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Adaptive Fuzzy Sliding-Mode Attitude Controller Design for Spacecrafts with Thrusters

Fu-Kuang Yeh
Department of Computer Science and Information Engineering, Chung Chou University of Science and Technology, Changhua, Taiwan, R.O.C.

1. Introduction

Space exploration has often been used as a proxy competition for geopolitical rivalries. The early era of space exploration was driven by a "space race" among might countries. Nowadays, many advanced nations have entered the space arena and competed with one another for the space technology development as they recognize the importance of space technologies for national strength. In general, physical exploration of space is conducted both by human spaceflights and by robotic spacecrafts. To promote the scientific standard of space vehicle performance, stability, and control, it requires analysis of the six degrees of freedom of the vehicle's flight. One is the translational motion in three dimensional axes; the other is the orientation about the vehicle's center of mass in these axes, known as pitch, roll and yaw. In this paper, the main goal is to propose an adaptive fuzzy sliding-mode controller for spacecrafts with thrusters to follow the predetermined trajectory in outer space by use of employing the spacecraft attitude control. By using fuzzy inference mechanism, the upper bounds of the lumped uncertainty can be estimated, and the adaptive theory with center adaption of membership functions is designed to estimate optimal upper bounds of the lumped uncertainty, respectively. For the above reasons, we know that the attitude control of spacecrafts for the space exploration is essential to successfully develop the space activities [1].

In order to deal with the nonlinear spacecraft attitude dynamics, we employ the quaternion representation [2-4] to model the equation of rotational motion and the time derivative of quaternion, so that the nonlinear attitude control is applicable. Using the quaternion representation, the global control effect can be fulfilled and the singularity problem, which will be faced with the discontinuity by the three- dimensional Euler's representation [5], can be avoided.

To cope with non-ideal factors surrounding the spacecraft under attitude control and to enhance the robustness property of the attitude tracking system, the sliding mode control has been employed by Hu et al. [6], which integrate both the command input shaping and sliding mode output feedback control techniques to investigate the vibration problem for a flexible spacecraft during attitude maneuvering, and by Yeh [7], which proposes two nonlinear attitude controllers for spacecrafts with thrusters to follow the predetermined trajectory to estimate parameters and eliminate disturbances. Servidia and Pena [8] present the attitude stabilization of a spacecraft using thrusters, considering from a practical point of view. Hu [9] proposes a dual-stage control system design scheme for rotational maneuvers and vibration stabilization of a flexible spacecraft in the presence of the parameter uncertainty and external disturbances as well as the control input saturation to actively suppress certain flexible modes. Xia et al. [10] present the adaptive law and the extended state observer for a spacecraft model that is nonlinear in dynamics with the inertia uncertainty and external disturbances to converge to the reference attitude states. Despite the popularity of such control technique, it is however well known that the chattering problem is worthy of more attention for the sake of practical deployment. Taking into consideration of the aforementioned reason, a guide to sliding mode control for practical implementation has been proposed by Young *et al*. [11]. Eker [12] proposes a second-order sliding mode control for uncertain plants using the equivalent control approach to improve the performance of control systems, in which second-order plant parameters are experimentally determined using input-output measured data.

A spacecraft equipped with thrusters can effectively control its acceleration direction [8, 13], which in turn implies that the maneuverability/controllability of the spacecraft can be greatly enhanced during the stage when the spacecraft is flying in the outer space; whereas Janhunen et al. [14] propose a space propulsion concept of electric solar wind sail using the natural solar wind dynamic pressure for producing spacecraft thrust.

Estimation theory is used to deal with estimating values of parameters based on measured/empirical data that have a disturbance component. A parameter estimation approach called adaptive control has been developed by Slotine [15, 16] to achieve accurate attitude tracking of a rigid spacecraft with large loads of unknown mass. Zou *et al.* [17] investigate the robust adaptive output feedback controller based on Chebyshev neural networks (CNN) for an uncertain spacecraft to counteract CNN approximation errors and external disturbances. Huang et al. [18] propose a robust adaptive PID-type controller incorporating a fuzzy logic system and a sliding-mode control action for compensating parameter uncertainties and the robust tracking performance. An adaptive fuzzy theory is generally employed to approximate unstructured uncertainties and dynamic disturbances, such as Tong et al. [19] discuss an adaptive fuzzy output feedback control approach for nonlinear systems to estimate unmeasured states; Islam and Liu [20] propose a robust adaptive fuzzy control system for the trajectory tracking control problem of robotic systems to approximate the certainty equivalent-based optimal controller and to cope with uncertainties.

In this paper, we investigate the adaptive fuzzy sliding-mode control for spacecrafts with thrusters, employing the fuzzy sliding-mode controller to estimate upper bounds of the lumped uncertainty, and the adaptive fuzzy sliding-mode controller with center adaption of membership functions to estimate optimal bounds of the lumped uncertainty, respectively. This paper is organized as follows. In Section 2, preliminaries for deriving three-degree-of-freedom attitude models of a spacecraft equipped with thrusters. In Section 3, we respectively propose the sliding-mode, the fuzzy sliding-mode and the adaptive fuzzy sliding-mode attitude controllers aiming for tracking the predetermined trajectory in outer space. For tracking realization, three simulation results incorporating the so-called quaternion-based attitude control are developed in Section 4. To demonstrate the superior property of the proposed attitude controllers, three numerical simulations are provided in that Section. Finally, conclusions are drawn in Section 5.

2. Equations of rotational motion for spacecrafts

Assume the spacecraft is a rigid body; therefore, the Euler's equation of rotational motion is adopted with the following general form as

$$J\dot{\Omega} = -\dot{J}\Omega - \Omega \times (J\Omega) + T_b + D \,, \tag{1}$$

where all the variables are defined in and please referring to the Nomenclature.

Assume that the movable nozzle is located at the center of the spacecraft tail, and the distance between the movable nozzle center and the spacecraft's center of gravity is ℓ. Furthermore, we also assume that the spacecraft is equipped with a number of thrusters on the surface near the center of gravity that will produce a pure rolling moment whose direction is aligned with the vehicle axis, X_b, referring to Fig. 1. Thus, the vector L_b, defined as the relative displacement from the spacecraft's center of gravity to the center of the movable nozzle, satisfies $|L_b| = \ell$. Note that J is the moment of inertia matrix for a spacecraft with respect to the body coordinate frame, and hence is a 3×3 symmetric matrix. After referring to Fig. 1 and Fig. 2, the torque exerted on the spacecraft can be expressed in the body coordinate frame as

$$T_b = L_b \times F_{Tb} + M_b = \ell n \begin{bmatrix} m_{bx} / \ell n \\ \sin d_p \\ -\cos d_p \sin d_y \end{bmatrix}, \tag{2}$$

where n is the magnitude of the movable nozzle thrust, d_p and d_y are the pitch angle and yaw angle, respectively, of the movable nozzle, $M_b = [m_{bx} \quad 0 \quad 0]^T$ is the aforementioned variable moment in the axial direction of the spacecraft, and F_{Tb} is the force produced by the movable nozzle in the body coordinate frame.

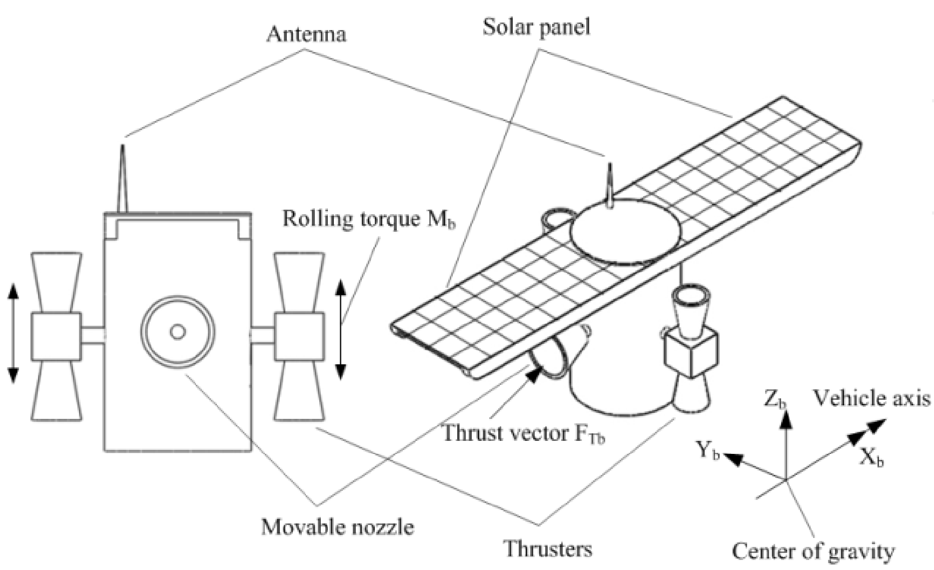


Fig. 1. Scheme of a spacecraft with the movable nozzle and fixed thrusters.

Let the rotation matrix B_b denote the transformation from the body coordinate frame to the inertial coordinate frame. Thus, the force exerted on the spacecraft observed in the inertial coordinate system is as follows:

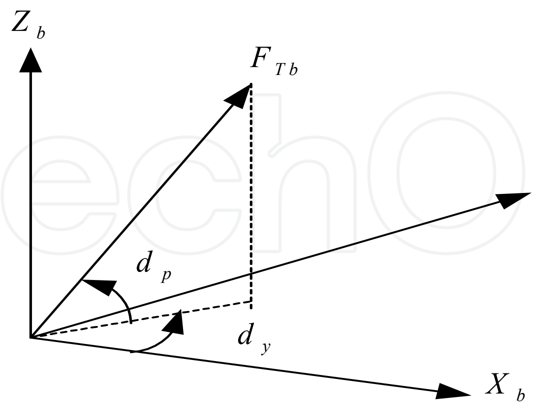


Fig. 2. Two angles of the movable nozzle in the body coordinate frame.

$$F_M = B_b F_{Mb} \ . \tag{3}$$

From Eqs. (1) and (2), the rotational motion model of a spacecraft can then be derived as

$$J\dot{\Omega} = -\dot{J}\Omega - \Omega \times (J\Omega) + \ell n \begin{bmatrix} m_{bx} / \ell n \\ \sin d_p \\ -\cos d_p \sin d_y \end{bmatrix} + D \ , \tag{4}$$

where $D = \begin{bmatrix} d_1 & d_2 & d_3 \end{bmatrix}^T$ is a disturbance vector in the body coordinate frame.

Generally speaking, the attitude of a rigid body may be described in various ways, and "quaternion" is one of the means. According to Euler's rotation theory [21], there exist a unit vector U and an angle ϕ such that U is perpendicular to the rotation plane with respect to a rotation angle ϕ. Thus, for any quaternion, it can be defined as four parameters $Q = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^T = \begin{bmatrix} \bar{Q}^T & q_4 \end{bmatrix}^T$ involving U and ϕ, i.e.,

$$\begin{aligned} \bar{Q} &= \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = U \sin(\phi/2), \\ q_4 &= \cos(\phi/2). \end{aligned} \tag{5}$$

In theory, it can be verified that the time derivative of a quaternion is a function of the corresponding angular velocity and the quaternion itself [2-4], i.e.,

$$\begin{aligned} \dot{\bar{Q}} &= \frac{1}{2}\langle \bar{Q} \times \rangle \Omega + \frac{1}{2} q_4 \Omega, \\ \dot{q}_4 &= -\frac{1}{2} \Omega^T \bar{Q} \end{aligned} \tag{6}$$

From Eqs.(1) and (6), the dynamic model of a spacecraft, treated as a rigid body, can be derived by differentiation of the angular velocity and the associated error quaternion as a function of the corresponding angular velocity and its error as well as the error quaternion itself, i.e.,

$$\begin{aligned} \dot{\bar{Q}}_e &= \frac{1}{2}\langle \bar{Q}_e \times \rangle \Omega_e + \frac{1}{2} q_{e4}\Omega_e \\ \dot{q}_{e4} &= -\frac{1}{2}\Omega_e^T \bar{Q}_e \\ J\dot{\Omega} &= -\dot{J}\Omega - \Omega \times (J\Omega) + T_b + D \end{aligned} \tag{7}$$

where $\Omega_e = \Omega - \Omega_d$ is the error between the angular velocity at the present attitude and the desired attitude, and T_b is the torque exerted on the spacecraft due to the movable nozzle and the rolling moment. Whereas the error quaternion $Q_e = \begin{bmatrix} q_{e1} & q_{e2} & q_{e3} & q_{e4} \end{bmatrix}^T = \begin{bmatrix} \bar{Q}_e^T & q_{e4} \end{bmatrix}^T$, $\bar{Q}_e^T = \begin{bmatrix} q_{e1} & q_{e2} & q_{e3} \end{bmatrix}$, is defined as the required rotation from the initial quaternion $Q = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^T$ to the desired quaternion $Q_d = \begin{bmatrix} q_{d1} & q_{d2} & q_{d3} & q_{d4} \end{bmatrix}^T$, and can be derived in matrix form [2-4] as

$$Q_e = \begin{bmatrix} q_{e1} \\ q_{e2} \\ q_{e3} \\ q_{e4} \end{bmatrix} = \begin{bmatrix} q_{d4} & q_{d3} & -q_{d2} & -q_{d1} \\ -q_{d3} & q_{d4} & q_{d1} & -q_{d2} \\ q_{d2} & -q_{d1} & q_{d4} & -q_{d3} \\ q_{d1} & q_{d2} & q_{d3} & q_{d4} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}, \tag{8}$$

and $\langle \bar{Q}_e \times \rangle = \begin{bmatrix} 0 & -q_{e3} & q_{e2} \\ q_{e3} & 0 & -q_{e1} \\ -q_{e2} & q_{e1} & 0 \end{bmatrix}$ is a skew-symmetric matrix.

3. Nonlinear attitude controller design

3.1 Sliding-mode attitude controller design

Considering the presence of model uncertainties, parameter variations, and disturbances, we recognized that the sliding mode control is an effectively robust controller for various applications; in this paper, we consider a sliding-mode control to eliminate all variation influences of a spacecraft during the whole flying course for the practical controller design. We first design a sliding-mode attitude controller, which can compensate for the adverse effect owing to spacecraft variations.

The principal procedure to verify the stability and robustness of the sliding-mode attitude tracking problem consists of the sliding and reaching conditions, and that will be given in detail as follows.

Step 1: Choose the sliding surface such that the sliding condition will be satisfied and hence the origin of the error dynamic is exponentially stable.

From the sliding mode theory, once the reaching condition is satisfied, the system is eventually forced to stay on the sliding surface, i.e.,

$$S = P\bar{Q}_e + \Omega_e = 0\,, \tag{9}$$

where P is a positive definite diagonal matrix. The system dynamics are then constrained by the following differential equations, referring to Eq. (7), as

$$\begin{aligned} \dot{\bar{Q}}_e &= -\frac{1}{2}\langle \bar{Q}_e \times \rangle P\bar{Q}_e - \frac{1}{2} q_{e4} P\bar{Q}_e, \\ \dot{q}_{e4} &= \frac{1}{2}\bar{Q}_e^T P\bar{Q}_e. \end{aligned} \tag{10}$$

Define another Lyapunov function $V_e(\bar{Q}_e) = \frac{1}{2}\bar{Q}_e^T\bar{Q}_e$, then

$$\begin{aligned} \dot{V}_e(\bar{Q}_e) &= \bar{Q}_e^T\dot{\bar{Q}}_e \\ &= -\frac{1}{2}\bar{Q}_e^T(\langle \bar{Q}_e \times \rangle P\bar{Q}_e + q_{e4}P\bar{Q}_e)\,, \\ &= -\frac{1}{2}q_{e4}\bar{Q}_e^T P\bar{Q}_e \end{aligned} \tag{11}$$

where $\bar{Q}_e^T \langle \bar{Q}_e \times \rangle = 0$.

Recalling that the quaternion definition q_{e4} in general has two possible values different only in sign and the sign can be arbitrarily chosen to meet the design convenience. For the sake of design and analysis, q_{e4} is selected as $q_{e4}\big|_{t=0} = c > 0$, and because $\dot{q}_{e4} = \frac{1}{2}\bar{Q}_e^T P\bar{Q}_e \geq 0$, hence we can conclude that q_{e4} is a positive and growing variable, i.e., $q_{e4}(t) \geq c$. By quaternion definition, a quaternion always satisfies the so-called unit-norm property. That is, $q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$, which implies that $1 \geq q_{e4}$, and hence $c \leq q_{e4}(t) \leq 1$, $\forall t \geq 0$, so that the following relation will hold as

$$-\frac{c}{2}\bar{Q}_e^T P\bar{Q}_e \geq \dot{V}_e(\bar{Q}_e) \geq -\frac{1}{2}\bar{Q}_e^T P\bar{Q}_e\,. \tag{12}$$

By Lyapunov stability theory, it can be proved that $\bar{Q}_e$ will be driven to zero when the system is constrained on the sliding mode dynamics and so will the error angular velocity Ω_e. For the above reason, the system origin $\left(\bar{Q}_e, \Omega_e\right) = \left(0_{3\times1}, 0_{3\times1}\right)$ of the ideal system can be verified to be exponentially stable.

Step 2: Design the controller such that the reaching condition is satisfied.

Let us define the sliding surface as above equation (9) shown as $S = P\bar{Q}_e + \Omega_e$, where $P = diag[p_1 \quad p_2 \quad p_3]$ is a 3×3 positive definite diagonal matrix. Here, we make an assumption that J is a symmetric and positive definite matrix, and let the Lyapunov function candidate be set as

$$V_s = \frac{1}{2} S^T J S \,, \tag{13}$$

where $V_s = 0$ only when $S = 0$. Then, the time derivative of V_s can be derived as

$$\dot{V}_s = S^T J \dot{S} + \frac{1}{2} S^T \dot{J} S \,. \tag{14}$$

Substituting Eq. (7) and the time derivative of Eq. (9) into the above equation (14), we have

$$\begin{aligned} \dot{V}_s &= S^T J \dot{S} + \frac{1}{2} S^T \dot{J} S \\ &= S^T \left[J\dot{\Omega} - J\dot{\Omega}_d + JP\dot{\bar{Q}}_e + \frac{1}{2}\dot{J}S \right] \\ &= S^T \left[-\dot{J}\Omega - \Omega \times (J\Omega) + T_b + D - J\dot{\Omega}_d + JP\left(\frac{1}{2}\langle \bar{Q}_e \times \rangle \Omega_e + \frac{1}{2} q_{e4} \Omega_e \right) + \frac{1}{2}\dot{J}S \right] \end{aligned} \tag{15}$$

Let the control torque input T_b be proposed as

$$T_b = -K_s S + \dot{J}_0 \Omega - \frac{1}{2}\dot{J}_0 S - J_0 P\left(\frac{1}{2}\langle \bar{Q}_e \times \rangle \Omega_e + \frac{1}{2} q_{e4}\Omega_e \right) + \Omega \times (J_0 \Omega) + J_0 \dot{\Omega}_d + \Lambda_s \,, \tag{16}$$

where $K_s = diag\begin{bmatrix} k_{s1} & k_{s2} & k_{s3} \end{bmatrix}$ is a positive definite diagonal matrix, and $\Lambda_s = \begin{bmatrix} \lambda_{s1} & \lambda_{s2} & \lambda_{s3} \end{bmatrix}^T$, $\lambda_{si} = -c_{si}(Q,\Omega,Q_d,\dot{Q}_d,\ddot{Q}_d) \cdot \mathrm{sgn}(s_i)$, with $\mathrm{sgn}(s_i) = \begin{cases} 1 & s_i > 0 \\ 0 & s_i = 0 \\ -1 & s_i < 0 \end{cases}$, $i = 1,2,3$, and $S = \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix}^T$ is a sliding surface. Let the external disturbance D and the induced 2-norm of $\Delta\dot{J}$ and ΔJ are all bounded, where $J = J_0 + \Delta J$, $\dot{J} = \dot{J}_0 + \Delta\dot{J}$. If the inequality condition shown below can be guaranteed

$$c_{si}(Q,\Omega,Q_d,\dot{Q}_d,\ddot{Q}_d) > \delta_i^{\max}\left(Q,\Omega,Q_d,\dot{Q}_d,\ddot{Q}_d\right) \geq |\delta_i| \,,\ i = 1,2,3 \,, \tag{17}$$

where

$$\begin{aligned} \bar{\delta} &= \begin{bmatrix} \delta_1 & \delta_2 & \delta_3 \end{bmatrix}^T \\ &= -\Delta\dot{J}\Omega - \Omega \times (\Delta J \Omega) + D - \Delta J \dot{\Omega}_d + \Delta J P\left(\frac{1}{2}\langle \bar{Q}_e \times \rangle \Omega_e + \frac{1}{2} q_{e4}\Omega_e \right) + \frac{1}{2}\Delta\dot{J}S \,' \end{aligned} \tag{18}$$

where bounding functions δ_i, $i = 1,2,3$ are obviously functions of Q, Ω, Q_d, $\dot{Q}_d$ and $\ddot{Q}_d$, then the exponential stability and robustness of the proposed controller for attitude tracking can be achieved.

It is evident that Eq. (15) becomes

$$
\begin{aligned}
\dot{V}_s &= -S^T K_s S - \sum_{i=1}^{3} |s_i| \left[c_{si} - \delta_i \operatorname{sgn}(s_i) \right] \\
&\le -S^T K_s S - \sum_{i=1}^{3} |s_i| \left[c_{si} - \delta_i^{\max} \right] \quad , \\
&\le -\sigma_{\min}(K_s) |S|^2 < 0
\end{aligned}
\tag{19}
$$

for $S \neq 0$, where $\sigma_{\min}(K_s)$ is the minimum eigenvalue of K_s, where K_s is a positive definite diagonal matrix as above mentioned. Therefore, the reaching and sliding conditions of the sliding mode $S = 0$ are guaranteed. As a result, the exponential stability and robustness of the sliding mode attitude controller can be achieved.

Remark 1: However, due to the existence of non-ideality in the practical implementation of the sign function $\operatorname{sgn}(s_i)$, the control law T_b in (16) always suffers from the chattering problem. To alleviate such undesirable phenomenon, the sign function can be simply replaced by the saturation function. The system is now no longer forced to stay on the sliding surface but is constrained within the boundary layer $|s_i| \le \varepsilon$, where ε is a small positive value. The cost of such substitution is a reduction in the accuracy of the desired performance.

To alleviate the chattering phenomenon, the saturation function may be employed to the control input of the sliding mode attitude control system. Consequently, the term $\Lambda_s = \begin{bmatrix} \lambda_{s1} & \lambda_{s2} & \lambda_{s3} \end{bmatrix}^T$ in Eq. (16) can be replaced by

$$
\lambda_{si} = -c_{si}(Q, \Omega, Q_d, \dot{Q}_d, \ddot{Q}_d) \cdot Sat(s_i, \varepsilon) , \tag{20}
$$

where $Sat(s_i, \varepsilon) = \begin{cases} 1 & s_i > \varepsilon \\ \dfrac{s_i}{\varepsilon} & |s_i| \le \varepsilon \\ -1 & s_i < -\varepsilon \end{cases}$, $i = 1, 2, 3$.

3.2 Fuzzy sliding-mode attitude controller design

Upper bounds of the lumped uncertainty, which includes the external disturbances and internal perturbations, for the sliding-mode control need to be decided before the controller is using. In general, upper bounds of the lumped uncertainty are difficult to be obtained in advanced by computing, but always by the method of try and error. In this section, a fuzzy sliding-mode controller is proposed, in which a fuzzy inference mechanism is used to estimate upper bounds of the lumped uncertainty. We recognized that the prior expert knowledge of the fuzzy inference mechanism, which can be used to estimate upper bounds of the lumped uncertainty, is available effectively.

Values of c_{s1}, c_{s2}, and c_{s3} in Eq. (17) can be estimated by the fuzzy inference mechanism. The control block diagram of the fuzzy sliding-mode controller is depicted as in Fig. 3. Based on fuzzy set theory, associated fuzzy sets involved in fuzzy control rules are defined and listed as follows:

PB: positive big; PM: positive medium; PS: positive small; ZE: zero;
NS: negative small; NM: negative medium; NB: negative big.

Here universes of discourse for inputs s_i, $\dot{s}_i$, $i = 1,2,3$ and outputs $\hat{c}_{si}$, $i = 1,2,3$ are assigned to be $[-50,\ \ 50]$, $[-6000,\ \ 6000]$, and $[-50,\ \ 50]$, respectively. Membership functions for the fuzzy sets corresponding to switching surfaces s_i, $i = 1,2,3$, its derivative $\dot{s}_i$, $i = 1,2,3$ and upper bounds of the lumped uncertainty $\hat{c}_{si}$, $i = 1,2,3$, are defined in Fig. 4.

Because the seven fuzzy subsets NB, NM, NS, ZE, PS, PM, and PB are used to divide every element of sliding surfaces s_i, $i = 1,2,3$ and its derivative $\dot{s}_i$, $i = 1,2,3$, respectively, the fuzzy inference mechanism contains 49 rules. The resulting fuzzy inference rules are given as the following Table 1.

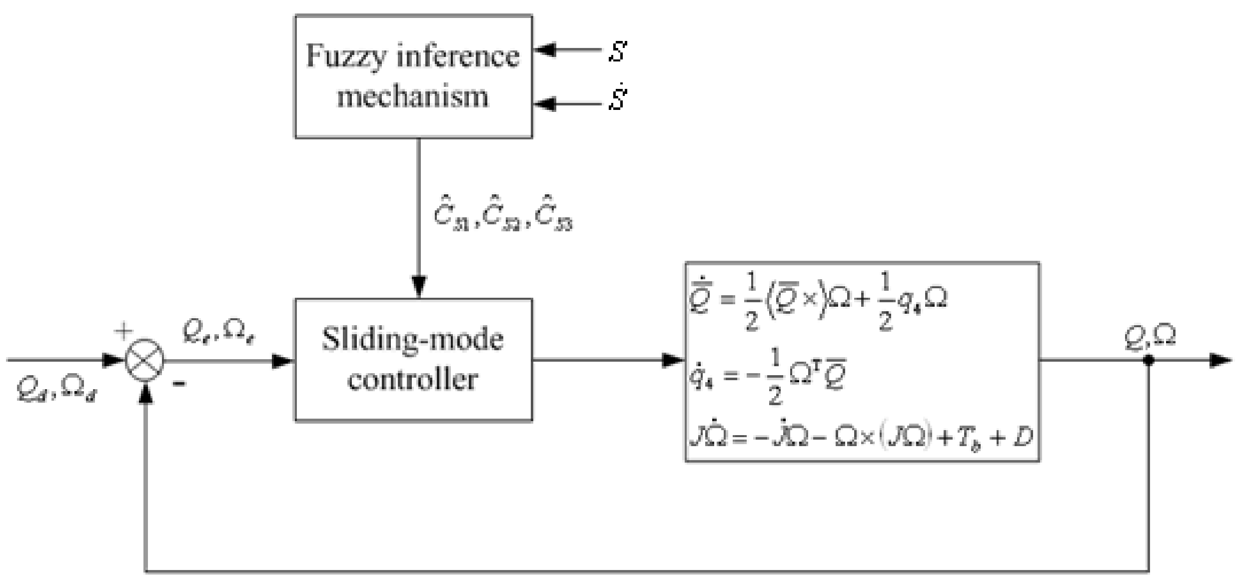


Fig. 3. Control block diagram of the fuzzy sliding-mode controller.

$\dot{s}_i$ \ s_i	NB	NM	NS	ZE	PS	PM	PB
PB	ZE	PS	PS	PM	PM	PB	PB
PM	NS	ZE	PS	PS	PM	PM	PB
PS	NS	NS	ZE	PS	PS	PM	PM
ZE	NM	NS	NS	ZE	PS	PS	PM
NS	NM	NM	NS	NS	ZE	PS	PS
NM	NB	NM	NM	NS	NS	ZE	PS
NB	NB	NB	NM	NM	NS	NS	ZE

Table 1. Rule base with 49 rules.

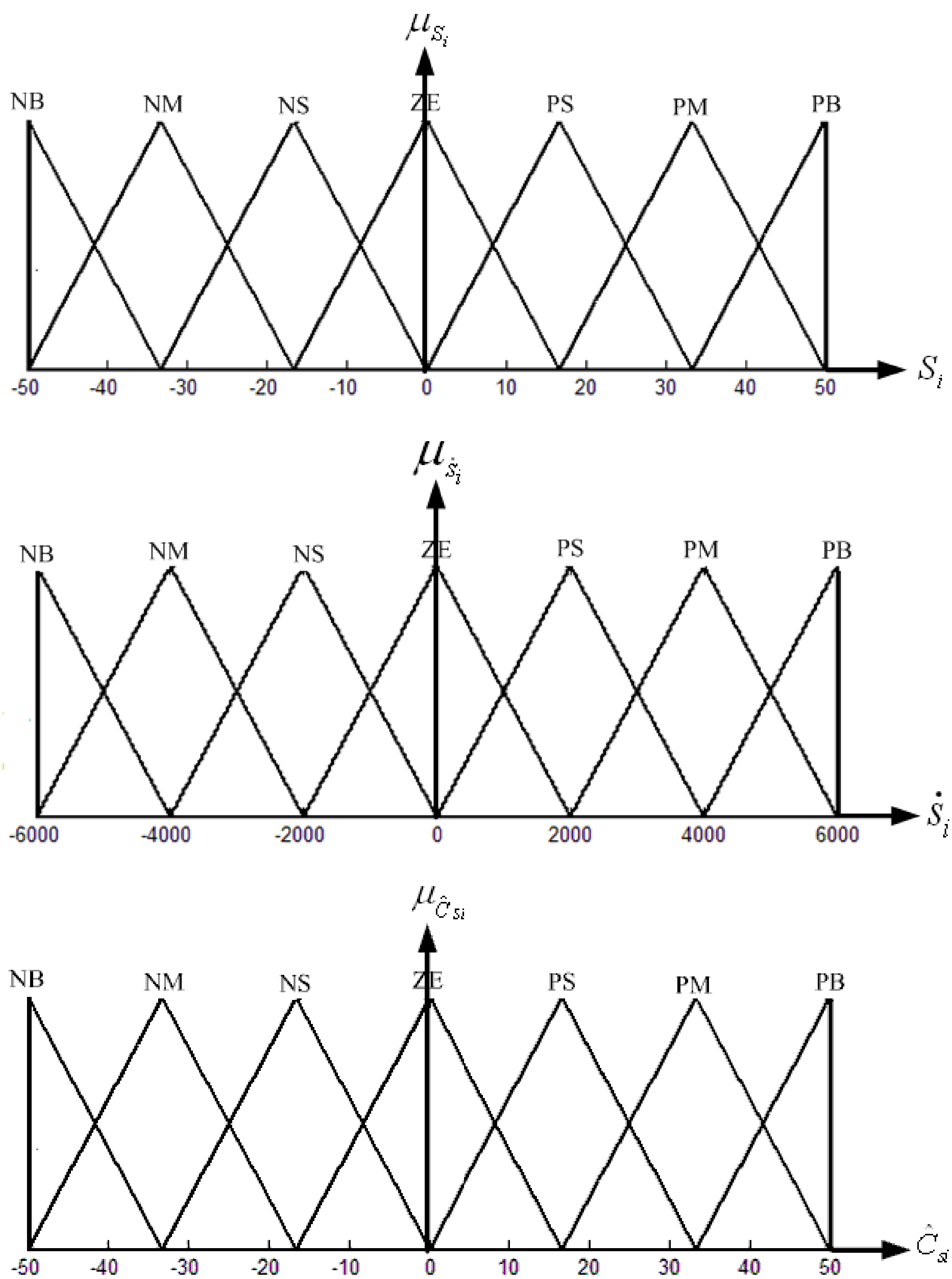


Fig. 4. Membership functions of fuzzy sets.

The fuzzy outputs $\hat{c}_{si}$, $i = 1,2,3$, can be calculated by the centre of area defuzzification as

$$\hat{c}_{si} = \left| \frac{\sum_{j=1}^{49} \omega_{ij} c_{ij}}{\sum_{j=1}^{49} \omega_{ij}} \right| = \left| \frac{\begin{bmatrix} c_{i1} & \cdots & c_{i49} \end{bmatrix} \begin{bmatrix} \omega_{i1} \\ \vdots \\ \omega_{i49} \end{bmatrix}}{\sum_{j=1}^{49} \omega_{ij}} \right| = \left| C_i^T W_i \right|, \ i = 1,2,3, \tag{21}$$

where $C_i^T = \begin{bmatrix} c_{i1} & \cdots & c_{i49} \end{bmatrix}$ is an adjustable parameter vector, c_{i1} through c_{i49} are the centre of the membership functions of $\hat{c}_{si}$, $W_i^T = \dfrac{\begin{bmatrix} \omega_{i1} & \cdots & \omega_{i49} \end{bmatrix}}{\sum_{j=1}^{49} \omega_{ij}}$ is a firing strength vector. Here, the absolute value of $C_i^T W_i$ in Eq. (21) is used to satisfy the requirement of estimating upper bounds, so that upper bounds of the lumped uncertainty are greater than or equal to zero, that is, $\hat{c}_{si} \geq 0$, $i = 1,2,3$.

The following Lemma is introduced to enable the fuzzy sliding-mode controller such that the reaching condition of the switching surface is satisfied. By Lyapunov stability theory, the fuzzy sliding-mode attitude controller of the spacecraft can be proved as an exponentially stable system. The sufficient conditions for successful stability effect are stated in the following lemma.

Lemma 1 Fuzzy sliding-mode attitude controller: Let the dynamic model of a spacecraft corresponding the angular velocity and the quaternion be given by Eqs. (1) and (6), and if the control torque input T_b is proposed as

$$T_b = -K_s S + \dot{J}_0 \Omega - \frac{1}{2} \dot{J}_0 S - J_0 P \left(\frac{1}{2} \left\langle \bar{Q}_e \times \right\rangle \Omega_e + \frac{1}{2} q_{e4} \Omega_e \right) + \Omega \times \left(J_0 \Omega \right) + J_0 \dot{\Omega}_d + \hat{\Lambda}_s , \tag{22}$$

where $K_s = diag\begin{bmatrix} k_{s1} & k_{s2} & k_{s3} \end{bmatrix}$ is a positive definite diagonal matrix, and $\hat{\Lambda}_s = \begin{bmatrix} \hat{\lambda}_{s1} & \hat{\lambda}_{s2} & \hat{\lambda}_{s3} \end{bmatrix}^T$, $\hat{\lambda}_{si} = -\hat{c}_{si}(C_i, W_i) \cdot \mathrm{sgn}(s_i)$, with $\mathrm{sgn}(s_i) = \begin{cases} 1 & s_i > 0 \\ 0 & s_i = 0 \\ -1 & s_i < 0 \end{cases}$, $i = 1,2,3$, and $S = \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix}^T$ is a sliding surface that are defined as Eq. (9). Let the external disturbance D and the induced 2-norm of $\Delta\dot{J}$ and ΔJ are all bounded, where $J = J_0 + \Delta J$, $\dot{J} = \dot{J}_0 + \Delta\dot{J}$. And values of $\hat{c}_{s1}$, $\hat{c}_{s2}$, and $\hat{c}_{s3}$ are chosen to be positive big enough and are computed by Eq. (21). Then the exponential stability and robustness of the fuzzy sliding-mode attitude control system can be achieved.

Proof: See the Appendix 1. From the proof in the Appendix 1, the exponential stability and robustness of the fuzzy sliding-mode attitude controller can be guaranteed, so that the spacecraft attitude tracking system can be achieved completely.

3.3 Adaptive fuzzy sliding-mode attitude controller design

Since the sliding-mode attitude controller for a spacecraft requires estimating upper bounds (constant values) of the lumped uncertainty, so that the lumped uncertainty during the entire flying course can be always eliminated. If upper bounds are not chosen appropriately, the high-gain problem will be suffered. That is, the control input torque is over-large definitely and the implementation cost is increased as well. Optimal upper bounds c_{si}, $i=1,2,3$, cannot be obtained exactly by pure sliding mode control owing to the unknown of uncertainties. Therefore, the adaptive fuzzy control algorithm based on the principle of sliding mode control is developed to estimate optimal upper bounds of the lumped uncertainty and to achieve the minimum control torque.

Assume there exists estimated upper bounds $\hat{c}_{si}$, $i=1,2,3$ which can achieve minimum control torque and satisfy the sliding-mode condition of the switching surface, errors between estimated and optimal upper bounds are shown as

$$\hat{c}_{si} - c_{si} = \tilde{c}_{si}\,,\ i=1,2,3\,, \tag{23}$$

where error upper bounds $\tilde{c}_{si}$, $i=1,2,3$ are small values. Here, estimated upper bounds can be computed as Eq. (21).

Assume there exists estimated upper-bound vector $\hat{C}_s = \begin{bmatrix}\hat{c}_{s1} & \hat{c}_{s2} & \hat{c}_{s3}\end{bmatrix}^T$, which can achieve minimum control torque and satisfy the sliding-mode condition of the switching surface, the error vector between the estimated and optimal upper-bound vectors are shown as

$$\hat{C}_s - C_s = \tilde{C}_s\,, \tag{24}$$

The following theorem is introduced to enable the adaptive fuzzy sliding-mode controller such that optimal upper bounds can be obtained. By Lyapunov stability theory, the adaptive fuzzy sliding-mode attitude controller of a spacecraft can be proved as an exponentially stable system. The sufficient conditions for the successful stability effect are stated in the following theorem.

Theorem 1 Adaptive fuzzy sliding-mode attitude controller: Let the dynamic model of a spacecraft corresponding the angular velocity and the quaternion be given by Eqs. (1) and (6), and if the control torque input T_b is proposed as

$$T_b = -K_s S + \dot{J}_0\Omega - \frac{1}{2}\dot{J}_0 S - J_0 P\left(\frac{1}{2}\left\langle \overline{Q}_e \times\right\rangle \Omega_e + \frac{1}{2}q_{e4}\Omega_e\right) + \Omega\times\left(J_0\Omega\right) + J_0\dot{\Omega}_d + \overline{\Lambda}_s\,, \tag{25}$$

where $K_s = diag\begin{bmatrix}k_{s1} & k_{s2} & k_{s3}\end{bmatrix}$ is a positive definite diagonal matrix, $\overline{\Lambda}_s = \begin{bmatrix}\overline{\lambda}_{s1} & \overline{\lambda}_{s2} & \overline{\lambda}_{s3}\end{bmatrix}^T$, $\overline{\lambda}_{si} = -c_{si}(\int s_i dt)\cdot \operatorname{sgn}(s_i)$, and $S = \begin{bmatrix}s_1 & s_2 & s_3\end{bmatrix}^T$ is a sliding surface. Let the external disturbance D and the induced 2-norm of $\Delta\dot{J}$ and ΔJ are all bounded, where $J = J_0 + \Delta J$, $\dot{J} = \dot{J}_0 + \Delta\dot{J}$, whereas $\hat{c}_{si}$, $i=1,2,3$ is estimated upper bounds that are assumed to be optimal, and the bounding function is shown as Eq. (18).

For adapting upper bounds of the lumped uncertainty, the adaptation laws can be let as

$$\dot{c}_{si} = -\Gamma_i \left| s_i \right|, \ i = 1,2,3, \tag{26}$$

where the optimal upper-bound vector is defined as $C_s = \begin{bmatrix} c_{s1} & c_{s2} & c_{s3} \end{bmatrix}^T$ and $\Gamma = diag(\Gamma_1, \Gamma_2, \Gamma_3)$ is a positive definite diagonal matrix. Then the exponential stability and robustness of the adaptively fuzzy sliding-mode attitude control system can be achieved.

Proof: To achieve the exponential stability and convergence of the spacecraft attitude tracking system, let us define the sliding surface as above equation (9) shown as $S = P\bar{Q}_e + \Omega_e$, where $P = diag[p_1 \quad p_2 \quad p_3]$ is a 3×3 positive definite diagonal matrix. Here, we also make an assumption that J is a symmetric and positive definite matrix, now we choose the Lyapunov-like function as

$$V = \frac{1}{2} S^T J S + \frac{1}{2} \tilde{C}_s^T \Gamma^{-1} \tilde{C}_s. \tag{27}$$

where $S = \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix}^T$ is the sliding surface, $\tilde{C}_s = \begin{bmatrix} \tilde{c}_{s1} & \tilde{c}_{s2} & \tilde{c}_{s3} \end{bmatrix}^T$ is the error upper-bound vector between the optimal upper-bound vector C_s and the estimated upper-bound vector $\hat{C}_s$, which elements are computed by Eq. (21), and $\Gamma^{-1} = diag(\Gamma_1^{-1}, \Gamma_2^{-1}, \Gamma_3^{-1})$ is a positive definite diagonal matrix. Then, the time derivative of V can be derived as

$$\dot{V} = S^T J \dot{S} + \frac{1}{2} S^T \dot{J} S + \tilde{C}_s^T \Gamma^{-1} \dot{\tilde{C}}_s. \tag{28}$$

Substituting Eqs. (1) and (6) and the time derivative of Eq. (9) into the above equation (28), we have

$$\begin{aligned} \dot{V} &= S^T \left[J\dot{\Omega} - J\dot{\Omega}_d + JP\dot{\bar{Q}}_e + \frac{1}{2}\dot{J}S \right] + \tilde{C}_s^T \Gamma^{-1} \dot{C}_s \\ &= S^T \left[-\dot{J}\Omega - \Omega \times (J\Omega) + T_b + D - J\dot{\Omega}_d + JP\left(\frac{1}{2}\langle \bar{Q}_e \times \rangle \Omega_e + \frac{1}{2} q_{e4} \Omega_e \right) + \frac{1}{2}\dot{J}S \right] + \tilde{C}_s^T \Gamma^{-1} \dot{C}_s \end{aligned} \tag{29}$$

where $\dot{\tilde{C}}_s = \dot{C}_s$, $\tilde{C}_s = C_s - \hat{C}_s$, here assume $\hat{C}_s$ is a slow-varying vector in which those elements are computed by Eq. (21), so that the first-time derivative of $\hat{C}_s$ can be neglected.

Let the control torque input T_b be proposed as Eq. (25), in which $\bar{\Lambda}_s = \begin{bmatrix} \bar{\lambda}_{s1} & \bar{\lambda}_{s2} & \bar{\lambda}_{s3} \end{bmatrix}^T$, $\bar{\lambda}_{si} = -c_{si}(\int s_i dt) \cdot \mathrm{sgn}(s_i)$, with $\mathrm{sgn}(s_i) = \begin{cases} 1 & s_i > 0 \\ 0 & s_i = 0 \\ -1 & s_i < 0 \end{cases}$, $i = 1,2,3$, where the values of c_{s1}, c_{s2}, and c_{s3} are optimal upper bounds. For adapting upper bounds of the lumped uncertainty, adaptation laws can be given as Eq. (26), and by the verification of fuzzy sliding-mode control in the Lemma 1, the inequality condition as Eq. (A.2) can be obtained. Bounding functions δ_i,

$i=1,2,3$ can also be defined as Eq. (18) and estimated upper bounds $\hat{c}_{si}$, $i=1,2,3$ can be computed by Eq. (21).

It is evident that Eq. (29) becomes

$$\begin{aligned}\dot{V} &= -S^T K_s S - \sum_{i=1}^{3} |s_i| \left[c_{si} - \delta_i \operatorname{sgn}(s_i) \right] - \sum_{i=1}^{3} |s_i| \left[\hat{c}_{si} - c_{si} \right] \\ &\le -S^T K_s S - \sum_{i=1}^{3} |s_i| \left[\hat{c}_{si} - \delta_i^{\max} \right] \\ &\le -\sigma_{\min}(K_s) |S|^2 < 0 \end{aligned} \tag{30}$$

for $S \ne 0$, where $\sigma_{\min}(K_s)$ is the minimum eigenvalue of K_s, where K_s is a positive definite diagonal matrix as above mentioned. Therefore, the reaching and sliding conditions of the sliding mode $S=0$ are guaranteed. As a result, the exponential stability and robustness of the adaptive fuzzy sliding mode attitude controller can be achieved.

Therefore, the global stability of the attitude tracking system is guaranteed by the proposed adaptive fuzzy sliding-mode controller. In the following, let us in detail verify the convergence of the states $\bar{Q}_e$ and Ω_e and of the parameters $\tilde{C}_s$.

First, the function V in Eq. (27) is a Lyapunov-like function, in our case simply a positive continuous function of time. Expression (30) shows that output errors converge to the sliding surface [16], so that $S = 0_{3\times 1}$. And because that a quaternion always satisfies the so-called unit-norm property, we have $\bar{Q}_e$, Ω_e are bounded. Let us now detail the proof itself. Since $\dot{V}$ is negative or zero and V is lower bounded, and because S, $\bar{Q}_e$, Ω_e are bounded. So, in turn, from Eq. (30), we have $\ddot{V} = -2S^T K_s \dot{S} - \sum_{i=1}^{3} \left[\hat{c}_{si} - \delta_i \operatorname{sgn}(s_i) \right] \dot{s}_i \operatorname{sgn}(s_i)$ exists and is bounded as a result that $\dot{S}$ is bounded, and referring to Eq. (5), we have that $\bar{Q}_e$, $\dot{\bar{Q}}_e$, and $\ddot{\bar{Q}}_e$ are all bounded, referring to reference [22], we have that $\dot{\Omega}_e$ is a function of $\ddot{\bar{Q}}_e$, such that $\dot{\Omega}_e$ is bounded. Therefore we can concluded that $\ddot{V}$ is uniformly continuous on $t \in [0, \infty)$. Consequently by Barbalat's lemma, $\dot{V}$ tends to zero as $t \to \infty$. This implies from Eq. (30) that $S \to 0$ as $t \to \infty$.

Now, given that S converges to zero only exponentially, the actual differential equation that governs $\bar{Q}_e$ by some exponentially decaying term can be given as

$$S = P\bar{Q}_e + \Omega_e = \varepsilon(t), \tag{31}$$

where $\varepsilon(t) = e^{-k_a t} S_0$ is an exponentially decaying function of time t, whereas $S_0 = P\bar{Q}_e + \Omega_e \big|_{t=0}$.

From Eqs. (7) and (31), the dynamics of $\bar{Q}_e$ are

$$\begin{aligned}\dot{\bar{Q}}_e &= \frac{1}{2}\langle\bar{Q}_e\rangle\Omega_e + \frac{1}{2}q_{e4}\Omega_e \\ &= \frac{1}{2}\left[\langle\bar{Q}_e\rangle\left(-P\bar{Q}_e+\varepsilon\right) + q_{e4}\left(-P\bar{Q}_e+\varepsilon\right)\right] \\ &= -\frac{1}{2}\left(\langle\bar{Q}_e\times\rangle P\bar{Q}_e + q_{e4}P\bar{Q}_e\right) + \frac{1}{2}\left(\langle\bar{Q}_e\times\rangle + q_{e4}I_{3\times3}\right)\varepsilon \\ &= f_1(\bar{Q}_e)+f_2(\bar{Q}_e)\end{aligned} , \tag{32}$$

where $f_1(\bar{Q}_e) = -\frac{1}{2}\left(\langle\bar{Q}_e\times\rangle P\bar{Q}_e + q_{e4}P\bar{Q}_e\right)$ and $f_2(\bar{Q}_e) = \frac{1}{2}\left(\langle\bar{Q}_e\times\rangle + q_{e4}I_{3\times3}\right)\varepsilon$. As indicated by Eq. (7), $\dot{\bar{Q}}_e = f_1(\bar{Q}_e)$ is an exponentially stable system, implying that the stability and convergence of Eq. (32) are governed by $f_2(\bar{Q}_e)$. If $f_2(\bar{Q}_e)\to 0$ exponentially as $t\to 0$, then the system is reduced to system Eq. (7), which is an exponentially stable system. From Eq. (31), ε is an exponentially decaying function, and because $\left|\frac{1}{2}\left(\langle\bar{Q}_e\times\rangle + q_{e4}I_{3\times3}\right)\right|$ is bounded, $\left|\frac{1}{2}\left(\langle\bar{Q}_e\times\rangle + q_{e4}I_{3\times3}\right)\right| \le \sigma$ for some constant $\sigma > 0$, $\left|f_2(\bar{Q}_e)\right| \le \sigma\varepsilon$ also becomes an exponentially decaying function, which now truly ensures the stability and convergence of Eq. (32).

To verify the convergence of the error parameter vector $\tilde{C}_s$, by definition in Lemma 1, due to estimated values $\hat{c}_{si} = \left|C_i^T W_i\right|$, $i=1,2,3$ being positive values, the estimated parameter vector $\hat{C}_s$ is a slow-varying vector, referring to Fig. 4. From Eq. (30), S is an exponentially decaying function vector of time t, we can obtain that the optimal vector C_s is a bounded vector by integrating Eq. (26). Finally, the bounded convergence of the error parameter vector $\tilde{C}_s = C_s - \hat{C}_s$ in Eq. (24) can be confirmed.

Remark 2: Similar to alleviate the chattering phenomenon, the saturation function may also be employed to the control input of the adaptive fuzzy sliding-mode control system. Consequently, the control torque T_b in Eq. (25) can be re-expressed as

$$\begin{aligned}T_b &= -K_s S + \dot{J}_0\Omega - \frac{1}{2}\dot{J}_0 S - J_0 P\left(\frac{1}{2}\langle\bar{Q}_e\times\rangle\Omega_e + \frac{1}{2}q_{e4}\Omega_e\right) \\ &+\Omega\times\left(J_0\Omega\right) + J_0\dot{\Omega}_d - C_s Sat(S,\varepsilon)\end{aligned} , \tag{33}$$

where $C_s Sat(S,\varepsilon) = \left[c_{s1}Sat(s_1,\varepsilon) \quad c_{s2}Sat(s_2,\varepsilon) \quad c_{s3}Sat(s_3,\varepsilon)\right]^T$, whereas

$$Sat(s_i,\varepsilon) = \begin{cases} 1 & s_i > \varepsilon \\ \dfrac{s_i}{\varepsilon} & \left|s_i\right| \le \varepsilon \\ -1 & s_i < -\varepsilon \end{cases}, \quad i = 1,2,3 .$$

Now the system will not be forced to strictly stay on the sliding surface but constrained within the boundary layer $\left|s_i\right| \le \varepsilon$.

4. Simulations

To validate the proposed attitude tracking problem, we present three nonlinear controllers respectively consisting of the sliding-mode attitude controller design in Section 3.1, the fuzzy sliding-mode attitude controller design in Section 3.2 and the adaptive fuzzy sliding-mode attitude controller design in Section 3.3 for a rigid spacecraft, so as to demonstrate the performance and effectiveness of the respective controller designs.

For simulation the initial conditions of quaternion and angular velocity are set as $Q(0) = \begin{bmatrix} 0.866 & -0.212 & 0.283 & 0.354 \end{bmatrix}^T$ and $\Omega(0) = [0 \quad 0 \quad 0]^T$, respectively. Furthermore, the desired values of the quaternion $Q_d(t) = \begin{bmatrix} 0.985 & 0.174\cos(0.2t) & 0.174\sin(0.2t) & 0 \end{bmatrix}^T$ with period 10π seconds, the angular velocity $\Omega_d(t) = 0_{3\times1}$, and its time derivative $\dot{\Omega}_d(t) = 0_{3\times1}$ are also be given. Of course, the setting elements of the desired quaternion must satisfy the unit-norm property of quaternion, i.e., $Q_d^T Q_d = 1$.

The above initial conditions of $\Omega_d(0) = 0_{3\times1}$ and $\dot{\Omega}_d(0) = 0_{3\times1}$ will have two principle advantages. One is to simplify the simulation procedure but not to lose the practical implementation; the other is to smoothly stabilize the spacecraft's flying in the outer space. On the other hand, the nominal part and the uncertain part of inertia matrix of a spacecraft is set as

$$J_0 = \begin{bmatrix} a_1 & a_4 & a_5 \\ a_4 & a_2 & a_6 \\ a_5 & a_6 & a_3 \end{bmatrix} = \begin{bmatrix} 950 & 10 & 5 \\ 10 & 600 & 30 \\ 5 & 30 & 360 \end{bmatrix}, \ \Delta J = \begin{bmatrix} 95 & 1 & 0.5 \\ 1 & 60 & 3 \\ 0.5 & 3 & 36 \end{bmatrix}.$$

And the variation and the variation uncertain part of the inertial matrix are set as

$$\dot{J}_0 = \begin{bmatrix} -0.95 & -0.01 & -0.005 \\ -0.01 & -0.6 & -0.03 \\ -0.005 & -0.03 & -0.36 \end{bmatrix}, \ \Delta\dot{J} = \begin{bmatrix} -0.095 & -0.001 & -0.0005 \\ -0.001 & -0.06 & -0.003 \\ -0.0005 & -0.003 & -0.036 \end{bmatrix}.$$

Further, we also consider the disturbance vector $D = \begin{bmatrix} d_1 & d_2 & d_3 \end{bmatrix}^T$, where the disturbances d_1, d_2, and d_3, which are containing real white Gaussian noises of power 1 dBW, given in Eq. (1), are shown in Fig. 3(e). In our simulation, some positive definite diagonal matrices of P (see Eq. (9)) for the sliding surface and K_s (see Eq. (16)) for the control torque input will also be shown as follows.

$$P = \begin{bmatrix} 120 & 0 & 0 \\ 0 & 120 & 0 \\ 0 & 0 & 120 \end{bmatrix}, \ K_s = \begin{bmatrix} 1400 & 0 & 0 \\ 0 & 1400 & 0 \\ 0 & 0 & 1400 \end{bmatrix},$$

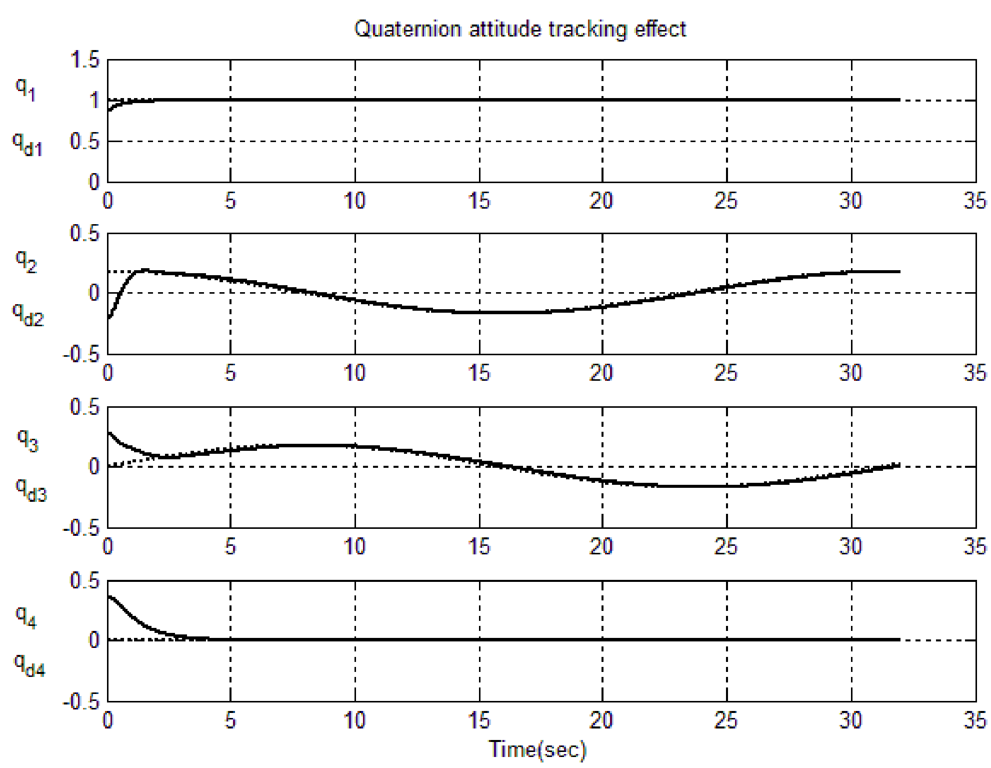
where parameters c_{s1}, c_{s2}, c_{s3} of the sliding-mode controller are appropriately set as $c_{s1} = c_{s2} = c_{s3} = 10$ (see Eq. (17)) to satisfy the better stability requirements of the overall system, such that external disturbances can be eliminated completely.

4.1 Simulation results of the sliding-mode attitude controller

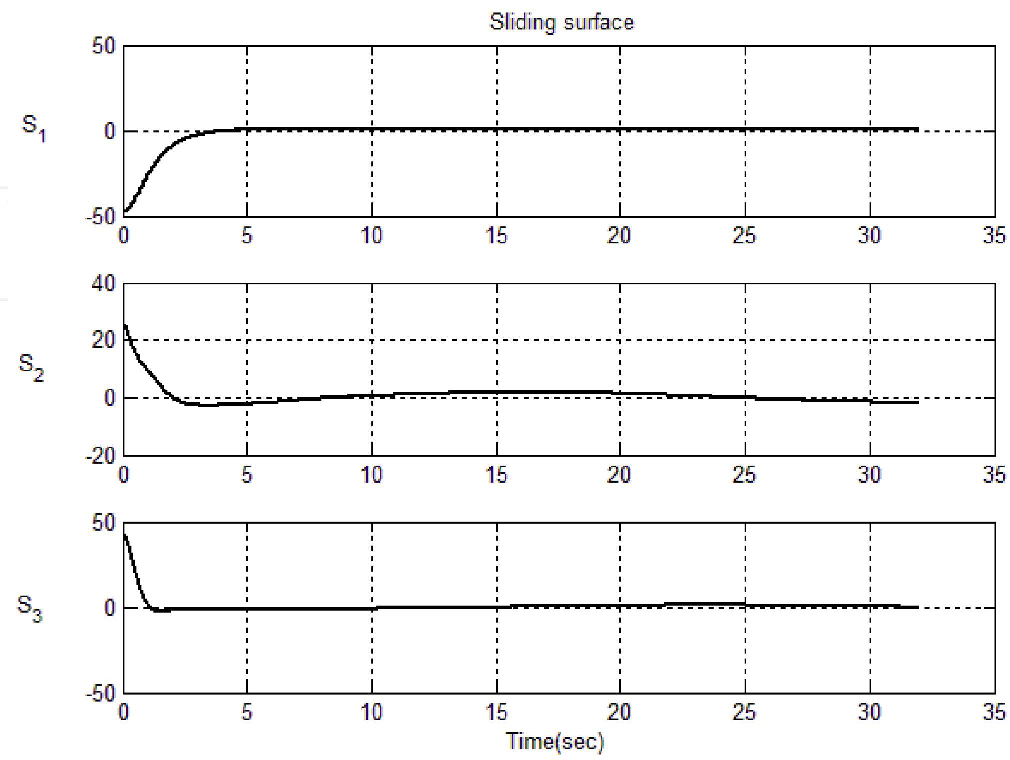
The appealing effect of the sliding-mode attitude control presented in quaternion form is given in Fig. 5(a), which shows the present attitude and the desired one simultaneously. The solid line in each sub-figure denotes the current quaternion of the spacecraft, where the dashed line denotes the desired attitude in Fig. 5(a), we can see that the current and desired attitudes are coincident with each other, that is, the attitude tracking effect is fulfilled after about 3 seconds. This is to show feasible of the conclusion from Eq. (19) and to show well results of attitude tracking of the spacecraft. Here, the robustness and effectiveness can simultaneously be demonstrated by results shown in Fig. 3 for the attitude control system. The sliding surface and the torque input of the sliding-mode controller with varied desired attitude are shown in Figs. 5(b) and 5(c), respectively to demonstrate the practical effect. Variations of the moment of inertia matrix J and disturbances D, which are used in the proposed three nonlinear controllers, are shown in Figs. 3(d) and 3(e), respectively.

4.2 Simulation results of the fuzzy sliding-mode attitude controller

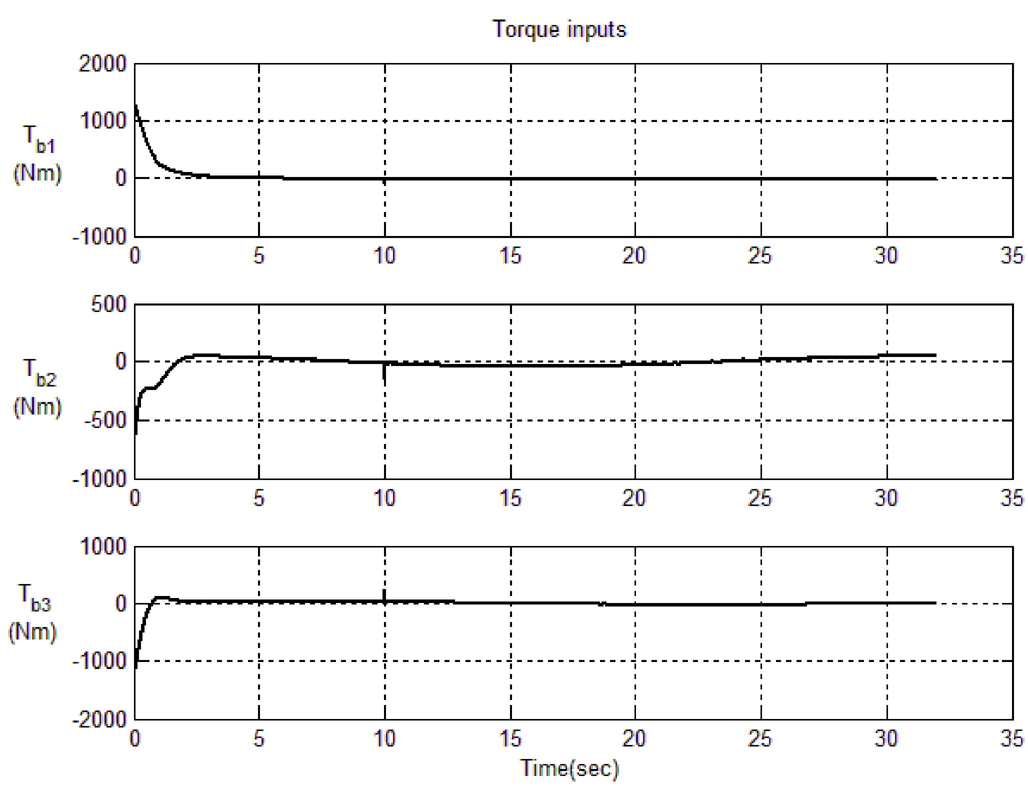
In this section, the revealing response of the fuzzy sliding-mode attitude control is given in Fig. 6(a), which also shows the present attitude and the desired one simultaneously. The solid line in each sub-figure denotes the current quaternion of the spacecraft, where the dashed line denotes the desired attitude in Fig. 6(a), we can see that the current and desired attitudes are coincident with each other. From Fig. 6(a), the attitude tracking effect is fulfilled almost totally after 3 seconds. This is to show feasible of the conclusion from Eq. (A.1) and to show well the results of attitude tracking of the spacecraft. The sliding surface and the torque input of the fuzzy sliding-mode controller with varied desired attitude are



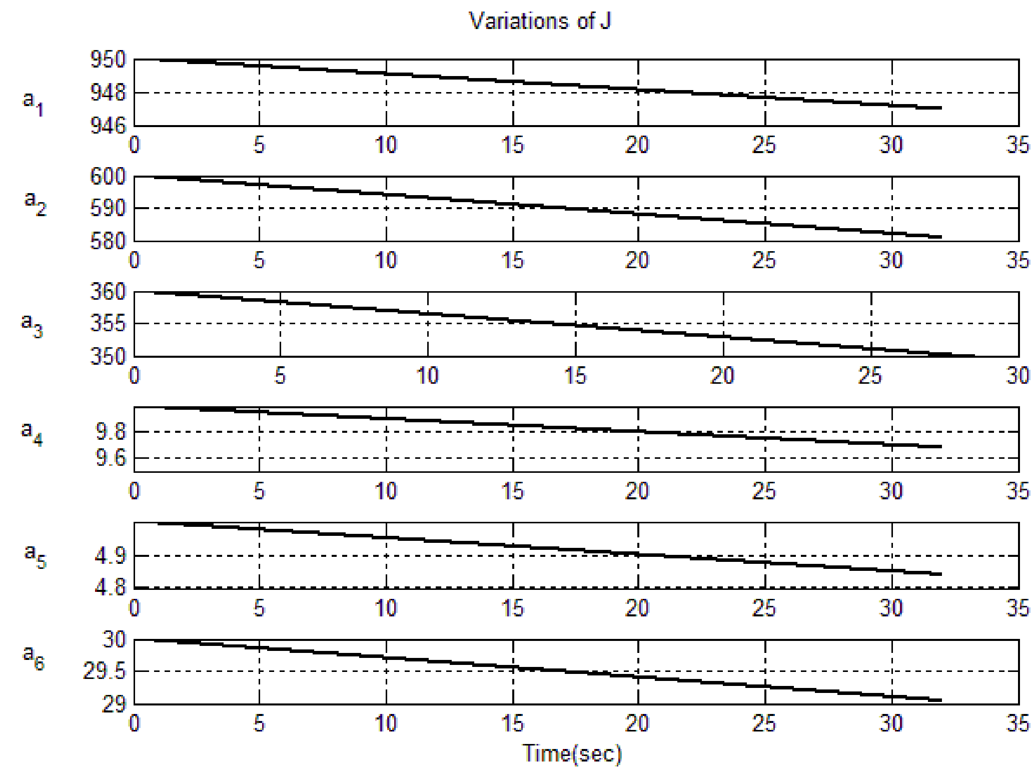
(a)


Sliding surface
S1
S2
S3
Time(sec)

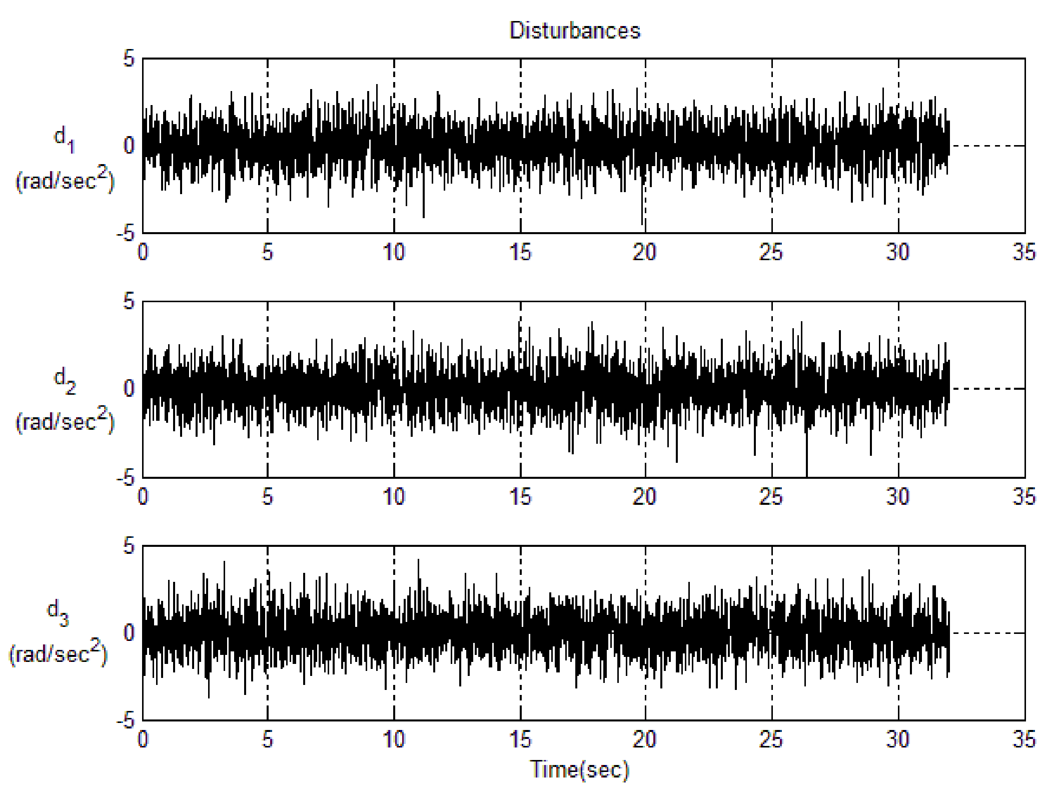
(b)


Torque inputs
Tb1 (Nm)
Tb2 (Nm)
Tb3 (Nm)
Time(sec)

(c)

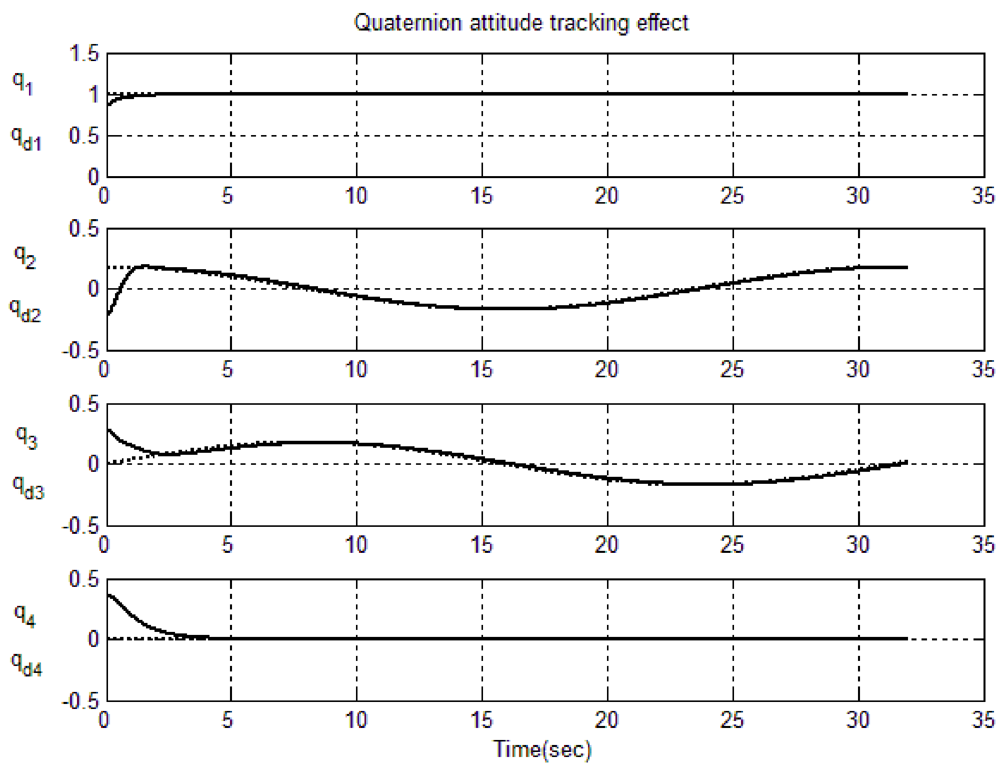


(d)

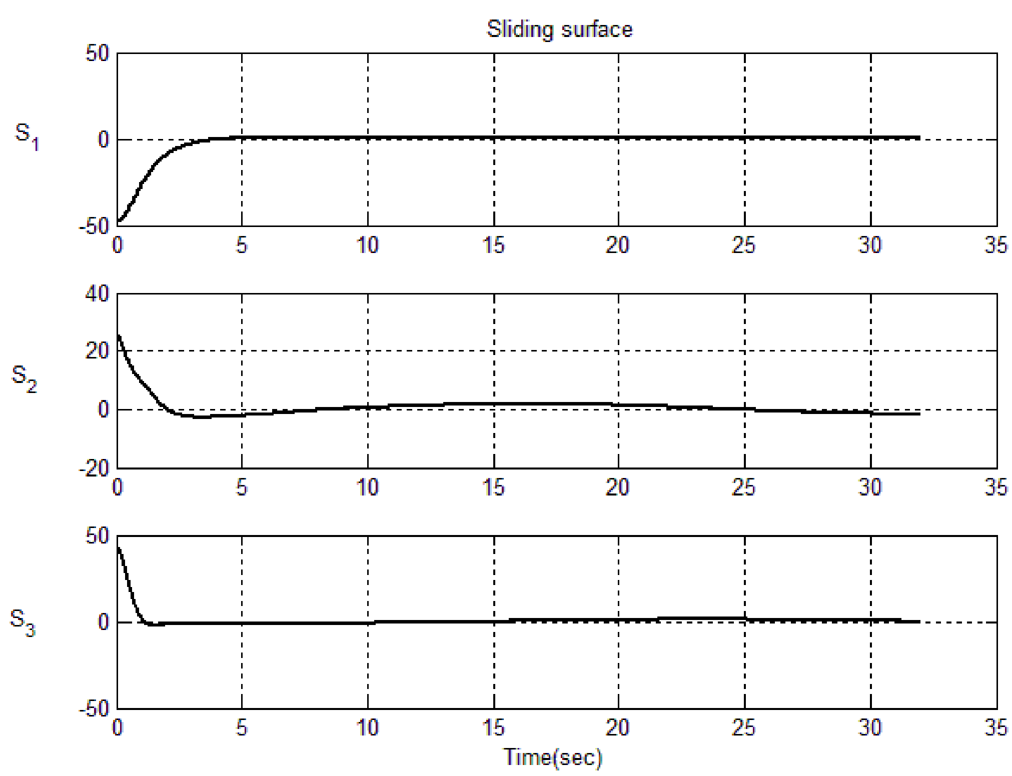


(e)

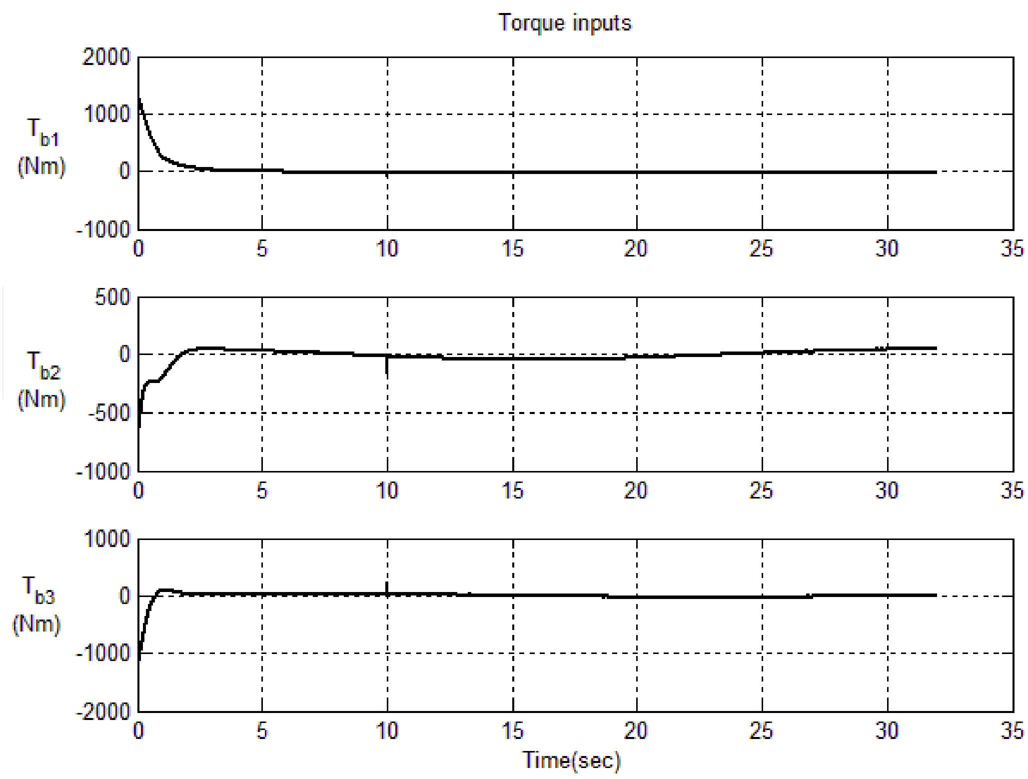
Fig. 5. Simulation results of (a) quaternion attitude tracking effects, (b) the convergence of sliding surface, (c) control torque inputs; (d) variations of J, and (e) real white Gaussian noises of power 1 dBW of the sliding-mode controller for spacecrafts.


Quaternion attitude tracking effect
q_1
q_{d1}
q_2
q_{d2}
q_3
q_{d3}
q_4
q_{d4}
Time(sec)

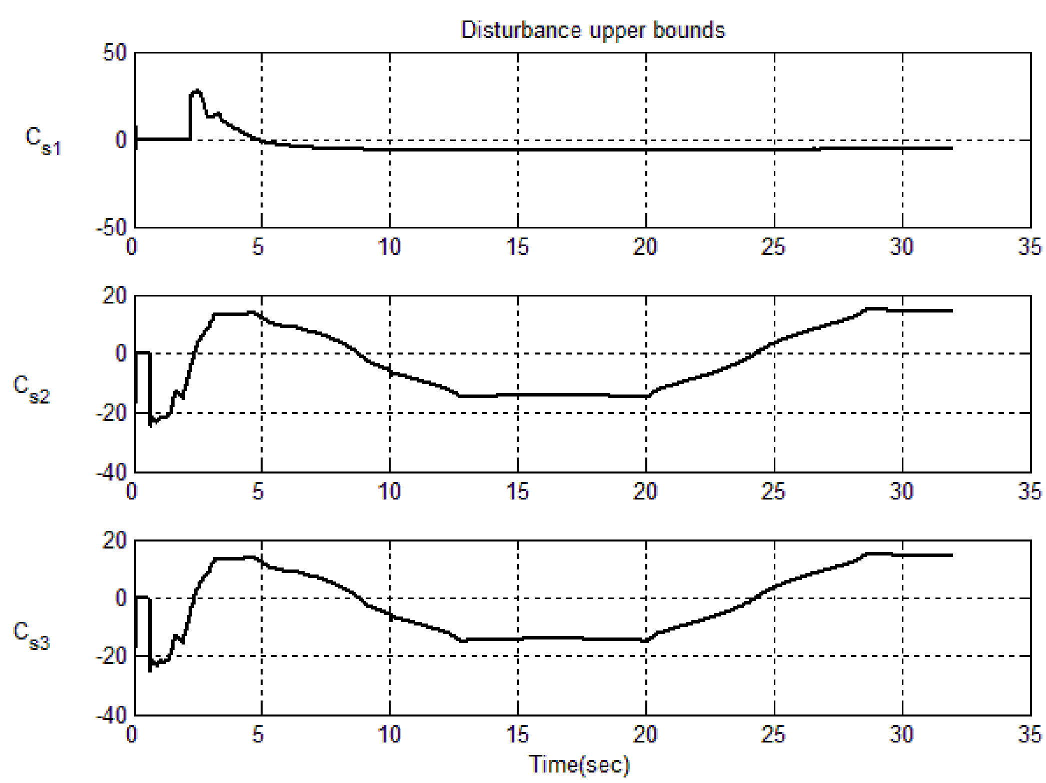
(a)


Sliding surface
S_1
S_2
S_3
Time(sec)

(b)



(c)



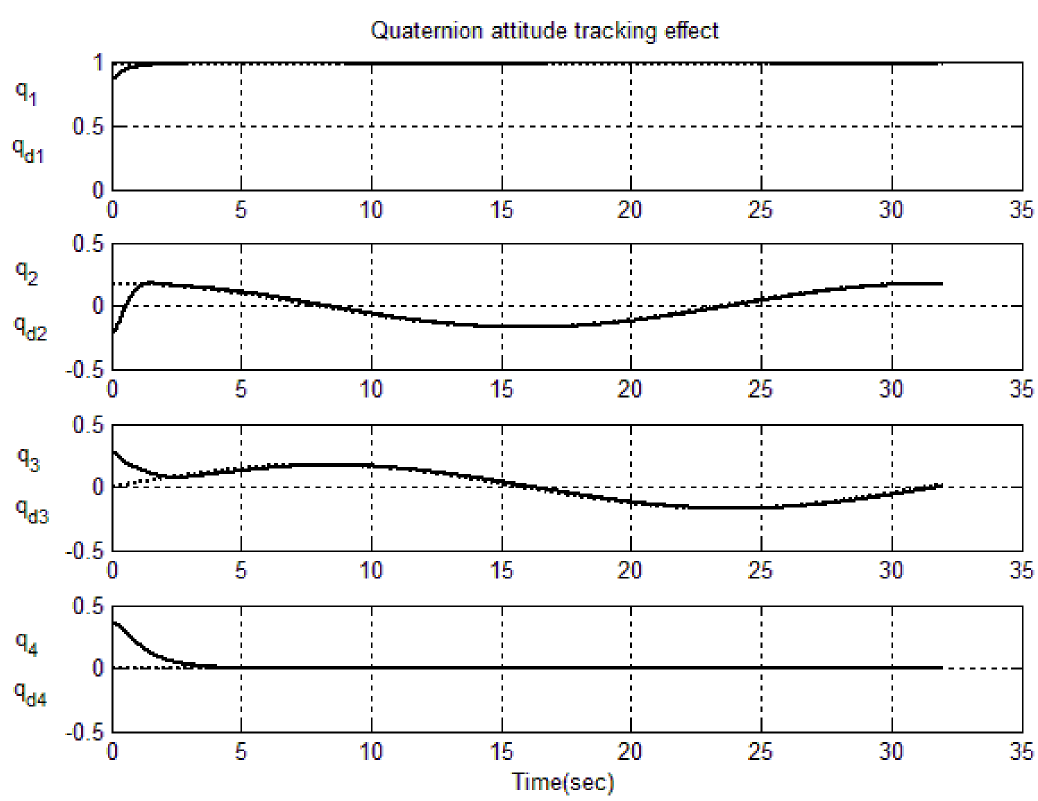
(d)

Fig. 6. Simulation results of (a) quaternion attitude tracking effects, (b) the convergence of sliding surface, (c) control torque inputs, and (d) estimated upper bounds of the lumped uncertainty of the fuzzy sliding-mode controller for spacecrafts.

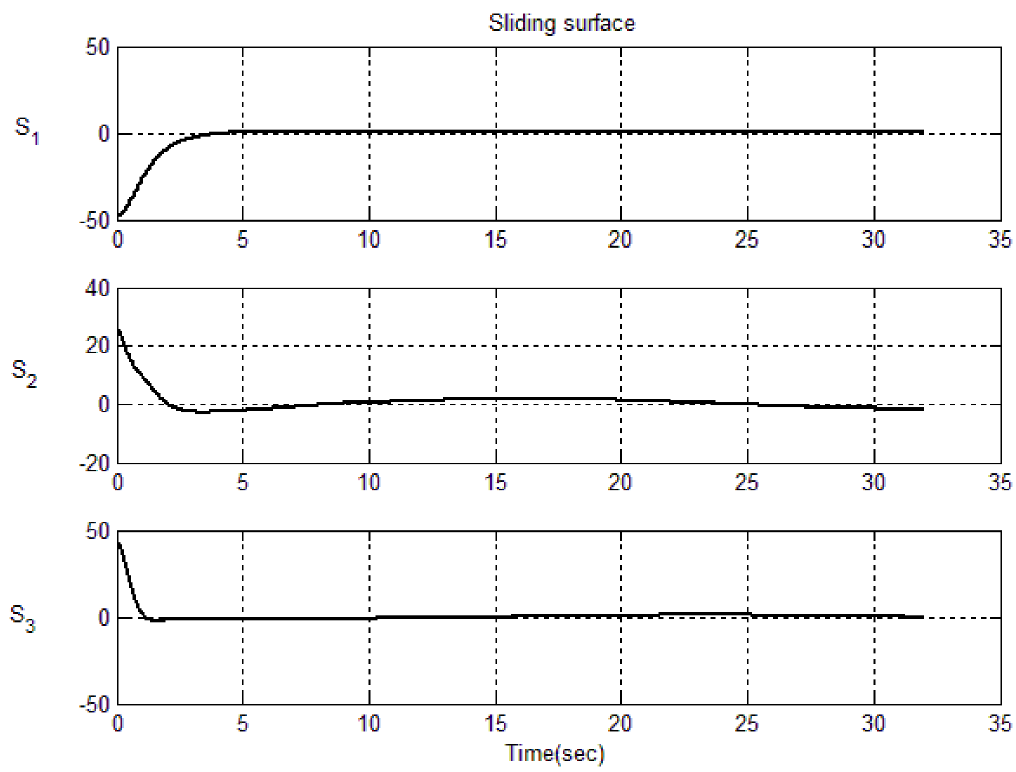
shown in Figs. 6(b) and 6(c), respectively to demonstrate the tracking effect. Slow-varying upper bounds of the lumped uncertainty and disturbances D are shown in Figs. 6(d) and 5(e), respectively.

4.3 Simulation results of adaptive fuzzy sliding-mode attitude controller

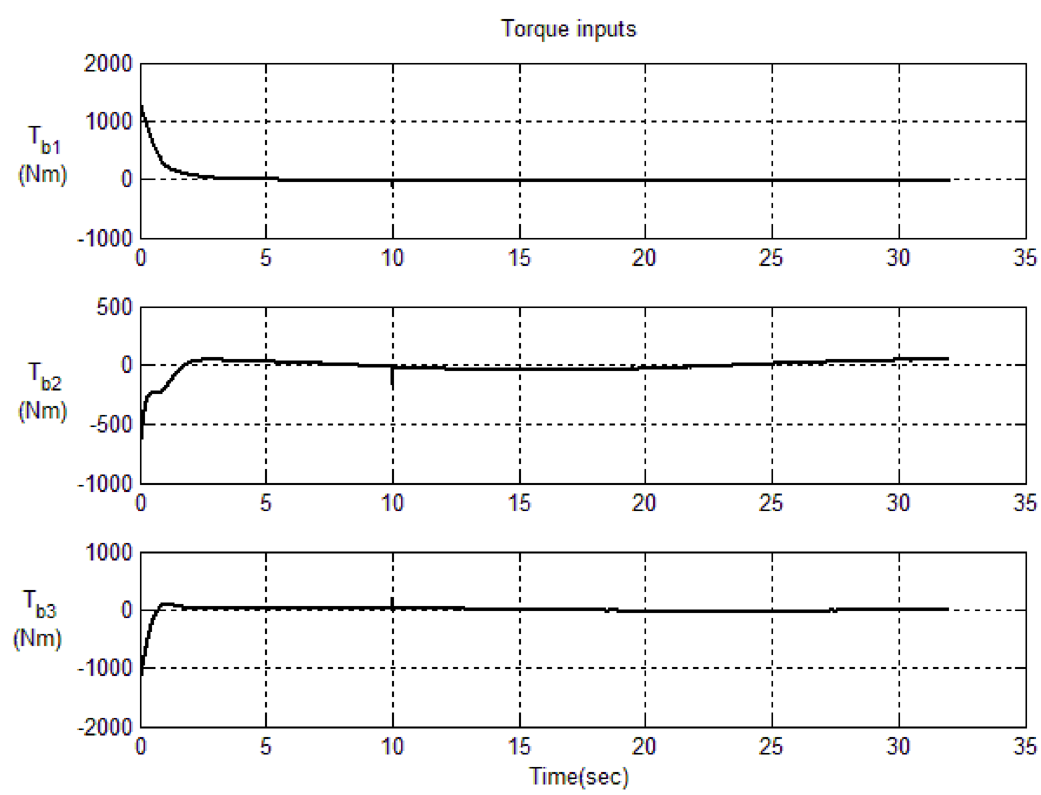
The revealing response of the adaptive fuzzy sliding-mode attitude control also presented in quaternion form is given in Fig. 7(a), which also shows the present attitude and the desired one simultaneously. The solid line in each sub-figure denotes the current quaternion of the spacecraft, where the dashed line denotes the desired attitude in Fig. 7(a), we can see the current and desired attitudes are coincident with each other. From Fig. 7(a), the attitude tracking effect can be seen to be fulfilled after about 3 seconds. This is to show feasible of the conclusion from Eq. (30) and to show well results of spacecraft attitude tracking. Here, the robustness and effectiveness can simultaneously be demonstrated by the results shown in Fig. 7 for the attitude control system. The sliding surface and the torque input of the adaptive fuzzy sliding-mode controller with varied desired attitude are shown in Figs. 7(b) and 7(c), respectively to demonstrate the exponential stability and convergence effect. The optimal and estimated upper bounds of the lumped uncertainty and disturbances D are shown in Figs. 7(d) and 5(e), respectively. Here the dashed line in each sub-figure denotes estimated upper bounds of the lumped uncertainty, where the solid line denotes optimal upper bounds of the lumped uncertainty in Fig. 7(d), we can see that optimal and estimated upper bounds are aligned with each other, that is, the tracking effect of the upper-bound



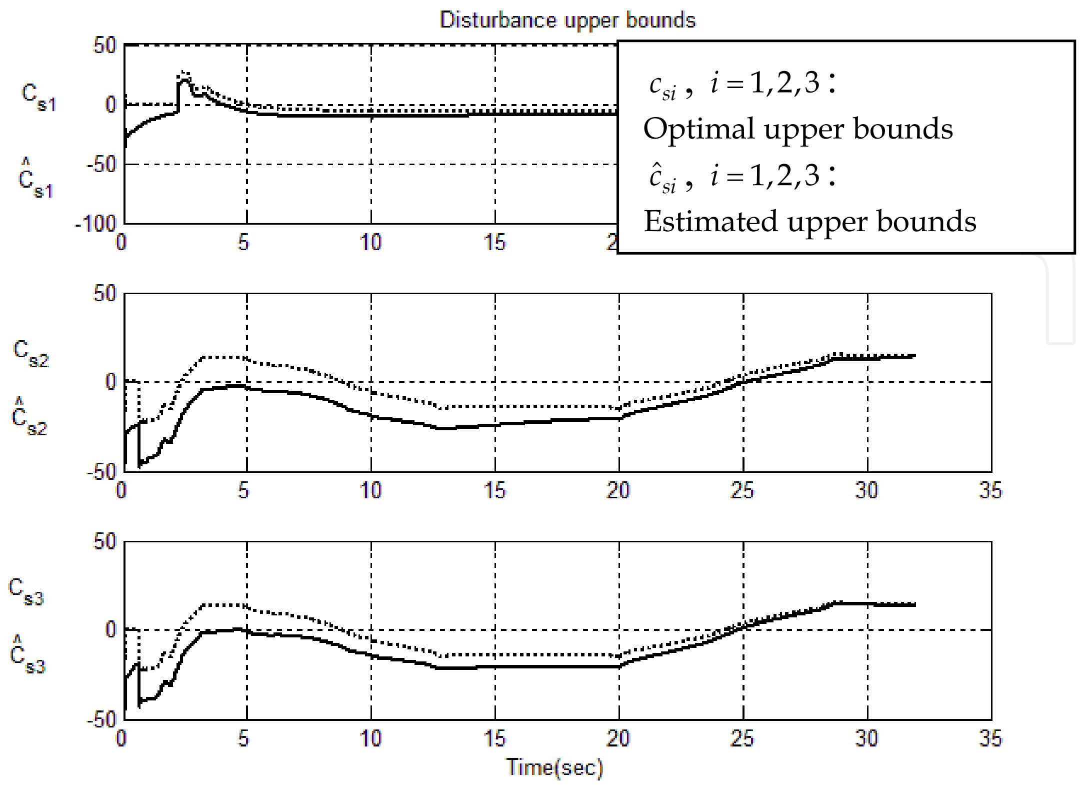
(a)


Sliding surface
S_1
S_2
S_3
Time(sec)

(b)


Torque inputs
T_{b1} (Nm)
T_{b2} (Nm)
T_{b3} (Nm)
Time(sec)

(c)



(d)

Fig. 7. Simulation results of (a) quaternion attitude tracking effects, (b) the convergence of sliding surface, (c) control torque inputs, and (d) optimal and estimated upper bounds of the lumped uncertainty of the adaptive sliding-mode controller for spacecrafts.

estimation can be fulfilled finally. Therefore, for upper-bound estimation the persistent exciting of a spacecraft is rich enough for the proposed controller.

5. Conclusions

Since the attitude control and system stability are the key for space technology, we address the nonlinear attitude controller designs of a spacecraft, respectively consisting of the sliding-mode, the fuzzy sliding-mode and the adaptive fuzzy sliding-mode attitude controllers. The fuzzy sliding-mode controller is designed to estimate upper bounds of the lumped uncertainty, and the adaptive fuzzy sliding-mode controller with center adaption of membership functions is designed to estimate optimal upper bounds of the lumped uncertainty, respectively. We prove the exponential stability for the proposed attitude controllers for external disturbances and inertia uncertainties through the aids of the Lyapunov stability analysis and the Barbalat's lemma.

Extensive simulations have been adopted to verify the feasibility for three attitude tracking controllers corresponding to the lumped uncertainty. The system performance and its stability can also be demonstrated by use of the aforementioned theoretical derivations and the realistic simulations.

6. Nomenclature

D Disturbance vector
d_p Pitch angle of propellant
d_y Yaw angle of propellant
F Thrust vector
J Moment of inertia matrix
J_0 Nominal part of J
ΔJ Variation of J
ℓ Distance between nozzle and center of gravity
$L_b = \begin{bmatrix} -\ell & 0 & 0 \end{bmatrix}^T$ Displacement vector
M Moment vector
n Magnitude of movable nozzle thrust
Q Quaternion
S Sliding surface
T Torque
Ω Angular velocity vector

7. Subscripts

b The body coordinate frame
d Desired
e Error
f Fuzzy sliding-mode controller
M Spacecraft
s Sliding-mode controller
T Thrust

8. Appendix 1

8.1 Proof of Lemma 1: Fuzzy sliding-mode attitude controller design

Proof: To achieve the exponential stability and convergence of the spacecraft attitude tracking system designed the fuzzy sliding-mode controller, the dynamic model of a spacecraft, the control torque input, and upper bounds of the lumped uncertainty are respectively defined as Eqs. (1), (6), (22), and (21). And if the Lyapunov function is defined as $V_f = \frac{1}{2} S^T J S$, then the time derivative of the Lyapunov function can be obtained as

$$\dot{V}_f = S^T J \dot{S} + \frac{1}{2} S^T \dot{J} S$$

$$= S^T \left[-\dot{J}\Omega - \Omega \times (J\Omega) + T_b + D - J\dot{\Omega}_d + JP\left(\frac{1}{2}\left\langle \bar{Q}_e \times \right\rangle \Omega_e + \frac{1}{2} q_{e4} \Omega_e \right) + \frac{1}{2} \dot{J} S \right]$$

$$= -S^T K_s S - \sum_{i=1}^{3} |s_i| \left[\hat{c}_{si} - \delta_i \operatorname{sgn}(s_i) \right]$$

$$\leq -S^T K_s S - \sum_{i=1}^{3} |s_i| \left[\hat{c}_{si} - \delta_i^{\max} \right]$$

$$\leq -\sigma_{\min}(K_s) |S|^2 < 0 , \tag{A.1}$$

where the values of $\hat{c}_{s1}$, $\hat{c}_{s2}$, and $\hat{c}_{s3}$ are chosen to be positive big enough and are computed by Eq. (21) to guarantee that the inequality condition shown below can be satisfied

$$\hat{c}_{si}(C_i, W_i) > \delta_i^{\max}\left(Q, \Omega, Q_d, \dot{Q}_d, \ddot{Q}_d\right) \geq |\delta_i| , \tag{A.2}$$

where bounding functions δ_i, $i = 1,2,3$ are shown as Eq. (18).

Here we assume that the external disturbance D and the induced 2-norm of $\Delta\dot{J}$ and ΔJ are all bounded, Therefore, the main goal of achieving exponential stability and robustness of the fuzzy sliding-mode attitude controller for the spacecraft attitude tracking system can be satisfied completely.

9. Acknowledgment

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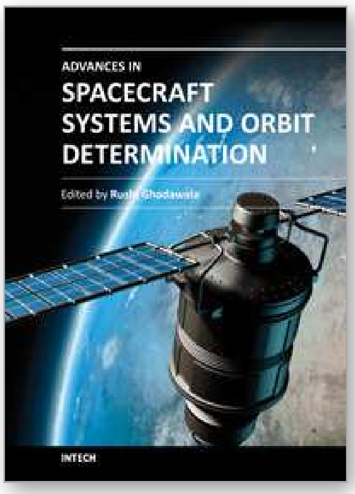

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