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The Pursuit of Evolutionary Particle Swarm Optimization

Hong Zhang
Department of Brain Science and Engineering,
Kyushu Institute of Technology
Japan

1. Introduction

Particle swarm optimization inspired with the social behavior in flocks of birds and schools of fish is an adaptive, stochastic and population-based optimization technique which was created by Kennedy and Eberhart in 1995 (9; 12). As one of the representatives of swarm intelligence (20), it has the distinctive characteristics: information exchange, intrinsic memory, and directional search in contrast to genetic algorithms (GAs) (14) and genetic programming (GP) (16). Due to ease of understanding and implementation, good expression and expandability, higher searching ability and solution accuracy, the technique has been successfully applied to different fields of science, technology, engineering, and applications for dealing with various large-scale, high-grade nonlinear, and multimodal optimization problems (22; 23).

Although the mechanism of a plain particle swarm optimizer (the PSO) (13) is simple to implement with only a few parameters, in general, it can provide better computational results in contrast to other methods such as machine learning, neural network learning, genetic algorithms, tabu search, and simulated annealing (1). Nevertheless, like other optimization methods, an essential issue is how to make the PSO efficiently in dealing with different kinds of optimization problems. And it is well-known that the systematic selection of the parameter values in the PSO is one of fundamental manners to the end, and the most important especially for establishing a policy which determines the PSO with high search performance.

However, in fact how to properly determine the values of parameters in the PSO is a quite attractive but hard subject especially for a detailed analysis of higher order (7). The cause is because the search behavior of the PSO has very high indeterminacy. Usually, these parameter values related to internal stochastic factors need to be adjusted for keeping search efficiency (5).

As new development and expansion of the technique of meta-optimization[1], the above issue already can be settled by the method of evolutionary particle swarm optimization (EPSO) (27), which provides a good framework to systematically estimate appropriate values of

[1] Meta-optimization, in general, is defined as the process of using an optimization algorithm to automatically search the best optimizer from all computable optimizers.

parameters in a particle swarm optimizer corresponding to a given optimization problem without any prior knowledge. Based on the use of meta-optimization, it could be expected to not only efficiently obtain an optimal PSO, but also to quantitatively analyze the know-how on designing it. According to the utility and reality of the method of the EPSO, further deepening meta-optimization research, i.e. dynamic estimation approach, is an indispensable and necessary step for efficiently dealing with any complex optimization problems.

To investigate the potential characteristics and effect of the EPSO, here we propose and study to use two different criteria: a temporally cumulative fitness function of the best particle and a temporally cumulative fitness function of the entire particle swarm respectively to evaluate the search performance of the PSO in an estimation process. The goal of the attempt is to supply the demand for diversification satisfying some different specification to the optimizer.

Needless to say, the search behavior and performance of the PSO closely relies on the determined values of parameters in the optimizer itself. For revealing the inherent characteristics of the obtained PSOs, we also propose an indicator to measure the difference in convergence of the PSOs estimated by respectively implementing each criterion. Due to verify the effectiveness of the proposed method and different characters of the obtained results, computer experiments on a suite of multidimensional benchmark problems are carried out.

The rest of the paper is organized as follows. Section 2 introduces the related work on this study. Section 3 describes basic mechanisms of the PSO and EPSO, two different criteria, and an indicator in detail. Section 4 shows the obtained results of computer experiments applied to a suite of multidimensional benchmark problems, and analyzes the respective character of the estimated PSOs with using each criterion. Finally Section 5 gives the conclusion and discussion.

2. Related work

Until now, many researchers have paid much attention to the issue, i.e. effectually obtaining the PSO with high search performance, and proposed a number of advanced algorithms to deal with it. These endeavors can be basically divided into two approaches: manual estimation and mechanical estimation shown in Figure 1.

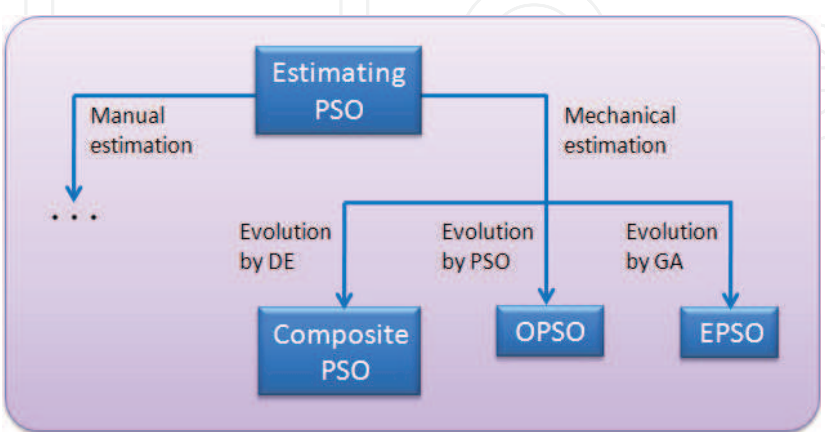

Fig. 1. Family of estimating PSO methods

Manual estimation is to try many values of parameters to find a proper set of parameter values in the PSO for dealing with various optimization problems reasonably well (2; 4; 10). Since its procedure belongs to a trial-and-error search, the computational cost is enormous, and the estimating accuracy is not enough.

In contrast to the above situation, mechanical estimation is to directly utilize evolutionary computation for achieving the task. A composite PSO (cPSO) (21) was proposed to estimate the parameter values of the PSO during optimization. In the cPSO, the differential evolution (DE) algorithm (24) is used to generate a difference vector of two randomly picked boundary vectors for parameter selection. In spite of the effect to the estimation, the internal stochastic factors in the DE have an enormous impact on the estimating process. Therefore, the recreation to obtain some similar results is difficult. This is the major shortcoming of the cPSO for certification.

In order to overcome the above mentioned weakness of instability in an estimation process, Meissner et al. proposed a method of optimized particle swarm optimization (OPSO) as an extension of the cPSO, which uses the PSO to deal with meta-optimization of the PSO heuristics (18). Zhang et al. independently proposed a method of evolutionary particle swarm optimization (EPSO) which uses a real-coded genetic algorithm with elitism strategy (RGA/E) to accomplish the same task (27). These methods are positive attempts of evolutionary computation applied for the design of the PSO itself, and give a marked tendency to deal with meta-optimization of analogous stochastic optimizers heuristics.

By comparing the mechanisms of both the OPSO and EPSO, we see that there are two big differences in achievement of estimating the PSO. The first one is on the judgment (selection) way used in evaluating the search performance of the PSO. The former uses an instantaneous fitness function and the PSO to estimation, and the latter uses a temporally cumulative fitness function and the RGA/E to estimation. The second one is on the estimating manner used in dealing with meta-optimization of the PSO heuristics.

Owing to the temporally cumulative fitness being the sum of an instantaneous fitness, fundamentally, the variation of the obtained parameter values, which comes from the stochastic influence in a dynamic evaluation process, can be vastly alleviated. According to this occasion, the use of the adopted criterion could be expected to give rigorous determination of the parameter values in the PSO, which will guide a particle swarm to efficiently find good solutions.

To investigate the potential characteristics of the EPSO, a temporally cumulative fitness function of the best particle and a temporally cumulative fitness function of the entire particle swarm are used for evaluating the search performance of the PSO to parameter selection. The former was reported in our previous work (27; 29). The latter is a proposal representing active behavior of entire particles inspired by majority decision in social choice for the improvement of the convergence and search efficiency of the entire swarm search (28).

The aim of applying the different criteria in estimating the PSO is to pursue the intrinsic difference and the inherent characters on designing the PSO with high search performance. For quantitative analysis to the obtained results, we also propose an indicator for judging the situation of convergence of the PSO, i.e. the different characteristics between the fitness value of the best particle and the average of fitness values of the entire swarm over time-step in search.

3. Basic mechanisms

For the sake of the following description, let the search space be N-dimensional, $\mathbf{S} \in \Re^N$, the number of particles in a swarm be P, the position of the ith particle be $\vec{x}^i = (x_1^i, x_2^i, \cdots, x_N^i)^T$, and its velocity be $\vec{v}^i = (v_1^i, v_2^i, \cdots, v_N^i)^T$, repectively.

3.1 The PSO

In the beginning of the PSO search, the particle's position and velocity are generated randomly, then they are updated by

$$\begin{cases} \vec{x}_{k+1}^i = \vec{x}_k^i + \vec{v}_{k+1}^i \\ \vec{v}_{k+1}^i = c_0 \vec{v}_k^i + c_1 \vec{r}_1 \otimes (\vec{p}_k^i - \vec{x}_k^i) + c_2 \vec{r}_2 \otimes (\vec{q}_k - \vec{x}_k^i) \end{cases}$$

where c_0 is an inertia coefficient, c_1 and c_2 are coefficients for individual confidence and swarm confidence, respectively. $\vec{r}_1$ and $\vec{r}_2 \in \Re^N$ are two random vectors in which each element is uniformly distributed over the interval $[0, 1]$, and the symbol $\otimes$ is an element-wise operator for vector multiplication. $\vec{p}_k^i (= arg \max_{j=1,\cdots,k} \{g(\vec{x}_j^i)\}$, where $g(\cdot)$ is the fitness value of the ith particle at k time-step) is the local best position of the ith particle up to now, and $\vec{q}_k (= arg \max_{i=1,2,\cdots} \{g(\vec{p}_k^i)\})$ is the global best position among the whole particle swarm. In the original PSO, $c_0 = 1.0$ and $c_1 = c_2 = 2.0$ are used (12).

To prevent particles spread out to infinity in the PSO search, a boundary value, v_{max}, is introduced into the above update rule to limit the biggest velocity of each particle by

$$\begin{cases} v_{k+1}^{ij} = v_{max}, \quad if \;\; v_{k+1}^{ij} > v_{max} \\ v_{k+1}^{ij} = -v_{max}, \; if \;\; v_{k+1}^{ij} < -v_{max} \end{cases}$$

where v_{k+1}^{ij} is the jth element of the ith particle's velocity $\vec{v}_{k+1}^i$.

For attaining global convergence of the PSO, the studies of theoretical analysis were minutely investigated (3; 5; 6). Clerc proposed a canonical particle swarm optimizer (CPSO) and analyzed its dynamical behavior. According to Clerc's constriction method, the parameter values in the equivalent PSO are set to be $c_0^\star = 0.7298$ and $c_1^\star = c_2^\star = 1.4960$. Since the value of the inertia coefficient $c_0^\star$ is less than 1.0, the CPSO has better convergence compared to the original PSO. Consequently, it is usually applied for solving many practice problems as the best parameter values to search (17).

Although the set of the parameter values, ($c_0^\star$, $c_1^\star$, $c_2^\star$), is determined by a rigid analysis in a low-dimensional case, it is hard to declare that these parameter values are whether the surely best ones or not for efficiently dealing with different kinds of optimization problems, especially in a high-dimensional case. To distinguish the truth of this fact, correctly obtaining the information on the parameter values of the equivalent PSO by evolutionary computation is expected to make clear.

3.2 The EPSO

In order to certainly deal with meta-optimization of the PSO heuristics, the EPSO is composed of two loops: an outer loop and an inner loop. Figure 2 illustrates a flowchart of the EPSO run. The outer loop is a real-coded genetic algorithm with elitism strategy (RGA/E) (26). The inner loop is the PSO. This is an approach of dynamic estimation. They exchange the necessary information each other during the whole estimating process. Especially, as information transmission between the loops in each generation, the RGA/E provides each parameter set of parameter values, $\vec{c}^j = (c_0^j, c_1^j, c_2^j)$ (the j-th individual in a population, $j \in J$, where J is the number of individuals), to the PSO, and the PSO returns the values of the fitness function, $F(c_0^j, c_1^j, c_2^j)$, corresponding to the given parameter set to the RGA/E. By the evolutionary computation, the RGA/E simulates the survival of the fittest among individuals over generations for finding the best parameter values in the PSO.

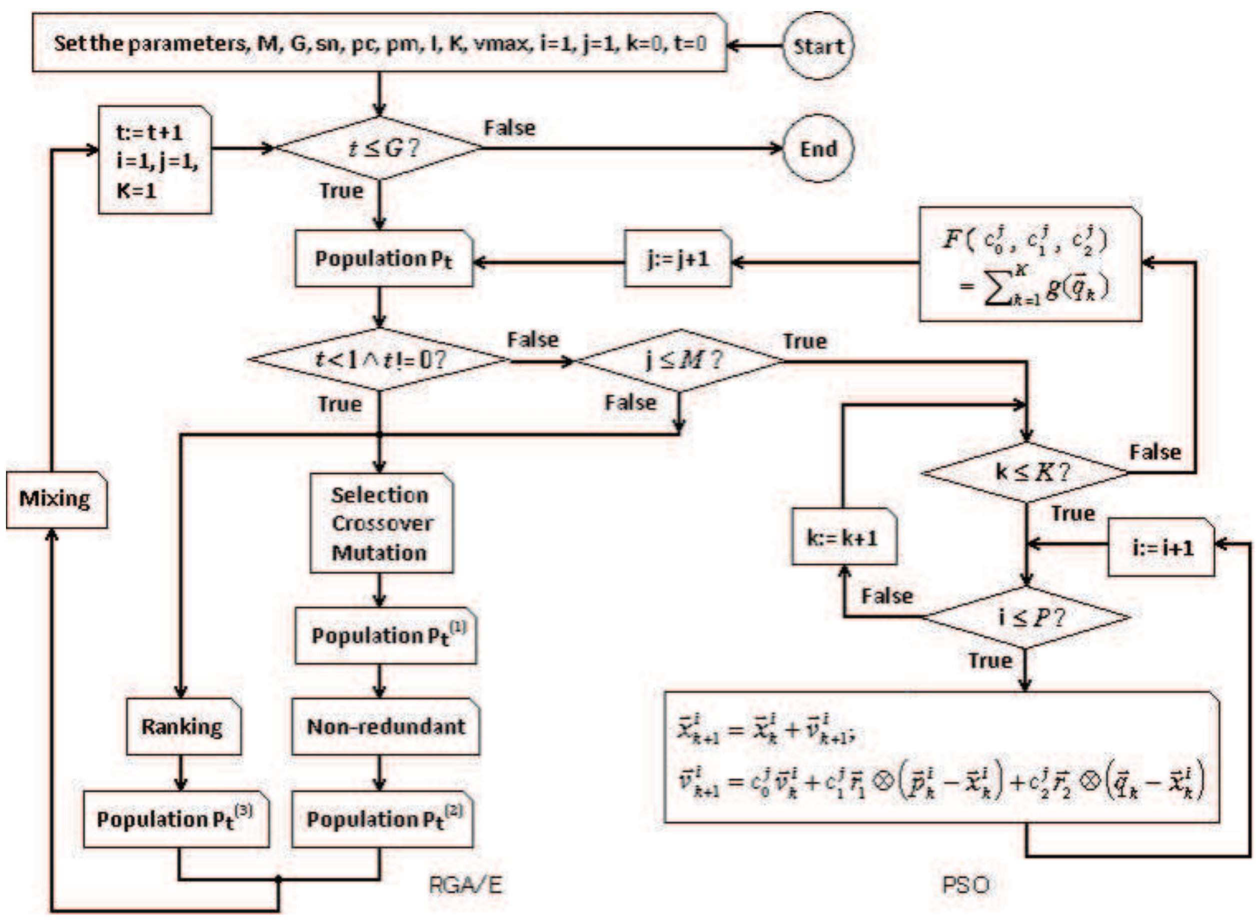

Fig. 2. A flowchart of the EPSO

As genetic operations in the RGA/E, roulette wheel selection, BLX-α crossover, random mutation, non-redundant strategy, and elitism strategy are used for efficiently finding an optimal individual (i.e. an optimal PSO) from the population of parameter values of the PSO. On being detailed, further refer to (33).

3.3 Two different criteria

To reveal the potential characteristics of the EPSO in estimation, two criteria are applied for evaluating the search performance of the PSO. The first criterion is a temporally cumulative fitness function of the best particle, which is defined as

$$F_1(c_0^j, c_1^j, c_2^j) = \sum_{k=1}^{K} g(\vec{q}_k)\big|_{c_0^j, c_1^j, c_2^j} \tag{1}$$

where K is the maximum number of iterations in the PSO run. The second criterion is a temporally cumulative fitness function of the entire particle swarm, which is defined as

$$F_2(c_0^j, c_1^j, c_2^j) = \sum_{k=1}^{K} \bar{g}_k\big|_{c_0^j, c_1^j, c_2^j} \tag{2}$$

where $\bar{g}_k = \sum_{i=1}^{P} g(\vec{x}_k^i)/P$ is the average of fitness values over the entire particle swarm at time-step k.

As an example, Figure 3 illustrates the relative evaluation between two pairs of the criteria, $\{g(\vec{q}_k), \bar{g}_k\}$ and $\{F_1, F_2\}$, during the evolutionary computation. It is clearly observed that the properties of the instantaneous fitness functions, $g(\vec{q}_k)$ and $\bar{g}_k$, are quite different. Namely, while the change of $g(\vec{q}_k)$ is monotonous increment, the change of $\bar{g}_k$ is non-monotonous increment with violent stochastic vibration. In contrast to this, the criteria, F_1 and F_2, are all monotonous increment with a minute vibration.

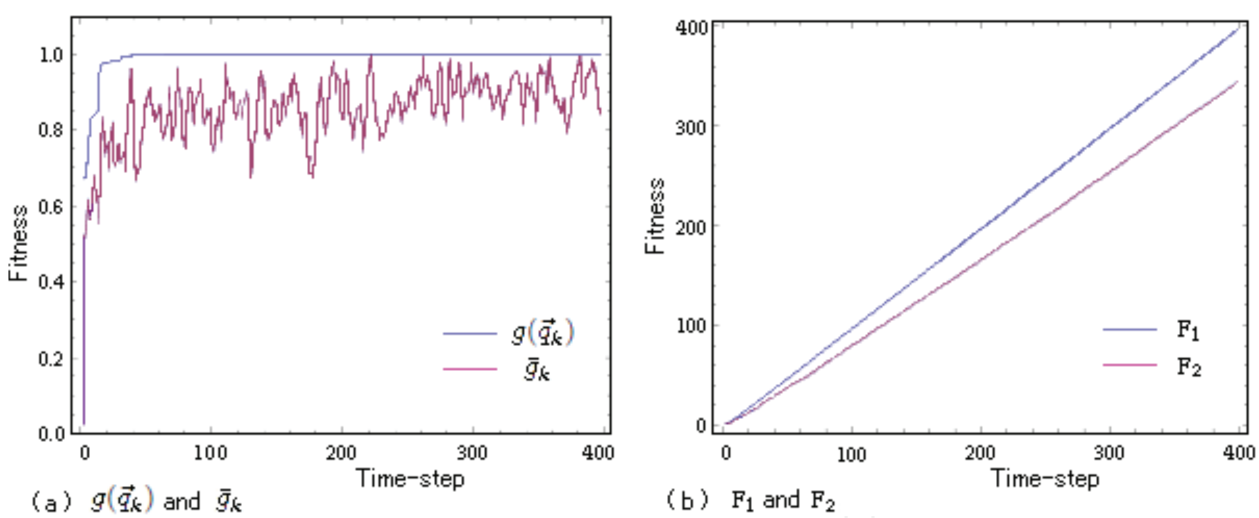

Fig. 3. Comparison of two pairs of the used fitness functions

Because both F_1 and F_2 are the sum of instantaneous fitness functions, $g(\vec{q}_k)$ and $\bar{g}_k$, over time-step, in theory, their variance is inversely proportional to the interval of summation. Thus, they could lead to vastly inhibit noise which comes from dynamic evaluation to the estimation. This property indicates that which of both F_1 and F_2 is well suitable for evaluating the search performance of the PSO, regardless of the difference in objects of evaluation themselves.

3.4 A convergence indicator

Looking from another viewpoint, the above difference in evaluational form can be considered as the disparity between the values of the temporally cumulative fitness function of the best particle and the average of fitness values over the entire particle swarm.

According to the concept of different characteristics, we propose to set the following convergence time-step, k_{max}, as a convergence indicator for measurement.

$$\forall k \geq k_{max}, \quad g(\vec{q}_k) - \bar{g}_k \leq \tau, \tag{3}$$

where τ is a positive tolerance coefficient.

It is clear that the shorter the convergence time-step is, the faster the convergence of particles is. Since most particles quickly converge on an optimal solution or a near-optimal solution, the convergence indicator, k_{max}, shows the conversion of difference of the different characteristics from increasing to decreasing, which representing a change of process, and indirectly records the index of diversity of the swarm over time-step in search.

4. Computer experiments

To facilitate comparison and analysis of the potential characteristics of the EPSO, the following suite of multidimensional benchmark problems (25) is used in the next experiments.

Sphere function:

$$f_{Sp}(\vec{x}) = \sum_{d=1}^{N} x_d^2$$

Griewank function:

$$f_{Gr}(\vec{x}) = \frac{1}{4000} \sum_{d=1}^{N} x_d^2 - \prod_{d=1}^{N} cos\Big(\frac{x_d}{\sqrt{d}}\Big) + 1$$

Rastrigin function:

$$f_{Ra}(\vec{x}) = \sum_{d=1}^{N} \Big(x_d^2 - 10\,cos(2\pi x_d) + 10\Big)$$

Rosenbrock function:

$$f_{Ro}(\vec{x}) = \sum_{d=1}^{N-1} 100\big(x_{d+1} - x_d^2\big)^2 + \big(1 - x_d\big)^2$$

The following fitness function in the search space, $\mathbf{S} \in (-5.12, 5.12)^N$, is defined by

$$g_\omega(\vec{x}) = \frac{1}{f_\omega(\vec{x}) + 1} \tag{4}$$

where the subscript, ω, stands for one of the followings: Sp (*Sphere*), Gr (*Griewank*), Ra (*Rastrigin*), and Ro (*Rosenbrock*). Since the value of each function, $f_\omega(\vec{x})$, at the optimal solution is zero, the largest fitness value, $g_\omega(\vec{x})$, is 1 for all given benchmark problems.

Figure 4 illustrates the distribution of each fitness function in two-dimensional space. It is clearly shown that the properties of each problem, i.e. the *Sphere* problem is an unimodal with axes-symmetry, the *Rosenbrock* problem is an unimodal with axes-asymmetry, and the *Griewank* and *Rastrigin* problems are multimodal with different distribution density and axes-symmetry.

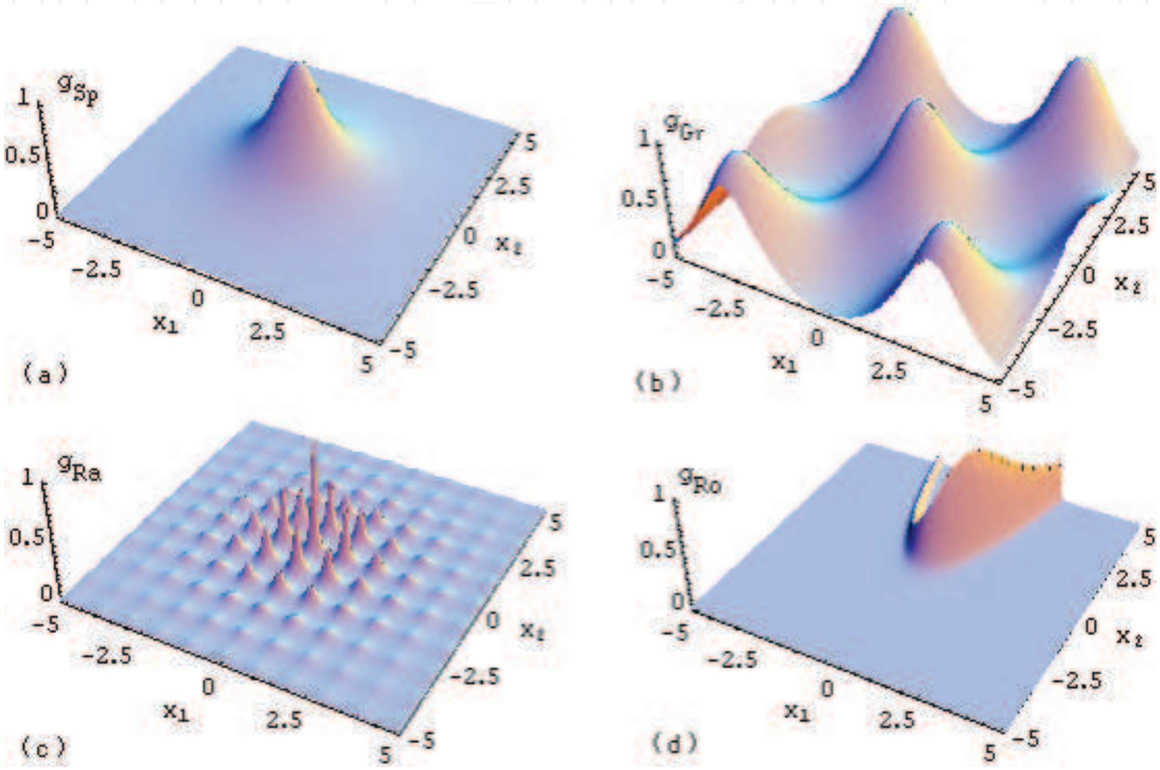

Fig. 4. Fitness functions corresponding to the given benchmark problems in two-dimensional space. (a) The *Sphere* problem, (b) The *Griewank* problem, (c) The *Rastrigin* problem, (d) The *Rosenbrock* problem.

4.1 Experimental condition

Table 1 gives the major parameters used in the EPSO run for parameter selection in the next experiments. As initial condition of the EPSO, positions of particles are set in random, and the corresponding velocities are set to zero.

Note that the constant, v_{max}, is used to arbitrarily limit the maximum velocity of each particle in search. Both non-redundant search and roulette wheel selection in genetic operations have not parameter to set. The smaller number of individuals, particles and iterations is chosen in order to acquire the balance between estimating accuracy and computing speed. As for the estimating accuracy, it can be guaranteed by repetitively taking average of the results.

On regarding the parameter setting for the genetic operations in the RGA/E, concretely, some experimental results reveal that bigger probability works better in generating superior

Parameters	Value
the number of individuals, J	10
the number of generations, G	20
the number of elite individuals, s_n	2
probability of BLX-2.0 crossover, p_c	0.5
probability of mutation, p_m	0.5
the number of particles, P	10
the number of iterations, K	400
the maximum velocity, v_{max}	5.12

Table 1. Major parameters in the EPSO run

individuals (33). This is the reason why the probability of crossover and mutation is set to 0.5 for efficient parameter selection.

4.2 Experimental results (1)

Computer experiments on estimating the PSO are carried out for each five-dimensional benchmark problem. It is to be noted that the appropriate values of parameters in the PSO are estimated under the condition, i.e. each parameter value is non-negative.

Based on the distribution of the resulting parameter values, $\hat{c}_0$, $\hat{c}_1$, and $\hat{c}_2$, within the top-twenty optimizers taken from the all obtained PSOs, they are divided into four groups, namely, a-type: $\hat{c}_0 = 0$, $\hat{c}_1 = 0$, $\hat{c}_2 > 0$; b-type: $\hat{c}_0 = 0$, $\hat{c}_1 > 0$, $\hat{c}_2 > 0$; c-type: $\hat{c}_0 > 0$, $\hat{c}_1 = 0$, $\hat{c}_2 > 0$; and d-type: $\hat{c}_0 > 0$, $\hat{c}_1 > 0$, $\hat{c}_2 > 0$. Doing this way is to adequately improve the accuracy of parameter selection, because each type of the obtained PSOs has stronger probability which solves the given benchmark problems regardless of the frequencies corresponding to them within the top-twenty optimizers.

Table 2 gives the resulting values of parameters in each type of the obtained PSOs, criterion values and frequencies. According to the statistical results, the following features and judgments are obtained.

1. The estimated PSOs are non-unique, and the parameter values in each optimizer are quite different from that in the original PSO or equivalent PSO.
2. The values of inertia coefficient, $\hat{c}_0$, and the coefficient for individual confidence, $\hat{c}_1$, could be zero, but the value of coefficient for swarm confidence, $\hat{c}_2$, is always non-zero, which plays an essential role in finding a solution to any given problem.
3. For the PSO in d-type cases, an overlapping phenomenon in each parameter value appears with the corresponding standard deviation (SD) in many cases. The variation of the respective SD indicates the adaptable range to each parameter value and the difficulty to obtain appropriate parameter value for handling the given problem.
4. For *Rastrigin* problem, both of $\hat{c}_1$ and $\hat{c}_2$ drastically exceed 1 in the criterion F_1 case. This suggests that the search behavior of the PSO is required to be more randomization extensively for enhancing the search performance to find an optimal solution or near-optimal solutions in search space. For the *Griewank* and *Rosenbrock* problems, $\hat{c}_1$

Problem	Dim.		Cumulative fitness	PSO	Parameter $\hat{c}_0$	$\hat{c}_1$	$\hat{c}_2$	Freq.
Sphere	5	F_1	395.3±0.7	*a*-type	0	0	2.4961±0.2468	20%
			–	*b*-type	–	–	–	–
			394.6±0.8	*c*-type	0.1975±0.0187	0	2.4665±0.3573	45%
			394.1±0.5	*d*-type	0.6770±0.2326	1.1293±0.0939	0.9375±0.6567	35%
		F_2	392.5±0.8	*a*-type	0	0	2.2990±0.1614	15%
			393.0±1.4	*b*-type	0	0.2397±0.1007	2.2867±0.1602	15%
			–	*c*-type	–	–	–	–
			392.2±0.5	*d*-type	0.4656±0.1514	0.9807±0.6100	1.3073±0.5850	70%
Griewank	5	F_1	–	*a*-type	–	–	–	–
			–	*b*-type	–	–	–	–
			396.8±0.0	*c*-type	0.1707±0.0000	0	0.6224±0.0000	5%
			396.6±0.6	*d*-type	0.5101±0.2669	2.0868±0.4260	1.0258±0.6117	95%
		F_2	–	*a*-type	–	–	–	–
			394.7±0.0	*b*-type	0	3.3247±0.0000	0.6994±0.0000	5%
			–	*c*-type	–	–	–	–
			394.8±0.8	*d*-type	0.4821±0.1911	1.2448±0.5229	1.6101±0.6596	95%
Rastrigin	5	F_1	–	*a*-type	–	–	–	–
			–	*b*-type	–	–	–	–
			396.0±0.0	*c*-type	1.0578±0.0000	0	82.171±0.0000	5%
			395.7±0.5	*d*-type	1.3459±0.5439	10.286±3.5227	24.929±21.857	95%
		F_2	230.0±20.9	*a*-type	0	0	3.8991±0.0681	100%
			–	*b*-type	–	–	–	–
			–	*c*-type	–	–	–	–
			–	*d*-type	–	–	–	–
Rosenbrock	5	F_1	–	*a*-type	–	–	–	–
			–	*b*-type	–	–	–	–
			298.4±3.7	*c*-type	0.6804±0.0000	0	2.1825±0.0000	10%
			317.1±18.8	*d*-type	0.9022±0.0689	1.3097±0.5619	0.7614±0.1689	90%
		F_2	–	*a*-type	–	–	–	–
			295.5±9.0	*b*-type	0	4.0370±0.5740	1.9494±0.1237	20%
			312.4±26.7	*c*-type	0.8033±0.0000	0	0.5165±0.0000	20%
			310.5±36.3	*d*-type	0.7042±0.0492	0.7120±0.3631	1.5028±0.6779	60%

Table 2. Estimated appropriate values of parameters in the PSO, cumulative fitness values, and frequencies in the top-twenty optimizers. The PSO in *a*-type: $\hat{c}_0 = 0$, $\hat{c}_1 = 0$, $\hat{c}_2 > 0$; The PSO in *b*-type: $\hat{c}_0 = 0$, $\hat{c}_1 > 0$, $\hat{c}_2 > 0$; The PSO in *c*-type: $\hat{c}_0 > 0$, $\hat{c}_1 = 0$, $\hat{c}_2 > 0$; in *d*-type: $\hat{c}_0 > 0$, $\hat{c}_1 > 0$, $\hat{c}_2 > 0$. The symbol "-" signifies no result corresponding to contain type of the PSO.

drastically exceeds 1 under the condition of $\hat{c}_0 = 0$. This suggests that there is a choice to adapt the spacial condition in using the criterion F_2 case for improving search performance of the PSO.

5. The average of the fitness values, F_1, is larger than that of F_2 except for the *Rosenbrock* problem. And the frequencies corresponding to the PSO in d-type are higher than other types for a majority given problems.

It is understood that the estimated PSOs related to each given benchmark problem are obtained by implementing the EPSO without any prior knowledge. The signification of the existence of the four types of the obtained PSOs reflects the possibility of problem-solving.

4.3 Performance analysis

For inspecting the results of the EPSO using two different criteria, we measure the search ability of each estimated PSO by the average of parameter values in Table 2, and show the obtained fitness values with 20 trials in Figure 5.

It is observed from Figure 5 that the search ability of the PSO estimated by using the criterion F_1 is superior to that by using the criterion F_2 except for the *Sphere* and *Griewank* problems. Therefore, the obtained results declare that the criterion F_1 is suitable for generating the PSO with higher adaptability in search compared with the criterion F_2. The cause is obvious, i.e. all of particles rapidly move in close to the global best position, $\vec{q}_k$, found by themselves up to now. About the fact, it can be confirmed by the following experiments. However, such improvement of the search performance of the entire particle swarm, in general, restricts active behavior of each particle, and will lose more chances for finding an optimal solution or near-optimal solutions.

For investigating the different characteristics, we measure the convergence time-step for each estimated PSO in d-type with the highest search ability in Figure 6. According to the different characteristics, for instance, the disparity between two criteria, i.e. $g(\vec{q}_k) - \bar{g}_k$, maximum tolerance, τ_{max}, and the convergence time-step, k_{max}, is shown in Figure 6.

In comparison with the difference between two criteria in the optimization, Table 3 gives the convergence time-step, k_{max}, of the original PSO, and the estimated PSO under the condition of the maximum tolerance, $\tau_{max}(= \max_{k=1\cdots K} \big(g(\vec{q}_k) - \bar{g}_k\big))$, corresponding to each given problem.

Problem	Convergence time-step, k_{max}		
	Original PSO	EPSO (F_1)	EPSO (F_2)
Sphere	236.1±95.63	8.100±2.268	7.200±2.375
Griewank	249.4±108.0	4.350±2.814	4.150±2.224
Rastrigin	363.4±92.40	224.2±152.3	99.15±52.83
Rosenbrock	397.4±2.370	34.15±7.862	25.72±8.672

Table 3. The convergence time-step for the original PSO and the estimated PSO.

Based on the results on the search performance (SP) and the convergence time-step (CT) in Table 3, the dominant relationship on their different characteristics is indicated as follows.

$$\text{SP:}\ \ \text{EPSO}(F_1) \succ \text{EPSO}(F_2) \succ \text{Original PSO}$$
$$\text{CT:}\ \ \text{EPSO}(F_2) \succ \text{EPSO}(F_1) \succ \text{Original PSO}$$

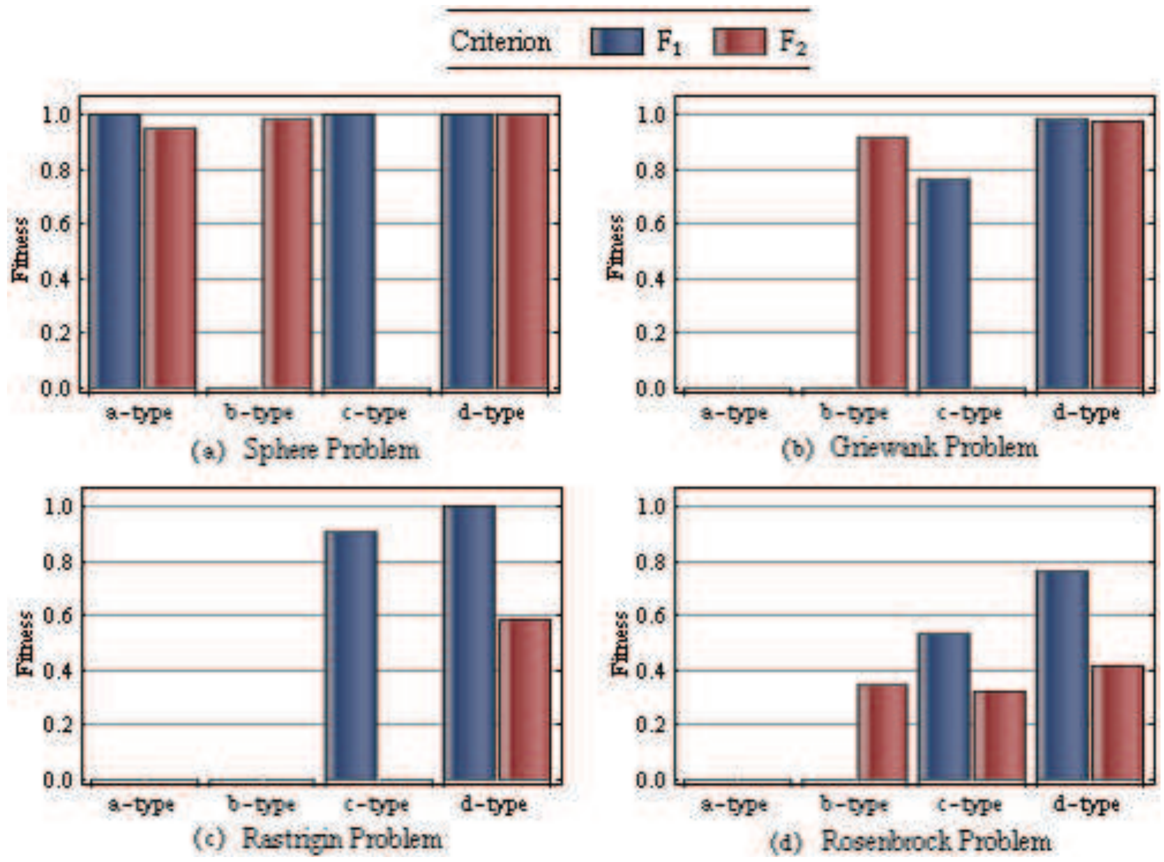

Fig. 5. The search ability of each estimated PSO

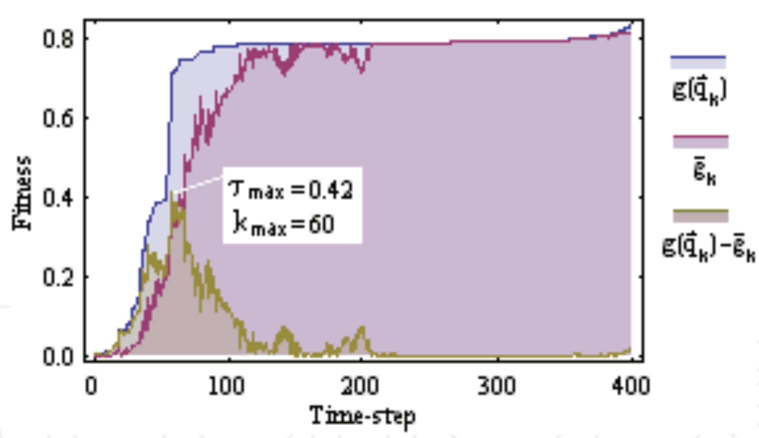

Fig. 6. The disparity in criterion, $g(\vec{q}_k) - \bar{g}_k$.

In comparison with both SP and CT, it is considered that the criterion F_1 well manages the trade-off between exploitation and exploration than that the criterion F_2 does. And the search performance of the original PSO is the lowest. These results indicate that these parameters, $c_0 = 1.0$ and $c_1 = c_2 = 2.0$, cannot manage the trade-off between exploration and exploitation in its heuristics well, so the original PSO is unreasonable for efficiently solving different optimization problems to conclude.

Table 4 gives the results of implementing the EPSO, the original PSO, the original CPSO, OPSO, and RGA/E. We can see that the search performance of the PSOs optimized by the

EPSO using the criterion F_1 is superior to that by the original PSO, the original CPSO, OPSO, and RGA/E for the given benchmark problems except the *Sphere* problem.

Problem	Dim.	Original PSO	Original CPSO	EPSO(F_1)	EPSO(F_2)	OPSO	RGA/E
Sphere	5	0.9997±0.0004	**1.0000**±0.0000	**1.0000**±0.0000	0.9830±0.0399	**1.0000**±0.000	0.9990±0.0005
Griewank	5	0.9522±0.0507	0.8688±0.0916	**0.9829**±0.0129	0.9826±0.0311	0.9448±0.0439	0.9452±0.0784
Rastrigin	5	0.1828±0.1154	0.6092±0.2701	**1.0000**±0.0000	0.6231±0.3588	0.2652±0.1185	0.9616±0.0239
Rosenbrock	5	0.4231±0.2208	0.6206±0.2583	**0.7764**±0.2723	0.5809±0.2240	0.3926±0.1976	0.3898±0.2273

Table 4. The obtained results of the EPSO, the original PSO, the original CPSO, OPSO, and RGA/E (the mean and the standard deviation of fitness values in each optimizer). The values in bold signify the best results for each problem.

Specially, the fact of what the search performance by the estimated PSO is superior to that by the original CPSO demonstrates the effectiveness of the proposed criteria, which emphasizes the importance of executing the EPSO to parameter selection.

4.4 Experimental results (2)

For further identifying the effectiveness of the EPSO, the following experiments are carried out for each benchmark problem in ten- and twenty-dimensional cases.

According to the better search performance corresponding to each type of the PSO in Section 4.3, Table 5 shows the obtained results of the PSO in d-type, their criterion values and frequencies. To demonstrate the search performance of these PSO in Table 5, Table 6 gives the obtained results for the EPSO using two different criteria, the original PSO, the original CPSO, OPSO, and RGA/E. Similar to the results of five-dimensional case in Table 4, it is confirmed that the search performance of the PSO optimized by the EPSO using the criterion F_1 is superior to that by the criterion F_2, and is also superior to that by the original PSO, the original CPSO, OPSO, and RGA/E for the given benchmark problems except for the *Rastrigin* problem.

Comparison with the values of parameters of the estimated PSO in different dimensional cases for the *Rastrigin* problem, we observe that the values of the estimated PSO, $\hat{c}_0$, are less than 1.0 in ten- and twenty-dimensional cases. Just as which the inertia coefficient is less than 1.0, so that the PSO cannot explore over a wide search space due to the origins of premature convergence and stagnation.

However, why the ideal results in five-dimensional case cannot be reappeared for dealing with same problem in ten- and twenty-dimensional cases, the causes may be associated with the experimental condition such as the number of generations $G = 20$, and iterations $K = 400$ of the EPSO run. Since they are too little, appropriate values of parameters in the PSO cannot be found without enough possibility in a bigger search space.

To testify the truth of the supposition, we tried to use the PSO in d-type by the criterion F_1 in Table 2 as a proxy for solving the ten- and twenty-dimensional *Rastrigin* problems. Under such circumstances, the resulting search performance of the EPSO with the criterion F_1 are

Problem	Dim.	Cumulative fitness		PSO	Parameter $\hat{c}_0$	$\hat{c}_1$	$\hat{c}_2$	Freq.
Sphere	10	F_1	380.8±1.9	d-type	0.8022±0.0224	1.6485±0.4327	0.7468±0.1453	100%
		F_2	375.9±2.6	d-type	0.7654±0.0468	1.3853±0.5210	0.8777±0.0439	95%
Griewank	10	F_1	389.9±1.4	d-type	0.7620±0.0016	1.5490±0.7157	0.7587±0.2100	95%
		F_2	386.4±1.4	d-type	0.7318±0.1111	1.3844±0.3688	1.2278±0.3945	100%
Rastrigin	10	F_1	59.79±14.8	d-type	0.5534±0.1462	2.1410±0.5915	2.0062±1.0027	25%
		F_2	32.61±5.8	d-type	0.3596±0.1740	3.3035±1.6040	1.2856±1.4118	55%
Rosenbrock	10	F_1	155.1±45.2	d-type	0.7050±0.2830	1.6254±0.8717	1.9030±0.5038	90%
		F_2	122.0±59.9	d-type	1.0159±0.0279	1.6045±0.4152	0.4983±0.1048	100%
Sphere	20	F_1	326.4±7.1	d-type	0.9091±0.0425	2.2427±0.1360	0.4249±0.0675	100%
		F_2	320.1±9.6	d-type	0.8860±0.0000	1.9482±0.1912	0.6693±0.1157	100%
Griewank	20	F_1	374.8±2.1	d-type	0.9717±0.0093	1.7877±0.2686	0.6989±0.1442	100%
		F_2	370.3±3.0	d-type	0.9738±0.0000	1.6542±0.3106	0.7064±0.0330	70%
Rastrigin	20	F_1	10.33±1.0	d-type	0.9776±0.0198	1.3934±0.2050	0.2179±0.0561	70%
		F_2	8.48±1.4	d-type	0.8920±0.0000	1.7465±0.4156	0.4155±0.2469	100%
Rosenbrock	20	F_1	10.49±1.7	d-type	0.9237±0.0000	1.9173±0.2636	0.8158±0.1274	100%
		F_2	10.93±2.3	d-type	0.8680±0.1128	0.9377±0.6782	1.0402±0.2969	100%

Table 5. Estimated appropriate values of parameters in the PSO, criterion values and frequencies in the top-twenty optimizers. The PSO in d-type: $\hat{c}_0 > 0$, $\hat{c}_1 > 0$, $\hat{c}_2 > 0$.

below.

$$\begin{cases} 0.7048 \pm 0.4536 \ in \ \ ten - dimensional \ \ case \\ 0.1160 \pm 0.3024 \ in \ \ twenty - dimensional \ \ case \end{cases}$$

We can see that the average of fitness values in each case is not only better than the old one in Table 6, but also is better than that of the RGA/E. Therefore, it is demonstrated that the above supposition is correct and the generality of the estimated result of the EPSO.

Problem	Dim.	Original PSO	Original CPSO	EPSO (F_1)	EPSO (F_2)	OPSO	RGA/E
Sphere	10	0.8481±0.0995	0.9518±0.2153	**0.9985**±0.0048	0.9599±0.1465	0.9980±0.0077	0.9957±0.0028
	20	0.0912±0.0662	0.2529±0.3654	**0.9791**±0.0512	0.9328±0.2132	0.6939±0.3131	0.9207±0.0290
Griewank	10	0.7290±0.1506	0.7025±0.1475	**0.9547**±0.0621	0.9282±0.1138	0.8236±0.1835	0.9136±0.1415
	20	0.6752±0.1333	0.6593±0.1653	**0.9174**±0.1657	0.9028±0.1565	0.8073±0.1742	0.8816±0.1471
Rastrigin	10	0.0600±0.0346	0.0336±0.0156	0.6319±0.0370	0.0936±0.0783	0.0321±0.0255	**0.6693**±0.2061
	20	0.0084±0.0019	0.0065±0.0010	0.0162±0.0075	0.0148±0.0046	0.0147±0.0033	**0.0844**±0.0292
Rosenbrock	10	0.0928±0.0423	0.0899±0.0763	**0.1467**±0.1694	0.1388±0.0811	0.0825±0.0719	0.1243±0.0650
	20	0.0012±0.0019	0.0070±0.0103	**0.0293**±0.0217	0.0193±0.0186	0.0084±0.0108	0.0108±0.0082

Table 6. The obtained results of the EPSO, the original PSO, the original CPSO, and RGA/E (the mean and the standard deviation of fitness values in each method). The values in bold signify the best results for each problem.

5. Conclusion and discussion

We presented the method of evolutionary particle swarm optimization which provides a good framework to effectually estimate appropriate values of parameters in the PSO corresponding to a given optimization problem. Two different criteria, i.e. a temporally cumulative fitness function of the best particle and a temporally cumulative fitness function of the whole particle swarm, are adopted to use for evaluating the search performance of the PSO without any prior knowledge.

According to the synthetic results of both the search performance and convergence time-step, it is confirmed that the criterion F_1 has higher adaptability in search than that by the criterion F_2. On the other hand, these experimental results also clearly indicated that the PSO with higher adaptability is available when we have a passionate concern for the behavior of the best particle in evaluation, and the PSO with faster convergence is available when we have a passionate concern for the behavior of the entire swarm in evaluation.

As well as we observed, specially the results of the PSO estimated by the criterion F_2 having higher convergence easily tend to be trapped in local minima. This phenomenon suggests that estimating the PSO alone is not enough, and that a valid effective method for alleviating premature convergence and stagnation is of necessity. We also tested how to obtain the PSO with high search performance in a high-dimensional case by using the knowledge obtained in low-dimensional case, and showed the effectiveness of the use of this way.

It is left for further study to investigate the relation between search ability and faster convergence. By obtaining the *Pareto* front of 2-objective optimization (8; 23), the know-how on designing the PSO can be generally interpreted not only at model selection level but also at multi-objective level.

Nevertheless, it is necessary to argue a method reduced name EPSO (19) as a supplementary explanation. The method was created by Miranda et al. in 2002 for improving the search performance of th PSO. Although the concepts of evolutionary computation such as selection and mutation are used to the PSO search process and the effect of adaptation could be obtained, its mechanism is similar to the cPSO (21) and is completely different from the EPSO described in Section 3.2.

Generally, the following three manners can be used for improving the search performance of the PSO. (1) Optimizing the PSO, i.e. rationally managing the trade-off between exploitation and exploration by adopting appropriate values of parameters in the PSO; (2) Enforcing the intelligence of the PSO search, i.e. practicing intellectual action in optimization; (3) Unifying the mentioned (1) and (2) manners for acquiring more efficiency to search. Needless to say, the third manner in particular is successful among them. This is because the search capability of the PSO can be easily improved by the combination of capacity and intellectuality. In recent years, a number of studies and investigations regarding the third manner are focused, and being accepted flourishingly (11; 15; 30–32).

Accordingly, it is also left for further study to still handle the above hard problems with powerful hybrid techniques such as blending a local search and the PSO search for further increasing search ability, and introducing the mechanism of diversive curiosity into the PSO for raising the search performance of a single particle swarm or even multiple particle swarms with hybrid and intelligent search (34) to exploration.

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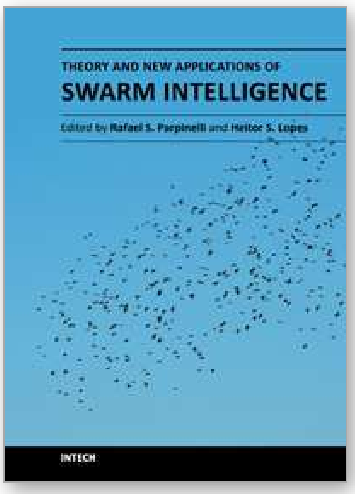

Theory and New Applications of Swarm Intelligence

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The field of research that studies the emergent collective intelligence of self-organized and decentralized simple agents is referred to as Swarm Intelligence. It is based on social behavior that can be observed in nature, such as flocks of birds, fish schools and bee hives, where a number of individuals with limited capabilities are able to come to intelligent solutions for complex problems. The computer science community have already learned about the importance of emergent behaviors for complex problem solving. Hence, this book presents some recent advances on Swarm Intelligence, specially on new swarm-based optimization methods and hybrid algorithms for several applications. The content of this book allows the reader to know more both theoretical and technical aspects and applications of Swarm Intelligence.

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Phone: +86-21-62489820
Fax: +86-21-62489821