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1. Introduction

Over the past few decades, the utility industry has suffered enormous financial losses because of vibration related problems in steam generators and heat exchangers. Cross-flow induced vibration due to shell side fluid flow around the tubes bundle of shell and tube heat exchanger results in tube vibration. This is a major concern of designers, process engineers and operators, leading to large amplitude motion or large eccentricities of the tubes in their loose supports, resulting in mechanical damage in the form of tube fretting wear, baffle damage, tube collision damage, tube joint leakage or fatigue and creep etc.

Most of the heat exchangers used in nuclear, petrochemical and power generation industries are shell and tube type. In these heat exchangers, tubes in a bundle are usually the most flexible components of the assembly. Because of cross-flow, tubes in a bundle vibrate. The general trend in heat exchanger design is towards larger exchangers with increased shell side velocities, to cater for the required heat transfer capacity, improve heat transfer and reduce fouling effects. Tube vibrations have resulted in failure due to mechanical wear, fretting and fatigue cracking. Costly plant shutdowns have lead to research efforts and analysis for flow-induced vibrations in cross-flow of shell side fluid. The risk of radiation exposure in steam generators used in pressurized water reactor (PWR) plants demand ultimate safety in designing and operating these exchangers.

(Erskine & Waddington, 1973) have carried out a parametric form of investigation on a total of nineteen exchanger failures, in addition to other exchangers containing no failures. They realized that these failures represent only a small sample of the many exchangers currently in service. The heat exchanger tube vibration workshop (Chenoweth, 1976) pointed out a critical problem i.e., the information on flow-induced vibration had mostly been withheld because of its proprietary nature.

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4University of Engineering & Technology, Lahore, Pakistan
Failure of heat exchanger tubes in a bundle due to flow-induced vibrations is a deep concern, particularly in geometrically large and highly rated units. Excessive tube vibration may cause failure by fatigue or by fretting wear. Each tube in a bundle is loosely supported at baffles, forming multiple supports often with unequal support spacing. Reactor components like heat exchanger tubes, fuel rods and piping sections may be modeled as beams on multiple supports. It is important to determine whether any of the natural frequencies be within the operating range of frequencies. Considerable research efforts have been carried out, which highlight the importance of the problem.

Tube natural frequency is an important and primary consideration in flow-induced vibration design. A considerable research has been carried out to calculate the natural frequencies of straight and curved (U-tubes) by various models for single and multiple, continuous spans, in air and in liquids for varying end and intermediate support conditions. (Chenoweth, 1976), (Chen & Wambsganss, 1974), (Shin & Wambsganss,1975), (Wambsganss, et al., 1974), (Weaver, 1993), (Brothman, et al., 1974), (Lowery & Moretti, 1975), (Elliott & Pick, 1973), (Jones, 1970), (Ojalvo & Newman, 1964) and (Khushnood et al., 2002), to name some who have carried out research and highlighted the importance of the calculation of natural frequencies of heat exchanger tubes in a bundle.

The dimensionless parameters required for modeling a system may be determined as follows (Weaver, 1993):

- Through non-dimensionalizing the differential equations governing the system behavior.
- From application of Buckingham Pi-theorem.
- This theorem only gives the number of $\pi$s, and not a calculation procedure. So we rely on (i) essentially.

(Shin & Wambsganss, 1975), and (Khushnood et al., 2000) gave the basics of model testing via dimensional analysis. (Blevins, 1977) has described non-dimensional variables such as geometry, reduced velocity, dimensionless amplitude, mass ratio, Reynolds number and damping factor as being useful in describing the vibrations of an elastic structure in a subsonic steady flow. However, other non-dimensional variables such as Mach number, capillary number, Richardson number, Strouhal number and Euler number are also useful in case effects such as surface tension, gravity, supersonic flow or vortex shedding are also considered.

It is generally accepted that the tube bundle excitation mechanisms are (Weaver, 1993, Pettigrew et al., 1991)

- Turbulent buffeting
- Vorticity excitation
- Fluid-elastic excitation
- Acoustic resonance

Turbulent buffeting cannot be avoided in heat exchangers, as significant turbulence levels are always present. Vibration at or near shedding frequency has a strong organizing effect on the wake. Vorticity or vortex shedding or periodic wake shedding is a discrete, periodic, and a constant Strouhal number phenomenon. Strouhal number is the proportionality constant between the frequency of vortex shedding and free stream velocity divided by
cylinder width. Fluid-elastic instability is by far the most dangerous excitation mechanism and the most common cause of tube failure. This instability is typical of self-excited vibration in that it results from the interaction of tube motion and flow. Acoustic resonance is caused by some flow excitation (possibly vortex shedding) having a frequency, which coincides with the natural frequency of the heat exchanger cavity.

With regard to dynamic parameters, including added mass and damping, the concept of added mass was first introduced by DuBuat in 1776 (Weaver, 1993). The fluid oscillating with the tube may have an appreciable affect on both natural frequency and mode shape. Added mass is a function of geometry, density of fluid and the size of the tube (Moretti & Lowry, 1976). Several studies including (Weaver, 1993, Lowery, 1995, Jones, 1970, Chen et al., 1994, Taylor et al., 1998, Rogers et al., 1984, Noghrehkar et al., 1995, Carlucci, 1980, Pettigrew et al., 1994, Pettigrew et al., 1986, Zhou et al., 1997) have targeted damping in heat exchanger tube bundles in single-phase and two-phase cross-flow. (Rogers et al., 1984) have given identification of seven separate sources of damping.

(Ojalvo & Newman, 1964) have presented design for out-of-plane and in-plane frequency factors for various modes. (Jones, 1970) carried out experimental and analytical analysis of a vibrating beam immersed in a fluid and carrying concentrated mass and rotary inertia. (Erskine & Waddington, 1973) conducted parametric form of investigation on a total of 19 exchanger failures along with other exchangers containing no failures, for comparative purpose, indicated the incompleteness of methods available till then and emphasized the need for a fully comprehensive design method. Finite element technique applied by (Elliott & Pick, 1973), concluded that the prediction of natural frequencies was possible with this method and that catastrophic vibrations might be prevented by avoiding matching of material and excitation frequencies. Lack of sufficient data to support comprehensive analytical description for several fundamentally different vibration excitation mechanisms for tube vibration have been indicated in Ontario Hydro Research Division Report (Simpson & Hartlen, 1974). The report also gives response in terms of mid-span amplitude to a uniformly distributed lift for a simply supported tube. A simple graphical method for predicting the in-plane and out-of-plane frequencies of continuous beams and curved beams on periodic, multiple supports with spans of equal length have been presented by (Chen & Wambgsanss, 1974). They have given design guidelines for calculating natural frequencies of straight and curved beams. (Wambgsanss, et al., 1974) have carried out an analytical and experimental study of cylindrical rod vibrating in a viscous fluid, enclosed by a rigid, concentric cylindrical shell, obtaining closed-form solution for added mass and damping coefficient. (Shin & Wambgsanss, 1975) have given information for making the best possible evaluation of potential flow-induced vibration in LMFR steam generator focusing on tube vibration. A simple computer program developed by (Lowery & Moretti, 1975), calculates frequencies of idealized support with multiple spans. (Chenoweth, 1976), in his final report on heat exchanger tube vibration, pointed out the slow progress and inadequacy of existing methods and a need for field data to test suitability of design procedures. It stressed the need for testing specially built and instrumented industrial-sized heat exchangers and wind tunnel based theories to demonstrate interaction of many parameters that contribute to flow-induced vibrations. (Rogers et al., 1984) have modeled mass and damping effects of surrounding fluid and also the effects of squeeze film damping. (Pettigrew et al., 1986) have treated damping of multi-span heat exchanger tubes in air and gases in terms of different
energy dissipation mechanisms, showing a strong relation of damping to tube support thickness.

(Price, 1995) has reviewed all known theoretical models of fluid-elastic instability for cylinder arrays subject to cross-flow with particular emphasis on the physics of instability mechanisms. Despite considerable difference in the theoretical models, there has been a general agreement in conclusions. (Masatoshi et al., 1997) have carried tests on an intermediate heat exchanger with helically coiled tube bundle using a partial model to investigate the complicated vibrational behavior induced by interaction through seismic stop between center pipe and tube bundle. They have indicated the effect of the size of gap between seismic stop and tube support of the bundle. (Botros & Price, 2000) have carried out a study of a large heat exchanger tube bundle of styrene monomer plant, which experienced severe fretting and leaking of tubes and considerable costs associated with operational shutdowns. Analysis through Computational Fluid Dynamics (CFD) and fluid-elastic instability study resulted in the replacement of a bundle with shorter span between baffles, and showed no signature of vibration over a wide range of frequencies. (Yang, 2000) has postulated that crossing-frequency can be used as a measure of heat exchanger support plate effectiveness. Crossing-frequency is the number of times per second the vibrational amplitude crosses the zero displacement line from negative displacement to positive displacement.

The wear of tube due to non-linear tube-to-tube support plate (TSP) interactions is caused by the gap clearances between the two interacting components. Tube wall thickness loss and normal work-rates for different TSP combination studies have been the target. Electric Power Research Institute (EPRI), launched an extensive program in early 1980's for analyses of fluid forcing functions, software development and studying linear and non-linear tube bundle dynamics. Other studies include (Rao et al., 1988), (Axisa & Izquierdo, 1992), (Payen et al., 1995), (Peterka, 1995), (Hassan et al., 2000), (Charpentier and Payen, 2000) and (Au-Yang, 1998).

Generally, there are three geometric configurations in which tubes are arranged in a bundle. These are triangular, normal square and rotated square. Relatively little information exists on two-phase cross-flow induced vibration. Not surprisingly as single-phase flow-induced vibration is not yet fully understood. Vibration in two-phase is much more complex because it depends upon two-phase flow regime and involves an important consideration, the \textit{void fraction}, which is the ratio of volume of gas to the volume of the liquid gas mixture. Two-phase flow experimentation is much more expensive and difficult to carry out usually requiring pressurized loops with the ability to produce two-phase mixtures of desired void fractions.

Two-phase flow research includes the models, such as, Smith Correlation (Smith, 1968), drift-flux model developed by (Zuber and Findlay, 1965), Schrage correlation (Schrage, 1988), and Feenstra model (Feenstra et al., 2000). (Frick et al., 1984) has given an overview of tube wear-rate in two-phase flow. (Pettigrew et al., 2000), (Mirza & Gorman, 1973), (Taylor et al., 1989), (Papp, 1988), (Wambsganss et al., 1992) and others have carried out potential research for vibration response. Earlier reviews on two-phase cross flow are provided by (Paidoussis, 1982), (Weaver & Fitzpatrick, 1988), (Price, 1995), and (Pettigrew & Taylor, 1994).

Two-phase cross-flow induced vibration in tube bundles of process heat exchangers and U-bend region of nuclear steam generators can cause serious tube failures by fatigue and fretting wear. Tube failures could force entire plant to shut down for costly repairs and suffering loss of production. Vibration problems may be avoided by thorough vibration
analysis. However, this requires an understanding of vibration excitation and damping mechanism in two-phase flow. A number of flow regimes (Table 1) can occur for a given boundary configuration, depending upon the concentration and size of the gas bubbles and on the mass flow rates of the two-phases. Two-phase (khushnood, et al., 2004) flow characteristics greatly depend upon the type of flow occurring.

<table>
<thead>
<tr>
<th>Flow Type</th>
<th>Average Void Fraction</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubble</td>
<td>~0.3</td>
<td>Some bubbles are present in liquid flow and move with the same velocity.</td>
</tr>
<tr>
<td>Slug</td>
<td>0.3-0.5</td>
<td>Liquid slugs flow intermittently.</td>
</tr>
<tr>
<td>Froth</td>
<td>0.5-0.8</td>
<td>More violent intermittent flow.</td>
</tr>
<tr>
<td>Annular</td>
<td>0.8-0.9</td>
<td>Mainly gas flow. Liquid adheres to the tube surface.</td>
</tr>
<tr>
<td>Mist</td>
<td>~0.9</td>
<td>Almost gas flow. Mist sometimes causes energy dissipation.</td>
</tr>
</tbody>
</table>

Table 1. Types of Flow in Two-Phases (khushnood, et al., 2004)

Vibration of tube in two-phase flow displays different flow regimes i.e., gas and liquid phase distributions, depending upon the void fraction and mass flux. It is known that four mechanisms are responsible for the excitation of tube arrays in cross-flow (Pettigrew, et al., 1991). These mechanisms are: turbulence buffeting, vortex shedding or Strouhal periodicity, fluid-elastic instability and acoustic resonance. Table 2 presents a summary of these vibration mechanisms for single cylinder and tube bundles for liquid, gas and liquid-gas two-phase flow respectively. Of these four mechanisms, fluid-elastic instability is the most damaging in the short term, because it causes the tubes to vibrate excessively, leading to rapid wear at the tube supports. This mechanism occurs once the flow rate exceeds a threshold velocity at which tubes become self-excited and the vibration amplitude rises rapidly with an increase in flow velocity.

<table>
<thead>
<tr>
<th>Flow Situation (Cross-Flow)</th>
<th>Fluid-Elastic Instability</th>
<th>Periodic Shedding</th>
<th>Turbulence Excitation</th>
<th>Acoustic Resonance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Cylinder</td>
<td>°</td>
<td>°</td>
<td>°</td>
<td>°</td>
</tr>
<tr>
<td>Liquid</td>
<td>°</td>
<td>*</td>
<td>*</td>
<td>°</td>
</tr>
<tr>
<td>Gas</td>
<td>°</td>
<td>Δ</td>
<td>Δ</td>
<td>°</td>
</tr>
<tr>
<td>Two-phase</td>
<td>°</td>
<td>°</td>
<td>°</td>
<td>°</td>
</tr>
<tr>
<td>Tube Bundle</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liquid</td>
<td>°</td>
<td>Δ</td>
<td>Δ</td>
<td>°</td>
</tr>
<tr>
<td>Gas</td>
<td>°</td>
<td>°</td>
<td>Δ</td>
<td>°</td>
</tr>
<tr>
<td>Two-phase</td>
<td>°</td>
<td>°</td>
<td>°</td>
<td>°</td>
</tr>
</tbody>
</table>

Table 2. Vibration Excitation Mechanisms (Pettigrew, et al., 1991)
Typically, researchers have relied on the Homogeneous Equilibrium Model (HEM) (Feentra et al., 2000) to define important fluid parameters in two-phase flow, such as density, void fraction and velocity. This model treats the two-phase flow as a finely mixed and homogeneous in density and temperature, with no difference in velocity between the gas and liquid phases. This model has been used a great deal because it is easy to implement and is widely recognized which facilitated earlier data comparison. Other models include Smith correlation (Smith, 1968), drift-flux model developed by (Zuber and Findlay, 1965), Schrage Correlation (Schrage, 1988), which is based on empirical data, and Feenstra model (Feentra et al., 2000), which is given in terms of dimensionless numbers.

Dynamic parameters such as added or hydrodynamic mass and damping are very important considerations in two-phase cross-flow induced vibrations. Hydrodynamic mass depends upon pitch-to-diameter ratio and decrease with increase in void fraction. Damping is very complicated in two-phase flow and is highly void fraction dependent. Tube-to-restraint interaction at the baffles (loose supports) can lead to fretting wear because of out of plane impact force and in-plane rubbing force. (Frick et al., 1984) has given an overview of the development of relationship between work-rate and wear-rate. Another important consideration in two-phase flow is the random turbulence excitation. Vibration response below fluid-elastic instability is attributed to random turbulence excitation.

(Pettigrew et al., 2000), (Mirza & Gorman, 1973), (Taylor et al.,1989), (Papp, 1988), and (Wambsganss et al., 1992) to name some, have carried out research for Root Mean Square (RMS) vibration response, encompassing spatially correlated forces, Normalized Power Spectral Density (NPSD), two-phase flow pressure drop, two-phase friction multiplier, mass flux, and coefficient of interaction between fluid mixture and tubes. More recently researchers have expanded the study to two-phase flow which occur in nuclear steam generators and many other tubular heat exchangers, a review of which was last given by (Pettigrew & Taylor, 1994). A current review on this topic is given by (Khushnood et al., 2004)

The use of Finite Element Method (FEM), Computational Fluid Dynamics (CFD) and Large Eddy Simulation (LES) have proved quite useful in analyzing flow-induced vibrations in tube bundles in recent years. Earlier on, only pressure drop and heat transfer calculations were considered as the basis of heat exchanger design. Recently, flow-induced tube vibrations have also been included in the design criteria for process heat exchangers and steam generators.

1.1 Regimes

(Kim et al., 2009) have carried flow induced vibrations (Experimental study of two circular cylinders in tandem arrangement) and examined three different experimental conditions both cylinders allowed to vibrate, the upstream cylinder is allowed to vibrate with the downstream cylinder fixed and downstream cylinder allowed to vibrate with upstream cylinder fixed. The results include five regimes depending upon $L/D$, fluctuating lift forces and vibration characteristics of the cylinder as given in Table 3.
<table>
<thead>
<tr>
<th>Regimes</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>$0.1 \leq \frac{L}{D} \leq 0.2$</td>
<td>$0.2 \leq \frac{L}{D} \leq 0.6$</td>
<td>$0.6 \leq \frac{L}{D} \leq 2.0$</td>
<td>$2.0 \leq \frac{L}{D} \leq 2.7$</td>
<td>$\frac{L}{D} \geq 2.7$</td>
</tr>
<tr>
<td>Response</td>
<td>Vibration absent</td>
<td>Violent vibrations of both</td>
<td>Convergent vibrations at $U_r \approx 6.7$</td>
<td>Vibration absent</td>
<td>Each vibrating like isolated cylinder at $U_r \approx 6.7$</td>
</tr>
<tr>
<td>Characteristics</td>
<td>Vibration amplitude is strongly dependant on whether upstream cylinder is fixed or vibrating</td>
<td>Upstream cylinder vibration is completely suppressed when downstream cylinder is fixed but the downstream cylinder is independent of upstream cylinder.</td>
<td>Downstream vibration is strongly dependant on upstream cylinder but upstream cylinder vibrations is insensitive to downstream cylinder.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Regimes of vibration for Circular cylinders tandem (Kim, et al., 2009)

2. Excitation mechanisms

2.1 Fluid-elastic instability

Fluid-elastic instability is by far the most dangerous excitation mechanism in heat exchanger tube bundles and the most common cause of tube failures. The forces associated with fluid-elastic instability exist only because of the motion of the body. Price, 1995 has presented comprehensive review on fluid-elastic instability of cylinder arrays in cross-flow. According to Price, the nature of fluid-elastic instability can be illustrated as a feedback mechanism between structural motion and the resulting fluid forces. A small structural displacement due to turbulence alters the flow pattern, inducing a change in fluid forces. This in turn leads to a further displacement, and so on. If the displacement increases (positive feedback), then fluid-elastic instability occurs. Three mechanisms (Price, 1995), which enable the cylinder to extract energy from flow:

- require a phase difference between cylinder displacement and fluid force generated.
- relies on there being at least two-degrees of freedom with a phase difference between them.
- because of non-linearities, the fluid force is hysteretic and its magnitude depends on the direction of cylinder motion.

A considerable theoretical and experimental research has been undertaken in the past three decades to arrive at a safe and reliable design criteria against fluid-elastic instability. The topic has been reviewed on regular basis from time to time by various researchers including (Paidoussis, 1980, 1981, 1987, 1987), (Chen, 1984, 1987, 1987, 1989), (Zukauskas et al., 1987), (Weaver & FitzPatrick, 1988), (Moretti, 1993) and (Price, 1995).
2.2 Fluid-elastic instability models

2.2.1 Jet switch model

(Roberts, 1962, 1966) considered both a single and a double row of cylinders normal to flow. His analysis was limited to in-flow motion (experiments indicated that instability was purely in the in-flow direction). Roberts assumed that the flow downstream of two-adjacent cylinders could be represented by two wake regions, one large and one small, and a jet between them as shown in Figure 1.

Fig. 1. Idealized model of the jet-flow between two cylinders in a staggered row of cylinders (Roberts, 1962).

Considering a downstream cylinder moving upstream; as the two cylinders cross, insufficient fluid flows in to the large wake region to maintain the entrainment, causing the wake to shrink and the jet to switch directions. Roberts has given the flow equation of motion for a cylinder or tube in a row.

\[
\frac{d^2 x}{d \tau^2} + 2 \zeta \frac{dx}{d \tau} + x = \frac{\rho V^2}{2m \omega_n^2} \left[ 0.717 \left( 1 - C_{pb}(x, \tau) \right) - 2 \left( \frac{\omega_n D}{V} \right) \left( 1 - C_{pb} \right) \frac{dx}{d \tau} \right]
\]

(1)

where \( C_{pb} \) is the base pressure coefficient, \( \tau \) is non-dimensional time \( (t \omega_n) \), \( D \) is the tube diameter, \( \omega_n \) is the natural frequency, \( x \) is the in-flow cylinder displacement, \( \zeta \) is the damping factor or ratio, \( \delta \) is the logarithmic decrement, and \( m \) is mass of the tube. Equation 1 was solved using the method of Krylov and Bogoliubov (Minorsky, 1947) giving \( V_c \), the velocity just sufficient to initiate limit cycle motion for any \( m \delta / \rho D^2 \). Neglecting unsteady terms and fluid damping, the solution reduces to

\[
\frac{V_c}{\omega_n \epsilon D} = K \left( \frac{m \delta}{\rho D^2} \right)^{0.5}
\]

(2)

where \( \epsilon \) is the ratio of fluid-elastic frequency to structural frequency, which is approximately 1.0 and \( \rho \) is the fluid density. This has the same form as the classical Conners equation (Blevins, 1979). Figure 2 presents Robert’s experimental data for pitch-to-dia. ratio \((P/D=1.5)\), showing a good agreement with this theoretical model.
2.2.2 Quasi-static models

Using a quasi-static analysis, (Connors & Parrondo, 1970) and later (Dalton & Helfinstine, 1971) developed the fluid-elastic instability prediction for cylinders (single row of cylinders) subjected to cross-flow. Connors measured the fluid forces instead of predicting these using a pitch to dia. ratio of $P/D=1.41$. He observed many different model patterns at instability, but suggested that the most dominant was elliptical motion (whirling). Using the measured fluid stiffness, Connors obtained energy balances in the in-and cross-flow directions, which must be satisfied simultaneously giving

$$\frac{V_{pc}}{f_n D} = K \left( \frac{m \delta}{\rho D^2} \right)^{0.5} \quad (3)$$

where $K$ is the so-called Connors constant, $f_n$ is the frequency of oscillation. $V_{pc}$ is the so-called pitch velocity given by

$$V_{pc} = \frac{VP}{P-D} \quad (4)$$

where $P$ being the centre-to-centre inter cylinder pitch

(Blevins, 1974) has derived Equation 3 by assuming that the fluid forces on any cylinder are due to relative displacements between itself and its neighboring cylinders. Later, (Blevins, 1979) modified his original analysis to account for flow dependent fluid damping giving

$$V_{pc} = \frac{VP}{P-D}$$
\[
\frac{V_{p\infty}}{f_{c\infty}D} = K \left[ \frac{m}{\rho D^2} 2\pi (\zeta_x \zeta_y)^{1/2} \right]^{1/2}
\]  \hspace{1cm} (5)

where \(\zeta_x\) and \(\zeta_y\) are total damping factors in the in-and cross-flow directions.

### 2.2.3 Inviscid model

Despite the obviously viscous nature of the interstitial flow through arrays of cylinders, the compactness of some arrays suggests that the cylinder wake regions are small, especially for normal triangular arrays with small \(P/D\) (Price, 1995). Hence under this assumption wake regions are neglected and flow is treated as inviscid. Many solutions based upon potential flow theory have been given, including (Dalton & Helfinstine, 1971), (Dalton, 1980), (Balsa, 1977), (Paidoussis et al., 1984), (Vander Hoot & Van Compen, 1984) and (Delaigue & Planchard, 1986). The results obtained from potential flow analyses are somewhat discouraging (Price, 1995). Recent flow visualizations suggest that even though the wake regions are small, the interstitial flow is more complex than that accounted for in these analyses.

### 2.2.4 Unsteady models

The unsteady models measure the unsteady forces on the oscillating cylinder directly. (Tanaka & Takahara, 1980, 1981) and (Chen, 1983) have given theoretical stability boundary for fluid-elastic instability as shown in Figures 3 and 4 respectively.

![Fig. 3. Theoretical stability boundary for fluid-elastic instability for an in-line square array, P/D=1.33, obtained by (Tanaka & Takahara, 1980, 1981).](www.intechopen.com)
2.2.5 Semi-analytical model

Out of many semi-analytical models, Figure 5 shows theoretical stability boundary for fluid-elastic instability obtained by (Lever & Weaver, 1986).

Fig. 5. Theoretical stability boundary for fluid-elastic instability obtained by (Lever & Weaver, 1986) for single flexible cylinder in a parallel triangular array with P/D=1.375.
2.2.6 Quasi-steady model

(Price, 1995) remarks that Fung and Blevins have concluded that quasi-steady fluid dynamics is valid provided \( \frac{V}{f_cD} \geq 10 \); however, experiments by Price, Paidoussis and Sychterz and others suggest that for closely spaced bodies the restriction on the use of quasi-steady fluid dynamics is much more severe than that suggested by Fung or Blevins. (Gross, 1975) carried out first quasi-steady analysis of cylinder arrays subjected to cross-flow concluding that instability in cylinder arrays is due to two distinct mechanisms: negative damping and stiffness controlled instability.

2.2.7 Computational fluid dynamic (CFD) models

The CFD solutions applicable to fluid-elastic instability and other problem areas of flow-induced vibrations are increasing. These include (Marn and Catton, 1991) and (Planchard & Thomas, 1993).

2.2.8 Non-linear models

The first non-linear model was given by (Roberts, 1962, 1966), who employed Krylov and Bogoliubov method (Minorsky, 1947) of averaging to solve the non-linear equations. Two-motivating forces have been remarked by (Price, 1995) for non-linear analyses. Firstly because of manufacturing tolerances and thermal constraints, there are likely to be small clearances between heat exchanger tubes and intermediate supports. Hence, large lengths of unsupported tubes, having very low natural frequencies. These low-frequencies may suffer from fluid-elastic instability at relatively low \( V_{pe} \). A second and more academic motivating force for these non-linear analysis has been to investigate the possibility of Chaotic behavior of tube motion.

2.2.9 Recent researches in fluid-elastic instability

A summary of some recent efforts on the analysis of fluid-elastic instability in heat exchanger and steam generator tube bundles is given in Table 4.

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Flow (phase)</th>
<th>Analysis type</th>
<th>Frequency</th>
<th>Span type</th>
<th>Model type</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Hassan et al., 2011)</td>
<td>Single phase</td>
<td>Simulation (linear/ non-linear)</td>
<td>Up to 90Hz approx.</td>
<td>Loosely supported multispans</td>
<td>Comparison with several time domains fluid force model</td>
<td>Tube supports interaction parameters Impact Force Contact rates Normal wave rate considered.</td>
</tr>
<tr>
<td>Researchers</td>
<td>Flow (phase)</td>
<td>Analysis type</td>
<td>Frequency</td>
<td>Span type</td>
<td>Model type</td>
<td>Remarks</td>
</tr>
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<td>------------------------------</td>
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<td>--------------------------</td>
<td>----------------------------</td>
<td>-------------------------------------------------------------------------</td>
</tr>
<tr>
<td>(Sim &amp; Park, 2010)</td>
<td>Two phase</td>
<td>Experimental test section</td>
<td>Frequency Range 8.25-12 Hz</td>
<td>Cantilevered flexible cylinders</td>
<td>Normal square tube bundles</td>
<td>Dimensionless flow velocity and mass-damping parameter considerations and fluid-elastic instability coefficients considerations</td>
</tr>
<tr>
<td>(Ishihara &amp; Kitayama, 2009)</td>
<td>Single phase</td>
<td>Experimental</td>
<td>Tube banks such as boilers and heat exchangers in power plant</td>
<td>Experimental</td>
<td>Onset of fluid-elastic instability and geometry relationship considerations</td>
<td></td>
</tr>
<tr>
<td>(Mitra et al., 2009)</td>
<td>Single &amp; two phase (Air-water &amp; air-steam flow)</td>
<td>Experimental</td>
<td>Frequency range 7.6 - 13.74 Hz</td>
<td>Fully flexible tube arrays and single flexible tube (Normal square array)</td>
<td>Displacement and damping mechanisms Critical flow velocity was found proportional to tube arrays</td>
<td></td>
</tr>
<tr>
<td>(Mahon &amp; Meskell, 2009)</td>
<td>Single phase</td>
<td>Experimental $P/D = 1.32$</td>
<td>Excitation frequency 6.62 Hz</td>
<td>Second array flexible tube with electromagnetic damper</td>
<td>Normal Triangular</td>
<td>Time delay considerations</td>
</tr>
<tr>
<td>(Hassan &amp; Hayden, 2008)</td>
<td>Single phase</td>
<td>Modeling and simulation</td>
<td>Up to 60 Hz</td>
<td>Critical velocity predictions dependent upon i.e. sensitive to both gap size and turbulence level</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Chung &amp; Chu, 2006)</td>
<td>Two phase Void Fraction 10-95%</td>
<td>Experimental $P/D = 1.633$</td>
<td>Strouhal number 0.15-0.19</td>
<td>Cantilevered straight tube bundles</td>
<td>Experimental</td>
<td>Hydrodynamic coupling effects consideration</td>
</tr>
</tbody>
</table>
Table 4. A summary of recent fluid-elastic instability research

2.3 Vorticity induced instability

Flow across a tube produces a series of vortices in the downstream wake formed as the flow separates alternatively from the opposite sides of the tube. This shedding of vortices produces alternating forces, which occur more frequently as the velocity of flow increases. For a single cylinder, frequency of vortex shedding \( f_{sv} \) is given below by a dimensionless Strouhal number \( S \).

\[
f_{sv} = \frac{SV}{D}
\]

where \( V \) is the flow velocity and \( D \) is the tube diameter. For a single cylinder, the vortex shedding Strouhal number is a constant with a value of about 0.2 (Chenoweth, 1993). Vortex shedding occurs for the range of Reynolds number \( 100 < \text{Re} < 5 \times 10^5 \) and \( > 2 \times 10^6 \) whereas it dies out in-between. The gap is due to a shift of the flow separation point in vortices in the intermediate transcritical Reynolds number range. Vortex shedding can excite tube vibration when it matches with the natural frequency of the tubes. For tube banks with vortex shedding, Strouhal number is not constant, but varies with the arrangement and spacing of tubes, typical values for in-line and staggered tube bundle geometries are given in (Karaman, 1912, Lienhard, 1966). Strouhal numbers for in-line tube banks are given in Figure 6.

The vortex shedding frequency can become locked-in to the natural frequency of a vibrating tube even when flow velocity is increased (Blevins, 1977). Earlier on, the mechanism of vortex shedding has been investigated by a number of researchers. These include Sipvack (Sipvack, 1946) and, Thomas and Kraus (Thomas & Kraus, 1964) who investigated the vortex shedding of two cylinders arranged parallel and perpendicular to flow direction respectively. Grotz and Arnold (Groth & Arnold, 1956) measured for the first time systematically the vortex shedding frequencies in in-line tube bank for various tube spacing ratios.

The cause of vorticity excitation has been disputed in literature (Owen, 1965), but recent studies of (Weaver, 1993) and, (Oengoren & Ziada, 1993) have confirmed its cause of existence as periodic vortex formation. Vorticity shedding can cause tube resonance in liquid flow or acoustic resonance of the tube bundles or acoustic resonance of the tube bundles’ containers in gas flows (Oengoren & Ziada, 1995). (Chen, 1990), (Zaida & Oengoren, 1992) and (Weaver, 1993) have summarized the recent research efforts targeted at improvement in Strouhal number charts for vortex shedding and acoustic resonance for in-line tube bundles.
(Oengoren and Ziada, 1992) have investigated the coupling between the acoustic mode and vortex shedding, which may occur near the condition of frequency coincidence. They have investigated the system response both in the absence and in the presence of a splitter plate, installed at the mid-height of the bundle to double the acoustic resonance frequencies and therefore double the Reynolds number at which frequency coincidence occurs. They have also investigated the effect of row number on vortex shedding and have carried out flow visualization in Reynolds number range of \( \leq 355000 \). Figure 7 is a typical example of the mechanism of vortex shedding from the tubes of the first two rows displaying a time series of symmetric and anti-symmetric patterns (Oengoren & Ziada, 1993).

(Liang et al., 2009) has addressed numerically the effect of tube spacing on vortex shedding characteristics of laminar flow past an inline tube arrays. The study employs a six row inline tube bank for eight pitch to diameter \( \frac{P}{D} \) ratios with Navier-Stokes continuity equation based unstructured code (validated for the case of flow past two tandem cylinders) (Axisa & Izquierdo, 1992). A critical spacing range between 3.0 and 3.6 is identified at which mean drag as well as rms lift and drag coefficients for last three cylinders attain maximum values. Also at critical spacing, there is 180° phase difference in the shedding cycle between successive cylinders and the vortices travel a distance twice the tube spacing within one period of shedding.

(Williamson & Govardhan, 2008) have reviewed and summarized the fundamental results and discoveries related to vortex induced vibrations with particular emphasis to vortex dynamics and energy transfer which give rise to the mode of vibrations. The importance of mass and damping and the concept of “critical mass”, “effective elasticity” and the relationship between force and vorticity. With reference to critical mass, it is concluded that
as the structural mass decreases, so the regime of velocity (non-dimensional) over which there is large amplitude of vibrations increases. The synchronizing regime become infinitely wide not simply when mass become zero but when a mass falls below special critical value when the numerical value depends upon the vibrating body shape.

(Williamson & Govardhan, 2000) present a large data set for the low branch frequency $f_{\text{lower}}$ plotted versus $m^*$ (mass ratio) yielding a good collapse of data on to single curve base equation 7.

$$f_{\text{lower}} = \sqrt{\frac{m^*+1}{m^* - 0.54}}$$  \hspace{1cm} (7)

This equation provides a practical and simple means to calculate the frequency attained by vortex induced vibrations. The critical mass ratio is given by

$$m_{\text{crit}}^* = 0.54 \pm 0.002$$  \hspace{1cm} (8)

Below which the lower branch of response can never be attained. With respect to combine mass-damping parameter’s capability to reasonably collapse peak amplitude data in Griffins plot, a number of parameters like stability parameter, Scrutom number and combined response parameter termed as Skop-Griffins parameter given by (S$_G$):

$$S_G = 2\pi 3S^2(m^*)$$  \hspace{1cm} (9)
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Where S stands for single vortices and $S_c$ is the Scruton number. (Hamakawa & Matsue, 2008) focused on relation between vortex shedding and acoustic resonance in a model (boiler plant) for tube banks to clarify the interactive characteristics of vortex shedding and acoustic resonance. Periodic velocity fluctuation due to vortex shedding was noticed inside the tube banks at the Reynolds number (1100-10000) without acoustic resonance and natural vortex shedding frequency of low gap velocities. Kumar et al., 2008 in their review stated that controlling or suppressing vortex induced vibrations is of importance in practical applications where active or passive control could be applied. (Paidoussis, 2006) specially addressed real life experiences in vortex induced vibrations and concludes with this mechanism in addition of other already clarified mechanisms of flow induced vibrations. Vortex induced vibrations of ICI nozzles and guide tubes in PWR for ICI thimble guiding into the core of the reactor to monitor reactivity may witness breakage of ICI nozzles resulting in strange noises experience in the reactor. Analysis of shedding frequencies confirmed the vortex induced vibrations to be the culprit partially due to the large values of varying lift coefficients and partially due to lock-in. (Hamakawa & Fukano, 2006) also focused vortex shedding in relation with the acoustic resonance in staggered tube banks and observe three Strouhal number (0.29, 0.22 and 0.19). In cases with no resonance inside tube banks, the last rows of tube banks and in both regimes respectively. The vortices of 0.29 and 0.22 components alternatively irregularly originated. (Pettigrew & Taylor, 2003) discussed and over viewed procedures and recommended design guidelines for periodic wave shedding in addition to other flow induced vibration considerations for shell and tube heat exchangers. It concludes that the fluctuating forces due to periodic wave shedding depends on the number of considerations like geometric configuration of tube bundles, its location, Reynolds number, turbulence, density of fluid and pitch to diameter $P/D$ ratio.

2.4 Turbulence excitation

Extremely turbulent flow of the shell-side fluid contains a wide spectrum of frequencies distributed around a central dominant frequency, which increases as the cross-flow velocity increases. This turbulence buffs the tubes, which extract energy from the turbulence at their natural frequency from the spectrum of frequencies present. When the dominant frequency for the turbulent buffeting matches the natural frequency, a considerable transfer of energy is possible leading to significant vibration amplitudes (Chenoweth, 1993). Turbulent flow is characterized by random fluctuation in the fluid velocity and by intense mixing of the fluid. Nuclear fuel bundles and pressurized water reactor (PWR) steam generators are existing examples (Hassan & Ibrahim, 1997).

Turbulence is by nature three-dimensional (Au-Yang, 2000). Large-Eddy Simulation, (LES) incorporated in three-dimensional computer codes has become one of the promising techniques to estimate flow turbulence. (Hassan & Ibrahim, 1997) & (Davis & Hassan, 1993) have carried out Large Eddy Simulation for turbulence prediction in two-and three-dimensional flows. The primary concern in turbulence measurements is how the energy spectrum or the power spectral density (PSD) of the eddies are distributed. The PSD of the...
velocity profile $E(n)$ is numerically equal to the square of the Fourier Transform of $U'(t)$, and is defined to be (Hassan & Ibrahim, 1997).

$$\overline{U'^2} = \int_{-\infty}^{\infty} E(n)dn$$

(10)

where $E(n)$ is the sum of power at positive and negative frequency $n$.

$$E(n) = \frac{4\pi^2|a(n)|^2}{T}$$

(11)

where $T$ is the time period over which integration is performed, and $a(n)$ is the Fourier Transform coefficient.

An important parameter of flow turbulence is the correlation function. The Lagrangian (temporal) auto-correlation over a time $T$ gives the length of time (past history) that is related to a given event (Hassan & Ibrahim, 1997).

$$(\text{Non-dimensional}) \quad R(\tau) = \frac{U'(t)U'(t+\tau)}{U'(t)U'(t)}$$

(12a)

$$(\text{Dimensional}) \quad R(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{+T} U'(t)U'(t+\tau)dt$$

(12b)

Physically $R(\tau)$ represents the average of the product of fluctuating velocity $U'$ values at a given time and at a time $\tau$ later. $R(\tau)$ gives information about whether and for how long the instantaneous value of $U'$ depends on its previous values. Cross-correlation curves can also be obtained as a function of the time delay to give the correlation between the velocities at consecutive separated location points (Owen, 1965).

$$R_{12}(\tau) = \frac{1}{T} \int_{-T}^{+T} U'_1(t)U'_2(t+\tau)dt$$

(13)

where $R_{12}$ gives the cross-correlation of the $U$-velocity component at 1- and 2- point locations.

Recently (Au-Yang, 2000) has reviewed the acceptance integral method to estimate the random vibration, Root Mean Square (RMS) of structures subjected to turbulent flow (random forcing function). The acceptance integral is given by:

$$J_{\alpha\beta}(\omega) = \frac{1}{T} \int_{x=0}^{x=L} \int_{x'=0}^{x=L} \phi_\alpha(x') [S_\beta(x', x', \omega)/S_\beta(x', x', \omega)] \phi_\beta(x') dx' dx$$

(14)

When $\alpha = \beta$, $J_{\alpha\alpha}$ is known as joint acceptance

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Yang obtained closed form solutions for the joint acceptances for two special cases of spring-supported and simply-supported beams. A review of turbulence in two-phase is presented by (Khushnood et al., 2003).

(Endres & Möller, 2009) present the experimental analysis of disturbance propagation with a fixed frequency against cross flow and its effect on velocity fluctuations inside the bank. It is concluded that continuous wavelet transforms of the signals. Figure 8 indicates the disturbance frequency to be showing steady behavior. Generally designing for enhanced heat exchange ratios in thermal equipments ignores the structural effects caused by turbulent flow.

![Continuous wavelet transforms of the signals at locations 0, 1 and 2. Tube bank with P/D = 1.26, vortex generator #2, Re_G = 6.46X10^4 (Endres & Möller, 2009).](image)

Fig. 8. Continuous wavelet transforms of the signals at locations 0, 1 and 2. Tube bank with P/D = 1.26, vortex generator #2, Re_G = 6.46X10^4 (Endres & Möller, 2009).

\[ J_{\alpha\alpha} = \text{Joint acceptance for } \alpha_{\alpha} \text{ mode} \]
\[ J_{\alpha\beta} = \text{Cross-acceptance} \]
\[ \ell = \text{Surface of 2-D structure of length of 1-D structure} \]
\[ x = \text{Position vector} \]
\[ S_p = \text{Double sided pressure power spectral density.} \]
\[ \phi_\alpha = \text{Mode shape function} \]
\[ \phi_\beta = \text{Mode shape function} \]
\[ \omega = \text{Frequency} \]
\[ \alpha, \beta = \text{Modal indices} \]
(Pascal-Ribot and Blanchet, 2007) proposed a formulation to collapse the dimensionless spectra of buffeting forces in a single characteristic curve and gives edge to the formulation over previously normalized models in terms of collapse of data.

\[
P_s = k \rho g \frac{\sigma}{\Delta \rho g} \left[ \alpha (1 - \alpha) \right]^2
\]

where \( \alpha \) is the void fraction.

\[
P_0 = k \rho g \frac{\sigma}{\Delta \rho g} \left\{ \alpha_c (1 - \alpha_c) \right\}^2 : \alpha_c = 0.4
\]

(Wang et al., 2006) concludes the physically realistic solutions for turbulent flow in a staggered tube banks can be realized by FLUENT (with 2-D Reynolds stress model).

Figure 10 shows the consistency of turbulence intensity contours obtained through standard wall function approach and non-equilibrium wall function approach whereas near-wall treatment model and near-wall turbulence model predicts much higher results (Wang, et al., 2006).
2.5 Acoustic resonance

Acoustic vibration occurs only when the shell-side fluid is a vapor or a gas. The characteristic frequency of acoustic vibration in a heat exchanger depends on some characteristic length, usually the shell diameter and the velocity of sound in shell-side fluid, $U_{\text{sound}}$. The acoustic frequency (Chenoweth, 1993) can be predicted by the following equation.

$$f_a = \frac{m U_{\text{sound}}}{2d}$$  \hspace{5em} (17)

where $m$ is the mode number (a dimensionless integer), and $d$ is the shell diameter. The lowest acoustic frequency is achieved when $m = 1$ and the characteristic length is the shell diameter. The acoustic frequencies of an exchanger can be excited by either vortex shedding or turbulent buffeting (Chenoweth, 1993). (Barrington, 1973) indicated that so...
long as the exciting frequencies are within 20% of an acoustic frequency, a loud sound may be produced. Acoustic vibration becomes destructive when it is in resonance with some component of exchanger. The acoustic frequencies of shell can be changed by inserting a detuning plate parallel to the direction of cross-flow to alter the characteristic length (Chenoweth, 1993). There are a number of published acoustic vibration criteria to predict strong acoustic vibration within a tube bank, including (Eisinger et al., 1994), (Groth & Arnold, 1956), (Chen, 1968), (Fitzpatrick, 1986), (Ziada et al.,) and Blevins (Blevins, 1990).

(Hanson & Ziada, 2011) have investigated the effects of acoustic resonance on the dynamic lift force acting on the central tube. Two effects of resonant sound field includes generation of “sound induced” dynamic lift because of resonant acoustic pressure distribution on the tube surface and synchronization of vorticity shedding. Sound enhancements coefficients and sound induced lift force development is carried through numerical solution. (Hanson et al., 2009) investigated aeroacoustic response of two side-by-side cylinders against cross flow. It is concluded that acoustic resonance synchronizes vortex shedding and eliminates bistable flow phenomenon. Vortex shedding is noticed a particular strouhal number which excites acoustic resonance. Figure 11 and figure 12 gives the pressure spectra for two side-by-side cylinders and aeroacoustic response of two side-by-side cylinders.

(Eisinger & Sullivan, 2007) considers strong acoustic resonance with acoustic pressure reading 165 dB for package boiler at near full load, suppression of resonance (lower frequency) through baffle covering with downstream section and the development of another resonance (higher frequency) in the unbaffled upstream section.

(Feenstra et al., 2006) carried out experimental investigation of the effects of width of test section for measuring the acoustic resonance with a small pitch rates staggered tubes. The conclusion was that the maximum acoustic pressure versus input energy parameter of Blevins and Bressler is not a reliable preditor and it over predicts.

Fig. 11. Pressure spectra for the two side-by-side cylinders for T/D=1.25 (Hanson, et al., 2011)
Fig. 12. Aeroacoustic response of the two side-by-side cylinders with $T/D = 1.25$ and $D = 21.8$ mm (Hanson, et al., 2011) (Eisinger & Sullivan, 2005) test results concluded that wide test sections gives the maximum acoustic pressure (lower acoustic mode) at $P = 53.4$ MPa which is 4.27 times greater than predicted by Blevins and Bressler.

3. Natural frequencies of tube vibration

In flow-induced vibration design of heat exchanger tube bundles, resonant conditions must be suppressed by ensuring separation of natural frequencies of the tubes and exciting frequencies (Shin & Wambsganss, 1975). A number of techniques are available for computing natural frequencies of straight, curved, and multiple span tubes. These tubes may be subjected to varying end conditions. Loose baffles act like “knife-edged rings” supports (Timoshenko, 1955). Tubes are rigidly fastened to the tube sheet and supported at intermediate points along their lengths by baffles or support plates. Some tubes in the centre may be supported by every baffle, whereas tubes that pass thorough baffle window may be supported by every second and third baffle. Table 5 (MacDuff & Feglar, 1957, Kissel, 1977) gives some of the formulas/techniques for estimating the natural frequencies of straight-and curved-or U-tubes.

3.1 Variables affecting tube natural frequencies

The tube natural frequencies are affected by tube-to-baffle hole clearance, axial stress, fins, span length, span shape (straight, U-tube), support type, temperature and tube vibration (Chenoweth, 1976, Elliot & Park, 1973, Pettigrew et al., 1986, Simpson et al., 1974, Tanaka & Takahara, 1981, TEMA Standards, 8th edition). At about one tenth of TEMA allowable clearance, the frequency is about 7% higher than that predicted for simple supports. For most exchangers, tube-to-hole clearance is not a significant parameter for controlling natural frequencies, but it may be important in damping and tube wear (Chenoweth, 1993). Due to manufacturing procedures, the tubes may be under a tension or compression axial loading. (Kissel, 1972) found a variation in natural frequency due to axial loading in a typical...
exchanger as much as ± 40%. The natural frequency varies as reciprocal of the span length squared (unsupported span). For finned tubes, effective diameter for the outside diameter should be used to find the area moment of inertia for natural frequency calculation (Chenoweth, 1993). Currently, software by TEMA (FIV) (TEMA Standards, 8th edition) is capable of predicting the natural frequencies.

<table>
<thead>
<tr>
<th>Formula/ Procedure</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ f_n = \left( \frac{1}{2\pi} \right)^{1/2} \lambda_n \left( \frac{EI}{m} \right) ]</td>
<td>Straight beams / single span</td>
</tr>
<tr>
<td>( n ) is the mode number and ( \lambda_n ) is a frequency factor which depends upon the end conditions</td>
<td></td>
</tr>
<tr>
<td>[ f_n = \frac{1}{2\pi} \lambda_n \left( \frac{1}{R\alpha} \right)^{1/2} ]</td>
<td>Curved beams/ single span</td>
</tr>
<tr>
<td>( \lambda_n ) is a frequency factor</td>
<td></td>
</tr>
<tr>
<td>( R ) is the radius of curvature and ( \alpha ) is the subtended angle</td>
<td></td>
</tr>
<tr>
<td>[ f_n = 59.55 \frac{C_n}{L^2} \left( \frac{EI}{M_c} \right)^{0.5} ]</td>
<td>U-tube curved</td>
</tr>
<tr>
<td>( C_n ) is the first mode U-tube constant</td>
<td></td>
</tr>
<tr>
<td>[ f_n = \frac{(\beta_n L)^2}{2\pi L^2} \left( \frac{EI_g}{W} \right)^{1/2} ]</td>
<td>Straight/multiple, free-free spans (1-5 span tests); idealized support conditions, ( (\beta_n L)^2 ) is eigen value</td>
</tr>
<tr>
<td>( \beta_n ) is an eigen value</td>
<td></td>
</tr>
<tr>
<td>[ f_n = \frac{3.13}{k^2} \left( \frac{C}{\gamma A} \right)^{1/2} ]</td>
<td>Clamped ring segments</td>
</tr>
<tr>
<td>( n ) is mode number; ( k ) is bending stiffness</td>
<td></td>
</tr>
<tr>
<td>( \gamma ) is specific weight; ( C ) is the torsional stiffness.</td>
<td></td>
</tr>
<tr>
<td>( A ) is cross-sectional area</td>
<td></td>
</tr>
<tr>
<td>[ f_n = \frac{C}{\gamma A} ]</td>
<td>Graphical in-plane and out of plane</td>
</tr>
<tr>
<td>( \lambda_n = \frac{n(n^2-1)}{\sqrt{1+k^2}} )</td>
<td>Straight/curved, single span / multiple span</td>
</tr>
</tbody>
</table>
4. Dynamic parameters

Added mass and damping are known to be dependent on fluid properties (in particular, fluid density and viscosity) as well as functions of component geometry and adjacent boundaries, whether rigid or elastic. Nuclear reactor components are typically immersed in a liquid coolant and are often closely spaced (Wambsganss, et al., 1974).

4.1 Added mass

(TEMA, 7th Edition, 1988) defines hydrodynamic mass as an effect which increases the apparent weight of the vibrating body due to the displacement of the shell-side flow resulting from motion of vibrating tubes, the proximity of other tubes within the bundles and relative location of shell-wall. The so-called "virtual mass" for a tube is composed of the mass of the tube, mass of the fluid contained in the tube and the inertia \( M' \) imposed by the surrounding fluid. This hydrodynamic mass \( M' \) is a function of the geometry, the density of the fluid, and the size of the tube. In an ideal fluid, it is proportional to the fluid density and to the volume of the tube (Moretti & Lowry, 1976), and hence may be expressed, per unit length as:

\[
\frac{M'}{L} = C_m \rho \pi r^2
\]  

(18)

where \( L \) is the tube length, \( r \) the radius of the tube and \( \rho \) is the mass per unit volume of the surrounding fluid, \( C_m \) is called the inertia coefficient which is a function of the geometry, and is discussed by Lamb (Lamb, 1932, 1945). If the moving tube is not infinitely long, the flow is three-dimensional and leads to smaller values of \( C_m \) (Moretti & Lowry, 1976). For a vibrating tube in a fluid region bounded by a circular cylinder, Stokes (Endres & Moller, 2009) has determined hydrodynamic mass per unit length as given by:

\[
m_h = C_m m_u
\]  

(19)
where \( C_m = \frac{R^2 + r^2}{R^2 - r^2} \). \( R \) is the outer radius of annulus and, \( m_a = \rho \pi r^2 \), where \( \rho \) is fluid density, and \( r \) is the tube radius.

(Wambsganss, et al., 1974) have published a study on the effect of viscosity on \( C_m \). Hydrodynamic mass \( M' \) for a tube submerged in water was determined by measuring its natural frequency, \( f_a \), in air and, \( f_w \), in water. Neglecting the density of air compared to water, the following equation may be obtained from beam equation (Moretti & Lowry, 1976).

\[
\frac{M'}{L} = \mu \left[ \left( \frac{f_a}{f_w} \right)^2 - 1 \right]
\]  

(20)

where \( \mu \) is the tube mass per unit length. The inertia coefficient, \( C_m \), can be obtained from Equation 18. Figure 13 gives the results showing the variation of \( C_m \) with pitch-to-diameter ratios (Wambsganss, et al., 1974).

4.2 Damping

System damping has a strong influence on the amplitude of vibration. Damping depends upon the mechanical properties of the tube material, the geometry of intermediate supports and the physical properties of the shell-side fluid. Tight tube-to-baffle clearances and thick baffles increase damping, as does very viscous shell-side fluid (Chenoweth, 1993). (Coit et al., 1974) measured log decrements for copper-nickel finned tubes of 0.032 in still air. The range of most of the values probably lies between 0.01 and 0.17 for tubes in heat exchangers (Chenoweth, 1993). From (Wambsganss, et al., 1974), damping can be readily obtained from the transfer function or frequency response curve as
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\[ \zeta = \frac{1}{2\sqrt{N^2 - 1}} \frac{\Delta f_n}{f_n} \]  

(21)

with \( \Delta f_n = f_N^{(1)} - f_N^{(2)} \),

where \( f_n \) is the resonant frequency and \( f_N^{(1)} \) and \( f_N^{(2)} \) are the frequencies at which the response is a factor \( \frac{1}{N^{th}} \) of resonant response.

(Lowery & Moretti, 1975), have concluded that damping is almost entirely a function of the supports. More complex support conditions (non-ideal end supports or intermediate supports with a slight amount of clearance) lead to values around 0.04. From analytical point of view, (Jones, 1970) has remarked that in most cases, the addition of damping to the beam equation re-couples its modes. Only a beam, which has, as its damping function, a restricted class of functions can be uncoupled. (Chen et al., 1994) have found the fluid damping coefficients from measured motion-dependent fluid forces. (Pettigrew et al., 1986, 1991) outlines the energy dissipation mechanisms that contribute to tube damping as given in Table 6:

<table>
<thead>
<tr>
<th>Type of damping</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structural</td>
<td>Internal to tube material</td>
</tr>
<tr>
<td>Viscous</td>
<td>Between fluid forces and forces transferred to fluid</td>
</tr>
<tr>
<td>Flow-dependent</td>
<td>Varies with flow regime.</td>
</tr>
<tr>
<td>Squeeze film</td>
<td>Between tube and fluid as tube approaches support</td>
</tr>
<tr>
<td>Friction</td>
<td>Coulomb damping at support</td>
</tr>
<tr>
<td>Tube support</td>
<td>Internal to support material</td>
</tr>
<tr>
<td>Two-phase</td>
<td>Due to liquid gas mixture</td>
</tr>
<tr>
<td>Thermal damping</td>
<td>Due to thermal load</td>
</tr>
</tbody>
</table>

Table 6. Energy dissipation mechanisms (Pettigrew et al., 1986, 1991)

4.3 Parameters influencing damping

(Pettigrew et al., 1986) further outlines the parameters that influence damping as given below:

The Type of tube motion. There are two principal types of tube motion at the supports, rocking motion and lateral motion. Damping due to rocking is likely to be less. Rocking motion is pre-dominant in lower modes. Dynamic interaction between tube and supports may be categorized in three main types, namely: sliding, impacting, and scuffing, which is impacting at an angle followed by sliding:

Effect of number of supports. The trend available in damping data referenced in (Pettigrew et al., 1986), when normalized give
where $\zeta_n$ is the normalized damping ratio, $N$ is the number of spans, and $\zeta$ is the damping ratio.

Effect of tube Frequency. Frequency does not appear to be significant parameter (Pettigrew et al., 1986).

Effect of vibration amplitude. There is no conclusive trend of damping as a function of amplitude. Very often, amplitude is not given in damping measurements (Pettigrew et al., 1986).

Effect of diameter or mass. Large and massive tubes should experience large friction forces and the energy dissipated should be large. However, the potential energy in the tube would also be proportionally large in more massive tubes. Thus, the damping ratio, which is related to the ratio of energy dissipated per cycle to the potential energy in the tube should be independent of tube size or mass (Pettigrew et al., 1986).

Effect of side loads. In real exchangers, side loads are possible due to misalignment of tube-supports or due to fluid drag forces. Side loads may increase or reduce damping. Small side loads may prevent impacting, and thus reduce damping, whereas large side loads may increase damping by increasing friction (Pettigrew et al., 1986).

Effect of higher modes. Damping appear to decrease with mode order, for mode order higher than the number of spans, since these higher modes involve relatively less interaction between tube and tube-support (Pettigrew et al., 1986).

Effect of tube support thickness. Referenced data in (Pettigrew et al., 1986) clearly indicates that support thickness is a dominant parameter. Damping is roughly proportional to support thickness. (Pettigrew et al., 1986) corrected the damping data line for support width less than 12.7mm such that

$$\zeta_{nc} = \zeta_n \left( \frac{12.7}{L} \right)$$

(23)

where $L$ is support thickness in mm and $\zeta_{nc}$ is the corrected normalized damping ratio.

Effect of clearance. For the normal range of tube-to-support diametral clearances (0.40mm-0.80mm), there is no conclusive trend in the damping data reviewed (Pettigrew et al., 1986).


Friction damping ratio in a multi-span tube (percentage)

(For liquid) $\zeta_F = 0.5 \left( \frac{N-1}{N} \right) \left( \frac{L}{l_m} \right)^{1/2}$

(24)

(For gas) $\zeta_F = 5.0 \left( \frac{N-1}{N} \right) \left( \frac{L}{l_m} \right)^{1/2}$

(25)
where $N$ is the number of tube spans, $L$ is the support thickness, $l_w$ is the characteristic span length usually taken as average of three longest spans.

**Viscous damping ratio**

Rogers simplified version of Chens' cylinder viscous damping ratio (percentage) of a tube in liquid.

$$\zeta_f = \frac{100\pi}{\sqrt{8}} \left( \frac{\rho D^2}{m} \right) \left( \frac{2\nu}{\pi f D^2} \right)^{1/2} \left[ 1 + \left( \frac{D}{D_e} \right)^3 \right] \left( 1 - \left( \frac{D}{D_e} \right)^3 \right)$$  \hspace{1cm} (26)

where $\rho$ is the fluid density, $m$ is the mass per unit length of tube (interior fluid and hydrodynamic mass), $D_e$ is the equivalent diameter to model confinement due to surrounding tubes, $D$ is the tube diameter, $f$ is the frequency of tube vibration and $\nu$ is the fluid kinematic viscosity. The term $S = \frac{\pi f D^2}{2\nu}$ is the Stoke number.

**Squeeze film damping ratio**


$$\zeta_s = \left( \frac{N-1}{N} \right) \left( \frac{2200}{f} \right) \left( \frac{\rho D^2}{m} \right) \left( \frac{L}{l_w} \right)^{1/2}$$  \hspace{1cm} (27)

**Support damping**

(Pettigrew, et al., 1991) has developed a semi-empirical expression to formulate support damping, using Mulcahys' theory (Mulcahy, 1980).

$$\zeta_s = \left( \frac{N-1}{N} \right) \left( \frac{2200}{f} \right) \left( \frac{\rho D^2}{m} \right) \left( \frac{L}{l_w} \right)^{0.6}$$  \hspace{1cm} (28)

According to TEMA standards, $\zeta$ is equal to greater of $\zeta_1$ and $\zeta_2$

(For shell-side liquids)

$$\zeta_1 = \frac{3.41d_o}{W_o f_n}$$  \hspace{1cm} (29)

$$\zeta_2 = \frac{0.012d_o}{W_o} \left( \frac{\rho_o \nu}{f_n} \right)^{1/2}$$  \hspace{1cm} (30)

where $\nu$ is the shell fluid velocity, $d_o$ is the outside tube diameter, $\rho_o$ is the density of shell-side fluid, $f_n$ is the fundamental frequency of tube span, and $W_o$ is the effective tube weight.

(For shell-side vapors)
$
\zeta = 0.314 \left( \frac{t_b}{N} \right)^{1/2} \frac{N - 1}{N} \left( \frac{l}{I} \right)
$  

(31)

where $N$ is the number of spans, $t_b$ is the baffle or support plate thickness, and $l$ is the tube unsupported span. A review of two-phase flow damping is presented by (Khushnood et al., 2003).

5. Damage numbers for collision and baffle damage (Chenoweth, 1976, Shin et al., 1975, Brothman et al., 1974)

Two types of vibration damage are prevalent in cross-flow regions of steam generators (Shin & Wambsganss, 1975).

- Tube-to-baffle impact.
- Tube-to-tube collision.

(Thorngren, 1970) deduced "damage numbers" for the two types of damage, based on the assumption that tube is supported by baffles and deflected by a uniformly distributed lift force. These damage numbers are given by following equations:

(Baffle damage number) $N_{BD} = \frac{d \rho V^2 l^2}{\beta S_m g_c A_m B_i}$  

(Collision damage number) $N_{CD} = \frac{0.625 d \rho V^2 l^4}{\beta^4 g_c A_m (d^2 + d_i^2) C_i E}$  

(32) \hspace{1cm} (33)

where

- $N_{BD} < 1$ for safe design.
- $N_{CD} < 1$ for safe design.
- $C_i =$ Maximum gap between tubes.
- $d =$ Tube outer diameter.
- $d_i =$ Tube inner diameter.
- $\beta =$ Tube-to-baffle-hole clearance factor.
- $E =$ Modulus of elasticity.
- $g_c =$ Gravitational constant.
- $B_i =$ Baffle thickness.
- $l =$ Length of tube between supports.
- $V =$ Free stream velocity.
- $A_m =$ Tube cross-sectional area $\left[ \frac{\pi}{4} (d^2 - d_i^2) \right]$ and
- $S_m =$ Maximum allowable fatigue stress [ASME Pressure Vessel code Sec. III].

Collision damage is usually predicted together with baffle damage, whereas the latter can be predicted without collision damage being indicated, i.e., baffle damage is important factor when appraising design (Erskine et al., 1973). (Burgreen et al., 1958) were the first to conduct an experiment to investigate vibration of tube for fluid flowing parallel to tube axis. (Quinn, 1962) and (Paidoussis, 1965) have developed analytical and empirical expressions respectively for peak amplitude. Paidoussis give the following expression:
Cross-Flow-Induced-Vibrations in Heat Exchanger Tube Bundles: A Review

\[
\frac{\Delta}{d} = \alpha_1 \frac{U^{1.6} e^{3.8} R_h^{0.25}}{I + U^2} \left( \frac{d_h}{d} \right)^{0.4} \beta^{2/3} \left( 5 \times 10^{-4} \times K_p \right) / (I + 4\beta)
\]

(34)

where

- \( K_p \) = Flow condition constant
- \( \Delta \) = Maximum vibration amplitude at mid-span
- \( d \) = Outer diameter of tube
- \( \alpha_1 \) = First mode beam eigen value of the tube
- \( U \) = Dimensionless flow velocity
- \( \epsilon = \frac{\ell}{d} \) = Tube length
- \( d_h \) = Hydraulic diameter
- \( R_h \) = Reynolds number
- \( I \) = Moment of inertia and
- \( \beta \) = Added mass fraction

Later on a number of expressions for peak and RMS amplitudes have been developed (Shin et al., 1975, Blevins, 1977).

6. Wear work-rates

In fretting wear, work-rate is defined as the rate of energy dissipation when a tube is in contact with its support. Energy is being dissipated through friction as the tube moves around in contact with its supports. A force (the contact force between tube and support) multiplied by a displacement (as the tube slides) results in work or dissipated energy required to move the tube (Taylor et al., 1998, Au-Yang, 1998). Normal work-rate \( W_n \) for different tube and tube support plate material combinations and different geometries (Au-Yang, 1998) is defined.

\[
W_n = \frac{1}{T} \int_0^T |F_n| \, dS
\]

(35)

where \( T \) is the total time, \( F_n \) is the normal contact force, and \( S \) is the sliding distance.

(Au-Yang, 1998) has assessed the cumulative tube wall wear after 5, 10, and 15, effective full power years of operation of a typical commercial nuclear steam generator, using different wear models.

The EPRI data reproduced from (Hofmann & Schettlet, 1989) in Figure 14 shows the wear volume against normal work-rate for the combination of Inconel 600 tube (discrepancy as plot shows J 600 whereas text indicates Inconel 600) and carbon tube support plate, a condition that applies to many commercial nuclear steam generators (Hofmann & Schettlet, 1989). Figure 15 shows the tube wall thickness loss against volumetric wear for different support conditions (Hofmann & Schettlet, 1989).
Fig. 14. Volumetric wear rate versus normal work-rate for different material combinations (Hofmann & Schettlet, 1989).

Fig. 15. Tube wall thickness loss versus volumetric wear for different support conditions, from Hofmann and Schettler (Hofmann & Schettlet, 1989).
(Payen et al., 1995) have carried predictive analysis of loosely supported tubes vibration induced by cross-flow turbulence for non-linear computations of tube dynamics. They have analyzed the gap effect and have concluded that wear work-rate decreases when the gap value increase at low velocities. (Peterka, 1995) has carried out numerical simulation of the tubes impact motion with generally assumed oblique impacts. (Charpentier and Payen, 2000) have carried out prediction of wear work-rate and thickness loss in tube bundles under cross-flow by a probabilistic approach. They have used Archard’s Law and wear correlation depending on the contact geometry, and have concluded that most sensitive parameters that affect the wear work-rate are the coefficient of friction, the radial gap and the spectral level of turbulent forces. (Paidoussis & Li, 1992) and (Chen et al., 1995) have studied the chaotic dynamics of heat exchanger tubes impacting on the generally loose baffle plates, using an analytical model that involves delay differential equations. They have developed a Lyapunov exponent technique for delay differential equations and have shown that chaotic motions do occur. They have performed analysis by finding periodic solutions and determining their stability and bifurcations with the Poincare map technique. Hopf bifurcation is defined as the loss of stable equilibrium and onset of amplified oscillation (Paidoussis & Li, 1992).

![Bifurcation Diagram](image)

Fig. 16. The bifurcation diagram (Paidoussis & Li, 1992, Chen et al., 1995)

A typical bifurcation diagram for the symmetric cubical model with \( P/D = 1.5 \), is given in Figure 16 showing dimensionless midpoint displacement amplitude in terms of dimensionless fluid velocity. Where \( U_H \) denotes critical \( U \) for Hopf bifurcation, \( U_P \), is the first post Hopf bifurcation, and \( U_{CH} \) denotes the onset of chaos. Total wear work rates against pitch velocity and mass flux have been given by (Taylor et al., 1995) and (Khushnood et al., 2003).
<table>
<thead>
<tr>
<th>Researchers</th>
<th>Salient fretting - wear features</th>
</tr>
</thead>
</table>
| (Rubiolo & Young, 2009)                 | • The evaluation of turbulence excitation is very challenging.  
• Identification of key wear factors that can be correlated to assembly operating conditions.  
• Functional dependence of wear damage against identified factors.  
• Grid cell clearance size and turbulence forces as key risk factors for PWR fuel assemblies.  
• Grid misalignment and cell tilts are less important.  
• Minimization of wear risk through modification in core loading.                                                                                                             |
| (Jong & Jung, 2008)                     | • Fretting wear in helical coil tubes steam generator.  
• Thermal-Hydraulic prediction through FEM.  
• Emphasis on the effects of number of supports, coil diameter and helix pitch on free vibration modes.  
• Design guidelines for designers and regulatory reviewers.                                                                                                                  |
| (Attia, 2006a)                          | • Investigation of fretting wear of Zr-2.5% Nb alloy.  
• Experimental setup includes special design fretting wear tribometers.  
• Fretting wear is initially dominated by adhesion and abrasion and then delamination and surface fatigue.  
• Volumetric wear loss decreased with number of cycles.                                                                                                                   |
| (Attia, 2006b)                          | • Fitness for service and life management against fretting fatigue.  
• Examples of fretting problems encountered in nuclear power plants.  
• Methodology to determine root cause.  
• Non-linearity of the problem and risk management.  
• Critical role of validation experimentally (long term) under realistic conditions and to qualify in-situ measurements of fretting damage non-destructive testing. |
| (Rubiolo, 2006)                         | • Probabilistic method of fretting wears predictions in fuel rods.  
• Non-linear vibration model VITRAN (Vibration Transient Analysis).  
• Numerical calculations of grid work and wear rates.  
• Monte carlo method applied for transient simulations (due to large variability of fuel assemble parameters).  
• Design preference of fuel rods.                                                                                                                                          |
| (Kim et al., 2006)                      | • A way toward efficient of restraining wear.  
• Increase in contact area through two different contours of spacer grid spacing.  
• Consideration of contact forces, slip displacement and wear scars on rods to explore mechanical damage phenomenon.  
• It concludes that the contact shape affects the feature and behavior of length, width and volumetric shape of wear.  
• A new parameter “equivalent depth” is introduced to represent wear severity.                                                                                               |

Table 7. Salient features of some recent researches on fretting wear Damage in Tube Bundles.
A generalized procedure to analyze fretting-wear process and its self-induced changes in properties of the system and flow chart for fretting fatigue damage prediction with the aid of the principles of fracture mechanics is presented in figure 17 & 18 respectively.

Fig. 17. System approach to the fretting wear process and its self-induced changes in the system properties (Attia, 2006a).

Fig. 18. Flow chart for the prediction of fretting fatigue damage, using fracture mechanics principles (Attia, 2006a).
7. Tube bundle vibrations in two-phase cross-flow

7.1 Modeling two-phase flow

Most of the early experimental research in this field relied on sectional models of tube arrays subjected to single-phase fluids such as air or water, using relatively inexpensive flow loops and wind tunnels. The cheapest and simplest approach to model two-phase flow is by mixing air and water at atmospheric pressure. However, air-water flows have a much different density ratio between phases than steam-water flow and this will affect the difference in the flow velocity between the phases. The liquid surface tension, which controls the bubble size, is also not accurately modeled in air-water mixtures. Table 8 gives the comparison of liquid and gas phase of refrigerants R-11, R-22 and air-water mixtures at representative laboratory conditions with actual steam-water mixture properties at typical power plant conditions (Feentra et al., 2000). This comparison reveals that the refrigerants approximate the liquid surface tension and liquid dynamic viscosity of steam-water mixtures more accurately than air-water mixtures.

<table>
<thead>
<tr>
<th>Property</th>
<th>R-11</th>
<th>Air-water</th>
<th>R-22</th>
<th>Steam-water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>40</td>
<td>22</td>
<td>23.3</td>
<td>260</td>
</tr>
<tr>
<td>Pressure (kPa)</td>
<td>175</td>
<td>101</td>
<td>1000</td>
<td>4690</td>
</tr>
<tr>
<td>Liquid Density (kg/m³)</td>
<td>1440</td>
<td>998</td>
<td>1197</td>
<td>784</td>
</tr>
<tr>
<td>Gas Density (kg/m³)</td>
<td>9.7</td>
<td>1.18</td>
<td>42.3</td>
<td>23.7</td>
</tr>
<tr>
<td>Liquid kinematic viscosity (μm²/sec)</td>
<td>0.25</td>
<td>1.0</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>Gas kinematic Viscosity (μm²/sec)</td>
<td>1.2</td>
<td>1.47</td>
<td>0.30</td>
<td>0.75</td>
</tr>
<tr>
<td>Liquid Surface Tension (N/m)</td>
<td>0.016</td>
<td>0.073</td>
<td>0.0074</td>
<td>0.0238</td>
</tr>
<tr>
<td>Density Ratio</td>
<td>148</td>
<td>845</td>
<td>28.3</td>
<td>33</td>
</tr>
<tr>
<td>Viscosity Ratio</td>
<td>0.20</td>
<td>0.70</td>
<td>0.47</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 8. Comparison of properties of air-water, R-22, and R-11 with steam-water at plant conditions (Feentra et al., 2000)

Typical nuclear steam generators such as those used in the CANDU design utilize more than 3000 tubes, 13mm in diameter, formed into an inverted U-shape. In the outer U-bend region, these tubes are subject to two-phase cross-flow of steam-water which is estimated to be of 20% quality. It is highly impractical and costly to perform flow-induced vibration experiments on a full-scale prototype of such a device so that small-scale sectional modeling is most often adopted. R-11 simulates the density ratio, viscosity ratio and surface tension of actual steam-water mixtures better than air-water mixtures and it also allows for localized phase change which air-water mixture does not permit. While more costly and difficult to use than air-water mixture, R-11 is a much cheaper fluid to model than steam-water because it requires 8% of the energy compared with water to evaporate the liquid and operating pressure is much lower, thereby reducing the size and cost of the flow loop (Feentra et al., 2000).

7.2 Representative published tests on two-phase flow across tube arrays

Table 6, an extension of period beyond 1993 (Pettigrew et al., 1973) presents a summary of salient features of the experimental tests performed on the three possible tube arrangements (triangular, normal square, and rotated square).
<table>
<thead>
<tr>
<th>Researchers</th>
<th>Fluid</th>
<th>Tube Array</th>
<th>Void Fraction</th>
<th>Tube Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Pettigrew et al., 1973)</td>
<td>Air-Water</td>
<td>Triangular/ Parallel. Square/ Rotated Square</td>
<td>10-20% (quality)</td>
<td>50</td>
</tr>
<tr>
<td>(Heilker &amp; Vincent, 1981)</td>
<td>Air-Water</td>
<td>Triangular/ Rotated Square</td>
<td>0.5 – 0.87</td>
<td>90</td>
</tr>
<tr>
<td>(Hara et al., 1981)</td>
<td>Air-Water</td>
<td>Single Tube</td>
<td>0.02 – 0.61</td>
<td>60</td>
</tr>
<tr>
<td>(Remy, 1982)</td>
<td>Air-Water</td>
<td>Square</td>
<td>0.65 – 0.85</td>
<td>10</td>
</tr>
<tr>
<td>(Nakamura et al., 1982)</td>
<td>Air-Water</td>
<td>Square/ Rotated Square</td>
<td>0.2 – 0.94</td>
<td>15</td>
</tr>
<tr>
<td>(Pettigrew et al., 1985)</td>
<td>Air-Water</td>
<td>Triangular/ Square</td>
<td>0.05-0.98</td>
<td>60</td>
</tr>
<tr>
<td>(Axisa et al., 1984)</td>
<td>Steam-Water</td>
<td>Square</td>
<td>0.52-0.98</td>
<td>11</td>
</tr>
<tr>
<td>(Nakamura et al., 1986)</td>
<td>Steam-Water</td>
<td>Square</td>
<td>0.75-0.95</td>
<td>12</td>
</tr>
<tr>
<td>(Hara, 1987)</td>
<td>Air-Water</td>
<td>Single/Row</td>
<td>0.01-0.5</td>
<td>5</td>
</tr>
<tr>
<td>(Goyder, 1988)</td>
<td>Air-Water</td>
<td>Triangular</td>
<td>0.5-0.8</td>
<td>34</td>
</tr>
<tr>
<td>(Gay et al., 1988)</td>
<td>Freon</td>
<td>Triangular</td>
<td>0.58-0.84</td>
<td>10</td>
</tr>
<tr>
<td>(Nakamura &amp; Fujita, 1988)</td>
<td>Air-Water</td>
<td>Square</td>
<td>0.02-0.95</td>
<td>60</td>
</tr>
<tr>
<td>(Funakawa et al., 1989)</td>
<td>Air-Water</td>
<td>Square/ Triangular</td>
<td>0.0-0.6</td>
<td>16</td>
</tr>
<tr>
<td>(Nakamura et al., 1990)</td>
<td>Steam-water</td>
<td>Square</td>
<td>0.33-0.91</td>
<td>17</td>
</tr>
<tr>
<td>(Axisa et al., 1990)</td>
<td>Steam-water</td>
<td>Square/ Triangular Parallel/ Triangular</td>
<td>0.52-0.99</td>
<td>11</td>
</tr>
<tr>
<td>(Papp &amp; Chen, 1994)</td>
<td>--</td>
<td>Normal triangular / Normal Square/ Parallel Triangular</td>
<td>25-98%</td>
<td>-</td>
</tr>
<tr>
<td>(Pettigrew, Taylor, Jong &amp; Currie, 1995)</td>
<td>Freon</td>
<td>Rotated triangular</td>
<td>40-90%</td>
<td>-</td>
</tr>
<tr>
<td>(Noghrrehkar et al., 1995)</td>
<td>Air-Water</td>
<td>Square fifth &amp; sixth row</td>
<td>0-90%</td>
<td>20</td>
</tr>
<tr>
<td>(Taylor et al., 1995)</td>
<td>Air-Water</td>
<td>U-bend tube bundle with 180° U-tubes parallel triangular configuration</td>
<td>0-90%</td>
<td>U-Tube radius 0.2</td>
</tr>
<tr>
<td>Marn &amp; Catton (Marn &amp; Catton, 1996)</td>
<td>Air-Water</td>
<td>Normal triangular, Parallel triangular, &amp; rotated square</td>
<td>5-99%</td>
<td>-</td>
</tr>
<tr>
<td>(Taylor &amp; Pettigrew, 2000)</td>
<td>Freon</td>
<td>Rotated triangular &amp; Rotated Square</td>
<td>50-98%</td>
<td>60</td>
</tr>
<tr>
<td>(Pettigrew et al., 2000)</td>
<td>Air-Water</td>
<td>Normal 30° &amp; rotated 60° triangular, Normal 90° &amp; rotated 45° square</td>
<td>0-100%</td>
<td>60</td>
</tr>
</tbody>
</table>
### Table 9. Representative Published Tests on Two-Phase Flow

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Fluid</th>
<th>Tube Array</th>
<th>Void Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Inada et al., 2000)</td>
<td>Air-Water</td>
<td>Square</td>
<td>0-70%</td>
</tr>
<tr>
<td>(Nakamera et al., 2000)</td>
<td>Freon</td>
<td>46x5 U-bend tubes, specification of actual westinghouse type-51 series steam generator.</td>
<td>(Not considered) based on Connors single-phase relation.</td>
</tr>
<tr>
<td>(Feentra et al., 2000)</td>
<td>R-11</td>
<td>Parallel/Triangular</td>
<td>0-0.99</td>
</tr>
<tr>
<td>(Pettigrew &amp; Taylor, 2003)</td>
<td>Steam-water</td>
<td>General overview</td>
<td>General overview</td>
</tr>
<tr>
<td>(Chung &amp; Chu, 2005)</td>
<td>Air-water</td>
<td>Normal square/rotated square</td>
<td>Void fraction 10-95%</td>
</tr>
<tr>
<td>(Parsad et al., 2007)</td>
<td>Steam-water</td>
<td>General overview</td>
<td>General overview</td>
</tr>
<tr>
<td>(Pascal-Ribot &amp; Blanchet, 2007)</td>
<td>Air-water</td>
<td>Rigid cylinder</td>
<td>10-80%</td>
</tr>
<tr>
<td>(Kakac &amp; Bon, 2008)</td>
<td>Steam-water</td>
<td>General overview</td>
<td>General overview</td>
</tr>
<tr>
<td>(Mitra et al., 2009)</td>
<td>Air-water / Steam-water</td>
<td>Normal square tube array suspended from piano wires</td>
<td>Void fraction 0-45%</td>
</tr>
<tr>
<td>(Sim &amp; Park, 2010)</td>
<td>--</td>
<td>Cantilevered flexible cylinder</td>
<td>3-38%</td>
</tr>
<tr>
<td>(Chu et al., 2011)</td>
<td>Air-water</td>
<td>U-tube rotated square</td>
<td>70-95%</td>
</tr>
</tbody>
</table>
7.3 Thermal hydraulic models

Considering two-phase flow, homogenous flow assumes that the gas and liquid phases are flowing at the same velocity, while other models for two-phase flow, such as drift-flux assume a separated flow model with the phases allowed to flow at different velocities. Generally the vapor flow is faster in upward flow because of the density difference.

7.3.1 The homogenous equilibrium model

Homogenous Equilibrium Model (HEM) treats the two-phase flow as finely mixed and homogeneous in density and temperature with no difference in velocity between the gas and liquid phases.

A general expression for void fraction $\alpha$, is given in (Feentra et al., 2000).

$$\alpha = \left[1 + S \frac{\rho_G}{\rho_L} \left(\frac{1}{x} - 1\right)\right]^{-1}$$

(36)

where $\rho_G$ and $\rho_L$ are the gas and liquid densities respectively and $S$ is the velocity ratio of the gas and liquid phase (i.e. $S = U_G / U_L$). The quality of the flow $x$ is calculated from energy balance, which requires measurement of the mass flow rate, the temperature of the liquid entering the heater, the heater power, and the fluid temperature in the test section. The HEM void fraction $\alpha_H$ is the simplest of the two-phase fluid modeling, whereby the gas and liquid phases are assumed to be well mixed and velocity ratio $S$ in Equation 36 is assumed to be unity. The average two-phase fluid density $\rho$ is determined by Equation 37.

$$\rho = a\rho_G + (1 - a)\rho_L$$

(37)

The HEM fluid density $\rho_H$ is determined using Equation 32 by substituting $\alpha_H$ in place of $\alpha$. The HEM pitch flow velocity $V_p$ is determined by

$$V_p = G_p / \rho_H$$

(38)

Where $G_p$=Pitch mass flux

7.3.2 Homogenous flow (Taylor & Pettigrew, 2000)

This model assumes no relative velocity between the liquid velocity $U_l$ and the gas velocity $U_g$. Slip $S$ between the two-phases is:

$$S = 1: U_h = U_g = U_l;$$

$$\varepsilon_g = \frac{j_g}{j_g + j_l}$$

(39)

where $U_h$ is the homogeneous velocity, $U_g$ is the gas phase velocity, $U_l$ is the liquid phase velocity, $\varepsilon_g$ is the homogenous void fraction, $j_g$ is superficial gas velocity and $j_l$ is the superficial liquid velocity.
7.3.3 Smith correlation

(Smith, 1968) assumes that kinetic energy of the liquid is equivalent to that of the two-phase mixture and a constant fraction \( k \) of liquid phase is entrained with the gas phase. The value \( k = 0.4 \) was chosen to correspond with the best agreement to experimental data for flow in a vertical tube. Using the Smith correlation, the slip is defined as follows.

\[
S = k + (1 - k) \left[ \frac{x \rho_l / \rho_g + k(1-x)}{x + k(1-x)} \right]^{1/2}
\]

(40)

where \( x \) is the mass quality, \( \rho_g \) is the density of the gas phase and \( \rho_l \) is the density of the liquid phase.

7.3.4 Drift-flux model

The main formulation of drift-flux model was developed by (Zuber and Findlay, 1965). This model takes into account both the two-phase flow non-uniformity and local differences of velocity between the two phases. The slip is defined as follows.

\[
S = \frac{(1 - \varepsilon_g)}{C_0 + \frac{U_g}{j}} - \varepsilon_g
\]

\[
C_0 = \frac{x}{1 - \frac{\rho_g}{\rho_l}} + \frac{\rho_g}{\rho_l}
\]

(41)

where \( \frac{U_g}{j} \) is averaged gas phase drift velocity.

\[
j = j_g + j_l = \dot{m} \left( \frac{x}{\rho_g + \frac{1-x}{\rho_l}} \right)
\]

(42)

Where \( \dot{m} \) is the mass flux.

The remaining two unknowns are empirical and (Lellouche et al., 1982) is used to estimate these.

\[
C_0 = \frac{L}{K_1 + (1-K_1)\varepsilon_g}
\]

(43)

7.3.5 Schrage correlation

The correlation by (Schrage, 1988) is based on empirical data from an experimental test section, which measures void fraction directly. This test section has two valves capable of isolating a part of the flow almost instantaneously.

The correlation is based on physical considerations and assumes two different hypotheses:
The Schrage correlation is as follows:

\[ \frac{\varepsilon_g}{\varepsilon_{gh}} = 1 + 0.123 F_r^{0.191} \ln x \]  

with

\[ F_r = \frac{\dot{m}}{\rho \sqrt{gD}} \]  

This correlation was established with an air-water mixture, but it remains valid for any other phase flow.

### 7.3.6 Feenstra model

In this model (Feentra et al., 2000), predicted velocity ratio of the phases is given by

\[ S = 1 + 25.7 (R_i \times \text{Cap})^{0.5} (P/D)^{-1} \]  

Where \( \text{Cap} \) is the capillary number and \( R_i \) is the Richardson number.

### 7.3.7 Comparison of void fraction models

The HEM greatly over-predicts the actual gamma densitometer void fraction measurement and the prediction of void fraction model by Feenstra et al., is superior to that of other models. It also agrees with data in literature for air-water over a wide range of mass flux and array geometry (Feentra et al., 2000). The main problem with using the HEM is that it assumes zero velocity ratios between the gas and liquid phases. This assumption is not valid in the case of vertical upward flow, because of significant buoyancy effects.

### 7.4 Dynamic parameters

#### 7.4.1 Hydrodynamic mass

Hydrodynamic mass \( m_h \) is defined as the equivalent external mass of fluid vibrating with the tube. It is related to the tube natural frequency \( f \) in two-phase mixture as discussed in (Carlucci & Brown, 1997) and is given below:

\[ m_h = m_t (f_g^2 / f)^2 - 1 \]  

where \( m_t \) is the mass of tube alone and \( f_g \) is the natural frequency in air.

Hydrodynamic mass depends on the pitch-to-diameter ratio of the tube, and is given by (Pettigrew et al., 1989)

\[ m_h = \left( \frac{\rho \pi d^2}{4} \right) \left[ \left( \frac{D_e}{d} \right)^2 + 1 \right] \left( \frac{(D_e/d)^2 - 1}{(D_e/d)^2 - 1} \right) \]  

where \( \rho \) is the two-phase mixture density.
\[ D_e / d = (0.96 + 0.5P / d)P / d , \text{ for a triangular bundle.} \]  \hspace{1cm} (49a)

\[ D_e / d = (1.07 + 0.56P / d)P / d , \text{ for a square bundle.} \]  \hspace{1cm} (49b)

where \( D_e \) is equivalent diameter to model confinement due to the surrounding tubes as given by (Rogers et al., 1984).

Early air-water studies (Carlucci, 1980) showed that added mass decreases with the void fraction as shown in Figure 19. It is also less than \((1 - \alpha)\), where \(\alpha\) is the void fraction. This deviation from expected \((1 - \alpha)\) line is caused by the air bubble concentrate at the flow passage center. Surprisingly added mass has attracted very little attention of researchers which is a potential avenue for future researches.

Fig. 19. Added mass as a function of void fraction (Carlucci, 1980).

### 7.4.2 Damping in two-phase

Subtracting the structural damping ratio from the total yields the two-phase fluid-damping ratio (Noghrehkar et al., 1995). Total damping includes structural damping, viscous damping and a two-phase component of damping as explained by (Pettigrew et al. 1994). The damping ratio increases as the void fraction increases and peaks at 60% (Carlucci, 1983), then the ratio decrease with \(\alpha\) (Figure 20). Damping also decreases as the vibration frequency increases (Pettigrew et al., 1985).
Damping in two-phase is very complicated. It is highly dependent upon void fraction and flow regime. The results for the two-phase component of damping can be normalized to take into account the effect of confinement due to surrounding tubes by using the confinement factor $C$ (Pettigrew et al., 2000). This factor is a reasonable formulation of the confinement due to $P / D$. As expected, greater confinement due to smaller $P / D$ increase damping. The confinement factor is given by equation below:

$$C = \frac{1 + (D / D_2)^3}{[1 - (D / D_2)^2]^2}$$  \hspace{1cm} (50)

### 7.5 Flow regimes

Many researchers have attempted the prediction of flow regimes in two-phase vertical flow. As yet, a much smaller group has examined flow regimes in cross-flow over tube bundles. Some of the first experiments were carried out by I.D.R. Grant (Collier, 1979) as it was the only available map at the time. Early studies in two-phase cross-flow used the Grant map to assist in identifying tube bundle flow regimes (Pettigrew et al., 1989) and (Taylor et al., 1989). More recently, Ulbrich & Mewes [180] performed a comprehensive analysis of available flow regime data resulting in a flow regime boundaries that cover a much larger
range of flow rates. They found that their new transition lines had an 86% agreement with available data. Their flow map is shown in Figure 21 by (Feenstra et al., 1990) with the flow regime boundary transitions in solid lines and the flow regimes identified with upper-case text. The dotted lines outline a previous flow regime map based on Freon-11 flow in a vertical tube from (Taitel et al., 1980).

Fig. 21. Flow regime map for vertically upward two-phase flow: From (Feenstra et al., 1986, Taitel et al., 1980), (Pettigrew et al., 1989), (Axisa, 1985), (Pettigrew et al., 1995), (Feenstra et al., 1995).

Almost every study of flow regimes in tube bundles has concluded that three distinct flow regimes exist. In fact, several studies have shown that these regimes can easily be identified by measuring the probability density function (PDF) of the gas component of the flow (Ulbrich & Mewes, 1997), (Noghrehkar et al., 1995) and (Lian et al., 1997).

7.6 Tube to restraint interaction (wear work-rate)

Significant tube-to-restraint interaction can lead to fretting wear. Large amplitude out-of-plane motion will result in large impact forces and in-plane motion will contribute to rubbing action. Impact force and tube-to-restraint relative motion can be combined to determine work-rate. Work-rate is calculated using the magnitude of the impact force and the effective sliding distance during line contact between the tube and restraint (Chen et al., 1995). The work-rate is given below in Equations 54 and 55.
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\[ W = \frac{1}{T_s} \int_{i=0}^{n} F_i dS_i \]  \hspace{1cm} (51)

\[ W = \frac{1}{T_s} \sum_{i=0}^{n} F_i \Delta S_i = \frac{1}{T_s} \sum_{i=0}^{n} \frac{F_i + F_{i+1}}{2} \Delta S_i \]  \hspace{1cm} (52)

where \( F_i \) is the instantaneous normal force, \( \Delta S_i \) is the sliding distance during line contact and \( n \) is the number of points discretized over the sample duration \( T_s \). As the work-rate increases, the effective wear rate increases and the operational life of the U-bend tube decreases. Implementation of the technology is described in detail by (Fisher et al., 1991). Measured values of wear work-rate for pitch velocity and mass flux (Chen et al., 1995) are presented in Figures 22a and 22b respectively. The effect of fluid-elastic forces is very evident in the measured work-rates.

It is interesting to note that at higher pitch velocities and/or mass fluxes, the wear work-rate does not increase. Further study is required to understand why the flow-rates do not affect the work-rates. This may be related to the fact that at high void fractions and high flow rates the random excitation forces are constant with increasing flow rate (Taylor, 1992).

![Fig. 22(a). Measured work-rate versus pitch velocity (Chen et al., 1995)](image-url)
7.7 Measurement of void fraction

In general, the surveyed research indicates two types of void fraction measurements (Feentra et al., 2000). The HEM void fraction and RAD void fraction. HEM refers to Homogeneous Equilibrium Model and RAD refers to Radiation Attenuation Method. The determination of fluid parameters (fluid density and flow velocity) are quite different when these two methods are used (Feentra et al., 2000). In RAD method (Feenstra et al., 2000, Wright & Bannister, 1970) gamma flux from radiation source which penetrates the test section will be attenuated by different amounts depending upon the average density of the two-phase flow. Void fraction $\alpha$ can be determined by interpolating the average density of the fluid between the benchmark measurements for one hundred percent liquid and gas according to the following equation.

$$\alpha = \ln\left(\frac{N}{N_L}\right) / \ln\left(\frac{N_G}{N_L}\right)$$

(53)

where $N$ represents the gamma counts obtained during an experimental trial, $N_L$ and $N_G$ are the reference counts obtained prior to the experiment for 100% liquid and 100% gas respectively. Gas phase velocity, $U_G$, and liquid phase velocity $U_L$ can be calculated by Equations below:

$$U_G = \frac{xG_p}{\alpha \rho_G}$$

(54)
where $G_p$ is the pitch mass flux.

A logical measure of an equivalent two-phase velocity, $V_{eq}$ is determined from averaging the dynamic head of the gas and liquid phases as given by equation below:

$$V_{eq} = \sqrt{\alpha \rho_G U_G^2 + (1 - \alpha) \rho_L U_L^2} / \rho$$  \hspace{1cm} (56)

8. Conclusions

Loss of Millions of Dollars through Cross-Flow-Induced-Vibrations related problems in steam generators and heat exchangers excitations has been a cause of major concern in process, power generation and nuclear industries. Flow-Induced Vibration pose a potential problem to designers, process engineers and plant operating and maintenance personnel. Such vibrations lead to motion of tubes in loose supports of baffles of tube bundles, resulting in mechanical damage, fretting wear, leaking and fatigue etc. Heat exchanger tubes are the most flexible components of the assembly. The risk of radiation exposure is always present in case of leakage in steam generator of PWR plants due to vibration related tube failures.

A number of design consideration have been reviewed in this chapter in order to achieve design improvements to support large scale heat exchangers with increased shell-side cross-flow-velocities. The prime consideration is the natural frequency of tubes in a bundle against cross-flow-induced-vibrations. Various analytical, experimental and computational techniques for straight & curved tubes have been discussed with reference to single and multiple spans and varying end and intermediate support conditions. Earlier, Flow-Induced-Vibration analysis was based upon the concept of two types of damage numbers (Collision damage and baffle damage). Discussion on these damage numbers and on the parameters that influence damping has been included.

Next consideration is the generally accepted following four tube bundle vibration excitation mechanisms (various models have been discussed & reviewed) including steady, unsteady, analytical, FEM based, CFD based, experimental, empirical correlation based, large eddy simulation (LES) based, linear and non-linear etc.

- **Turbulent Buffeting** - It can not be avoided in Heat Exchangers and is caused due to turbulence.
- **Vorticity Excitation** - Vortex shedding or periodic wake Shedding
  Self excited vibration resulting from interaction of tube motion and flow is the most dangerous excitation mechanism.
- **Fluid-Elastic Instability** - Caused by some flow excitation having frequency which coincides with natural frequency.
- **Acoustic Resonance** - 

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Dynamic parameters like added mass and damping which are function of geometry, density of fluid and tube size have been targeted by a number of researches in single-phase and two-phase flow. These researches have identified seven separate sources of damping which have been highlighted.

Tube wear due to non-linear tube-to-tube support plate interactions caused by gap clearances between interacting components resulting in thickness loss and normal wear work-rates have been reviewed. Chaotic dynamics of tubes impacting generally on loose baffle plates with consideration of stability and bifurcation have been discussed.

Two-phase Cross-Flow-Induced-Vibrations in tube bundles of process heat exchanger and U-bend region of Nuclear Steam generators can cause serious tube factures by fatigue and fretting wear. Solution to such problems require understanding of vibration excitation and damping mechanism in two-phase flow. This further requires consideration of different flow regimes which characterize two-phase flow. The discussion includes the most important parameter which is void fraction, various thermal-hydraulic models, dynamic parameters, wear work-rates, void fraction measurement and application of TEMA/ASME and other codes have been reviewed. In conclusion the objective of this chapter is to suggest improvements in the design guidelines from the available researches to use the related equipment at optimal performance level.

9. Acknowledgements

We are deeply indebted to University of Engineering & Technology, Taxila – Pakistan, PASTIC, Islamabad – Pakistan and College of EME NUST, Rawalpindi – Pakistan for providing financial, administrative and technical support. We sincerely appreciate the support provided by Mr. Zahid Iqbal, Mr. Riffat Iqbal and Mr. Muhammad Shafique in finalizing the manuscript.

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This book covers various topics, from thermal-hydraulic analysis to the safety analysis of nuclear power plant. It does not focus only on current power plant issues. Instead, it aims to address the challenging ideas that can be implemented in and used for the development of future nuclear power plants. This book will take the readers into the world of innovative research and development of future plants. Find your interests inside this book!

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