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Electronic Structures of Tetragonal ABX₃: Role of the B-X Coulomb Repulsions for Ferroelectricity and Piezoelectricity

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1. Introduction

Since Cohen proposed an origin for ferroelectricity in perovskites (ABX₃) [1], investigations of ferroelectric materials using first-principles calculations have been extensively studied [2-20]. Currently, using the pseudopotential (PP) methods, most of the crystal structures in ferroelectric ABX₃ can be precisely predicted. However, even in BaTiO₃, which is a well-known ferroelectric perovskite oxide with tetragonal structure at room temperature, the optimized structure by the PP methods is strongly dependent on the choice of the Ti PPs as illustrated in Fig. 1; preparation for Ti 3s and 3p semicore states in addition to Ti 3d, 4s, and 4p valence states is essential to the appearance of the tetragonal structure. This is an important problem for ferroelectricity, but it has been generally recognized for a long time that this problem is within an empirical framework of the calculational techniques [21].

Figure 1. Illustration of the choice of Ti 3s and 3p states in pseudopotentials.
It is known that ferroelectric state appears when the long-range forces due to the dipole-dipole interaction overcome the short-range forces due to the Coulomb repulsions. Investigations about the relationship between the Ti-O Coulomb repulsions and the appearance of ferroelectricity in ATiO$_3$ ($A$ = Ba, Pb) were reported both theoretically and experimentally. Theoretically, Cohen first proposed the hybridization between Ti 3d state and O 2p state (Ti 3d-O 2p) as an origin for ferroelectricity in BaTiO$_3$ and PbTiO$_3$ [1]. On the other hand, we investigated [20] the influence of the Ti-O Coulomb repulsions on Ti ion displacement in tetragonal BaTiO$_3$ and PbTiO$_3$ where O$_z$ denotes the O atom to the z-axis (Ti is displaced to the z-axis). Whereas the hybridization between Ti 3d state and O 2p state stabilize Ti ion displacement, the strong Coulomb repulsions between Ti 3s and 3p$^z$ states and O 2p$^z$ states do not favourably cause Ti ion displacement. Experimentally, on the other hand, Kuroiwa et al. [22] showed that the appearance of ferroelectric state is closely related to the total charge density of Ti-O bonding in BaTiO$_3$. As discussed above, investigation about a role of Ti 3s and 3p states is important in the appearance of the ferroelectric state in tetragonal BaTiO$_3$, in addition to the Ti 3d-O 2p hybridization as an origin of ferroelectricity [1].

It seems that the strong B-X Coulomb repulsions affect the most stable structure of ABX$_3$. It has been well known that the most stable structure of ABX$_3$ is closely related to the tolerance factor $t$,

$$t = \frac{r_A + r_X}{\sqrt{2} (r_B + r_X)},$$  \hspace{1cm} (1)

where $r_A$, $r_B$, and $r_X$ denote the ionic radii of $A$, $B$, and $X$ ions, respectively [23]. In general ferroelectric ABX$_3$, the most stable structure is tetragonal for $t \geq 1$, cubic for $t = 1$, and rhombohedral or orthorhombic for $t \leq 1$. In fact, BaTiO$_3$ with $t = 1.062$ shows tetragonal structure in room temperature. However, recently, BiZn$_{0.5}$Ti$_{0.5}$O$_3$ (BZT) with $t = 0.935$ was experimentally reported [24] to show a tetragonal PbTiO$_3$-type structure with high $c/a$ ratio (1.211). This result is in contrast to that of BiZn$_{0.5}$Mg$_{0.5}$O$_3$ (BMT) with $t = 0.939$, i.e., the most stable structure was reported to be the orthorhombic or rhombohedral structure [25, 26]. Several theoretical papers of BZT have been reported [4-6], but the role of the Zn-O Coulomb repulsions in the appearance of the tetragonal structure has not been discussed sufficiently.

Piezoelectric properties in ABX$_3$ are also closely related to the crystal structure. Investigations of the relationship between piezoelectric properties and the crystal structure of ABX$_3$ by first-principles calculations have been extensively studied [2-19]. Moreover, phenomenological investigations of the piezoelectric properties have been also performed [27, 28]. However, it seems that the piezoelectric properties in the atomic level have not been sufficiently investigated. Therefore, further theoretical investigation of the relationship between piezoelectric properties and the crystal structure of ABX$_3$, especially the B-X Coulomb repulsions, should be needed.

Recently, we investigated the roles of the Ti-O Coulomb repulsions in the appearance of a ferroelectric state in tetragonal BaTiO$_3$ by the analysis of a first-principles PP method [11-15]. We investigated the structural properties of tetragonal and rhombohedral BaTiO$_3$ with two kinds of Ti PPs, and propose the role of Ti 3s and 3p states for ferroelectricity. We also inves-
tigated the role of the Zn-O Coulomb repulsions in the appearance of a ferroelectric state in tetragonal BZT [10, 13]. Moreover, we also investigated the structural, ferroelectric, and piezoelectric properties of tetragonal $ABX_3$ and discussed the piezoelectric mechanisms based on the $B$-$X$ Coulomb repulsions [12, 14, 15, 18, 19].

In this chapter, based on our recent papers and patents [10-19], we discuss a general role of $B$-$X$ Coulomb repulsions for the appearance of the ferroelectric state in $ABX_3$. Then, we also discuss the relationship between the $B$-$X$ Coulomb repulsions and the piezoelectric properties of tetragonal $ABX_3$.

2. Methodology

The calculations for $ABX_3$ were performed using the ABINIT code [29], which is one of the norm-conserving PP (NCPP) methods. Electron-electron interaction was treated in the local-density approximation (LDA) [30]. Pseudopotentials were generated using the OPIUM code [31]:

i. In BaTiO$_3$, 5s, 5p and 6s electrons for Ba PP, and 2s and 2p electrons for O PP were treated as semicore or valence electrons, respectively. Moreover, in order to investigate the role of Ti 3s and 3p states, two kinds of Ti PPs were prepared: the Ti PP with 3s, 3p, 3d and 4s electrons treated as semicore or valence electrons (Ti3spd4s PP), and that with only 3d and 4s electrons treated as valence electrons (Ti3d4s PP). In both PPs, the differences between the calculated result and experimental one are within 1.5 % of the lattice constant and within 10 % of the bulk modulus in the optimized calculation of bulk Ti. The cutoff energy for plane-wave basis functions was set to be 50 Hartree (Hr). The number of atoms in the unit cell was set to be five, and a 6×6×6 Monkhorst-Pack $k$-point mesh was set in the Brillouin zone of the unit cell. Positions of all the atoms were optimized within the framework of the tetragonal ($P4mm$) or rhombohedral ($R3m$) structure.

ii. In BZT and BMT, 5d, 6s, and 6p electrons for Bi PP, and 2s and 2p electrons for O PP were treated as semicore or valence electrons, respectively. Moreover, in order to investigate the roles of Zn and Ti 3s and 3p states, and Mg 2s and 2p states, two types of PPs were prepared: the PPs with only Zn and Ti 3d and 4s states, and Mg 3s states, considered as valence electrons (Case I), Zn and Ti 3s, 3p, 3d, and 4s states, and Mg 2s, 2p, and 3s states considered as semicore or valence electrons (Case II). The cutoff energy for plane-wave basis functions was set to be 70 Hr for Case I and 110 Hr for Case II. A 4×4×4 Monkhorst-Pack $k$-point mesh was set in the Brillouin zone of the unit cell. The calculated results can be discussed within 0.02 eV per formula unit (f.u.) using the above conditions. The present calculations were performed for the monoclinic, rhombohedral, and $A_\alpha$, $C_\alpha$, and $G$-type tetragonal structures. The number of atoms in the unit cell was set to be 10 for the rhombohedral and monoclinic structures, and 20 for the $A_\alpha$, $C_\alpha$, and $G$-type tetragonal structures. Positions of all the atoms were optimized within the framework of the rhombohedral ($R3$), monoclinic ($Pm$), and tetragonal ($P4mm$) structures.
iii. Relationship between the $B$-$X$ Coulomb repulsions and the piezoelectric properties in tetragonal $ABX_3$ is investigated. The pseudopotentials were generated using the opium code [31] with semicore and valence electrons (e.g., Ti3spd4s PP), and the virtual crystal approximation [32] were applied to several $ABX_3$.

Spontaneous polarizations and piezoelectric constants were also evaluated, due to the Born effective charges [33]. The spontaneous polarization of tetragonal structures along the [001] axis, $P_3$, is defined as

\[ P_3 = \sum_k \frac{e\varepsilon c}{\Omega} Z_{33}^*(k) u_3(k), \]  

where $e$, $c$, and $\Omega$ denote the charge unit, lattice parameter of the unit cell along the [001] axis, and the volume of the unit cell, respectively. $u_3(k)$ denotes the displacement along the [001] axis of the $k$th atom, and $Z_{33}^*(k)$ denotes the Born effective charges [33] which contributes to the $P_3$ from the $u_3(k)$.

The piezoelectric $e_{3j}$ constant is defined as

\[ e_{3j} = \left( \frac{\partial P_3}{\partial \eta_{3j}} \right)_{u_3} + \sum_k \frac{e\varepsilon c}{\Omega} Z_{33}^*(k) \left( \frac{\partial u_3(k)}{\partial \eta_{jj}} \right) \quad (j = 3, 1), \]  

where $e$ and $\Omega$ denote the charge unit and the volume of the unit cell. $P_3$ and $c$ denote the spontaneous polarization of tetragonal structures and the lattice parameter of the unit cell along the [001] axis, respectively. $u_3(k)$ denotes the displacement along the [001] axis of the $k$th atom, and $Z_{33}^*(k)$ denotes the Born effective charges which contributes to the $P_3$ from the $u_3(k)$. $\eta_3$ denotes the strain of lattice along the [001] axis, which is defined as $\eta_3 = (c - c_0)/c_0$; $c_0$ denotes the $c$ lattice parameter with fully optimized structure. On the other hand, $\eta_1$ denotes the strain of lattice along the [100] axis, which is defined as $\eta_1 = (a - a_0)/a_0$; $a_0$ denotes the $a$ lattice parameter with fully optimized structure. The first term of the right hand in Eq. (3) denotes the clamped term evaluated at vanishing internal strain, and the second term denotes the relaxed term that is due to the relative displacements.

The relationship between the piezoelectric $d_{33}$ constant and the $e_{33}$ one is

\[ d_{33} = \sum_{j=1}^{6} s_{3j}^E \times^T (e_{3j}), \]  

where $s_{3j}$ denotes the elastic compliance, and `$^T$` denotes the transposition of matrix elements. The suffix $j$ denotes the direction-indexes of the axis, i.e., 1 along the [100] axis, 2 along the [010] axis, 3 along the [001] axis, and 4 to 6 along the shear directions, respectively.
3. Results and discussion

3.1. Ferroelectricity

3.1.1. Role of Ti 3s and 3p states in ferroelectric BaTiO$_3$

Figures 2(a) and 2(b) show the optimized results for the ratio $c/a$ of the lattice parameters and the value of the Ti ion displacement ($\delta_{Ti}$) as a function of the $a$ lattice parameter in tetragonal BaTiO$_3$, respectively. Results with arrows are the fully optimized results, and the others results are those with the $c$ lattice parameters and all the inner coordination optimized for fixed $a$. Note that the fully optimized structure of BaTiO$_3$ is tetragonal with the Ti3spd4s PP, whereas it is cubic with the Ti3d4s PP. This result suggests that the explicit treatment of Ti 3s and 3p semicore states is essential to the appearance of ferroelectric states in BaTiO$_3$.

![Figure 2](image-url)

The calculated results shown in Fig. 2 suggest that the explicit treatment of Ti 3s and 3p semicore states is essential to the appearance of ferroelectric states in BaTiO$_3$. In the following, we investigate the role of Ti 3s and 3p states for ferroelectricity from two viewpoints.

One viewpoint concerns hybridizations between Ti 3s and 3p states and other states. Figure 3(a) and 3(b) shows the total density of states (DOS) of tetragonal BaTiO$_3$ with two Ti PPs. Both results are in good agreement with previous calculated results [7] by the full-potential linear augmented plane wave (FLAPW) method. In the DOS with the Ti3spd4s PP, the energy “levels”, not bands, of Ti 3s and 3p states, are located at -2.0 Hr and -1.2 Hr, respectively. This result suggests that the Ti 3s and 3p orbitals do not make any hybridization but only give Coulomb repulsions with the O orbitals as well as the Ba orbitals. In the DOS with the Ti3d4s PP, on the other hand, the energy levels of Ti 3s and 3p states are not shown because Ti 3s and 3p states were treated as the core charges. This result means that the Ti 3s and 3p orbitals cannot even give Coulomb repulsions with the O orbitals as well as the Ba orbitals.
Another viewpoint is about the Coulomb repulsions between Ti 3s and 3p\(_x\) states and O\(_x\) 2s and 2p\(_x\) states in tetragonal BaTiO\(_3\). Figure 4(a) and 4(b) show two-dimensional electron-density contour map on the \(xz\)-plane. These are the optimized calculated results with \(a\) fixed to be 3.8 Å, and the electron density in Fig. 4(a) is quantitatively in good agreement with the experimental result [22]. The electron density between Ti and O ions in Fig. 3(a) is larger than that in Fig. 4(b), which suggests that Ti ion displacement is closely related to the Coulomb repulsions between Ti 3s and 3p\(_x\) states and O\(_x\) 2s and 2p\(_x\) states; the Ti-O Coulomb repulsion is an important role in the appearance of the ferroelectric state in BaTiO\(_3\).

The present discussion of the Coulomb repulsions is consistent with the previous reports. A recent soft mode investigation [8] of BaTiO\(_3\) shows that Ba ions contribute little to the appearance of Ti ion displacement along the [001] axis. This result suggests that Ti ion displacement is closely related to the structural distortion of TiO\(_6\) octahedra. In the present calculations, on the other hand, the only difference between BaTiO\(_3\) with the Ti3spd4s PP and with the Ti3d4s PP is the difference in the expression for the Ti 3s and 3p states, i.e., the explicit treatment and including core charges. However, our previous calculation [20] shows that the strong Coulomb repulsions between Ti 3s and 3p\(_x\) states and O\(_x\) 2s and 2p\(_x\) states do not favor Ti ion displacement along the [001] axis. This result suggests that the Coulomb repulsions between Ti 3s and 3p\(_x\) states and O\(_x\) 2s and 2p\(_x\) states would contribute to Ti ion displacement along the [001] axis, and the suggestion is consistent with a recent calculation [9] for PbTiO\(_3\) indicating that the tetragonal and ferroelectric structure appears more favorable as the \(a\) lattice constant decreases.
Considering the above investigations, we propose the mechanism of Ti ion displacement as follows: Ti ion displacement along the $z$-axis appears when the Coulomb repulsions between Ti 3s and 3p$_x$($y$) states and O$_x$($y$) 2s and 2p$_x$($y$) states, in addition to the dipole-dipole interaction, overcome the Coulomb repulsions between Ti 3s and 3p$_z$ states and O$_z$ 2s and 2p$_z$ states. An illustration of the Coulomb repulsions is shown in Fig. 5(a). In fully optimized BaTiO$_3$ with the Ti3spd4s PP, the Ti ion can be displaced due to the above mechanism. In fully optimized BaTiO$_3$ with the Ti3d4sPP, on the other hand, the Ti ion cannot be displaced due to the weaker Coulomb repulsions between Ti and O$_x$($y$) ions. However, since the Coulomb repulsion between Ti and O$_z$ ions in BaTiO$_3$ with the Ti3d4s PP is also weaker than that in BaTiO$_3$ with the Ti3spd4s PP, the Coulomb repulsions between Ti and O$_z$ ions in addition to the log-range force become comparable to the Coulomb repulsions between Ti and O$_x$ ions both in Ti PPs, as the $a$ lattice parameter becomes smaller. The above discussion suggests that the hybridization between Ti 3d and O$_z$ 2s and 2p$_z$ stabilizes Ti ion displacement, but contribute little to a driving force for the appearance of Ti ion displacement.
It seems that the above proposed mechanism for tetragonal BaTiO₃ can be applied to the mechanism of Ti ion displacement in rhombohedral BaTiO₃, as illustrated in Fig. 5(b). The strong isotropic Coulomb repulsions between Ti 3s and 3px (y, z) states and Ox (y, z) 2s and 2px (y, z) states yield Ti ion displacement along the [111] axis. On the other hand, when the isotropic Coulomb repulsions are weaker or stronger, the Ti ion cannot be displaced and therefore it is favoured for the crystal structure to be cubic.

\[ \frac{V}{V_{\text{opt}}} \lesssim 0.9 \text{ or } \gtrsim 1.3, \]

the isotropic Coulomb repulsions are weaker or stronger, and the Ti ion cannot be displaced along the [111] axis and therefore the crystal structure is cubic for both Ti PPs. For \( 0.9 \leq \frac{V}{V_{\text{opt}}} \leq 1.3 \), on the other hand, the isotropic Coulomb repulsions are strong enough to yield Ti ion displacement for both Ti PPs. However, since the magnitude of the isotropic Coulomb repulsion is different in the two Ti PPs, the properties of the 90-α degree and \( \delta_{\text{Ti}} \) are different quantitatively.

Figure 5. Illustrations of the proposed mechanisms for the Coulomb repulsions between Ti 3s and 3p states and O 2s and 2p states in BaTiO₃: (a) anisotropic Coulomb repulsions between Ti 3s and 3pₓ (y, z) states and Oₓ (y, z) 2s and 2pₓ (y, z) states, and between Ti 3s and 3pᵢ states and O₂ 2s and 2pᵢ states, in the tetragonal structure. (b) Isotropic Coulomb repulsions between Ti 3s and 3pₓ (y, z) states and Oₓ (y, z) 2s and 2pₓ (y, z) states, in the rhombohedral structure [11].

Let us investigate the structural properties of rhombohedral BaTiO₃. Figures 6(a) and 6(b) show the optimized results of the 90-α degree and \( \delta_{\text{Ti}} \) as a function of fixed volumes of the unit cells in rhombohedral BaTiO₃, respectively, where α denotes the angle between two lattice vectors. In these figures, α denotes the angle between two crystal axes of rhombohedral BaTiO₃, and \( \delta_{\text{Ti}} \) denotes the value of the Ti ion displacement along the [111] axis. Results with arrows are the fully optimized results; \( V_{\text{opt}} \) denote the volume of the fully optimized unit cell with the Ti₃spd₄s PP. The other results are those with all the inner coordination optimized with fixed volumes of the unit cells. The proposal mechanisms about the Coulomb repulsions seem to be consistent with the calculated results shown in Fig.6: For \( V/V_{\text{opt}} \leq 0.9 \) or \( \gtrsim 1.3 \), the isotropic Coulomb repulsions are weaker or stronger, and the Ti ion cannot be displaced along the [111] axis and therefore the crystal structure is cubic for both Ti PPs. For \( 0.9 \leq V/V_{\text{opt}} \leq 1.3 \), on the other hand, the isotropic Coulomb repulsions are strong enough to yield Ti ion displacement for both Ti PPs. However, since the magnitude of the isotropic Coulomb repulsion is different in the two Ti PPs, the properties of the 90-α degree and \( \delta_{\text{Ti}} \) are different quantitatively.
3.1.2. Role of Zn 3s, 3p and 3d states in ferroelectric BiZn$_{0.5}$Ti$_{0.5}$O$_3$

As discussed in Sec. 3.1.1, the Coulomb repulsions between Ti 3s and 3p states and O 2s and 2p states have an important role in the appearance of the ferroelectric state in tetragonal BaTiO$_3$. In this subsection, we discuss the role of Zn 3d ($d^{10}$) states in addition to 3s and 3p states for ferroelectricity in tetragonal BZT.

Table 1 shows a summary of the optimized results of BZT in Cases I and II. $\Delta E_{\text{total}}$ denotes the difference in total energy per f.u. between the rhombohedral and other structures. Although the lattice constant in each structure except the rhombohedral one seems to be quantitatively similar in both cases, properties of $\Delta E_{\text{total}}$ are different. In Case I, the rhombohedral structure is the most stable, which is in disagreement with the experimental result [24]. In Case II, on the other hand, the monoclinic structure, which is the “pseudo-C-type-tetragonal” structure, is the most stable. Unfortunately, this result seems to be in disagreement with the experimental result [24], but is in good agreement with the recent calculated result [6]. Note that the magnitude of $\Delta E_{\text{total}}$ in Case II is markedly smaller than that in Case I. In contrast to BZT, the rhombohedral structure is the most stable structure in both cases in BMT, which is consistent with the experimental result [26].

Figures 7(a) and 7(b) show two-dimensional electron density contour maps of the C-type tetragonal BZT in Cases I and II, respectively. The Coulomb repulsion of Zn-O in Case II is larger than that in Case I, and the Coulomb repulsion favorably causes Zn ion displacement to O in Case II. This result is consistent with Sec. 3.1.1. In contrast to the properties of Zn-O bonding, the inner coordination of the Ti ion is similar in both cases, although the electron densities are markedly different. This result suggests that the Coulomb repulsion magnitude of Ti-O is the same as that of Ti-O in small Ti-O bonding (≈ 1.8 Å), in both Cases I and II. Figures 7(c) and 7(d) show two-dimensional electron density contour maps of the C-type tet-
ragonal BMT in Cases I and II, respectively. Although the electron densities in both cases are markedly different, the inner coordination of the Mg ion are similar. This result suggests that the Coulomb repulsion between Mg and O is not strong sufficiently for inducing Mg ion displacement even in Case II.

<table>
<thead>
<tr>
<th>Structure</th>
<th>a (Å)</th>
<th>c (Å)</th>
<th>c/a</th>
<th>α (deg.)</th>
<th>ΔE&lt;sub&gt;total&lt;/sub&gt; (eV/f.u.)</th>
</tr>
</thead>
<tbody>
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<td>A-type Tetra.</td>
<td>3.748</td>
<td>4.579</td>
<td>1.222</td>
<td>90</td>
<td>0.316</td>
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<tr>
<td>C-type Tetra.</td>
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<td>4.784</td>
<td>1.300</td>
<td>90</td>
<td>0.240</td>
</tr>
<tr>
<td>G-type Tetra.</td>
<td>3.725</td>
<td>4.574</td>
<td>1.300</td>
<td>90</td>
<td>0.158</td>
</tr>
<tr>
<td>Monoclinic</td>
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<td>4.741</td>
<td>1.269</td>
<td>90</td>
<td>0.193</td>
</tr>
<tr>
<td>Rhombohedral</td>
<td>5.560</td>
<td>1</td>
<td>59.9</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Experiment [24]</td>
<td>3.822</td>
<td>4.628</td>
<td>1.211</td>
<td>90</td>
<td>---</td>
</tr>
</tbody>
</table>

\[(a)\]

Table 1. Summary of the optimized results of BZT in (a) Case I and (b) Case II. a and c denote the lattice parameters, and α and β denote angles between two lattice axes. ΔE<sub>total</sub> denotes the difference in total energy per f.u. between the rhombohedral and other structures [10].

Finally in this subsection, we discuss the difference in the electronic structures between the C-type tetragonal and the monoclinic BZT. Figures 8(a) and 8(b) show the electron density contour maps of the C-type tetragonal BZT and that of the monoclinic BZT in Case II, respectively. This result suggests that the strong Coulomb repulsion between Zn and O<sub>2</sub> causes the small Zn ion displacement in the [110] direction in the monoclinic BZT, which makes the Coulomb repulsion of Zn-O<sub>2</sub> weaker than that in the C-type tetragonal BZT. As a result, this small Zn ion displacement makes the monoclinic BZT more stable than the C-type tetragonal structure.
Figure 7. Two-dimensional electron density contour maps of monoclinic (a) BZT in Case I, (b) BZT in Case II, (c) BMT in Case I, and (d) BMT in Case II. The electron density increases as color changes from blue to red via white. Contour curves are drawn from 0.2 to 2.0 e/Å$^3$ with increments of 0.2 e/Å$^3$ [10].

Figure 8. Two-dimensional electron density contour maps of BZT in Case II (a) C-type tetragonal and (b) monoclinic. The electron density increases as color changes from blue to red via white. Contour curves are drawn from 0.2 to 2.0 e/Å$^3$ with increments of 0.2 e/Å$^3$ [10].
3.2. Piezoelectricity

3.2.1. Role of the Ti-O Coulomb repulsions in tetragonal piezoelectric SrTiO$_3$ and BaTiO$_3$

As discussed in Sec. 3.1, the Coulomb repulsions between Ti 3s and 3p$_{x,y}$ states and O $(\eta)$ 2s and 2p$_{x,y}$ states have an important role in the appearance of the ferroelectric state in tetragonal SrTiO$_3$. In this subsection, we discuss the role of the Ti-O Coulomb repulsions for piezoelectric SrTiO$_3$ and BaTiO$_3$.

Figures 9(a) shows the optimized results for $c - c_{\text{lab}}$ as a function of the $a$ lattice parameters in tetragonal SrTiO$_3$ and BaTiO$_3$, where $c_{\text{lab}}$ denotes the $c$ lattice parameter in cubic SrTiO$_3$ and BaTiO$_3$, respectively. These results are the fully optimized results and the results with the $c$ lattice parameters and all the inner coordination optimized for fixed $a$. The fully optimized parameters of SrTiO$_3$ ($a = 3.84$ Å: cubic) and BaTiO$_3$ ($a = 3.91$ Å and $c = 4.00$ Å: tetragonal) are within 2.0 $\%$ in agreement with the experimental results in room temperature. Figure 9(b) shows the evaluated results for $P_y$ as a function of the $a$ lattice parameters in tetragonal SrTiO$_3$ and BaTiO$_3$, where $P_y$ which is evaluated by Eq. (2), denotes the spontaneous polarization along the [001] axis. Note that the tetragonal and ferroelectric structures appear even in SrTiO$_3$ when the fixed $a$ lattice parameter is compressed to be smaller than the fully-optimized $a$ lattice parameter. As shown in Figs. 9(a) and 9(b), the tetragonal and ferroelectric structure appears more favorable as the fixed $a$ lattice parameter decreases, which is consistent with previous calculated results [9, 11]. The results would be due to the suggestion discussed in the previous section that the large Coulomb repulsion of Ti-O bonding along the [100] axis (and the [010] axis) is a driving force of the displacement of Ti ions along the [001] axis, i.e., the large Coulomb repulsion along the [100] axis (and the [010] axis) is essential for the appearance of the tetragonal structure.

![Figure 9](image-url)  
Figure 9. Optimized calculated results as a function of a lattice parameters in compressive tetragonal SrTiO$_3$ and BaTiO$_3$: (a) $c - c_{\text{lab}}$ and (b) $P_y$, i.e., spontaneous polarization along the [001] axis [12].

In the following, we use $c - c_{\text{lab}}$ as a functional parameter, because $c - c_{\text{lab}}$ is closely related to $\eta$. Figures 10(a) and 10(b) shows the piezoelectric properties of $e_{33}$ and $e_{31}$ as a function of $c - c_{\text{lab}}$ in tetragonal SrTiO$_3$ and BaTiO$_3$. The value $c - c_{\text{lab}}$ is optimized value as shown in Fig. 9(a) and $e_{33}$ and $e_{31}$ are evaluated values in their optimized structures. Note that $e_{33}$ become...
larger at \(c - c_{\text{ob}} = 0\), especially in SrTiO\(_3\). These properties seem to be similar to the properties around the Curie temperatures in piezoelectric ABO\(_3\); Damjanovic emphasized the importance of the polarization extension as a mechanism of larger piezoelectric constants in a recent paper [28]. Contrary to \(e_{33}\), on the other hand, the changes in \(e_{31}\) are much smaller than the changes in \(e_{33}\), but note that \(e_{31}\) shows negative in SrTiO\(_3\) while positive in BaTiO\(_3\).

As expressed in Eq. (3), \(e_{33}\) is the sum of the contributions from the clamped term and the relaxed term. However, it has been generally known that the contribution to \(e_{33}\) from the clamped term is much smaller than that from the relaxed term; in fact, the absolute values of the \(e_{33}\) clamped terms are less than 1 C/m\(^2\) in both SrTiO\(_3\) and BaTiO\(_3\). We therefore investigate the contributions to the relaxed term of \(e_{33}\) and \(e_{31}\) in detail. As expressed in Eq. (3), the relaxed terms of \(e_{33}\) are proportional to the sum of the products between the \(Z_{31}\) (\(k\)) and \(\partial u_{33}(k)/\partial \eta_{1}\); \(j = 3\) or \(1\) values. Let us show the evaluated results of \(Z_{31}\) (\(k\)), \(\partial u_{33}(k)/\partial \eta_{1}\), and \(\partial u_{33}(k)/\partial \eta_{3}\) in the following. Figures 11(a) and 11(b) show the \(Z_{31}\) (\(k\)) values in SrTiO\(_3\) and BaTiO\(_3\), respectively. Properties of the \(Z_{31}\) (\(k\)) values are quantitatively similar in both SrTiO\(_3\) and BaTiO\(_3\). Therefore, the difference in the properties of \(e_{33}\) and \(e_{31}\) between SrTiO\(_3\) and BaTiO\(_3\) must be due to the difference in the properties of \(\partial u_{33}(k)/\partial \eta_{1}\). Figures 12(a) and 12(b) show the \(\partial u_{33}(k)/\partial \eta_{1}\) values in SrTiO\(_3\) and BaTiO\(_3\), respectively. In these figures, O\(_3\) and O\(_5\) denote oxygen atoms along the [100] and [001] axes, respectively. Clearly, the absolute values of \(\partial u_{33}(k)/\partial \eta_{1}\) are different in between SrTiO\(_3\) and BaTiO\(_3\). On the other hand, Figs. 13(a) and 13(b) show the \(\partial u_{33}(k)/\partial \eta_{3}\) values in SrTiO\(_3\) and BaTiO\(_3\), respectively. The absolute values of \(\partial u_{33}(k)/\partial \eta_{3}\), especially for Ti, O\(_3\), and O\(_5\) are different in between SrTiO\(_3\) and BaTiO\(_3\). As a result, the quantitative differences in \(e_{33}\) and \(e_{31}\) between SrTiO\(_3\) and BaTiO\(_3\) are due to the differences in the contribution of the \(\partial u_{33}(k)/\partial \eta_{1}\) and \(\partial u_{33}(k)/\partial \eta_{3}\) values, respectively.
Let us discuss the reasons of the quantitative differences in $e_{33}$ between SrTiO$_3$ and BaTiO$_3$. Figure 14(a) shows the difference between the Ti-O$_3$ distance ($R_{TiO_3}$) and the sum of the $r_{Ti}$...
and \( r_{\text{ox}} (r_{\text{Ti}} + r_{\text{Ox}}) \) along the [001] axis as a function of \( c - c_{\text{sub}} \). Note that \( R_{\text{Ti-Ox}} \) is smaller than \( r_{\text{Ti}} + r_{\text{Ox}} \) in both \( \text{SrTiO}_3 \) and \( \text{BaTiO}_3 \). However, the difference in absolute value between \( R_{\text{Ti-Ox}} \) and \( r_{\text{Ti}} + r_{\text{Ox}} \) in \( \text{SrTiO}_3 \) is smaller than the difference in \( \text{BaTiO}_3 \) for \( 0 \leq c - c_{\text{sub}} \leq 0.20 \). This result suggests that the Ti-O Coulomb repulsion along the [001] axis in \( \text{SrTiO}_3 \) is smaller than that in \( \text{BaTiO}_3 \) and that therefore the Ti ion of \( \text{SrTiO}_3 \) can be displaced more easily along the [001] axis than that of \( \text{BaTiO}_3 \). This would be a reason why the absolute values of \( \partial u_0(k)/\partial \eta_1 \) of Ti and O ions in \( \text{SrTiO}_3 \) are larger than that in \( \text{BaTiO}_3 \). Figure 14(b) shows the difference between the A-O distance \( (R_{\text{A-O}}) \) and the sum of \( r_\text{A} \) and \( r_{\text{Ox}} (r_\text{A} + r_{\text{Ox}}) \) on the (100) plane as a function of \( c - c_{\text{sub}} \) where the values of the ionic radii are defined as Shannon’s ones [23]. Note that \( R_{\text{A-O}} \) is smaller than \( r_\text{A} + r_{\text{Ox}} \) in both \( \text{SrTiO}_3 \) and \( \text{BaTiO}_3 \). However, the difference in absolute value between \( R_{\text{Sr-Ox}} \) and \( r_\text{A} + r_{\text{Ox}} \) in \( \text{SrTiO}_3 \) is much smaller than the difference in \( \text{BaTiO}_3 \) for \( 0 \leq c - c_{\text{sub}} \leq 0.20 \). This result suggests that the Sr-O Coulomb repulsion on the (100) plane in \( \text{SrTiO}_3 \) is much smaller than the Ba-O Coulomb repulsion in \( \text{BaTiO}_3 \) and that therefore Sr and O ions of \( \text{SrTiO}_3 \) can be displaced more easily along the [001] axis than that of Ba and O ions of \( \text{BaTiO}_3 \). This would be a reason why the absolute values of \( \partial u_0(k)/\partial \eta_1 \) of Sr and O ions in \( \text{SrTiO}_3 \) are larger than those of Ba and O ions in \( \text{BaTiO}_3 \).

![Diagram](image.png)

**Figure 14.** Evaluated values as a function of \( c - c_{\text{sub}} \) in optimized tetragonal \( \text{SrTiO}_3 \) and \( \text{BaTiO}_3 \): (a) difference between the Ti-O distance \( (R_{\text{Ti-Ox}}) \) and \( r_{\text{Ti}} + r_{\text{Ox}} \), (b) difference between the A-O distance \( (R_{\text{A-O}}) \) and \( r_\text{A} + r_{\text{Ox}} \). \( R_{\text{Sr-Ox}}, R_{\text{Ba-Ox}}, R_{\text{A-O}} \) in \( \text{ATiO}_3 \) are also illustrated. Note that all the ionic radii are much larger and that A and Ti ions are displaced along the [001] axis in actual \( \text{ATiO}_3 \) [12].

Finally, in this subsection, we discuss the relationship between \( \partial u_0(k)/\partial \eta_1 \) and \( c - c_{\text{sub}} \) in detail. Figure 15(a) shows the differences in the total energy \( (\Delta E_{\text{total}}) \) as a function of \( \eta_0(Ti) \). In
this figure, the properties of SrTiO₃ with ηᵢ = 0.011, SrTiO₃ with ηᵢ = 0.053 and fully optimized BaTiO₃ as a reference, are shown. Calculations of $E_{\text{total}}$ were performed with the fixed crystal structures of previously optimized structures except Ti ions. Clearly, the magnitude of $u_s(Ti)$ at the minimum points of the $\Delta E_{\text{total}}$ and the depth of the potential are closely related to the spontaneous polarization $P_s$ and the Curie temperature ($T_c$), respectively. On the other hand, $e_{13}$ seems to be closely related to the deviation at the minimum points of the $\Delta E_{\text{total}}$. Figure 15(b) shows illustrations of $\Delta E_{\text{total}}$ curves with deviations at the minimum points of the $\Delta E_{\text{total}}$ values, corresponding to the $\Delta E_{\text{total}}$ curves of SrTiO₃ in Fig. 15(a). Clearly, as ηᵢ becomes smaller, the deviated value at the minimum point of the $\Delta E_{\text{total}}$ values becomes smaller, i.e., the Ti ion can be displaced more favourably. On the other hand, as shown in Fig. 12(a), the absolute value of $\partial u_s(Ti)/\partial \eta_i$ becomes larger as ηᵢ becomes smaller.

Therefore, the Ti ion can be displaced more favourably as the deviated value at the minimum point of the $\Delta E_{\text{total}}$ values becomes smaller. The relationship between $e_{13}$ and $\partial u_s(Ti)/\partial \eta_i$ is discussed in Sec. 3.2.3.

Figure 15. a) $\Delta E_{\text{total}}$ as a function of $u_s(Ti)$ in tetragonal SrTiO₃ and BaTiO₃. (b) Illustration of the $\Delta E_{\text{total}}$ curves in tetragonal SrTiO₃ with ηᵢ = 0.011 and SrTiO₃ with ηᵢ = 0.053 with deviations at the minimum point of $\Delta E_{\text{total}}$ [14].

3.2.2. Proposal of new piezoelectric materials

The previous discussion in Sec. 3.2.1 suggests that the piezoelectric properties of $e_{13}$ are closely related to the $B$-$X$ Coulomb repulsions in tetragonal $ABX_3$. In the viewpoint of the change of the $B$-$X$ Coulomb repulsions, we recently proposed new piezoelectric materials [16, 17], i.e., BaTi₁₋ₓNiₓO₃ and Ba(Ti₁₋ₓNbₓ)O₃(ScₓNdx).

It has been known that BaNiO₃ shows the 2H hexagonal structure as the most stable structure in room temperature. Moreover, the ionic radius of Ni²⁺ (d⁹) with the low-spin state in 2H BaNiO₃ is 0.48 Å, which is much smaller than that of Ti⁴⁺ (d⁰), 0.605 Å, in BaTiO₃. Therefore, due to the drastic change in the (Ti₁₋ₓNiₓ)O Coulomb repulsions in tetragonal BaTi₁₋ₓNiₓO₃, the $e_{13}$ piezoelectric values are expected to be larger than that in tetragonal BaTiO₃, especially around the morphotropic phase boundary (MPB). Figure 16(a) shows the total-en-
nergy difference $\Delta E_{\text{total}}$ between 2H and tetragonal structures of BaTi$_{1-x}$Ni$_x$O$_3$ as a function of $x$. The most stable structure changes at $x = 0.26$. Figure 16(b) shows $c - c_{\text{cub}}$ as a function of $x$. The $c - c_{\text{cub}}$ value shows 0 around $x = 0.26$, which suggests the appearance of the MPB, i.e., the $e_{33}$ piezoelectric value shows a maximum at $x = 0.26$.

Figure 16. (a) $\Delta E_{\text{total}}$ (total-energy difference between 2H and tetragonal structures), and (b) $c - c_{\text{cub}}$ of the tetragonal structure, as a function of $x$ in BaTi$_{1-x}$Ni$_x$O$_3$ [16]. For $0.26 \leq x \leq 1$, the tetragonal structure is not the most stable one.

Another proposal is tetragonal Ba(Ti$_{1-3z}$Nb$_{3z}$)(O$_{1-z}$N$_z$)$_3$ [17]. Due to the change of (Ti$_{1-3z}$Nb$_{3z}$)-(O$_{1-z}$N$_z$) Coulomb repulsions, the $e_{33}$ piezoelectric values are expected to be larger than that in tetragonal BaTiO$_3$. Recent experimental paper reported that the most stable structure of BaNbO$_2$N is cubic in room temperature [34]. Contrary to the experimental result, however, our calculations suggest that the tetragonal structure will be more stable than the cubic one, as shown in Fig. 17(a). Figure 17(b) shows $c - c_{\text{cub}}$.
as a function of $x$. The $c - c_{\text{cub}}$ value shows almost 0 at $x = 0.12$. Although the MPB does not appear in tetragonal Ba(Ti$_{1.3}$Nb$_3$)(O$_{1.5}$N$_2$)$_3$, the $e_{33}$ piezoelectric values are expected to show a maximum at $x = 0.12$.

3.2.3. Piezoelectric properties of in tetragonal $ABX_3$

In the following, we discuss the role of the $B$-$X$ Coulomb repulsions in piezoelectric $ABX_3$.

Figures 18(a) and 18(b) show the piezoelectric properties of $e_{33}$ as a function of the value $c - c_{\text{cub}}$ in tetragonal $ABX_3$, where $c_{\text{cub}}$ denotes the $c$ lattice parameter in cubic $ABX_3$. $c - c_{\text{cub}}$ is a closely related parameter to $\eta$. For $ABX_3$: SrTiO$_3$, BaTiO$_3$, and PbTiO$_3$ with the $c$ lattice parameter and all the inner coordination optimized for fixed $a$, and BaTi$_1$Ni$_3$O$_7$ ($0 \leq x \leq 0.05$), Ba(Ti$_{1.3}$Nb$_3$)(O$_{1.5}$N$_2$)$_3$ ($0 \leq z \leq 0.125$), Ba$_{1-x}$Sr$_x$TiO$_3$ ($0 \leq y \leq 0.5$), BaTi$_1$Zr$_{1-x}$O$_3$ ($0 \leq x \leq 0.06$), and BiM'O$_3$ ($M' = Al, Sc$) with fully optimized, were prepared [15]. Note that $e_{33}$ becomes larger as $c - c_{\text{cub}}$ becomes smaller and that the trend of $e_{33}$ is almost independent of the kind of $A$ ions. Moreover, note also that $e_{33}$ of BaTi$_1$Ni$_3$O$_7$ and that of Ba(Ti$_{1.3}$Nb$_3$)(O$_{1.5}$N$_2$)$_3$ show much larger values than the other $ABX_3$.

Figure 18. $e_{33}$ as a function of $c - c_{\text{cub}}$ for different scales [15].

Let us discuss the relationship between $\partial u_i(k)/\partial \eta_j$ and $c - c_{\text{cub}}$ in BaTi$_1$Ni$_3$O$_7$ and BaTiO$_3$ in the following. Figures 19(a) and 19(b) show the $\partial u_i(k)/\partial \eta_j$ values. In these figures, O$_2$ and O$_3$ denote oxygen atoms along the [100] and [001] axes, respectively. Clearly, the absolute values of $\partial u_i(k)/\partial \eta_j$ in BaTi$_1$Ni$_3$O$_7$ are much larger than those in BaTiO$_3$. Moreover, in comparison with Fig. 18, properties of $e_{33}$ are closely related to those of $\partial u_i(k)/\partial \eta_j$. Figure 20(a) shows the difference between $R_{4,0O}$ and $r_8 + r_{10}$ along the [001] axis, and Fig. 20(b) shows the difference between $R_{4,0O}$ and $r_4 + r_{10}$ on the (100) plane for several $ABO_3$ as a function of $c - c_{\text{cub}}$. Clearly, the difference between $R_{4,0O}$ and $r_8 + r_{10}$ is closely related to $e_{33}$ shown in Fig. 18, rather than the difference between $R_{4,0O}$ and $r_4 + r_{10}$. Moreover, note that the difference in absolute value between $R_{4,0O}$ and $r_8 + r_{10}$ in BaTi$_1$Ni$_3$O$_7$ is much smaller than that in BaTiO$_3$. This result suggests that the (Ti$_{1.3}$-Ni$_3$)-O$_2$ Coulomb repulsion along the [001] axis in BaTi$_1$Ni$_3$O$_7$ is much smaller than the Ti-O Coulomb repulsion in BaTiO$_3$ and that therefore Ti$_{1.3}$Ni$_3$ ion of BaTi$_1$Ni$_3$O$_7$ can be displaced more easily along the [001] axis than Ti ion of
BaTiO$_3$. This must be a reason why the absolute value of $\delta \omega(k)/\delta \eta$ of Ti$_{1-x}$Ni$_x$ and O$_2$ ions in BaTi$_{1-x}$Ni$_x$O$_3$ is larger than those in BaTiO$_3$.

Figure 20. $\delta \omega(k)/\delta \eta$ as a function of $c - c_{\text{sub}}$: (a) BaTi$_{1-x}$Ni$_x$O$_3$ and (b) BaTiO$_3$ [15].

Figure 21(a) shows $\Delta E_{\text{total}}$ as a function of the displacement of the Ti$_{1-x}$Ni$_x$ ions with fixed crystal structures of fully-optimized BaTi$_{1-x}$Ni$_x$O$_3$. Calculations of $E_{\text{total}}$ were performed with the fixed crystal structures of previously optimized structures except Ti$_{1-x}$Ni$_x$ ions. The deviated value at the minimum point of $\Delta E_{\text{total}}$, i.e., $\partial(\Delta E_{\text{total}})/\partial \theta(Ti_{1-x}Ni_x)$, becomes
smaller as $x$ becomes larger. Moreover, both $e_{33}$ and $\partial u_3(Ti_1, Ni_1)/\partial \eta_3$ become larger as $x$ becomes larger, as shown in Figs. 18 and 19. This result is consistent with the result of SrTiO$_3$ shown in Fig. 15(a).

Let us discuss the above reasons in the following. $\partial (\Delta E_{\text{total}})/\partial u_3(Ti_1, Ni_1)$ can be written as

$$\frac{\partial \Delta E_{\text{total}}}{\partial u_3(Ti_1, Ni_1)} = \left( \frac{\partial \Delta E_{\text{total}}}{\partial \eta_3} \right) \left( \frac{\partial u_3(Ti_1, Ni_1)}{\partial \eta_3} \right)^{-1}. \quad (5)$$

As shown in Fig. 21(b), $\partial (\Delta E_{\text{total}})/\partial \eta_3$ is almost constant, and therefore, $\partial (\Delta E_{\text{total}})/\partial u_3(Ti_1, Ni_1)$ is almost proportional to $(\partial u_3(Ti_1, Ni_1)/\partial \eta_3)^{-1}$, i.e.,

$$\left( \frac{\partial \Delta E_{\text{total}}}{\partial u_3(Ti_1, Ni_1)} \right) \propto \left( \frac{\partial u_3(Ti_1, Ni_1)}{\partial \eta_3} \right)^{-1}. \quad (6)$$

On the other hand, according to Eq. (3), $e_{33}$ becomes larger as $\partial u_3(Ti_1, Ni_1)/\partial \eta_3$ becomes larger. This is a reason why $e_{33}$ becomes larger as $\partial (\Delta E_{\text{total}})/\partial u_3(Ti_1, Ni_1)$ becomes smaller. This result is consistent with the result of SrTiO$_3$ discussed in Sec. 3.2.1.

![Figure 21](image.png)

Figure 21. (a) $\Delta E_{\text{total}}$ as a function of $u_3(Ti_1, Ni_1)$ in BaTi$_{1-x}$Ni$_x$O$_3$. Results with $x = 0.05$, 0.04, and 0 are shown. Dashed lines denote guidelines of $\partial (\Delta E_{\text{total}})/\partial u_3(Ti_1, Ni_1)$ for each $x$. (b) $\Delta E_{\text{total}}$ as a function of $\eta_3$ for BaTi$_{1-x}$Ni$_x$O$_3$ and BaTiO$_3$ [15].

Finally, we comment on the difference in the properties between $e_{33}$ and $d_{33}$ in tetragonal $ABX_3$. Figures 22(a) and 22(b) show the piezoelectric properties of $d_{33}$ as a function of $c - c_{\text{cub}}$. Note that the trend of $d_{33}$ is closely dependent on the kind of $A$ ions. This result is in contrast with the trend of $e_{33}$ as shown in Fig. 18. As expressed in Eq. (4), $d_{33}$ is closely related to the elastic compliance $s_{ij}$ as well as $e_{33}$. In fact, the absolute value of $s_{ij}$ in BiRX$_3$ or PbBX$_3$ is generally larger than that in $ABX_3$ with alkaline-earth $A$ ions. This result must be due to the larger Coulomb repulsion of Bi-X or Pb-X derived from 6s electrons in Bi (Pb) ion.
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4. Conclusion

We have discussed a general role of the B-X Coulomb repulsions for ferroelectric and piezoelectric properties of tetragonal ABX₃, based on our recent papers and patents. We have found that both ferroelectric state and piezoelectric state are closely related to the B-X Coulomb repulsions as well as the B-X ones, as illustrated in Fig. 23(a). Moreover, as illustrated in Fig. 23(b), we have also found that εᵢ is closely related to the deviation at the minimum point of the ΔEᵢ.
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