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# PID-Like Controller Tuning for Second-Order Unstable Dead-Time Processes

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## 1. Introduction

Several processes encountered in various fields of engineering exhibit an inherently unstable behaviour coupled with time delays. To approximate the open loop dynamics of such systems for the purpose of designing controllers, many of these processes can be satisfactorily described by unstable transfer function models. The most widely used models of this type is the unstable first order plus dead-time (UFOPDT) and the unstable second order plus dead-time (USOPDT) transfer function models, which take into account dead times that might appear in the model, due to measurement delay or due to the approximation of higher order dynamics of the process, by a simple transfer function model.

Research on tuning methods of two or three-term controllers for unstable dead-time processes has been very active in the last 20 years. The most widely used feedback schemes for the control of such processes are the Proportional-Integral-Differential (PID) controller with set-point filter (Jung et al, 1999; Lee et al, 2000), the Pseudo-Derivative Feedback (PDF) or I-PD controller (Paraskevopoulos et al, 2004), and the Proportional plus Proportional–Integral–Derivative (P-PID) controller (Jacob & Chidambaram, 1996; Park et al, 1998). These control schemes are identical in practice, provided that the parameters of the controllers and of the pre-filters needed in some cases are selected appropriately. Controller tuning for unstable dead-time processes has been performed according to several methods, the most popular of them being various modifications of the Ziegler-Nichols method (De Paor & O' Malley, 1989; Venkatashankar & Chidambaram, 1994; Ho & Xu, 1998), several variations of the direct synthesis tuning method (Jung et al, 1999; Prashanti & Chidambaram, 2000; Paraskevopoulos et al, 2004; Padma Sree & Chidambaram, 2004), the ultimate cycle method (Poulin & Pomerleau, 1997), the pole placement method (Clement & Chidambaram, 1997), the method based on the minimization of various integral criteria, the Internal Model Control (IMC) tuning method (Rotstein & Lewin, 1991; Lee et al, 2000; Yang et al, 2002; Tan et al, 2003), the optimization method (Jhunjhunwala & Chidambaram, 2001; Visioli, 2001), the two degrees of freedom method (Huang & Chen, 1997; Liu et al, 2005; Shamsuzzoha et al, 2007), etc. (see the work (O'Dwyer, 2009), and the references cited therein). Moreover, due to the wide practical acceptance of the gain and phase margins (GPM) in characterizing system robustness, some tuning methods for unstable dead-time models, based on the satisfaction of GPM specifications, have also been reported (Ho & Xu, 1998; Fung et al, 1998;

Wang & Cai, 2002; Lee & Teng, 2002; Paraskevopoulos et al, 2006). The vast majority of the tuning methods mentioned above refer to the design of controllers for UFOPDT models and less attention has been devoted to USOPDT models (Lee et al, 2000; Rao & Chidambaram, 2006). Usually these models are further simplified to second order ones without delay, or to UFOPDT models, in order to design controllers for this type of processes. However, this simplification is not possible when the time delay of the system and/or the stable dynamics (stable time constant) are significant.

The aim of this work is to present a variety of innovative tuning rules for designing PID-like controllers for USOPDT processes. These tuning rules are obtained by imposing various specifications on the closed-loop system, such as the appropriate assignment of its dominant poles, the satisfaction of several time response criteria (like the fastest settling time and the minimization of the integral of squared error), as well as the simultaneous satisfaction of stability margins specifications. In particular, the development of the proposed tuning methods relying on the assignment of dominant poles as well as on time response criteria is performed on the basis of the fact that (under appropriate selection of the derivative term), the delayed open loop response of a 3rd order system, with poles equal to the three dominant poles of the closed loop system, is identical to the closed loop step response of an USOPDT system. Simple numerical algorithms are, then, used to obtain the solution of the tuning problem. To reduce the computational effort and to obtain the controller settings in terms of the process parameters (a fact that permits on-line tuning), the obtained solution is further approximated by analytical functions of these parameters. Moreover, in the case of the method that relies on the satisfaction of stability margin specifications, the controller parameters are obtained using iterative algorithms, whose solutions, in a particular case, are further approximated quite accurately by analytic functions of the process parameters. The obtained approximate solutions have been obtained using appropriate curve-fitting optimization techniques. Furthermore, the admissible values of the stability robustness specifications for a particular process are also given in analytic forms. Finally, the tuning rules proposed in this work, are applied to the control of an experimental magnetic levitation system that exhibits highly nonlinear unstable behaviour. The experimental results obtained clearly illustrate the practical efficiency of the proposed tuning methods.

## 2. PID-like controller structures for USOPDT processes

The three main feedback configurations applied in the extant literature in order to control unstable processes with time delay are depicted in Fig. 1 (see Jacob & Chidambaram, 1996; Park et al, 1998, Paraskevopoulos et al, 2004). As it can easily verified, the loop transfer functions obtained by these control schemes are identical, provided that the following relations hold

$$\begin{aligned} \overline{K}'_C &= \overline{K}_C(\overline{\tau}_I + \overline{\tau}_D) / \overline{\tau}_I = (1+\overline{k}_c)\overline{k}_{c,i} = \overline{K}_P \\ \overline{\tau}'_I &= \overline{\tau}_I + \overline{\tau}_D = \overline{\tau}_i(1+\overline{k}_c) / \overline{k}_c = \overline{K}_P / \overline{K}_I \\ \overline{\tau}'_D &= \overline{\tau}_D\overline{\tau}_I / (\overline{\tau}_I + \overline{\tau}_D) = \overline{\tau}_d\overline{k}_c / (1+\overline{k}_c) = \overline{K}_D / \overline{K}_P \end{aligned} \tag{1}$$

$$
\begin{aligned}
G_{F,P-PID}(s) &= G_{F,PID}\frac{\left(\overline{\tau}_I s+1\right)\left(\overline{\tau}_D s+1\right)}{\overline{\tau}_i\overline{\tau}_d s^2+\overline{\tau}_i s+1} \\
G_{F,PDF}(s) &= G_{F,PID}\left(\overline{\tau}_I s+1\right)\left(\overline{\tau}_D s+1\right)
\end{aligned} \tag{2}
$$

where $\overline{K}'_C$, $\overline{\tau}'_I$ and $\overline{\tau}'_D$ are the three controller parameters of the conventional PID controller in its parallel form. In the case of the series PID controller, the pre-filter $G_{F,PID}$ is used in order to cancel out all or some of the zeros introduced by the controller and to smoothen the set-point step response of the closed loop system. The pre-filters $G_{F,P\text{-}PID}$ and $G_{F,PDF}$ are the equivalent pre-filters of the corresponding control schemes. Note that, the pre-filter $G_{F,PDF}$ can be used only when the reference input is a known and differentiable signal. Therefore, is seldom used in real practice. From Fig. 1, one can easily recognize that in the case of regulatory control the three control schemes are identical when the controller parameters are chosen as suggested by (1), even if there are no pre-filters used. Moreover, one can also see that the stability properties of the closed loop system are not affected, in any case, by the respective pre-filter used, which is applied here, only to filter the set point and to prevent excessive overshoot in closed-loop responses to set-point changes, which are common in the case of unstable time-delay systems (Jacob & Chidambaram, 1996). Thus, the loop transfer functions obtained for the above three alternative control schemes are identical.

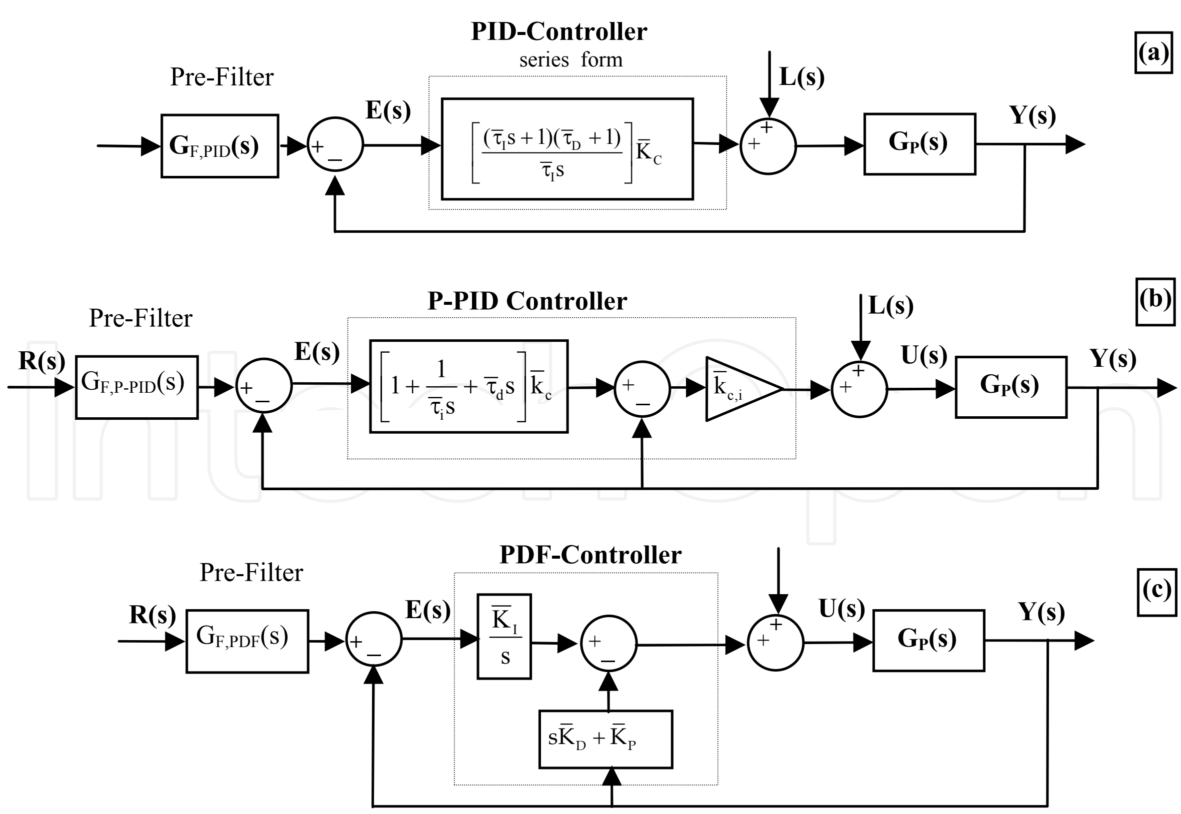



Fig. 1. Equivalent three-term controller schemes with appropriate pre-filters: (a) The series PID controller, (b) The PDF (or I-PD) controller, and (c) The P-PID controller.

| Original Parameters | Normalized Parameters | Original Parameters | Normalized Parameters |
|---|---|---|---|
| $\bar{\tau}_U$ | $\tau_U=1$ | $\omega$ | $w=\omega\bar{\tau}_U$ |
| $\bar{\tau}_S$ | $\tau_S=\bar{\tau}_S/\bar{\tau}_U$ | $s$ | $\hat{s}=s\bar{\tau}_U$ |
| $\bar{d}$ | $d=\bar{d}/\bar{\tau}_U$ | $\bar{K}$ | $K=1$ |
| $\bar{\tau}_I$ | $\tau_I=\bar{\tau}_I/\bar{\tau}_U$ | $\bar{K}_C$ | $K_C=\bar{K}\bar{K}_C$ |
| $\bar{\tau}_D$ | $\tau_D=\bar{\tau}_D/\bar{\tau}_U$ | | |

Table 1. Normalized vs. original system parameters.

In the sequel, our focus of interest is the design of PID-like controllers when applied to control USOPDT process, with the following transfer function model

$$G_P(s)=\frac{\bar{K}\exp\left(-\bar{d}s\right)}{\left(\bar{\tau}_S s+1\right)\left(\bar{\tau}_U s-1\right)} \tag{3}$$

where $\bar{K}$, $\bar{d}$, $\bar{\tau}_S$ and $\bar{\tau}_U$ are the process gain, the time delay and the stable and unstable time constants, respectively. In order to simplify the analysis and in order to facilitate comparisons, all system and controller parameters are normalized with respect to $\bar{\tau}_U$ and $\bar{K}$. Thus, the original process and controller parameters are replaced with the dimensionless parameters shown in Table 1.

Observe now that, the loop transfer function of an USOPDT system in connection with a PID-like controller, is given by

$$G_L(\hat{s})=\frac{K_C\left(\tau_I\hat{s}+1\right)\left(\tau_D\hat{s}+1\right)\exp\left(-d\hat{s}\right)}{\tau_I\hat{s}\left(\tau_S\hat{s}+1\right)\left(\hat{s}-1\right)} \tag{4}$$

while, using the pre-filter $G_{F,\,PID}(\hat{s})=(\tau_I\hat{s}+1)^{-1}$, the closed-loop transfer function becomes

$$G_{CL}(\hat{s})=\frac{K_C(\tau_D\hat{s}+1)\exp(-d\hat{s})}{\tau_I\hat{s}(\tau_S\hat{s}+1)(\hat{s}-1)+K_C(\tau_I\hat{s}+1)(\tau_D\hat{s}+1)\exp(-d\hat{s})} \tag{5}$$

Relations (2) and (5) are next elaborated for the derivation of the tuning methods proposed in this work.

## 3. Frequency domain analysis of closed-loop USOPDT processes

The argument and the magnitude of the loop transfer function (4) are given by

$$\varphi_L(w)=-3\pi/2-dw-\mathrm{atan}(w)-\mathrm{atan}(\tau_S w)+\mathrm{atan}(\tau_I w)+\mathrm{atan}(\tau_D w) \tag{6}$$

$$A_L(w)=\left|G_L(jw)\right|=K_C\frac{\sqrt{1+\left(\tau_I w\right)^2}\sqrt{1+\left(\tau_D w\right)^2}}{\tau_I w^2\sqrt{1+w^2}\sqrt{1+\left(\tau_S w\right)^2}} \tag{7}$$

It is not difficult to recognize that the Nyquist plot of the $G_L(\hat{s})$ has tow crossover points with the real axis, which determine the critical (or crossover) frequencies $w_{min}$ and $w_{max}$, and the critical gains $K_{C,min}=1/A_L(w_{min})$ and $K_{C,max}=1/A_L(w_{max})$. These crossover frequencies are obtained as the solutions of the equation $\varphi_L(w_C)=-\pi$, or equivalently, of the equation

$$-\pi/2-dw_C+atan(w_C)+atan(\tau_I w_C)+atan(\tau_D w_C)-atan(\tau_S w_C)=0 \tag{8}$$

when the values of the *atan* function are assigned in the range $(-\pi/2, \pi/2)$. Having computed $w_{min}$ and $w_{max}$, one can determine the acceptable values for the controller gain $K_C$, for which the closed-loop system is stable. In particular $K_{C,min}<K_C<K_{C,max}$, where, with subscript "$M$" used for either "min" or "max"

$$K_{C,M}=\frac{\tau_I w_M\sqrt{1+w_M^2}\sqrt{1+\left(\tau_S w_M\right)^2}}{\sqrt{1+\left(\tau_I w_M\right)^2}\sqrt{1+\left(\tau_D w_M\right)^2}} \tag{9}$$

We next define the increasing gain margin $GM_{inc}$, the decreasing gain margin $GM_{dec}$ and the gain margin product of the closed-loop system as follows

$$GM_{inc}=K_{C,max}/K_C \;,\;\; GM_{dec}=K_C/K_{C,min} \tag{10}$$

$$GM_{prod}=GM_{inc}GM_{dec}=K_{C,max}/K_{C,min} \tag{11}$$

Obviously for the closed loop system to be stable $GM_{inc}$ and $GM_{dec}$ should be grater than one. Note that, the largest the values of $GM_{prod}$, the more robust the system becomes with respect to the gain uncertainty, if the controller gain $K_C$ is appropriately selected. Furthermore, the phase margin of the closed loop system is defined by $PM=\varphi_L(w_G)+\pi$, where $w_G$ is the frequency at which $A_L(w_G)=1$. From (7), one can easily conclude that $w_G$ is given by the maximum real root of the equation

$$\tau_I^2\tau_S^2\omega_G^6+\left(\tau_I^2\tau_S^2+\tau_I^2-K_C^2\tau_I^2\tau_D^2\right)\omega_G^4+\left[\tau_I^2-K_C^2\left(\tau_I^2+\tau_D^2\right)\right]\omega_G^2-K_C^2=0 \tag{12}$$

In order to obtain the maximum phase margin for given $d$, $\tau_S$, $\tau_I$ and $\tau_D$, the controller gain $K_C$ should be selected as

$$K_C=\frac{\tau_I w_p\sqrt{1+w_p^2}\sqrt{1+\left(\tau_S w_p\right)^2}}{\sqrt{1+\left(\tau_I w_p\right)^2}\sqrt{1+\left(\tau_D w_p\right)^2}} \tag{13}$$

where $w_p$ is the frequency at which the argument of the loop transfer function is maximized. From (6), one can easily conclude that $w_p$ is given by the solution of $d\varphi_L/d\omega\,\big/_{w=wp}=0$, or equivalently of the equation

$$-d+\frac{1}{1+w_p^2}+\frac{\tau_I}{1+\tau_I^2 w_p^2}+\frac{\tau_D}{1+\tau_D^2 w_p^2}-\frac{\tau_S}{1+\tau_S^2 w_p^2}=0 \tag{14}$$

that results in a fourth order linear equation with respect to $w_p^2$, with only one acceptable positive real root. Substituting $w_p$ in (6), the respective maximum argument $\varphi_L(w_p)$ is calculated.

When the maximum phase margin is zero, then the closed-loop system (with the appropriate selection of $K_C$) is marginally stable. The solution of $max(PM(d,\tau_I,\tau_D,\tau_S))=0$, yields the acceptable values of the controller parameters $\tau_I$ and $\tau_D$, which render the close-loop system stable. Obviously these values depend on the rest of the system parameters. From (8) and for $\tau_I \rightarrow \infty$, one can easily verify that $w_{min}=0$ and $\varphi_L(0)=-\pi$. If, at $w_{min}=0$, the derivative of $\varphi_L$ is positive, then, it is obvious that the system has a maximum phase margin grater than zero and can be stabilized with the appropriate $K_C$. With this observation, using (14), one can easily verify that, for $\tau_D > \tau_{D,min} \equiv 1-d-\tau_S$, the closed-loop system can be stabilized. Note here that, when $\tau_S \leq 1$, $\tau_{D,min}$ is also the smallest $\tau_D$ that renders the closed-loop system stable, while when $\tau_S > 1$, the system can be stabilized with smaller values of $\tau_D$. Moreover, although the function $\varphi_L(\tau_D)$ is strictly increasing, the function $GM_{prod}(\tau_D)$ is not strictly monotonous. In fact, there exists a very large value of $\tau_D$ for which $GM_{prod}(\tau_D)=1$ and the system is no longer stabilizable. In the case where $\tau_I \rightarrow \infty$, then $K_{C,min}=1$. Solving the equation $K_{C,min}(\tau_D)=1$, one can determine the maximum value of $\tau_D$, say $\tau_{D,max}$, for which the system can be stabilized. Unfortunately, the solution of $K_{C,min}(\tau_D)=1$ involves nonlinear equations that can only be solved using iterative algorithms. A simple and quite accurate approximate solution for $\tau_{D,max}$ has been obtained through fitting, using the optimization toolbox of MATLAB® and is given by

$$\hat{\tau}_{D,\max} \approx 0.85 + \tau_S \left(-0.46 + 1.5 / d\right) \tag{15}$$

The maximum normalized error of this approximation is 6%, when $0.1 < \tau_D < 10$ and $0.01 < d < 0.9$. In general, it is plausible to obtain a stable closed-loop system by selecting $\tau_{D,min} < \tau_D < \tau_{D,max}$. In real practice, when $\tau_D$ is close to $\tau_{D,min}$ or $\tau_{D,max}$, the stability region of the closed-loop system is very small. After extensive search, it has been found that a more suitable range for the selection of $\tau_D$ is the following

$$\tau_S \leq \tau_D \leq \tau_S + d/2 \tag{16}$$

When $\tau_D$ is selected in the range defined by (16), very large $PM_{max}$ and $GM_{prod}$ can be obtained. Moreover, with this selection the functions $max(PM(\tau_I))$ and $GM_{prod}(\tau_I)$ are strictly increasing with respect to $\tau_I$. This is a very useful property for the design of PID-like controllers for USOPDT processes. It is worth noticing, at this point, that in order to tune PID-like controllers for USOPDT processes one can distinguish three cases depending on the values of $d$ and $\tau_S$. In the case where $\tau_S < 0.1$ the PID-type controllers can be tuned using tuning rules for UFOPDT systems, assuming that the new normalized dead time is equal to $d+\tau_S$. On the other hand, if $\tau_S > 10$, then it is possible to tune the PID-type controller assuming that the system is a second order one with no time delay. In this particular case, the inverse of the eigen-frequency of the closed loop system (without delay) must be at least five times larger than the time delay of the USOPDT system. Finally, in the case where $0.1 < \tau_S < 10$, the above approximate solutions do not provide accurate results, and it is recommended to use the more accurate tuning rules presented in the following Sections.

## 4. Controller tuning by assigning the closed-loop system dominant poles

A first method of tuning PID-like controllers for USOPDT processes is based on the appropriate placement of the dominants poles of the closed-loop system. This method is designated here as the DPC method, since it relies on the satisfaction of dominant poles criteria. In order to systematically present the DPC method, we start by selecting the derivative time constant $\tau_D$ equal to the lowest value in the range defined by (16). That is, $\tau_D = \tau_S$. With this selection, relations (4) and (5) take the forms

$$G_L(\hat{s}) = \frac{K_C\left(\tau_I\hat{s}+1\right)\exp\left(-d\hat{s}\right)}{\tau_I\hat{s}\left(\hat{s}-1\right)} \tag{17}$$

$$G_{CL}(\hat{s}) = \frac{K_C\exp(-d\hat{s})}{\tau_I\hat{s}(\hat{s}-1)+K_C(\tau_I\hat{s}+1)\exp(-d\hat{s})} \tag{18}$$

Clearly, in this case, the closed-loop transfer function has no zeroes. Note also that, if initially $\tau_S$>>*1*, then, the controller parameter $\tau_D$ takes very large values, a fact that is not desirable, for reasons of noise amplification. Unfortunately, as suggested by (16), in this case, large values of $\tau_D$ are inevitable and an appropriate filtered derivative should be considered.

Let us now select the controller gain $K_C$ as the geometric middle point of the two ultimate gains, $K_{C,min}$ and $K_{C,max}$, of the closed loop system, that is

$$K_C = \sqrt{K_{C,\min}K_{C,\max}} \tag{19}$$

Note that this selection of $K_C$ provides the same robustness against both increasing and decreasing parametric uncertainty of the system gain. This is particularly useful for systems with large values of $d$ (i.e. $d>0.3$) where the region of stability is reduced significantly (Paraskevopoulos et al, 2006).

On the basis of (17), the two ultimate gains are, in this case, given by

$$K_{C,\min} = \frac{\tau_I w_{\min}\sqrt{1+w_{\min}^2}}{\sqrt{1+(\tau_I w_{\min})^2}} \quad , \quad K_{C,\max} = \frac{\tau_I w_{\max}\sqrt{1+w_{\max}^2}}{\sqrt{1+(\tau_I w_{\max})^2}} \tag{20}$$

In (20), $w_{min}$ and $w_{max}$ are the two critical frequencies given by the two solutions of the equation (8), when $\tau_D=\tau_S$ and when the values of the atan function are assigned in the range $(-\pi/2,\pi/2)$. For given $d$, the solution of (8), for $\tau_D=\tau_S$, exists only if $\tau_I$ is larger than a critical value $\tau_{I,min}(d)$ (Paraskevopoulos et al, 2006). Since there are no analytical solutions for (8), two very accurate approximations for $w_{min}$ and $w_{max}$ that are obtained by using optimization techniques are proposed here. These approximations are

$$\begin{aligned} \hat{w}_{\min}(d,\tau_I) &= f_{w_{\min}}(d,\tau_I)\sqrt{\frac{1}{\tau_I - d(1+\tau_I)}} \\ \hat{w}_{\max}(d,\tau_I) &= f_{w_{\max}}(d,\tau_I)\frac{\pi}{2d}\frac{\left(\tau_I - 0.9463(\tau_I+1)d\right)}{\left(\tau_I - 0.5609(\tau_I+1)d\right)} \end{aligned} \tag{21}$$

$$f_{w_{\min}}(d,\tau_I) = 1 + \frac{\left(0.006+0.03\mathrm{d}/(1.14\text{-}\mathrm{d})\right)\hat{\tau}_{I,\min}}{\left(0.973+0.05/(1-d)\right)\tau_I - \hat{\tau}_{I,\min}}$$

$$f_{w_{\max}}(d,\tau_I) = \left(1 + 0.22d^4\right)\left[1 + \left(0.1 - 0.3\sqrt{d}\right)\left(\hat{\tau}_{I,\min} / \tau_I\right)^2\right] \quad (22)$$

where $\hat{\tau}_{I,\min}$ is an approximation of $\tau_{I,min}$, given by

$$\hat{\tau}_{I,\min}(d) = \left(0.0029\text{-}0.0682\sqrt{\mathrm{d}} + 1.4941\mathrm{d}\right) / (1.003\text{-}\mathrm{d})^2 \quad (23)$$

The normalized errors of the ultimate gains, defined by $\tilde{K}_{C,\min} = (K_{C,\min} - \hat{K}_{C,\min}) / K_{C,\min}$ and $\tilde{K}_{C,\max} = (K_{C,\max} - \hat{K}_{C,\max}) / K_{C,\max}$, where $\hat{K}_{C,\max}$ and $\hat{K}_{C,\min}$ are the approximations of $K_{C,max}$ and $K_{C,min}$, respectively, obtained using (21), never exceed 2.2% for $d \le 0.9$ and $\tau_I > 1.2\,\hat{\tau}_{I,\min}$. Moreover the normalized error relative to $\hat{\tau}_{I,\min}$ never exceeds 1.4% for $d \le 0.9$.

Since, here $\tau_D = \tau_S$, and $K_C$ is obtained according to relations (19)-(23) as a function of $\tau_I$, in order to tune a PID-like controller it only remains to specify $\tau_I$. In the present Section, we propose to select the controller parameter $\tau_I$, in order to maximize the real part of the slowest dominant pole (i.e. the pole with the smallest real part). This way the resulting closed loop system will have a very fast settling time and, at the same time, a very smooth (non-oscillatory) response.

In order to obtain a pole-zero description of (18), the exponential term in (18) is approximated by the relation

$$\exp(-d\hat{s}) = \lim_{n\to\infty}\left[(d/n)\hat{s} + 1\right]^{-n} \quad (24)$$

From (24), it can be easily recognized that the exponential term $exp(-d\hat{s})$ is equivalent to an infinite number of poles at $\hat{s} = -n/d + j0$. A typical example of the root locus of (18) is shown in Fig. 2 (for $d=0.5$, $n=25$, $K_C$ given by (19) and $1.1\tau_{I,min} < \tau_I < 10\tau_{I,min}$). From this figure, it becomes clear that, there exist three dominant poles that are responsible for the shape of the closed-loop system response. The rest of the poles contribute only to the delay of the response. Extensive simulation analysis (for $0<d<0.9$, $\tau_I > \tau_{I,min}$ and $K_{C,min} < K_C < K_{C,max}$) shows that the step response of an USOPDT system controlled by a PID-like controller (when $\tau_D = \tau_S$) cannot be easily distinguished from that of a 3rd order system with the same dominant poles and the same initial delay, when $n>20$ in (24). This fact is illustrated in Fig. 3.

| Range of d | Estimated $\tau_I(d)$ | M.N.E. |
|---|---|---|
| $0<d<0.17$ | $3.06\sqrt{d} + 4.19d - 12.66d^2$ | 1.5% |
| $0.17<d<0.9$ | $\left(3.47\sqrt{d} - 2.9d + 8.37d^2 + 18.28d^5\right)(0.95 - d)^{-1}$ | 2% |

Table 2. Approximate expressions of $\tau_I(d)$ for the DPC method.

In order to solve the tuning problem presented above, MATLAB® control toolbox was used to estimate the poles of a 27th order closed loop system ($n=25$ in (24)). Moreover, a simple algorithm based on the dissection method was used to find the value of $\tau_I$ that maximizes the real part of the slowest dominant pole. Since this procedure cannot be applied on-line due to its computational burden, the function $\tau_I(d)$ obtained by the DPC method has been approximated by analytical functions $\hat{\tau}_I(d)$. The parameters involved in these functions have been estimated using the optimization toolbox of MATLAB®, in order to minimize the maximum normalized error (M.N.E.), defined by $\tilde{\tau}_I = (\tau_I - \hat{\tau}_I)/\tau_I$. These approximate expressions are given in Table 2, together with their maximum normalized error. The response obtained by the DPC method can be distinguished as follows: For *d<0.157* the method gives three real dominant poles (the two slowest are identical) and the response approximates that of a critical second-order system response. For *d>0.157* the method gives two complex and one real poles all with the same real part (see also Fig. 4).

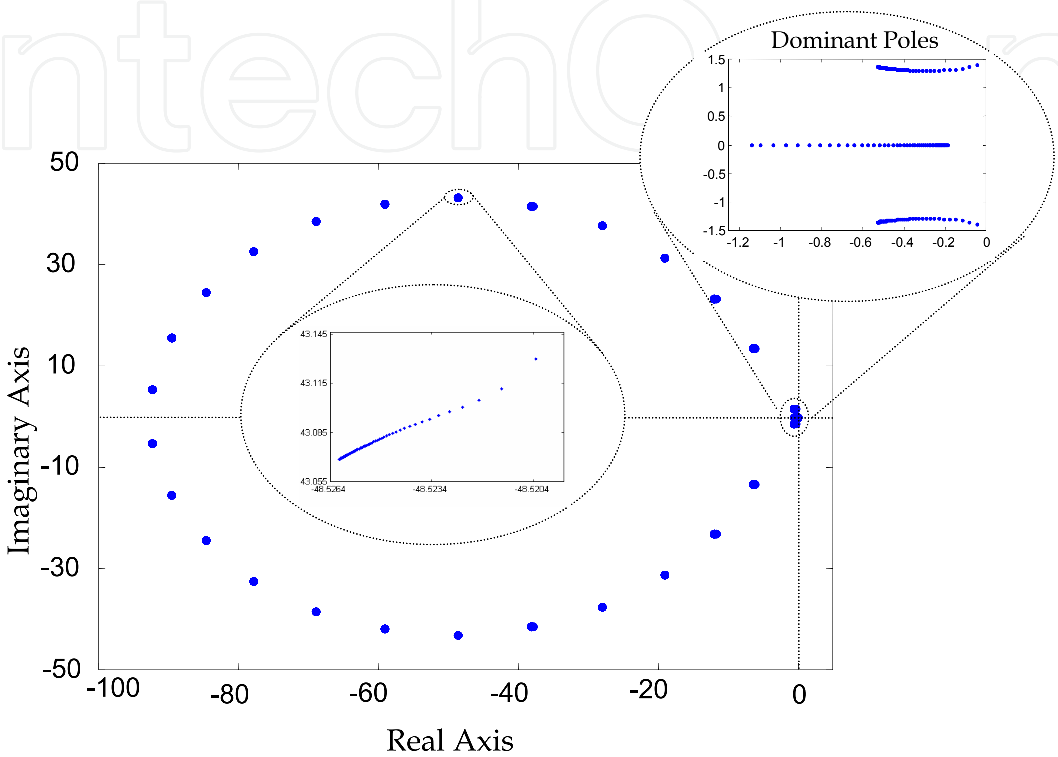



Fig. 2. A typical root locus of (18) for $d=0.5$, $n=25$, $1.1\tau_{I,min}<\tau_I<10\tau_{I,min}$ and $K_C$ given by (19).

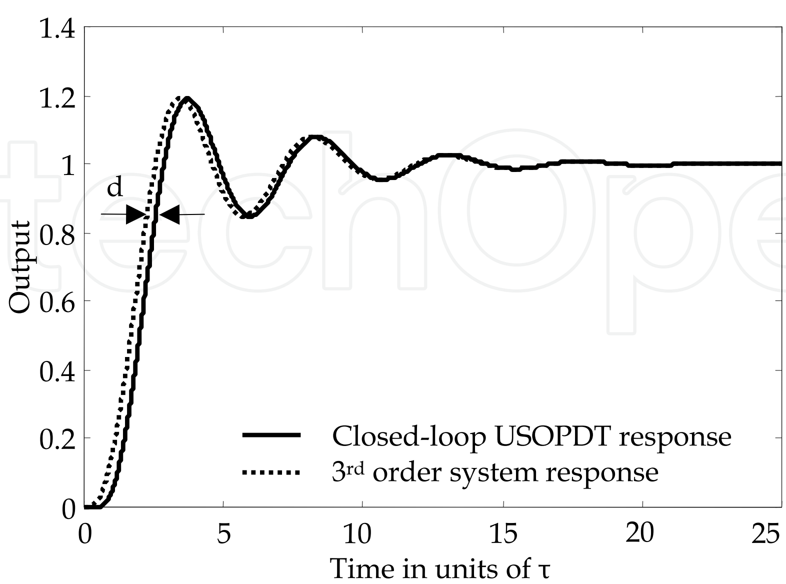



Fig. 3. A typical closed loop set-point step response of the USOPDT process and the response of the 3rd order system.

## 5. Controller tuning based on closed-loop time-response criteria

In this Section, we consider again that $\tau_D=\tau_S$ as well as that $K_C$ is obtained through (19)-(23), and we present three alternative methods for the selection of the parameter $\tau_I$. These methods are based on some very useful closed-loop set-point step response criteria.

A first, widely used, criterion for tuning PID-like controllers is the fastest settling time (FST) method. In the case of an oscillatory response, the settling time is usually estimated from the envelope of the response. Since for systems with time delay the closed-loop response is not known in analytical form, to estimate here the envelope of the response, we use the response of a third-order system having the dominant poles of the closed-loop USOPDT system. In particular, the response of a third order system, with two complex poles ($p_{I,1}=a+jb$ and $p_{I,2}=a-jb$) and one real pole ($p_R$), is given by

$$y(t)=1-\left[e^{-\zeta w_0 t}\left(A\cos(w_n t)+B\sin(w_n t)\right)+Ce^{-p_R t}\right] \tag{25}$$

where $w_0=\sqrt{a^2+b^2}$, $\zeta=a/w_0$, $w_n=w_0\sqrt{1-\zeta^2}$, $A=p_R(-p_R+2\zeta w_0)/D$, $B=p_R w_0(-\zeta p_R+2\zeta^2 w_0-w_0)/(Dw_n)$, $C=-\mathrm{w}_0^2/\mathrm{D}$ and $D=-\mathrm{p}_\mathrm{R}^2+2\mathrm{p}_\mathrm{R}\zeta\mathrm{w}_0-\mathrm{w}_0^2$. The two envelopes (top and bottom) of (25) are given by

$$y_{g1,2}(t)=1\pm\left[e^{-\zeta w_0 t}\left(\sqrt{A^2+B^2}\right)+Ce^{-p_R t}\right] \tag{26}$$

Therefore, for the application of the FST method, a simple algorithm based on the dissection method, is used to estimate the value of parameter $\tau_I$ that minimizes the time $t_{stl}$ required for obtaining $\left|1-y_{g1}(t_{stl})\right|=0.01$.

A second criterion, on the basis of which the tuning of the PID-like controller is performed, stems from the need to provide the fastest possible set-point step response of the closed loop system with a maximum overshoot of 1% (OPOS method). Also in this case a search algorithm is used to estimate the smallest value of the parameter $\tau_I$ (and hence the fastest response) for which the maximum of $y(t)$, given by (25), is smaller than 1.01 for all $t>0$.

Finally, the third method is based on the minimization of the integral of squared errors due to a unit step change in the set point (ISE-Sp method). The first part of the response, for $t<d$, can not be affected by the controller. Hence, for the optimization problem of minimizing the integral of squared errors, one can use the response obtained by (25). The integral of $(1-y(t))^2$ can then be calculated analytically, and it is given by

$$\begin{aligned}ISE_{Sp}&=\int_0^\infty\left(1-y(t)\right)^2dt=\frac{C^2}{2p_R}+2C\frac{A(\zeta w_0+p_R)+Bw_n}{p_R^2+w_0^2+2\zeta w_0 p_R}\\&+\left[A^2(1+\zeta^2)+B^2(1-\zeta^2)+2AB\zeta\sqrt{1-\zeta^2}\right]\left(4w_0\zeta\right)^{-1}\end{aligned} \tag{27}$$

Then, using (27) in combination with a simple search algorithm, the parameter $\tau_I$ that minimizes the value of $ISE_{Sp}$ can be estimated.

All three methods presented above cannot be applied on-line because of the excessive computational burden required to calculate the values of the three dominant poles. For this reason, the parameter $\tau_I$ obtained by the application of these methods, is next calculated for all values of $d<0.9$ and the function $\tau_I(d)$ is approximated using the optimization toolbox of MATLAB®. The resulting approximations $\hat{\tau}_I(d)$ are given in Table 3. The M.N.E. in the estimation of the function $\tau_I(d)$ is less than 2.8%, for all these approximations. This error in $\tau_I$ does not produce a significant change in the response of the closed loop system.

| Method | Range of d | Estimated $\tau_I(d)$ | M.N.E. |
|---|---|---|---|
| FST | $0<d<0.17$ | $0.017+0.42\sqrt{d}+8.08d$ | 1.5% |
| | $0.17<d<0.9$ | $\dfrac{3.26\sqrt{d}-1.96d+5.55d^2+15.47d^5}{0.96-d}$ | 2% |
| OPOS | $0<d<0.9$ | $\dfrac{2.29\sqrt{d}+0.69d+2.29d^2+15.07d^5}{0.96-d}$ | 2.8% |
| ISE-Sp | $0<d<0.9$ | $\dfrac{0.1\sqrt{d}+2.47d+2.78d^2+5.59d^5}{0.95-d}$ | 2.7% |

Table 3. Estimates of $\tau_I(d)$ for the tuning methods based on closed-loop time-domain criteria.

| Method | $d$=0.1 | $d$=0.5 | $d$=0.9 |
|---|---|---|---|
| DPC | -12.61, -2.502±j0.175 | -0.425, -0.412±j1.312 | -0.0377, -0.0377±j0.412 |
| FST | -12.949, -2.326±j1.641 | -0.516, -0.368±j1.302 | -0.0550, -0.0291±j0.411 |
| OPOS | -12.964, -2.318±j1.675 | -0.556, -0.349±j1.299 | -0.0609, -0.0262±j0.410 |
| ISE-Sp | -14.765, -1.378±j4.231 | -0.785, -0.237±j1.298 | -0.0883, -0.0129±j0.409 |

Table 4. Locations of dominant poles for some typical examples.

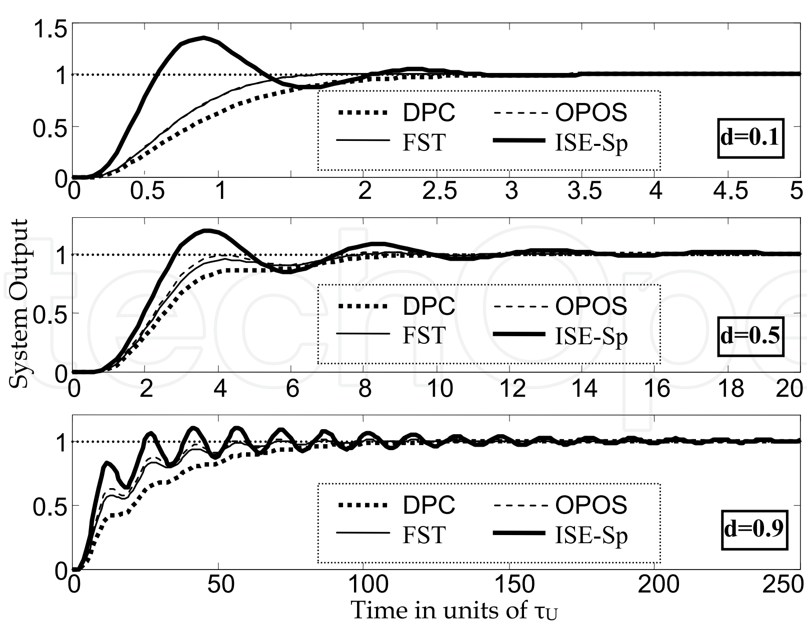



Fig. 4. Characteristic set-point step responses obtained by the proposed tuning methods.

For example, when $\hat{\tau}_I(d)$ is used instead of $\tau_I$, to apply the FST method, the maximum normalized error in the settling time is less than 0.5%.

In Table 4, the locations of the three dominant poles of the closed loop system are given in the case where the normalized dead time takes the values 0.1, 0.5 and 0.9, for all methods presented above. The corresponding closed loop responses obtained from a unit change in the set-point are illustrated in Fig. 4. From these responses and the locations of the dominant poles reported in Table 4, one can easily recognize that the FST and the OPOS methods provide us controllers with similar performance. Moreover, the response obtained when the ISE_Sp method is used is the fastest, although very oscillatory. Finally, in the case where the DPC method is used, the response obtained is sluggish and smooth. Moreover, since this method yields a large value of $\tau_I$, it provides a very robust controller.
Table 5 presents a stability robustness comparison with other existing PID tuning methods. In particular, the tuning methods presented in Sections 4 and 5 are compared with the R&L method with $\lambda=2.2$ (Rotstein & Lewin, 1991), the P&M method (De Paor and O'Malley, 1989), the H&X method with specifications $A_m=1.3$ and $\varphi_m=10^o$ (Ho & Xu, 1998), the P&P method based on the ITAE criterion (Poulin & Pomerleau, 1997) and the J&C method based on the IMC tuning rule with $\lambda=2.5$ (Jacob & Chidambaram, 1996), in the special case where $d=0.5$ and $\tau_S=1$. Table 5 presents the increasing and decreasing gain margins $GM_{inc}$ and $GM_{dec}$ as well as the phase margin $PM$. Moreover, it presents the maximum simultaneous multiplicative uncertainty $A_a$ of all system parameters (i.e. when the system parameters $d$, $\tau_S$, $K$ are increased by $A_a$ and $\tau_U$ is decreased by $A_a$) and the maximum multiplicative uncertainty $A_d$ of the time delay (i.e. when only $d$ is increased by $A_d$), for which the closed loop system remains stable. The results presented in Table 5 show that the DPC method provides more robust controllers than most other methods (except the J&C method with $\lambda=2.5$, that gives a significantly slower response in both set point tracking and regulatory control). The aim of the other three methods, presented in this Section, is to provide faster responses and hence they provide lesser robustness. Finally, it is worth noticing that all the other methods used in robustness comparison are not applicable in cases where $d>0.7$.

| Method | $K_C$ | $\tau_I$ | $\tau_D$ | $PM_{(rad)}$ | $GM_{inc}$ | $GM_{dec}$ | $a_a$ | $a_d$ |
|---|---|---|---|---|---|---|---|---|
| DPC | 1.618 | 8.150 | 1 | 0.172 | 1.469 | 1.462 | 1.101 | 1.268 |
| FST | 1.622 | 6.948 | 1 | 0.155 | 1.446 | 1.436 | 1.091 | 1.240 |
| OPOS | 1.623 | 6.539 | 1 | 0.148 | 1.436 | 1.425 | 1.088 | 1.225 |
| ISE-SP | 1.632 | 4.834 | 1 | 0.107 | 1.372 | 1.353 | 1.064 | 1.163 |
| R&L ($\lambda$=2.2) | 2.116 | 10.24 | 0.902 | 0.087 | 1.173 | 1.860 | 1.043 | 1.103 |
| P&M | 1.357 | 6.960 | 1 | 0.133 | 1.729 | 1.202 | 1.103 | 1.288 |
| H&X | 1.518 | 6.543 | 1 | 0.148 | 1.536 | 1.332 | 1.095 | 1.255 |
| P&P | 1.798 | 8.431 | 1 | 0.154 | 1.325 | 1.631 | 1.082 | 1.204 |
| J&C ($\lambda$=2.5) | 1.573 | 9.495 | 1 | 0.188 | 1.528 | 1.443 | 1.113 | 1.307 |

Table 5. Robustness performance comparison with other existing tuning methods.

## 6. Controller tuning based on closed-loop stability margins specifications

When a PID-like controller is used to control an USOPDT process, it is possible, in some cases, to simultaneously satisfy the design specifications $GM_{dec}$, $GM_{inc}$, and $PM$ exactly. The PID-like controller sought can be found from the solution of the system of equations (8)-(14). Unfortunately, this system of equations is too complicated to be solved on-line and it is not always solvable. Furthermore, the solution might not be appropriate or useful, especially if the derivative term is too large. For this reason, we propose here, to select a priori the derivative term $\tau_D$ of the controller, on the basis of the designer's knowledge relative to the process. If there are no restrictions imposed by the process, then it is recommended to select $\tau_D$ as large as possible in the range proposed by (16). This way, the resulting closed-loop system has the fastest possible response, for both, the set-point tracking and the load attenuation case, a well as the smallest possible maximum error in the case of regulatory control. Having selected $\tau_D$, as previously mentioned, three methods are then proposed, in order to tune the rest of the controller parameters.

### 6.1 The Phase Margin (PM) tuning method

In the case where, the only specification for the closed loop system is the desired phase margin $PM^{des}$, then it is recommended to tune the PID-like controller in such a way that this single specification is achieved at the maximum phase margin corresponding to the frequency $w_p$, namely, when $w_G=w_p$. This way, the integral reset time $\tau_I$ is the smallest possible that satisfies the specification and, hence, the obtained controller provides the fastest possible response, for both set-point tracking and regulatory control. The main steps of this tuning method are the following:

#### 6.1.1 The PM algorithm

*Step 1.* Given the system parameters $d$, $\tau_S$, the controller derivative term $\tau_D$ and the phase margin specification $PM^{des}$, set initially $\tau_I=0$.
*Step 2.* With this value of $\tau_I$, calculate $w_p$ as the maximum real root of (14).
*Step 3.* Select the new value of $\tau_I$ from the solution of $PM^{des}=\varphi_L(w_p)+\pi$, with respect to $\tau_I$, which is given by

$$\tau_I = w_p^{-1}\tan\left[PM^{des}+\frac{\pi}{2}+dw_p+at\mathrm{an}(\tau_S w_p)-at\mathrm{an}(w_p)-at\mathrm{an}(\tau_D w_p)\right] \tag{28}$$

*Step 4.* Repeat Steps 2 and 3 until convergence.
*Step 5.* With known $\tau_I$, calculate the corresponding frequency $w_p$ from (14) and the controller gain $K_C$ from (13). This completes the method.
The above algorithm converges to the correct solution, if such a solution exists, i.e. if for given $d$, $\tau_S$, $\tau_D$ there exists a value of $\tau_I$ for which $PM(d,\tau_S,\tau_D,\tau_I)=PM^{des}$.

### 6.2 The Gain Margin (GM) tuning method

This method is applicable in the case where the specifications of the closed loop system are described in the form of increasing and decreasing gain margins ($GM_{inc,des}$ and $GM_{dec,des}$). To present the method, two iterative algorithms for the calculation of the crossover frequencies $w_{min}$ and $w_{max}$ are first presented.

#### 6.2.1 The $w_{min}$ algorithm

*Step 1.* Start with an initial estimate for $w_{min}$. An appropriate value for fast convergence is

$$w_{\min}^{init} = \sqrt{\left[\tau_I - d(1-\tau_I)\right]^{-1}} \tag{29}$$

*Step 2.* Calculate the error of this approximation using the relation

$$e_r = -\pi / 2 - dw_{\min}^{init} + at\,\text{an}(w_{\min}^{init}) + at\,\text{an}(\tau_I w_{\min}^{init}) + at\,\text{an}(\tau_D w_{\min}^{init}) - at\,\text{an}(\tau_S w_{\min}^{init}) \quad (30)$$

*Step 3.* Take the new value of $w_{min}$ as $w_{\min}^{new} = w_{\min}^{old}(1 - e_r)$.

*Step 4.* Repeat Steps 2 and 3 until a convergence.

#### 6.2.2 The $w_{max}$ algorithm

*Step 1.* Start with a very large initial estimate of $w_{max}$, say $w_{\max}^{init} = 10^4$.

*Step 2.* Using (8), calculate the new value of $w_{max}$ as

$$w_{\max}^{new} = d^{-1}\left[-\frac{\pi}{2} + at\,\text{an}(w_{\max}^{old}) + at\,\text{an}(\tau_I w_{\max}^{old}) + at\,\text{an}(\tau_D w_{\max}^{old}) - at\,\text{an}(\tau_S w_{\max}^{old})\right] \quad (30)$$

*Step 3.* Repeat Steps 2 and 3 until convergence.

These two algorithms always converge to the correct values of $w_{min}$ and $w_{max}$, if for given $d$, $\tau_S$, $\tau_D$ and $\tau_I$ there exists a solution of (8), with respect to $w_C$, when the *atan* function takes values in the range $(-\pi/2,\pi/2)$. We are now able to present the main steps of proposed GM tuning method.

#### 6.2.3 The GM algorithm

*Step 1.* Given the system parameters $d$, $\tau_S$, the controller derivative term $\tau_D$ and the desired gain matrix product $GM_{prod,des}$, solve $max(PM(d,\tau_I,\tau_D,\tau_S))=0$ to obtain $\tau_{I,min}$.

*Step 2.* Set $\tau_{I,1}= \tau_{I,min}$ and $\tau_{I,2}= 10^3\tau_{I,min}$.

*Step 3.* Take the new value of $\tau_I$ as the average of $\tau_{I,1}$ and $\tau_{I,2}$, i.e. $\tau_I=(\tau_{I,1}+\tau_{I,2})/2$.

*Step 4.* Calculate the values of $w_{min}$ and $w_{max}$ using the $w_{min}$ Algorithm and the $w_{max}$ Algorithm, respectively, for the obtained $\tau_I$, and obtain $K_{C,min}$ and $K_{C,max}$ from (9).

*Step 5.* Calculate the value of $GM_{prod}$ from (11).

*Step 6.* If $GM_{prod}<GM_{prod,des}$ or $w_{min}\leq 0$ or $w_{max}\leq 0$, then $\tau_{I,1}=\tau_I$ or else $\tau_{I,2}=\tau_I$.

*Step 7.* Repeat Steps 3 to 6 until convergence.

*Step 8.* The controller gain is evaluated from either $K_C=K_{C,max}/G_{inc,des}$ or $K_C=K_{C,min}G_{dec,des}$. This completes the algorithm.

The above algorithm converges to the correct solution, if such a solution exists, i.e. if for given $d$, $\tau_S$, $\tau_D$ there exists a value of $\tau_I$ for which $GM_{prod}(d,\tau_S,\tau_D,\tau_I)=GM_{prod,des}$.

### 6.3 The Phase and Gain Margin (PGM) tuning method

If the derivative term is a priori selected, then it is not possible, in general, to simultaneously satisfy the specifications on $GM_{dec}$, $GM_{inc}$, and $PM$ exactly, with the remaining two free controller parameters. This is due to the fact that, it is not possible to assign three independent specifications with only two independent controller parameters, namely $K_C$ and $\tau_I$. Indeed, with the controller parameters $K_C$ and $\tau_I$ obtained from the GM Algorithm, in order to satisfy $GM_{dec}$ and $GM_{inc}$, then a specific value of the phase margin $PM(d,K_C,\tau_I,\tau_D)$ is obtained, and, hence, in this case the phase margin cannot be selected independently. Keeping these in mind, we propose here a tuning method, in order to achieve simultaneous, although not exact, satisfaction of all three specifications $PM$, $GM_{dec}$ and $GM_{inc}$. This method is based on the tuning methods presented in the previous two subsections. The basic steps, for the selection of the parameters of a PID-like controller that satisfy all three specifications, are the following:

### 6.3.1 The PGM algorithm

*Step 1.* For the selected value of $\tau_D$, check if there exists a value of $K_C$ that is able satisfy all three specifications, when $\tau_I \rightarrow \infty$.

*Step 2.* Calculate the two controllers obtained by the PM and the GM methods. If the controller with the largest value of $\tau_I$ satisfies all three specifications, then this is the controller sought. In the opposite case continue with *Step 3*.

*Step 3.* Assume that $K_{C,PM}$ and $\tau_{I,PM}$ are the controller parameters obtained form the application of the PM tuning method and $K_{C,GM}$ and $\tau_{I,GM}$ are the controller parameters obtained from the GM tuning method. Then, if none of these two controllers satisfy all specifications, check which controller gives the largest gain $K_C$, and distinguish the following two cases:

1. If $K_{C,PM}>K_{C,GM}$, then in order to satisfy all specifications with the smallest value of $\tau_I$, gradually increase $\tau_I$ (starting from the $max(\tau_{I,GM},\tau_{I,PM})$), while maintaining the same increasing gain margin $GM_{inc}$ (by selecting $K_C=K_{C,max}(d,\tau_S,\tau_I,\tau_D)/GM_{inc,des}$), until the phase margin specification is also satisfied.
2. If $K_{C,PM}<K_{C,GM}$, then gradually increase $\tau_I$ (starting from the $max(\tau_{I,GM},\tau_{I,PM})$), while maintaining the same decreasing gain margin $GM_{dec}$ (by selecting $K_C= K_{C,min}(d,\tau_S,\tau_I,\tau_D)GM_{dec,des}$), until the phase margin specification is also satisfied.

This completes the algorithm.

Although there are several ways to select the controller parameters in order to satisfy all three specifications (although not exactly), the method presented here is preferred, because it requires the smallest computational effort, since for a given $\tau_I$, the phase margin can be calculated exactly without the use of iterative algorithms (using (12) and $PM=\varphi_L(w_G)+\pi$). It is noted here that, in all PID tuning methods presented above, if the response obtained is too oscillatory (due to the small value of $\tau_I$), then, by increasing the value of $\tau_I$, the damping of the closed-loop system increases. From the analysis presented in Section 3, it becomes clear that, when $\tau_I$ is increased, the resulting closed-loop system is more robust, and hence all the stability robustness specifications are still satisfied (although not exactly).

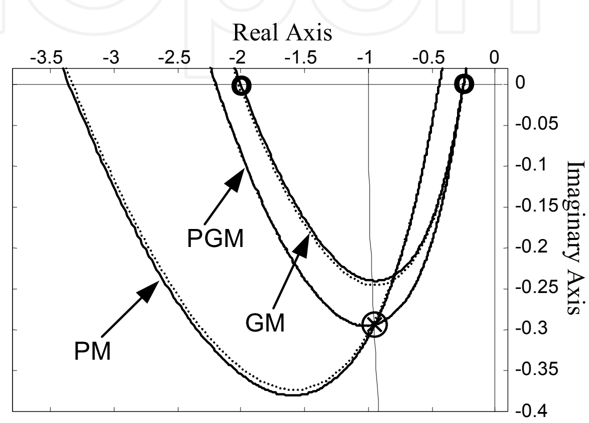
### 6.4 Simplification of the tuning rules for on-line tuning

The tuning rules presented in the previous sections can significantly be simplified, in the case where $\tau_D=\tau_S$. In this case, the loop transfer function is given by (17), and the solutions of the algorithms presented in Subsections 6.1.1 and 6.2.1-6.2.3, can easily be approximated with satisfactory accuracy for all systems with $0<d<0.9$. In particular, the solutions for $w_{min}$ and $w_{max}$, can be approximated by relations (21)-(23). Note that, here, $\hat{\tau}_{I,\min}(d)$ is an accurate approximation of the smallest value of the integral term $\tau_I$, for which (8) has a solution, when $\tau_D=\tau_S$, and when the atan function takes values in the range $(-\pi/2, \pi/2)$. Table 6 summarizes useful approximations of some other parameters involved in the aforementio-ned algorithms. Note that the maximum normalized errors for the parameters $K_{C,min}$ and $K_{C,max}$, when their estimates are obtained by (20), using $\hat{w}_{\min}$ and $\hat{w}_{\max}$ as given by (21), never exceed 2.2% for $d \le 0.9$ and $\tau_I>1.2\,\hat{\tau}_{I,\min}$.

In Table 7, numerical applications of the PM, GM and PGM tuning methods are presented for three processes with normalized dead time 0.1, 0.5 and 0.9. The controller parameters obtained from the application of these tuning methods are presented in the left section of Table 7 for both the exact $(K_C, \tau_I)$ and the approximated controller parameters ($\hat{K}_C$, $\hat{\tau}_I$). In the right section of Table 7, the polar plots of the resulting closed-loop systems are presented. Solid and dashed lines are used for the exact and the approximate controller, respectively. The gain margin specifications are indicated by the symbol ‘**o**’ and the point on the unit circle which determines the phase margin specification is indicated by the symbol ‘⊗’. From all these polar plots, it becomes obvious that the approximate solution is very accurate and in most cases cannot be distinguished from the exact solution. Note that, since the proposed tuning methods provide a controller that satisfies the required stability robustness specifications with significant accuracy, it is possible to design a closed loop system with any desired design specifications. The most robust (but slow) closed loop system possible (when $\tau_D$=$\tau_S$) can be obtained when $PM_{des}\rightarrow PM_{max}$ or when $GM_{prod,des}\rightarrow GM_{pred,max}$ (i.e. $\tau_I\rightarrow\infty$), while it is possible to design a faster but less robust system with less conservative stability margins specifications.

| Function | Approximation | MNE | Valid Range $d<0.9$ and |
|---|---|---|---|
| $GM_{prod,\max}(d)$ | $\dfrac{\pi}{2d\left[1+0.4085d/(1-0.2864d)\right]}$ | 3% | |
| $\tau_I(d,PM^{des})$ | $\hat{\tau}_{I,\min}(d)\left[1+f_{PM}(d)\dfrac{PM^{des}/PM_{\max}(d)}{1-\left[PM^{des}/PM_{\max}(d)\right]}\right]$ | 5% | $PM^{des}>0.2PM_{max}$ |
| $\tau_I(d,GM_{prod,des})$ | $\hat{\tau}_{I,\min}(d)\left(1+0.65\dfrac{A^{d+1}}{1-A}\right)+g(d)$ | 3% | $GM_{prod,des}>1+0.2\times(GM_{prod,max}-1)$ |
| $f_{PM}(d)=(-0.0153+0.436\sqrt{d}+0.632d)/d$ , $A=\dfrac{\sqrt{GM_{prod,des}}-1}{\sqrt{GM_{prod,\max}(d)}-1}$<br>$g(d)=10^{-2}[-0.18+5\sqrt{d}-32d+75d^2-51d^3+(-2.3d^2+3d^4)/(1-d)^3]$ | | | |

Table 6. Approximations of parameters involved in the PM, GM and PGM algorithms, when $\tau_D$=$\tau_S$.

| Specifications | | | Controller parameters | | Nyquist Plots |
|---|---|---|---|---|---|
| $d$=0.1 , $\tau_D$= $\tau_S$ | $PM$=0.3, $GM_{inc}$=4, $GM_{dec}$=2 | PM Method | $K_C=5.2293$<br>$\hat{K}_C=5.2170$ | $\tau_I=0.3010$<br>$\hat{\tau}_I=0.2980$ |  |
| | | GM Method | $K_C=3.0225$<br>$\hat{K}_C=3.0400$ | $\tau_I=0.3184$<br>$\hat{\tau}_I=0.3216$ | |
| | | PGM Method | $K_C=3.1333$<br>$\hat{K}_C=3.1411$ | $\tau_I=0.3598$<br>$\hat{\tau}_I=0.3597$ | |

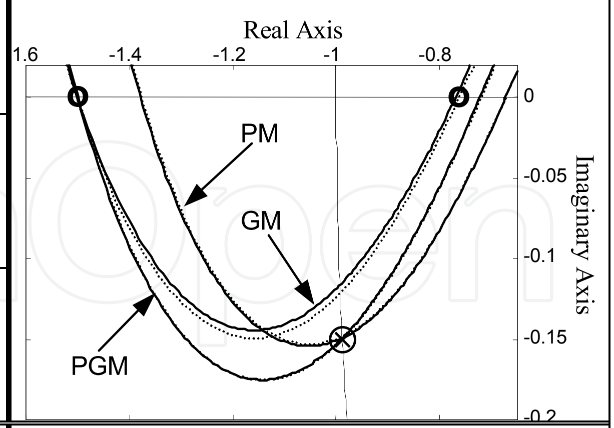
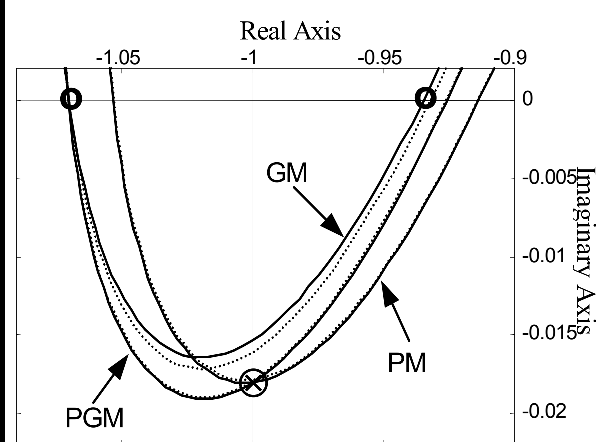
| Specifications | | | Controller parameters | | Nyquist Plots |
|---|---|---|---|---|---|
| $d$=0.5 , $\tau_D=\tau_S$ | $PM$=0.15, $GM_{inc}$=1.3, $GM_{dec}$=1.5 | PM Method | $K_C = 1.5690$<br>$\hat{K}_C = 1.5688$ | $\tau_I = 6.5667$<br>$\hat{\tau}_I = 6.5160$ |  |
| | | GM Method | $K_C = 1.7581$<br>$\hat{K}_C = 1.7505$ | $\tau_I = 5.5286$<br>$\hat{\tau}_I = 5.7115$ | |
| | | PGM Method | $K_C = 1.6933$<br>$\hat{K}_C = 1.6916$ | $\tau_I = 6.6907$<br>$\hat{\tau}_I = 6.9608$ | |
| $d$=0.9 , $\tau_D=\tau_S$ | $PM$=0.018, $GM_{inc}$=1.07, $GM_{dec}$=1.07 | PM Method | $K_C = 1.0602$<br>$\hat{K}_C = 1.0602$ | $\tau_I = 777.17$<br>$\hat{\tau}_I = 744.56$ |  |
| | | GM Method | $K_C = 1.0811$<br>$\hat{K}_C = 1.0795$ | $\tau_I = 511.24$<br>$\hat{\tau}_I = 590.60$ | |
| | | PGM Method | $K_C = 1.0756$<br>$\hat{K}_C = 1.0759$ | $\tau_I = 971.4$<br>$\hat{\tau}_I = 930.70$ | |

Table 7. Some characteristic numerical examples of the proposed tuning methods reported in Section 6.

## 7. Application to an experimental magnetic levitation system

In this section the tuning methods presented above will be applied to the experimental magnetic levitation system shown in Figure 5. This experimental system is a popular gravity-biased one degree of freedom magnetic levitation system in which an electromagnet exerts attractive force to levitate a steel ball. The dynamics of the MagLev system can be described by the following simplified state space model (Yang & Tateishi, 2001)

$$dx / dt = v \ , \ \ dv / dt = g - (c / M)\left[ i^2 / (x_\infty + x)^2 \right] \tag{31}$$

where $x$, $v$ and $M$ are the air gap (vertical position), the velocity and the mass of the steel ball respectively, $g$ is the gravity acceleration, $i$ is the coil current, $c$ and $x_\infty$ are constants that are determined by the magnetic properties of the electromagnet and the steel ball. Moreover the coil of the electromagnet has an inductance $L$ and a total resistance $R$.

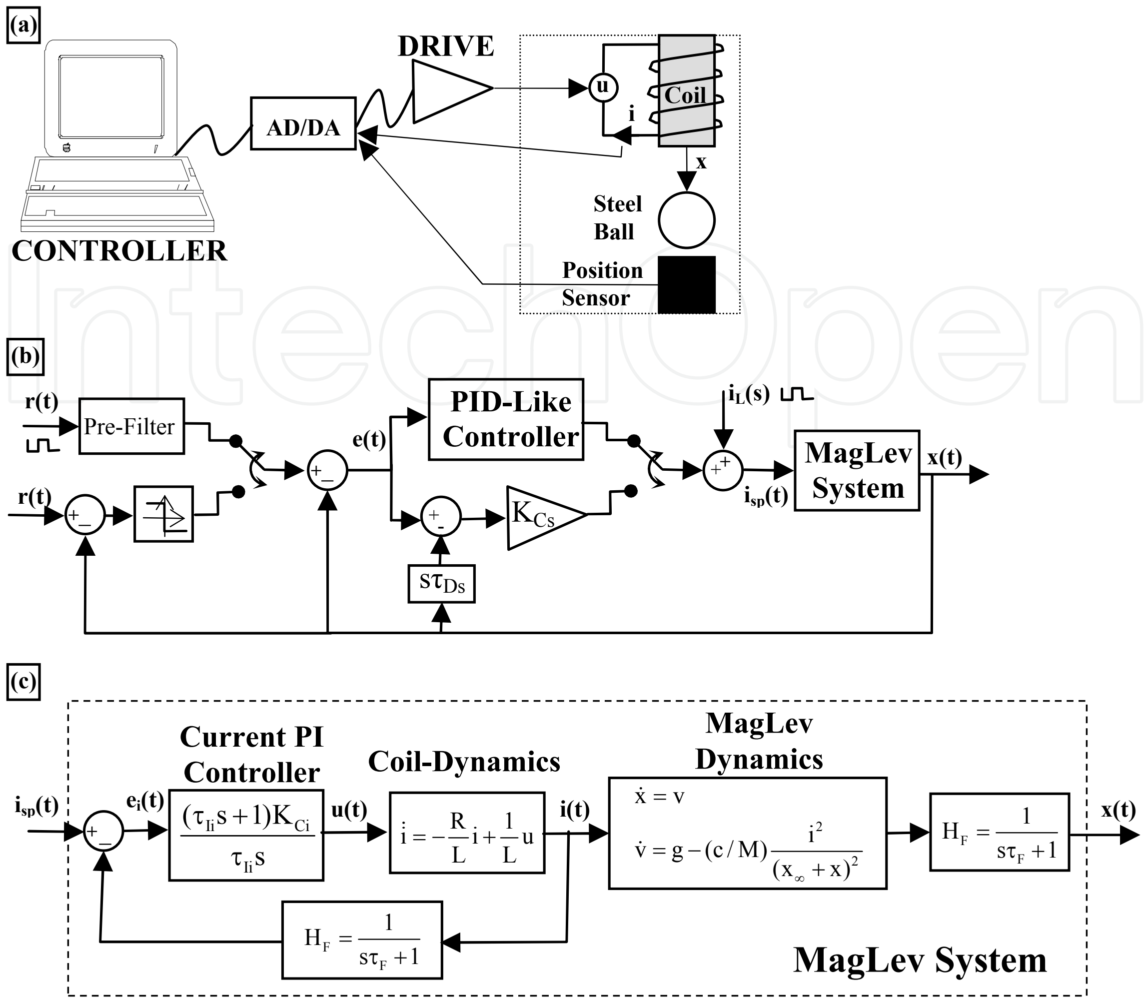



Fig. 5. MagLev system diagrams: (a) Schematic diagram, (b) Control diagram and (c) Block diagram.

Linearizing (31) about an operating point $x_0$, the following second order transfer function for the MagLev system is obtained

$$H_1^M(s) = \frac{K_m}{(\tau_{Um}s-1)(\tau_{Sm}s+1)} \tag{32}$$

where, $K_m$, $\tau_{Um}$, and $\tau_{Sm}$ are the gain, the unstable and the stable time constants of the system given by

$$K_m = \sqrt{c/(Mg)} \quad , \quad \tau_{Um} = \tau_{Sm} = \sqrt{0.5\left(x_\infty + x_0\right)/g} \tag{33}$$

For the MagLev system used in the following experiments the current $i$ is controlled by a PI controller (see Figure 5c). Moreover, to reduce measurement noise additional first order filters with time constants $\tau_F$ are used for the measurement of $x$ and $i$ (Figure 5c). The unmodelled dynamics of the current control loop, the measurement filters and the dynamics of the electrical circuitry (amplifiers, drivers etc.) is modelled here as a time delay $d_m$. Therefore, the complete transfer function of the linearized MagLev system is given by

$$H_1^M(s) = \frac{K_m \exp(-d_m s)}{(\tau_{Um}s-1)(\tau_{Sm}s+1)} \tag{34}$$

The model parameters $c$ and $x_\infty$ are obtained from measurements of the steady state value of the coil current (which is given by $i_0 = \sqrt{g(M/c)(x_\infty + x_0)^2}$ ), for several values of $x_0$ (3mm<$x_0$<11mm), using a stabilizing PID controller. Since the model parameters $K_m$, $\tau_{Um}$ and $\tau_{Sm}$ can be obtained from (33), to identify the time delay $d_m$ of the system, a single closed loop relay-feedback experiment can be used. The control diagram for this experiment is shown in Figure 5b. Using a PD stabilizing controller with derivative time $\tau_{Ds}$=$\tau_{Um}$, one can easily verify that $d_m$ is given by

$$d_m = \omega_C^{-1} a\sin\left[\left(\tau_{Um}\omega_C K_m K_{Cs}\right) / (\tau_{Um}^2 \omega_C^2 + 1)\right] \tag{35}$$

where $\omega_C$ is the ultimate frequency of the closed loop system, which is measured by the relay experiment. The values of the model parameters for the linearized system given by (33), about the operating point $x_0$=7mm, are listed in Table 8 together with the parameters of the PI-current controller and the time constant of the two measurement filters used. It is noted here that the selection of the filter time constant $\tau_F$ and the gains of the PI current controller are performed intentionally in order to produce a significant time delay to the MagLev system. Finally, it is mentioned that the sampling intervals for all experiments is chosen as $\tau_{st}$=0.5ms, which is fast enough to assume a continuous-time system.

| | |
|---|---|
| **Physical parameters** | |
| $M$=0.068 Kg , $g$=9.81 m/sec$^2$ , $c$=8.068 $\cdot 10^{-5}$Hm, $x_\infty$=0.00215m , $L$=0.4125 , $R$=11Ω | |
| **Linearized Model parameters** (around $x_0$=0.007m) | |
| $K_m$=0.008474 m/A , $\tau_{Um}$= $\tau_{Sm}$ =0.0216 sec, $d_m$=0.01037 sec , $i_0$=1.08 A | |
| **Current controller and measurement filter parameters** | |
| $K_{Ci}$=200 , $\tau_{Ii}$=1 , $\tau_F$=0.005 | |
| **Parameters of the designed PID controller** | |
| OPOS | $K_{Cm}$=196.7 , $\tau_{Im}$ =0.1273 , $\tau_{Dm}$ =0.0216 |
| ISE-Sp | $K_{Cm}$=197.9 , $\tau_{Im}$=0.0936 , $\tau_{Dm}$ =0.0216 |
| DPC | $K_{Cm}$=196.1 , $\tau_{Im}$=0.1565 , $\tau_{Dm}$=0.0216 |
| FST | $K_{Cm}$=196.5 , $\tau_{Im}$=0.1346 , $\tau_{Dm}$=0.0216 |
| GM | $K_{Cm}$=118.5 , $\tau_{Im}$= 0.428 , $\tau_{Dm}$ =0.0216 |
| PM | $K_{Cm}$= 147.5 , $\tau_{Im}$=0.1162 , $\tau_{Dm}$=0.0216 |

Table 8. System and controller parameters for the experiments in the MagLev system.

A series of experiments have been performed by applying all four methods reported in Sections 4 and 5 to the MagLev system. In Fig. 6, the set-point and load step responses around the operating point $x_0$=7mm are presented. In particular, in Figs 6a and 6b, the response of the MagLev to a pulse waveform with amplitude 1 mm and period 5 sec is shown in the case where the PID controller is tuned using the OPOS and ISE-Sp methods, respectively. Fig. 6c shows the tracking response in the case where the DPC method is used. In this case the amplitude of the pulse waveform used as reference input is 7mm (from 3.5mm to 10.5mm). Finally, Fig. 6d shows the regulatory control response, in the case where the FST method is used with a change in the system input (current set-point) produced by a pulse waveform with amplitude 0.2A (i.e. 20% change in the steady state value of the coil current). Fig. 6 verifies the efficiency and good performance of the proposed methods. As expected, the ISE-Sp method provides the fastest response, but with an overshoot of about 20%. The FST and OPOS methods produce very smooth and fast regulatory and set-point tracking responses. Finally, the DPC method provides a very robust controller that can control the MagLev system in a large operating region. However, this controller provides a rather sluggish response.

As a second application of the proposed tuning methods, a robust PID controller is designed in order to guarantee a stable closed loop system in a wide operating region (3.5mm< $x$<10.5mm) and in the case of ±20% uncertainty in the parameters $c$, $x_\infty$ and 10% uncertainty in the time delay $d_m$. The problem of converting the parametric uncertainties into gain and phase margin specifications is a very complicated problem that remains unsolved, in the general case. Here, in order to select appropriate specifications for the design of the controller, the following observations are made: (a) From (8), it is clear that the uncertainty in the model parameters $\tau_{Um}$ and $\tau_{Sm}$ (which depend on $x_\infty$ and $x_0$) does not affect the argument of the loop transfer function. The only term which influences the phase uncertainty is the uncertainty in the identification of the time delay. (b) Assuming that $\tau_I > 5\tau_{I,min}$, (this assumption is in accordance with our desire to design a very robust controller as suggested in the work (Paraskevopoulos et al, 2006)) one can easily verify from $\hat{\tau}_{I,\min}(d)$ (given in Table 6), that a ±10% change in $d_m$ produces a change in $\omega_{min}$ and $\omega_{max}$ smaller than

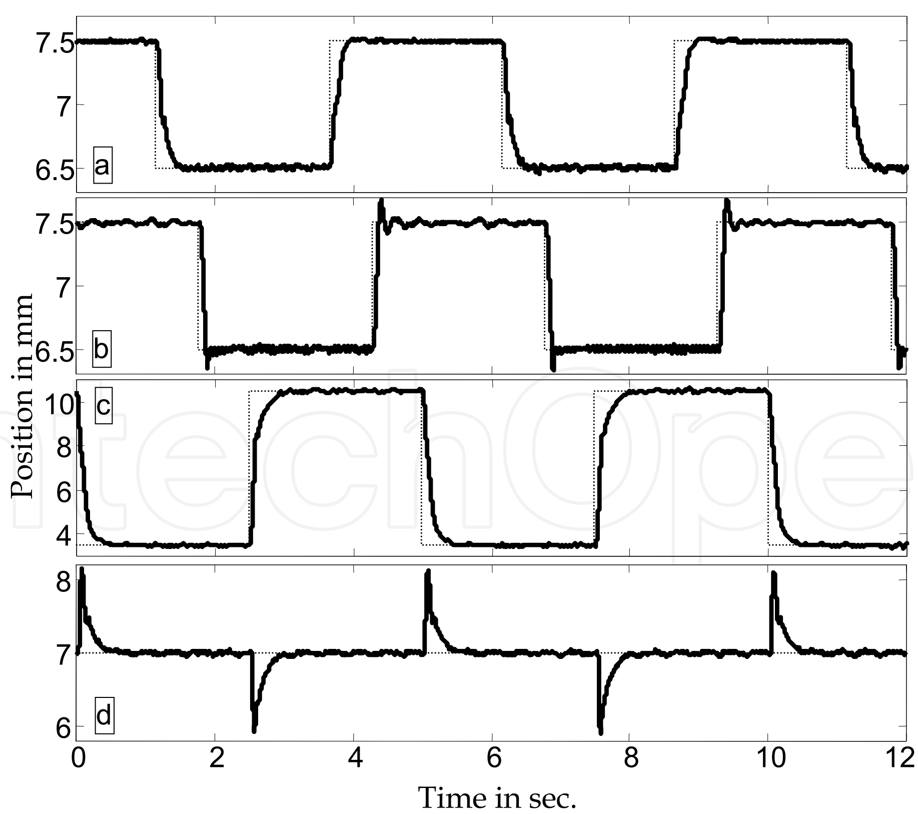



Fig. 6. Experimental MagLev position responses. Set-point tracking response: (a) using OPOS method, (b) using ISE-Sp method, (c) using DPC method. (d) Regulatory control step response using FST method (current load disturbance amplitude 20% or 0.2 A).

5% and 15%, respectively. (c) The magnitude of the loop transfer function is affected by all parameters, as well as the operating point. The two extreme worst cases are obtained when $d$ and $c$ are maximized and $x_0$, $x_\infty$ are minimized (scenario A) and when $d$, $x_0$, $x_\infty$ are maximized and $c$ is minimized (scenario B). From scenario A, we obtain the smallest maximum ultimate gain $min(K_{C,max})$, while from scenario B, we obtain the largest minimum ultimate gain $max(K_{C,min})$. Obviously, for the closed loop system to be stable under the assumed uncertainty and for the whole desired operating region, there must be $min(K_{C,max}) > max(K_{C,min})$. Based on the above observation one can easily verify that for $\tau_I > 5\tau_{I,min}$, the inequalities $min(K_{C,max}) > 0.53K_{C,max,0}$ and $max(K_{C,min}) < 1.2K_{C,min,0}$, must hold, where $K_{C,max,0}$ and $K_{C,min,0}$ are the nominal values of $K_{C,max}$ and $K_{C,min}$ at the operating point $x_0$=0.007m, i.e. the case where there is no uncertainty. To guarantee stability, the increasing and decreasing gain margins must be selected grater than 1/0.53 and 1.2, respectively.
Based on the above results and observations, in order to tune the PID controller, the GM tuning method is next applied with specifications $GM_{inc}$=2 and $GM_{dec}$=1.25. The obtained controller gains are listed in Table 8. The Nyquist plots for the two extreme scenarios A and B and for the nominal system, using the obtained robust controller, are shown in Fig. 7, which verifies that the closed loop system is always stable.

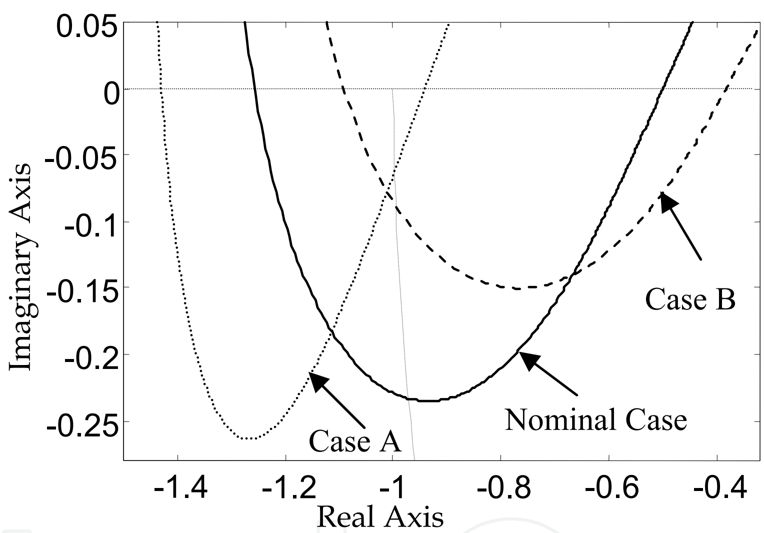



Fig. 7. Nyquist plots of the MagLev system using the robust controller designed with the GM method.

For the experimental application, a pre-filter with transfer function $G_{F,PID}(s)=1/(s\tau_I+1)$ is used in order to cancel the zero introduced by the PID controller. With this filter excessive overshoots in the set-point step response of the system are avoided. The experimental results obtained are presented in Figure 8. The set-point step response from 3.5mm to 10.5mm is shown in Figure 8a. This response is rather slow due to the small value of $K_{Cm}$ and the very large value of $\tau_{Im}$ ($\tau_{Im}/\tau_{Um}$=19.81=9.728$\tau_{I,min}$). This is more evident in the regulatory control case, around the operating point $x_0$=7mm, shown in Figure 8b. This response is obtained from a change in the system input (current set-point) produced by a pulse waveform with amplitude 0.2A (or 20% change in i).
A faster controller can be designed if the desired operating region is smaller under the assumption of the same parameter uncertainties as in the previous application. In this

case, the PM tuning method is used with a specification $PM_{des}$=0.15 rad. The obtained controller is presented in Table 6. Figures 9a and 9b, show the set-point step response from 6.5mm to 7.5mm and the regulatory control around the operating point $x_0$=7mm using the new controller. Clearly, the obtained responses are significantly faster, as it was expected from the design of the PID controller (smaller $\tau_{Im}$, larger $K_{Cm}$). Moreover, in the case of regulatory control the maximum error produced in the present case is significantly smaller (at least three times smaller) than the maximum error produced when the robust controller is used.

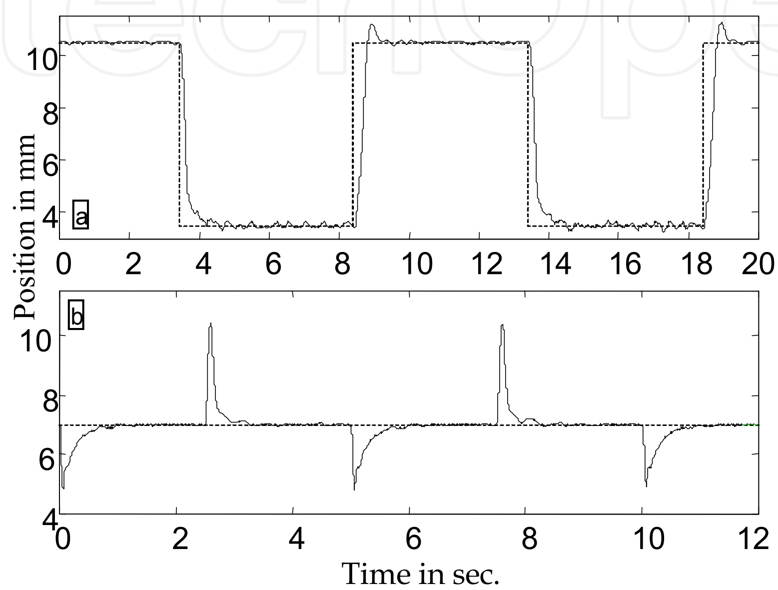



Fig. 8. Position response of MagLev system using the robust controller designed with the GM-method: (a) Set-point response and (b) Load step response (current load disturbance amplitude 20% or 0.2 A).

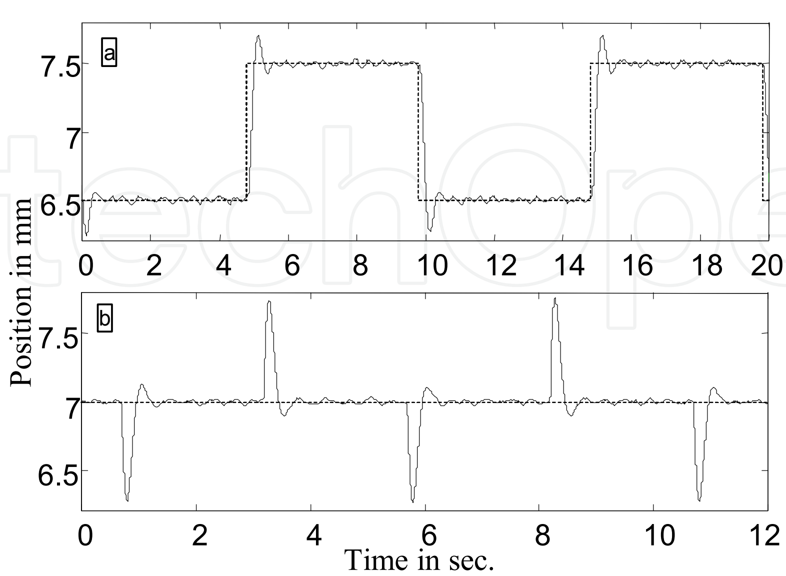



Fig. 9. Position response of MagLev system using a fast controller designed with the PM-method. Other legend as in Fig. 8.

## 8. Conclusions

New methods for tuning PID-like controllers for USOPDT systems have been developed in this work. These methods are based on various criteria, such as the appropriate assignment of the dominant poles of the closed-loop system, the attainment of various time-domain closed-loop characteristics, as well as the satisfaction of gain and phase margins specifications of the closed-loop system. In the general case, where the derivative action of the controller is selected arbitrarily, the tuning methods require the use of iterative algorithms for the solution of nonlinear systems of equations. In the special case where the controller derivative time constant is selected equal to the stable time constant of the system, the solutions of the nonlinear system of equations involved in the tuning methods are given in the form of quite accurate analytic approximations and, thus, the iterative algorithms can be avoided. In this case the tuning methods can readily be used for on-line applications. The proposed tuning methods have successfully been applied to the control of an experimental magnetic levitation system that is modelled as an USOPDT process. The obtained experimental results verify the efficiency of the proposed tuning methods that provide a very satisfactory performance of the closed-loop system.

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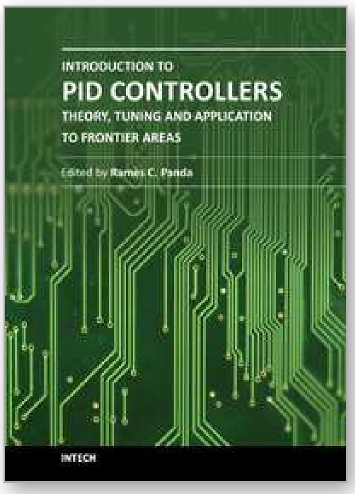


**Introduction to PID Controllers - Theory, Tuning and Application to Frontier Areas**
Edited by Prof. Rames C. Panda

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This book discusses the theory, application, and practice of PID control technology. It is designed for engineers, researchers, students of process control, and industry professionals. It will also be of interest for those seeking an overview of the subject of green automation who need to procure single loop and multi-loop PID controllers and who aim for an exceptional, stable, and robust closed-loop performance through process automation. Process modeling, controller design, and analyses using conventional and heuristic schemes are explained through different applications here. The readers should have primary knowledge of transfer functions, poles, zeros, regulation concepts, and background. The following sections are covered: The Theory of PID Controllers and their Design Methods, Tuning Criteria, Multivariable Systems: Automatic Tuning and Adaptation, Intelligent PID Control, Discrete, Intelligent PID Controller, Fractional Order PID Controllers, Extended Applications of PID, and Practical Applications. A wide variety of researchers and engineers seeking methods of designing and analyzing controllers will create a heavy demand for this book: interdisciplinary researchers, real time process developers, control engineers, instrument technicians, and many more entities that are recognizing the value of shifting to PID controller procurement.