_5$ and $A_6$ are constants.

If time of the process is long enough ($t \to \infty$), the equilibrium moisture content can be calculated by:

$$M_e = M_0 + A_5$$

(19)

Linearization of Eq. (18) gives:
It can be deduced from Eq. (11) (for absorption or adsorption), (17) and (20) that \( A_1 = A_2 / A_3 = 1 / (A_5 A_6) \), \( A_2 = 1 / A_3 = 1 / A_5 \) and therefore \( A_3 = A_5 \) and \( A_4 = 1 / A_6 \).

The Singh-Kulshrestha [37] model has been used in [16] for describing water absorption of wheat starch, whey protein concentrate, and whey protein isolate.

Wesołowski in [38] developed the following empirical, three parameter, exponential equation to describe rehydration of apples:

\[
\frac{m(t)}{m_d} = A(B - e^{-C t})
\]  

(21)

where \( A \), \( B \), and \( C \) are constants.

For a long enough time, equilibrium value is given by:

\[
\left( \frac{m(t)}{m_d} \right)_{t \to \infty} = A \cdot B
\]

(22)

Equation (21) was also verified when mass has been replaced with moisture content [38,39].

The model proposed by Witrowa-Rajchert in [40] is as follows:

\[
\frac{m(t)}{m_d} = A + B \left( 1 - \frac{1}{1 + B C t} \right)
\]

(23)

where \( A \), \( B \), and \( C \) are constants.

It is an empirical, three parameter, non-exponential model. For a long enough time, equilibrium value is given by:

\[
\left( \frac{m(t)}{m_d} \right)_{t \to \infty} = A + B
\]

(24)

Equation (23) was also verified for the increase of moisture content and volume. Discussed model has been applied for describing the rehydration of apples, carrots, parsleys, potatoes, and pumpkins [40,41].
The probabilistic Weibull model was described first by Dr. Walodi Weibull to represent the distribution of the breaking strength of materials and later to describe the behaviour of systems or events that have some degree of variability [14]. For drying and rehydration processes a two parameter, exponential equation based on the Weibull model is as follows:

\[
\frac{M - M_0}{M_e - M_0} = 1 - \exp \left[ - \left( \frac{t}{\alpha} \right)^{\beta} \right]
\] (25)

where \( \alpha \) is the scale parameter (s) and \( \beta \) is the dimensionless shape parameter. The scale parameter \( \alpha \) is a kinetic coefficient. It defines the rate of the moisture uptake process and represents the time needed to accomplish approximately 63% of the moisture uptake process. Different values of \( \alpha \) lead to a very different curves: for instance, the higher its value, the slower the process at short times. The shape parameter is a behaviour index, which depends on the process mechanism [42]. Although the Weibull model is empirical one, it was demonstrated recently that the Weibull distrubution has a solid theoretical basis, stemming from physical principles [6].

The Weibull model was found to yield good results in the description of rehydration of a variety of dried foods such as \textit{Boletus edulis} mushrooms [4], \textit{Morchella esculenta} (morel) [14], oranges [43], and ready-to-eat breakfast cereal [42]. Cunha et al. in [44] correlated the scale parameter \( \alpha \) value by means of exponential equation according to an Arrhenius type relationship (Eq. (12)).

Marques et al. in [35] modified the Weibull model obtaining the following form of equation:

\[
\frac{m(t)}{m_d} = \left( \frac{m(t)}{m_d} \right)_{t \to \infty} + \left[ 1 - \left( \frac{m(t)}{m_d} \right)_{t \to \infty} \right] \exp \left[ - \left( \frac{t}{\alpha} \right)^{\beta} \right]
\] (26)

and applied it for modelling of dried tropical fruits rehydration.

Marabi et al. in [6] modified the Weibull model obtaining the following form of equation:

\[
\frac{M - M_0}{M_e - M_0} = 1 - \exp \left[ - \left( \frac{t}{\alpha'} \right)^{\beta} \right]
\] (27)

and

\[
\alpha' = \frac{I_2^2}{D_{\text{calc}}}
\] (28)
where $R_g$ is the constant and is a characteristic of the geometry utilized. Marabi et al. in [6] applied Eq. (27) for modelling rehydration of carrots.

The rate of rehydration can be obtained from the semi-empirical first order kinetic model [45]:

$$\frac{dM}{dt} = -k(M - M_e)$$

(30)

At zero time $M$ is equal $M_0$, the moisture content of the dry material, and Eq. (30) is integrated to give the following expression:

$$\frac{M - M_e}{M_0 - M_e} = \exp(-kt)$$

(31)

The Arrhenius equation (Eq. (12)) can be employed to describe the temperature dependence of rehydration rate constant $k$ [45-47].

The first order kinetic model has been reported to adequately describe the rehydration of various dried products such as apples, potatoes, carrots, bananas, pepper, garlic, mushrooms, onion, leeks, peas, corn, pumpkins, and tomatoes [1], chickpeas [46], soybeans [47], and tamarind seeds [45].

Misra and Brooker in [48] developed the following empirical, exponential model

$$\frac{M - M_e}{M_0 - M_e} = \exp(-kt^n)$$

(32)

(where $n$ is constant) and applied it for modelling the rewetting of dried corn. Equation (32) has been successfully used by Shatadal et al. in [49] to describe the rewetting of dried canola.

Mizuma et al. in [50] modified the first order kinetic model obtaining two form of equations:

$$\frac{dx}{dt} = k(1-x)^n$$

(33)

and

$$\frac{dx}{dt} = k(1-x)^n(x + a)$$

(34)
Studies revealed that rehydration is a multifaceted mass transfer process, and uptake is governed by several mechanisms of liquid imbibition in porous media. Rehydration of dried plant tissues is a very complex phenomenon involving different transport mechanisms, including molecular diffusion, convection, hydraulic flow, and capillary flow. One or more mechanisms may occur simultaneously during water or other medium imbibition into a dry food sample [51].

Theoretical models take into account the process basic physical principles. Physically based modelling requires in depth process understanding. As evaluation of some physical properties and complex process interrelationships are very difficult to quantify, the efficiency of these models is typically limited to approximations [51].

Theoretical models describing water absorption in foods are mostly based on the diffusion of water through a porous medium, therefore they assume that liquid water sorption by plant tissue is a diffusion controlled process. If water transport is assumed to take place by diffusion, then the process of rehydration can be described using Fick’s second law:

$$\frac{\partial M}{\partial t} = \nabla (D \nabla M)$$

(36)

In order to solve the differential equation (36) the following simplifying assumptions were adopted mostly in the literature:

• the initial moisture content in the solid is uniform (the initial condition):

$$M_{i,0} = M_0$$

(37)

• the water diffusion coefficient is constant,

• moisture gradient at the centre of the solid is zero,

• the sample geometry remains constant during the rehydration process,

• external resistance to heat and mass transfer is negligible, i.e. the sample surface attains saturation (equilibrium) moisture content instantaneously upon immersion in absorption media (the boundary condition of the first kind):
heat transfer is more rapid than mass transfer, so that the process can be assumed isothermal.

Biological materials before drying are cut into small pieces, mostly slices or cubes. They can also be spherical in shape. Therefore Eq. (36) applied to the description of the rehydration of dried material (with the simplifying assumptions mentioned above) takes the following form:

for an infinite plane (slices):

$$\frac{\partial M}{\partial t} = D \frac{\partial^2 M}{\partial x^2}$$  \hspace{1cm} (39)

for a finite cylinder (slices):

$$\frac{\partial M}{\partial t} = D \left( \frac{\partial^2 M}{\partial r^2} + \frac{1}{r} \frac{\partial M}{\partial r} + \frac{\partial^2 M}{\partial z^2} \right)$$

$$\left( t > 0; 0 < r < R_c; -h < z < +h \right)$$  \hspace{1cm} (40)

for a cube:

$$\frac{\partial M}{\partial t} = D \left( \frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} + \frac{\partial^2 M}{\partial z^2} \right)$$

$$\left( t > 0; -R_c < x < +R_c; -R_c < y < +R_c; -R_c < z < +R_c \right)$$  \hspace{1cm} (41)

for a sphere:

$$\frac{\partial M}{\partial t} = D \left( \frac{\partial^2 M}{\partial r^2} + \frac{2}{r} \frac{\partial M}{\partial r} \right)$$

$$\left( t > 0; 0 < r < R_c \right)$$  \hspace{1cm} (42)

(t>0; 0<r<R_c) The initial conditions (Eq. (37)) are following:

for an infinite plane

$$M(x,0) = M_0 = \text{const}$$  \hspace{1cm} (43)
for a finite cylinder

\[ M(r, z, 0) = M_0 = \text{const} \quad (44) \]

for a cube

\[ M(x, y, z, 0) = M_0 = \text{const} \quad (45) \]

for a sphere

\[ M(r, 0) = M_0 = \text{const} \quad (46) \]

The boundary conditions of the first kind (Eq. (38)) take the following form:

for an infinite plane

\[ M(\pm R, t) = M_{\infty} = \text{const} \quad (47) \]

for a finite cylinder

\[ M(R_c, z, t) = M_{\infty} = \text{const} \quad (48) \]

\[ \frac{\partial M(0, z, t)}{\partial r} = 0, \; M(0, z, t) \neq \infty \quad (49) \]

\[ M(r, h, t) = M_{\infty} = \text{const} \quad (50) \]

\[ \frac{\partial M(r, 0, t)}{\partial z} = 0 \quad (51) \]

for a cube

\[ M(\pm R_c, y, z, t) = M_{\infty} = \text{const} \quad (52) \]

\[ M(x, \pm R_c, z, t) = M_{\infty} = \text{const} \quad (53) \]

\[ M(x, y, \pm R_c, t) = M_{\infty} = \text{const} \quad (54) \]

for a sphere
An analytical solution of: (i) Eq. (39) at the initial and boundary conditions given by Eqs. (43) and (47), (ii) Eq. (40) at the initial and boundary conditions given by Eqs. (44) and (48)-(51), (iii) Eq. (41) at the initial and boundary conditions given by Eqs. (45) and (52)-(54), and (iv) Eq. (42) at the initial and boundary conditions given by Eqs. (46) and (55)-(56) with respect to mean moisture content as a function of time, take the following form [52]:

for an infinite plane

\[
\frac{M(t) - M_c}{M_0 - M_c} = \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \exp \left[ -\frac{\pi^2(2n+1)^2}{4} \frac{Dt}{R_c^2} \right] \]

(57)

for a finite cylinder

\[
\frac{M(t) - M_c}{M_0 - M_c} = \frac{32}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{\mu_n^2} \sum_{m=0}^{\infty} \frac{1}{(2m+1)^2} \exp \left[ -\frac{\mu_n^2Dt}{R_c^2} \right] \sum_{p=0}^{\infty} \frac{1}{(2p+1)^2} \exp \left[ -\frac{\pi^2(2m+1)^2}{4} \frac{Dt}{h^2} \right] \]

(58)

where \( \mu_n \) are the roots of the Bessel equation of the first kind of zero order

\[
I_0(\mu_n) = 0
\]

(59)

for a cube

\[
\frac{M(t) - M_c}{M_0 - M_c} = \frac{512}{\pi^2} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{p=0}^{\infty} \frac{1}{(2n+1)(2m+1)(2p+1)} \exp \left[ -\frac{\pi^2}{4} \frac{(2n+1)^2 + (2m+1)^2 + (2p+1)^2}{R_c^2} \frac{Dt}{R_c^2} \right] \]

(60)

for a sphere

\[
\frac{M(t) - M_c}{M_0 - M_c} = \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp \left[ -\frac{n^2\pi^2Dt}{R_c^2} \right] \]

(61)

In order to take into account the necessary number of terms of the series, thirty terms are routinely used in the calculations [4,14], although Sanjuán et al. [11] stated that taking 6-7 terms
can be enough. The moisture diffusion coefficient $D$, also termed effective diffusivity, is an apparent value that comprises all the factors involved in the process. This coefficient is often assumed to be temperature-dependent according to an Arrhenius type relationship (Eq. (12)) [11, 53, 54]. Fick’s equation was also solved considering that the effective diffusivity is moisture-dependent [4]. In that case, the diffusion model cannot be solved analytically. The finite element method (FEM) was used in order to identify the parameters. The relationship between the moisture diffusion coefficient and the moisture content considered was:

$$D = \exp(a + bM)$$  \hspace{1cm} (62)

where $a$ and $b$ are the parameters.

Equation (57) has been reported to adequately describe the rehydration of slices of various dried products such as *Boletus edulis* mushrooms [4], broccoli stems [11], carrots [2, 8], and *Morchella esculenta* (morel) [14]. Bilbao-Sáinz et al. in [34] stated that Fick’s equation of diffusion (Eq. (58)) was not suitable to model the sorption data of apples (var. Granny Smith). Equation (61) was found to yield good results in the description of rehydration of dried amaranth grains [53], dried date palm fruits [54], and dried soybeans [47].

The mathematical model of rehydration developed by Górnicki in [55] is based on the general theory of mass and heat transfer laws. The model assumes that mass transfer in plant tissue is a diffusion controlled process. The model allows for determination of temperature distribution and concentration distribution of both dry matter and water in time and space inside rehydrated material. The developed model takes into account changeable boundary conditions and changes of material geometry. Six methods of determination of mass transfer coefficients were proposed. The proposed model has been reported to adequately describe the rehydration of slices and cubes both parsley and apples (var. Idared).

Few researches recently embarked on a new approach, which is motivated by the recognition that rehydration of dry food particulates could not be explained and/or modeled solely by a Fickian mechanism. Mechanisms, such as water imbibition, capillarity and flow in porous media, were suggested and are considered relevant for describing the ingress of water into the dried food particulates [51].

Lee et al. in [56] described the rehydration process of freeze-dried fruits (avocado, kiwi fruit, apple, banana, and potato) based on capillary movement of water in the fruit samples. The movement of water through the dried material was assumed to follow capillary motion as described by the Lucas-Washburn equation. The following assumptions were made: (i) the food structure may be simplified as to consist of multi-individual pores, (ii) one dimensional flow, (iii) steady state flow, (iv) fully developed flow, (v) Newtonian fluid with negligible inertia effects, and the following equation was obtained:

$$\frac{dh(t)}{dt} = \frac{k_1}{h(t)} - k_2$$  \hspace{1cm} (63)
where

\[ k_1 = \frac{\gamma \cos \Theta}{4\eta} \text{ and } k_2 = \frac{r^2 \rho g}{8\eta} \] (64)

Lee et al. in [56] stated that the parameter \( k_1 \) (m/s) will be the dominant factor of the initial rate of rehydration while the parameter \( k_2 \) (m s^{-1}) will become significant during the final state as rehydration approaches equilibrium.

Other researches started to apply the capillary imbibition theory to model the rehydration of foods. Weerts et al. in [57-59] utilized a capillary flow approach to model the temperature and anisotropy effects during the rehydration of tea leafs. Saguy et al. in [60] studied the kinetics of water uptake of freeze-dried carrots and stated that water imbibition followed the general Lucas-Washburn equation. Utilizing different liquid media highlighted, however, the need for model improvement overcoming several discrepancies mainly related to the utilization of a simple “effective” cylindrical capillary and a constant contact angle.

Consequently, due to the complexity of water transport into porous media, the need for further research necessary for the development of the theory and model for the application of capillary imbibition is emphasized [60]. Saguy et al. in [51] elaborated a list of recommended future studies in this field.

### 3. Discussion of some results of modelling of mass transfer kinetics during rehydration of dried apple cubes

The authors’ own results of research are presented in this chapter.

Ligol variety apples used in this study were acquired in local market. The apples were washed in running tap water, hand peeled and the seeds were removed, and then cut into 10 mm cubes thickness using specially cutting machine. Samples were dried on the same day. The fluidized bed drying was carried out using the laboratory dryer constructed in the Department of Fundamental Engineering, Faculty of Production Engineering, Warsaw University of Life Sciences, Warsaw, Poland. The drying chamber consists of a column, which is a Plexiglas cylinder of 12 cm in diameter and 180 cm in height. Drying conditions were 60°C of temperature and 6 m s^{-1} of air velocity. Prior to placing the sample in the drying chamber, the system was run for about one hour to obtain steady conditions. Once the air temperature and fluidization velocity had stabilized, the sample was put into the fluidized bed dryer and the drying begins. Drying was continued until there was no weight change. Experiments were replicated three times. Dried apple cubes were stored in airtight glass containers after dehydration until they were used in the rehydration experiments.

The dried apple sample was rehydrated by immersion in distilled water at 20°C. The ratio of the volume of apple cubes to that of the medium (water) was maintained at 1:25. An initial
amount of 10 g of dried apples was used in each trial. The following measurements were replicated three times under laboratory conditions: (i) dry matter of solid changes of the examined samples during rehydration, (ii) volume changes of the examined samples during rehydration, (iii) mass changes of the examined samples during rehydration. Rehydration times were 10, 20, 30, 50, 60, 90, 120, 180, 240, 300 and 360 min. At these specified intervals, samples were carefully removed, blotted with paper towel to remove superficial water, and weighted. Dry matter of solid was determined according to AOAC standards [61]. The mass of samples during rehydration and dry matter of samples were weighted with the electronic scales WPE-300 (RADWAG, Radom, Poland). Maximum relative error was 0.1%. The volume changes of apple cubes during rehydration were measured by buoyancy method using petroleum benzine. Maximum relative error was 5%.

Plot for the variation in mass, dry matter of solid, and volume with time during rehydration are shown in Fig. 1, 2, and 3, respectively. It can be seen from Fig. 1 and 3 that moisture uptake increases with increasing rehydration time, and the rate is faster in the initial period of rehydration and decreased up to the saturation level. This initial period of high water uptake can be attributed to the capillaries and cavities near the surface filling up rapidly [4,62]. As water absorption proceeds, rehydration rates decline due to increased extraction rates of soluble materials [63]. Similar trends have been reported in the previous studies [24,28,64]. It can be observed from Fig. 2 that solute loss increases with increasing rehydration time, and the rate is faster in the initial period of rehydration and decreased up to the saturation level. The explanation of such a course of variation in dry matter of solid with time can be the following. There is an initial steep decrease in solid content because of a high rate of mass transfer (solid gradient). As the solute concentration equilibrated with the environment, the rate of change of solid dry matter is substantially reduced [16]. Similar findings have been noted in the previous studies [2,8,40,55].

The course of rehydration characteristics of apple cubes was described with the following models: the Peleg model (Eq. (7)) [27], the Pilosof-Boquet-Batholomai model (Eq. (15)) [36], the Singh-Kulshrestha model (Eq. (18)) [37], and Witrowa-Rajchert model (Eq.(23)) [40]. The mentioned models were applied for the description of the increase in mass and volume, and the decrease in dry matter of solid. Mass transfer kinetics during rehydration of apple cubes was also modelled using theoretical model based on Fick’s second law (Eq. (60)). The variation of dry matter of solid with time and moisture content was described with this model. The goodness of fit of the tested models to the experimental data was evaluated with the determination coefficient \( R^2 \), the root mean square error (RMSE), and reduced chi-square \( \chi^2 \). The higher the \( R^2 \) value, and lower the RMSE and \( \chi^2 \) values, the better is the goodness of fit [15,28]. In this study, the regression analyses were done using the STATISTICA routine.

Coefficients of the chosen empirical models and the results of the statistical analyses are given in Table 1.

As can be seen from the statistical analysis results, generally high determination coefficient \( R^2 \) were observed for all considered empirical models. The values of RMSE and \( \chi^2 \) are comparable for all models, although it can be noticed that Witrowa-Rajchert model [40] gave the lowest values of RMSE and \( \chi^2 \). It turned out from the statistical analyses that the Witrowa-
Figure 1. Variation in mass with time during rehydration of apple cubes immersed in distilled water at 20ºC

Figure 2. Variation in dry matter of solid with time during rehydration of apple cubes immersed in distilled water at 20ºC
Rajchert model [40] can be considered as the most appropriate. Taking into account values of determination coefficient $R^2$ it can be, however, stated that all considered models may be assumed to represent the rehydration characteristics. The equilibrium mass and dry matter of solid obtained from the models are in good agreement with the experimental data but only Witrowa-Rajchert model [40] gave appropriate value of equilibrium volume.

Diffusion coefficients estimated from Fick’s second law for a cube (Eq. (60)) and the results of the statistical analyses are given in Table 2. Diffusion coefficients are considered constant and because the cubes dimensions changed during the rehydration, four kinds of variables were identified: $D/R_{c}^2$, $D_1$ for $R_c=10$ mm, $D_2$ for mean dimension of cube according to time, and $D_3$ for mean dimension of cube according to moisture content. It can be noticed that diffusion model described the mass transfer kinetics during rehydration of dried apple cubes well. The determined values of $D/R_{c}^2$ (1.37·10$^{-5}$ s$^{-1}$ and 2.64·10$^{-5}$ s$^{-1}$) were found to be lower than the reported in the literature for mushrooms: 4.9·10$^{-4}$ s$^{-1}$ and 7.9·10$^{-4}$ s$^{-1}$ [4,14]. The determined values of mass diffusion coefficients was found to be between 2.78·10$^{-10}$ m$^2$s$^{-1}$ and 6.60·10$^{-10}$ m$^2$s$^{-1}$. These values are within the general range of 10$^{-12}$-10$^{-8}$ m$^2$s$^{-1}$ (mostly about 10$^{-10}$ m$^2$s$^{-1}$) for food materials [65,66].

The swelling of dried apple cubes during rehydration was also described using following formula:

$$\frac{V}{V_r} = \left(\frac{M}{M_r}\right)^n$$

(65)
<table>
<thead>
<tr>
<th>Variable</th>
<th>Model name</th>
<th>Eq. no.</th>
<th>Model parameters</th>
<th>Equilibrium value</th>
<th>$R^2$</th>
<th>RMSE</th>
<th>$\chi^2$</th>
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<tbody>
<tr>
<td>Mass</td>
<td>Peleg [27]</td>
<td>7</td>
<td>$A_1=0.017407$</td>
<td>39.104</td>
<td>0.986810</td>
<td>0.970836</td>
<td>1.001431</td>
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<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Pilosof-Boquet-Batholomai [36]</td>
<td>15</td>
<td>$A_1=29.10352$</td>
<td>39.407</td>
<td>0.986810</td>
<td>0.986810</td>
<td>0.970836</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$A_2=0.506620$</td>
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</tr>
<tr>
<td></td>
<td>Singh-Kulshrestha [37]</td>
<td>18</td>
<td>$A_1=29.10352$</td>
<td>39.104</td>
<td>0.986810</td>
<td>0.986810</td>
<td>0.970836</td>
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<tr>
<td></td>
<td>Witrowa-Rajchert [40]</td>
<td>23</td>
<td>$A_1=1.102427$</td>
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<td>0.987488</td>
<td>0.987488</td>
<td>0.867144</td>
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<td>$B=2.838299$</td>
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<td></td>
<td></td>
<td></td>
<td>$C=0.622424$</td>
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<td></td>
</tr>
<tr>
<td>Dry matter of solid</td>
<td>Peleg [27]</td>
<td>7</td>
<td>$A_1=-0.04470$</td>
<td>3.332</td>
<td>0.985213</td>
<td>0.204884</td>
<td>0.044601</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>$A_2=-0.14997$</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Pilosof-Boquet-Batholomai [36]</td>
<td>15</td>
<td>$A_1=-6.66822$</td>
<td>3.332</td>
<td>0.985213</td>
<td>0.204884</td>
<td>0.044601</td>
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<td></td>
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<td></td>
<td>Singh-Kulshrestha [37]</td>
<td>18</td>
<td>$A_1=-6.66679$</td>
<td>3.333</td>
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<td>$A_2=3.358309$</td>
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</tr>
<tr>
<td></td>
<td>Witrowa-Rajchert [40]</td>
<td>23</td>
<td>$A_1=0.932323$</td>
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<td>$C=-4.268212$</td>
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<tr>
<td>Volume</td>
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<td>7</td>
<td>$A_1=0.020528$</td>
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<td>0.762512</td>
<td>0.617763</td>
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<td>$A_2=0.033683$</td>
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<tr>
<td></td>
<td>Pilosof-Boquet-Batholomai [36]</td>
<td>15</td>
<td>$A_1=29.71020532$</td>
<td>39.710</td>
<td>0.992033</td>
<td>0.762450</td>
<td>0.617663</td>
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<td></td>
<td></td>
<td></td>
<td>$A_2=0.611001699$</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Singh-Kulshrestha [37]</td>
<td>18</td>
<td>$A_1=29.71820$</td>
<td>39.718</td>
<td>0.992039</td>
<td>0.762449</td>
<td>0.617660</td>
</tr>
<tr>
<td></td>
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<td>$A_2=1.634618$</td>
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</tr>
<tr>
<td></td>
<td>Witrowa-Rajchert [40]</td>
<td>23</td>
<td>$A_1=1.018117$</td>
<td>52.156</td>
<td>0.992155</td>
<td>0.740390</td>
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<td>$B=1.317555$</td>
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<td></td>
<td></td>
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<td>$C=1.191287$</td>
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Table 1. Coefficients of the chosen empirical models and the results of the statistical analyses

<table>
<thead>
<tr>
<th>Variable</th>
<th>$D/D_r$</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$R^2$</th>
<th>RMSE</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry matter of solid</td>
<td>$2.6407\times 10^{-5}$</td>
<td>$6.6017\times 10^{-10}$</td>
<td>$6.1702\times 10^{-10}$</td>
<td>$5.3756\times 10^{-10}$</td>
<td>0.985583</td>
<td>0.034884</td>
<td>0.000996</td>
</tr>
<tr>
<td>Moisture content</td>
<td>$1.3661\times 10^{-5}$</td>
<td>$3.4153\times 10^{-10}$</td>
<td>$3.1921\times 10^{-10}$</td>
<td>$2.7811\times 10^{-10}$</td>
<td>0.99637</td>
<td>0.057613</td>
<td>0.003265</td>
</tr>
</tbody>
</table>

Table 2. Diffusion coefficients (m$^2$/s$^{-1}$) estimated from the Fick’s second law for a cube (Eq. (60)) and the results of the statistical analyses
Such a formula has been used in the literature for modelling drying shrinkage [67]. The model showed a very good fit to the experimental swelling data with a high value of the determination coefficient $R^2=0.98920$ and low values of root mean square error $\text{RMSE}=0.01780$ and reduced chi-square $\chi^2=0.00032$. The estimated value of swelling coefficient $n=0.293$.

The coefficients shown in Table 1 were calculated by fitting experimental data to four chosen empirical models. As it was stated in chapter 2.2.1, the coefficients of the Peleg model, the Pilosof-Boquet-Batholomai model, and the Singh-Kulshrestha model are connected between themselves. Table 3 shows the model parameters estimated using these interdependences.

It turned out from the comparison of the results of calculations presented in Tables 1 and 3 that the values of coefficients determined using both methods are almost the same. It can be stated therefore that there is a similarity of these three considered models and their predictive abilities for rehydration of dried apple cubes are identical. The same results have been obtained by Sopade et al. in [16] for describing water absorption of wheat starch, whey protein concentrate, and whey protein isolate. Shittu et al. in [68] observed, however, differences in discussed models predictive ability for the hydration of African breadfruit seeds.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model name</th>
<th>Equation no.</th>
<th>Coefficients estimated using coefficients from the following model</th>
<th>Model parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>Peleg [27]</td>
<td>7</td>
<td>Pilosof-Boquet-Batholomai [36] $A_1=0.017407518$</td>
<td>$A_2=0.034360111$</td>
</tr>
<tr>
<td></td>
<td>Peleg [27]</td>
<td>7</td>
<td>Singh-Kulshrestha [37] $A_1=0.017407518$</td>
<td>$A_2=0.034360111$</td>
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<tr>
<td></td>
<td>Pilosof-Boquet-Batholomai [36]</td>
<td>15</td>
<td>Singh-Kulshrestha [37] $A_3=29.10351515$</td>
<td>$A_4=0.506619958$</td>
</tr>
<tr>
<td>Dry matter of solid</td>
<td>Peleg [27]</td>
<td>7</td>
<td>Pilosof-Boquet-Batholomai [36] $A_1=0.044699038$</td>
<td>$A_2=0.149964985$</td>
</tr>
<tr>
<td></td>
<td>Peleg [27]</td>
<td>7</td>
<td>Singh-Kulshrestha [37] $A_1=0.044664539$</td>
<td>$A_2=0.149997317$</td>
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<tr>
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<td>Pilosof-Boquet-Batholomai [36]</td>
<td>15</td>
<td>Singh-Kulshrestha [37] $A_3=-6.666785913$</td>
<td>$A_4=0.29776892$</td>
</tr>
<tr>
<td>Volume</td>
<td>Peleg [27]</td>
<td>7</td>
<td>Pilosof-Boquet-Batholomai [36] $A_1=0.020565381$</td>
<td>$A_2=0.033658468$</td>
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<tr>
<td></td>
<td>Peleg [27]</td>
<td>7</td>
<td>Singh-Kulshrestha [37] $A_1=0.020585496$</td>
<td>$A_2=0.033649414$</td>
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<tr>
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<td>Pilosof-Boquet-Batholomai [36]</td>
<td>15</td>
<td>Singh-Kulshrestha [37] $A_3=29.71819917$</td>
<td>$A_4=0.611763873$</td>
</tr>
</tbody>
</table>

Table 3. Coefficients of the Peleg model [27], the Pilosof-Boquet-Batholomai model [36], and the Singh-Kulshrestha model [37] estimated using the interdependences between the coefficients.
4. Conclusions

Four empirical and one theoretical models were investigated for their suitability to describe the mass transfer kinetics during rehydration of dried apple cubes. The determination coefficient, root mean square error, and reduced chi-square method were estimated for all models considered to compare their goodness of fit the experimental rehydration data. All models described the rehydration characteristics of dried apple cubes satisfactorily ($R^2 > 0.9852$). The empirical Witrowa-Rajchert model [40] and the theoretical model based on Fick’s second law can be considered as the most appropriate. Theoretical models give an explanation of the phenomena occurring during rehydration but are difficult in application compared to empirical models. Therefore, if the description of rehydration curves is only needed it is better to apply empirical models. Such need occurs especially in food industry.

The determined values of mass diffusion coefficient was found to be between $2.78 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$ and $6.60 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$. These values are within the general range for food materials.

Acknowledgements

The authors are grateful for the financial support from research project No. N N313 780940 from the Polish National Science Centre.

Nomenclature

- $a, b$ - constants (Eq.(62))
- $A, B$ - constants (Eqs. (21), (22), (23), and (24))
- $A$ - surface area ($\text{m}^2$)
- $C$ - constant (Eqs. (21) and (23))
- $A_0$ - constant (Eqs. (12))
- $A_1$ - constant (Eqs. (7), (8), (9), (11), (12), (13), and (14))
- $A_2$ - constant (Eqs. (7), (8), (10), (11), (13), and (14))
- $A_3$ - constant (Eqs. (15), (16), and (17))
- $A_4$ - constant (Eqs. (15), and (17))
- $A_5$ - constant (Eqs. (18), (19), and (20))
- $A_6$ - constant (Eqs. (18), and (20))
- $a$ - constant (Eq. (34))
a, b, c - equation coefficients (Eq. (2))

$E_a$ - activation energy (J mol$^{-1}$)

$D$ - mass diffusion coefficient (effective diffusivity) (m$^2$s$^{-1}$)

$D_{calc}$ - calculated diffusion coefficient (m s$^{-1}$)

$D_{eff}$ - effective diffusion coefficient (m s$^{-1}$)

$g$ - gravitational constant (m s$^{-2}$)

$h$ - half of cylinder height, high of liquid rise (m)

$k$ - rehydration rate constant (s$^{-1}$)

$k_1$ - constant (Eq. (64)) (m$^2$s$^{-1}$)

$k_2$ - constant (Eq. (64)) (m$^2$s$^{-1}$)

$L$ - characteristic dimension (m)

$M$ - moisture content (dry basis)

$M_e$ - equilibrium moisture content (dry basis)

$m$ - mass kg

$n$ - constant (Eqs. (32), (33), and (34))

$n$ - swelling coefficient (Eq. (65))

$r$ - mean pore radius (m)

$r, x, y, z$ - coordinates (m)

$R$ - universal gas constant (J K$^{-1}$ mol$^{-1}$)

$R_p$ - half of plane or cube thickness, cylinder radius, sphere radius (m)

$R_e$ - constant in Eq.(29)

$RMSE$ - root mean square error

$R^2$ - coefficient of determination

$s$ - dry matter content (kg d.m. kg$^{-1}$)

$T$ - temperature (K)

$t$ - time (s)

$x$ - water absorption ratio (Eqs. (33), (34), (35))

$V$ - volume (m$^3$)

$\alpha$ - constant (Eqs. (25) and (26))

$\alpha'$ - constant (Eqs. (27) and (28))
β - constant (Eqs. (25), (26), and (27))

γ - surface tension (N m⁻¹)

η - fluid viscosity (Pa s)

θ - advancing liquid constant angle (rad)

ρ - liquid density (kg m⁻³)

χ² - reduced chi-square

Subscripts

0 - initial

A - outer surface of body

d - dried

o - before drying

r - rehydrated

Author details

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Faculty of Production Engineering, Warsaw University of Life Sciences, Poland

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