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Description of Field States with Correlation Functions and Measurements in Quantum Optics

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1. Introduction

Modern physics deals with the consistent quantum concept of electromagnetic field. Creation and annihilation operators allow describing pure quantum states of the field as excited states of the vacuum one. The scale of its changes obliges to use statistical description of the field. Therefore the main object for full description of the field is a statistical operator (density matrix). Field evolution is reflected by operator equations. If the evolution equations are formulated in terms of field strength operators, their general structure coincides with the Maxwell equations. At the same time from the point of view of experiments only reduced description of electromagnetic fields is possible. In order to analyze certain physical situations and use numerical methods, we have the necessity of passing to observable quantities that can be measured in experiments. The problem of parameters, which are necessary for non-equilibrium electromagnetic field description, is a key one for building the field kinetics whenever it is under discussion. The field kinetics embraces a number of physical theories such as electrodynamics of continuous media, radiation transfer theory, magnetic hydrodynamics, and quantum optics. In all the cases it is necessary to choose physical quantities providing an adequate picture of non-equilibrium processes after transfer to averages. It has been shown that the minimal set of parameters to be taken into account in evolution equations included binary correlations of the field. The corresponding theory can be built in terms of one-particle density matrices, Wigner distribution functions, and conventional simultaneous correlation functions of field operators. Obviously, the choice depends on traditions and visibility of phenomenon description. Some methods can be connected due to relatively simple relations expressing their key quantities through one another. The famous Glauber's analysis (Glauber, 1966) of a quantum detector operation had resulted in using correlation functions including positive- and negative-frequency parts of field operator amplitudes in the quantum optics field. Herewith the most interesting properties of field states are described with non-simultaneous correlation functions. Various approaches in theoretical and experimental research into field correlations are compared in the present chapter.

Our starting point is investigation of the Dicke superfluorescence (Dicke, 1954) on the basis of the Bogolyubov reduced description method (Akhiezer & Peletminskii, 1981). It paves the way to constructing the field correlation functions. We can give a relaxation process picture in different orders of the perturbation theory. The set of correlation functions providing a rather full description of the superfluorescence phenomenon obeys the set of differential equations. The further research into the correlation properties of the radiated field requires establishing the connection with the behavior of Glauber functions of different orders.

2. Electromagnetic field as an object of quantum statistical theory

A statistical operator ρ of electromagnetic field should take into account the whole variety of field modes and statistical structure abundance for each of them. Proceeding from the calculation convenience provided by using coherent states $|z\rangle$ of field modes, the Glauber-Sudarshan representation for the statistical operator of field (Klauder & Sudarshan, 1968) footholds in physics. We refer to the following view of this diagonal representation

$$\rho = \int d^2 z P\left(z, z^*\right) | z\rangle\langle z | \tag{1}$$

where $P(z, z^*)$ is so called P-distribution ($z \equiv \{z_{\alpha k}\}$ and these variables are numbered by polarization α and wave vector k of the field modes). Since coherent states form an overcrowded basis in the state space of the mode with the completeness condition

$$\frac{1}{\pi}\int d^2 z \,| z\rangle\langle z |= \hat{1} \,, \tag{2}$$

the most general representation for the statistical operator should include not only projection operators $| z\rangle\langle z |$, but also more general operator products $| z\rangle\langle z' |$. Nevertheless it can be shown (Glauber, 1969; Kilin, 2003) that a P-distribution can be obtained as a two-dimensional Fourier transformation of the generating functional

$$F\left(u, u^*\right) = \mathrm{Sp}\rho e^{\sum_{\alpha k} u_{\alpha k} c^+_{\alpha k}} e^{-\sum_{\alpha k} u^*_{\alpha k} c_{\alpha k}} \tag{3}$$

which is a generating one for all normally ordered field moments and can be calculated directly with an arbitrary statistical operator ρ. Here we use standard notation of quantum electrodynamics: $c^+_{\alpha k}$, $c_{\alpha k}$ are Bose amplitudes (creation and annihilation operators) of the field.

So we can use the representation (1) in all cases when the Fourier integral for (3) exists. Such situation embraces a great variety of states that are interesting for physicists. More general cases reveal themselves in singularities of the P-distribution, the representation (1) still being prospective for using if the P-distribution can be expressed via generalized functions of slow growth, i.e. δ-function and its derivatives. The term "P-distribution" is relatively conventional: function $P(z, z^*)$ is a real but non-positive one. Nevertheless, the field state description with the Glauber-Sudarshan P-distribution remains the most demonstrative and consumable. For example, a proposed definition of non-classical states of electromagnetic field (Bogolyubov (Jr.) et al., 1988) uses the expression (1) for the statistical operator. A state is referred to as non-classical one if one of two requirements is obeyed: either average number of photons in a mode is less than 1, or P-function is not positively determined or has singularity that is higher than the δ-function.

For a multi-mode field the statistical operator takes the form of a direct product of one-mode statistical operators. In Schrödinger picture the Liouville equation

$$\partial_t \rho(t) = -\frac{i}{\hbar}[\hat{H}, \rho(t)] \tag{4}$$

describes the evolution of an arbitrary physical system. In the case when electromagnetic field interacting with matter is under consideration the problem is reduced to the correct account of the matter influence, so some kinds of effective Hamiltonians may appear in an analogue of (4) for the statistical operator of field. Evolution description in Heisenberg picture seems to be closer to the classical one. We come to operator Maxwell equations for field operators with terms corresponding to the matter influence and demanding some kind of material equations.

More graphic way to describing the electromagnetic field, its states, and their evolution is using correlation functions of different types, i.e. averaged values of physical quantities characterizing the field. The problem of choosing them will be discussed below.

3. Correlation functions provided by methods of quantum optics

Conventional classical optics was very restricted in measuring the parameters of fields. All conclusions about properties of light including its polarization properties were drawn from measurements of light intensity, i.e. from values of some quadratic functions of the field (Landau & Lifshitz, 1988). Naturally, we speak now about transversal waves in vacuum. Regarding a wave, close to a monochromatic one, we use slowly varying complex amplitude $E_{0n}(t)$ for its description:

$$E_n = E_{0n}(t) e^{-i\omega t} . \tag{5}$$

Partially polarized light is characterized with the tensor of polarization

$$J_{mn} = \overline{E_{0m} E_{0n}^*} \tag{6}$$

where m and n corresponds to two possible directions of polarization and quick oscillations of field are neglected. Averaging is performed over time intervals or (in the case of statistically stable situation) in terms of probabilities. A sum of diagonal components of J_{mn} is a real value that is proportional to the field intensity (the energy flux density in the wave in our case). Note that the discussion of field correlation functions by Landau in the earlier edition of the mentioned book was one of the first in the literature.

A rather full analysis of the classical measurement picture is given in (Klauder & Sudarshan, 1968). It should be mentioned that real field parameters are obtained from complex conjugated values in this approach. Transition to the quantum electromagnetic theory (Scully & Zubairy, 1997) is connected with substitution of operator structures with creation and annihilation operators instead of complex conjugated functions and coming to positive- and negative-frequency parts of field operators. Such expressions will be shown later on.

Physical picture of field parameter registration in the quantum case can be reduced to the problem of photon detection. An ideal detector should have response that is independent of radiation frequency and be small enough in comparison with the scale of field changes. Generally accepted analysis of quantum photon detector (Glauber, 1965; Kilin, 2003) is based on using an atom in this role and regarding the operator of field-atom interaction in the electric dipole approximation

$$\hat{V} = -\hat{p}_n \cdot \hat{E}_n(x)$$

with $\hat{p}_n$ standing for the operator of the electric dipole moment of an atom localized in a point with a radius-vector x (we shall denote in such a simple way a three-dimensional spatial vector). The quantum theory derives the total probability w of atom transition from a definite initial ground state $|g\rangle$ to an arbitrary final excited one $|e\rangle$ belonging to the continuous spectrum during the time interval from t_0 to t on the basis of Dirac's nonstationary perturbation theory in the interaction picture (Kilin, 2003)

$$w = \int_{t_0}^{t} d\tau \int_{t_0}^{t} d\tau' \sum_{mn} R_{mn}(\tau - \tau') G_{mn}^{(1,1)}(x, \tau'; x, \tau) \tag{7}$$

where $R_{mn}(\tau - \tau')$ is a function of detector sensitivity and

$$G_{mn}^{(1,1)}(x_1, t_1; x_1', t_1') \equiv \langle \hat{E}_m^{(-)}(x_1, t_1) \hat{E}_n^{(+)}(x_1', t_1') \rangle \tag{8}$$

is field correlation function of the first order (we use the notation $\langle \hat{A} \rangle \equiv \mathrm{Sp}\, \rho \hat{A}$ for an arbitrary operator $\hat{A}$). Here and further we use standard expressions for operators of the vector potential, electric and magnetic field in the Coulomb gauge (Akhiezer A. & Berestetsky V., 1969)

$$\hat{A}_n(x) = c \sum_{k\alpha} \left(\frac{2\pi\hbar}{\omega_k V} \right)^{1/2} e_{\alpha k n} (c_{\alpha k} + c^{+}_{\alpha, -k}) e^{ikx} ; \tag{9}$$

$$\hat{E}_n(x) = i \sum_{k\alpha} \frac{(2\pi\hbar\omega_k)^{1/2}}{V^{1/2}} e_{\alpha k n} (c_{\alpha k} - c^{+}_{\alpha, -k}) e^{ikx} , \tag{10}$$

$$\hat{B}_n(x) = i \sum_{k\alpha} \frac{(2\pi\hbar\omega_k)^{1/2}}{V^{1/2}} \varepsilon_{nlm} \tilde{k}_l e_{\alpha k m} (c_{\alpha k} + c^{+}_{\alpha, -k}) e^{ikx}$$

In these formulas $e_{\alpha k n}$ are vectors of the circular polarization ($e_{\alpha k n} k_n = 0$), $\tilde{k}_l \equiv k_l / k$, $\omega_k = ck$, V is field volume. Field operators in (8) are the positive- and negative-frequency parts of electric field operator in the picture of interaction

$$\hat{E}_n(x, t) = \hat{E}_n^{(+)}(x, t) + \hat{E}_n^{(-)}(x, t) , \tag{11}$$

$$\hat{E}_n^{(+)}(x,t) \equiv i\sum_{k\alpha} \frac{(2\pi\hbar\omega_k)^{1/2}}{V^{1/2}} e_{\alpha kn} c_{\alpha k} e^{i(kx-\omega_k t)}, \qquad \hat{E}_n^{(-)}(x,t) = \hat{E}_n^{(+)}(x,t)^+ .$$

The correlation function of detector sensitivity in the suggestion that matrix elements of the dipole moment operator between the ground and excited states (so called dipole moment of transition) $\langle e | \hat{p}_n | g \rangle \equiv p_n$ are independent of a final state takes the form

$$R_{mn}(\tau-\tau') = \pi \frac{\nu}{\hbar^2} p_m^* p_n \delta(\tau-\tau') \equiv s_{mn}\delta(\tau-\tau') \tag{12}$$

where ν stands for the spectral density of states in the continuous spectrum. It is expedient to notice that the dependence of matrix elements of electric dipole moment on time in the interaction picture results in positive- and negative-frequency parts of field operators appearing in calculated averages.

It follows from (7) and (12) that the rate of counting for the considered model of an ideal photon detector makes

$$p(t) = \frac{dw}{dt} = \sum_{mn} s_{mn} G_{mn}^{(1,1)}(x,t;x,t) \tag{13}$$

The problem of correlation of modes with different polarizations is a complicated one from the point of view of quantum measurements. So in most cases theoretical consideration goes to the presence of polarization filter. For such case the correlation (13) takes the form

$$p(t) = sG^{(1,1)}(x,t;x,t) \equiv s\langle E^{(-)}(x,t) E^{(+)}(x,t)\rangle, \qquad \hat{E}(x,t) \equiv \hat{E}_n(x,t) e_n \tag{14}$$

confirming that an ideal detector measures a correlation function of the first order with coinciding space-time arguments, i.e. field intensity in a fixed point (e_n is polarization vector depending on the filter).

Correlation properties of radiation manifest themselves in interference experiments. The well-known Young scheme with signals from two apertures interfering can be analyzed in quantum terms. Schematically, we regard (in accordance with Huygens-Fresnel principle) a field value in an observation point x at some time t as a linear combination of field parameters in aperture points x_1 and x_2 at proper time moments. Using our previous considerations concerning quantum detectors, we put down, for example, for negative-frequency part of the electric field strength for a fixed field polarization

$$\hat{E}^{(-)}(x,t) = \alpha_1 \hat{E}^{(-)}(x_1,t_1) + \alpha_2 \hat{E}^{(-)}(x_2,t_2) \tag{15}$$

where $t_{1,2} = t - s_{1,2}/c$ and $s_{1,2} = |x_{1,2} - x|$; α_1 and α_2 are determined by the system geometry. Thus for readings of an ideal detector placed in x we obtain an expression including an interference term

$$2\,\mathrm{Re}\,\alpha_1\alpha_2^* \langle \hat{E}^{(-)}(x_1,t_1) \hat{E}^{(+)}(x_2,t_2)\rangle .$$

The most important conclusion at this stage is possibility of measuring a correlation function of the first order defined by (8) with arbitrary arguments on the basis of the Young scheme and one photon detector. The stability of the statistical situation is suggested, thus function (8) is transformed into the function of $t_1' - t_1$. So, using polarization filters after apertures, we obtain a scheme for measuring a correlation function (8) in the most general form.

We see that optical measurements with one quantum detector lead to considering a correlation function of the first order (8) with necessity. In order to obtain information about more complex correlation properties of electromagnetic fields, we should consider a more complicated model problem corresponding to the scheme of the famous pioneer experiments of Hanbury Brown and Twiss (Hanbury Brown & Twiss, 1956). We suppose that two ideal detectors of photons are located in points x_1 and x_2; optical shutters are placed in front of the detectors. The shutters are opened at the time moment t_0 and closed at the moments t_1 and t_2. Calculation of probability of photon absorption in each detector gives the following result

$$w^{(2)} = \int_{t_0}^{t_1} d\tau_1 \int_{t_0}^{t_2} d\tau_2 \int_{t_0}^{t_1} d\tau_1' \int_{t_0}^{t_2} d\tau_2' R_{m_1 n_1}(\tau_1 - \tau_1') R_{m_2 n_2}(\tau_2 - \tau_2') G^{(2,2)}_{m_1 m_2, n_1 n_2}(x_1, \tau_1; x_2, \tau_2; x_1, \tau_1'; x_2, \tau_2') \quad (16)$$

where $R_{mn}(\tau - \tau')$ is a sensitivity correlation function determined by (12) and a correlation function of the second order

$$G^{(2,2)}_{m_1 m_2, n_1 n_2}(y_1, y_2; y_1', y_2') = \langle \hat{E}^{(-)}_{m_1}(y_1) \hat{E}^{(-)}_{m_2}(y_2) \hat{E}^{(+)}_{n_1}(y_1') \hat{E}^{(+)}_{n_2}(y_2') \rangle \quad (17)$$

is introduced (we use here an abbreviated notation $y = (x, t)$). In the above-considered case of a broadband detector the rate of coinciding of photon registrations by two detectors makes

$$p^{(2)} = \frac{\partial^2 w^{(2)}}{\partial t_1 \partial t_2} = s_{m_1 n_1} s_{m_2 n_2} G^{(2,2)}_{m_1 m_2, n_1 n_2}(x_1, t_1; x_2, t_2; x_1, t_1; x_2, t_2) \quad (18)$$

with detector parameters s_{mn} introduced in (12). Therefore the Hanbury Brown–Twiss experimental scheme with registering the coincidence of photon absorption by two detectors obtaining signals from the divided light beam with a delay line in front of one of detectors provides measuring of the correlation function of the second order (17) if each detector operates with a certain polarization of the wave.

Generalizations of the Hanbury Brown–Twiss coincidence scheme for the case of *N* detectors are considered as obvious. The rate of *N*-fold coincidences is connected with a correlation function of *N*th order. The analysis of ideal quantum photon detector operation and coincidence scheme by Glauber has elucidated the nature of field functions measured via using the noted schemes – they are functions built with the set of normally ordered operators

$$\langle \hat{E}^{(-)}_{m_1}(y_1) \ldots \hat{E}^{(-)}_{m_M}(y_M) \hat{E}^{(+)}_{n_1}(y_1) \ldots \hat{E}^{(+)}_{n_M}(y_M) \rangle \quad (19)$$

in the case of M detectors. At last, the most general set of normally ordered correlation functions introduced by Glauber (Glauber, 1963) looks like

$$G^{(M,N)}_{m_1...m_M,n_1...n_N}(y_1...y_M,y'_1...y'_N) \equiv \langle \hat{E}^{(-)}_{m_1}(y_1)...\hat{E}^{(-)}_{m_M}(y_M)\hat{E}^{(+)}_{n_1}(y'_1)...\hat{E}^{(+)}_{n_N}(y'_N)\rangle \ . \tag{20}$$

Functions (20) equal to zero usually at $M \neq N$ except very special states with broken symmetry (Glauber, 1969). Such function complex provides the most full description of the field correlation properties. In this picture taking into account magnetic field amplitudes is not necessary since they are simply connected with electric field amplitudes for each mode of electromagnetic field. Notice that the electric-dipole mechanism of absorption really dominates in experiments.

Method of photon counting corresponds to the general ideas of statistical approach; in its terms a number of quantum optics phenomena is described adequately, so the term "quantum optics" is used mainly as "statistical optics". Traditional terminology concerning correlation properties of light is based on the notion "coherence". In scientific literature coherences of the first and second orders are distinguished. It can be substantiated that, for example, the visibility of interference fringes in the Young scheme is determined by the coherence function of the first order that is a normalized correlation function of the first order (Scully & Zubairy, 1997)

$$g^{(1)}(x_1,x_2,\tau) = \frac{\langle \hat{E}^{(-)}(x_1,t)\hat{E}^{(+)}(x_2,t+\tau)\rangle}{\sqrt{\langle \hat{E}^{(-)}(x_1,t)\hat{E}^{(+)}(x_1,t)\rangle\langle \hat{E}^{(-)}(x_2,t+\tau)\hat{E}^{(+)}(x_2,t+\tau)\rangle}} \ . \tag{21}$$

Similarly to (21), the photon grouping effect is determined by the coherence function of the second order

$$g^{(2)}(x,\tau) = \frac{\langle \hat{E}^{(-)}(x,t)\hat{E}^{(-)}(x,t+\tau)\hat{E}^{(+)}(x,t)\hat{E}^{(+)}(x,t+\tau)\rangle}{\langle \hat{E}^{(-)}(x,t)\hat{E}^{(+)}(x,t)\rangle\langle \hat{E}^{(-)}(x,t+\tau)\hat{E}^{(+)}(x,t+\tau)\rangle} \tag{22}$$

Coherences of higher orders (Bogolyubov (Jr.) et al., 1988) can be introduced in the same way. We shall refer to Glauber functions (20) as the main means of field description in quantum optics. Differences between time arguments play the decisive role in the physical interpretation of functions. Taking into account all difficulties and conditions for measurements, functions of lower orders are really urgent for experimental work.

4. Superfluorescence in Dicke model as an important example of collective quantum phenomena

The Dicke model of a system of great quantity of two-level emitters interacting via electromagnetic field (Dicke, 1954) is a noticeable case of synergetics in statistical system behavior during the relaxation processes. Its research history is very informative. R. Dicke came to the conclusion about superradiant state formation proceeding from the analysis of symmetry of quantum states of emitters described with quasispin operators. For long time equilibrium properties of the Dicke model were under discussion and the possibility of phase transition has been established; it was associated with field states in lasers. At the next step it has become clear that self-organizing takes place in the dynamical process and presents some kind of a "dynamical phase transition" (Bogolyubov (Jr.) & Shumovsky, 1987). N excited atoms come to coordinated behavior without the mechanism of stimulated emission and a peak of intensity, proportional to N^2, appeared for modes that were close to the resonant one in a direction determined by the geometry of the system (Banfi & Bonifacio, 1975). So we have a way of coherent generation that is alternative to the laser one. This way can be used hypothetically in X- and γ-ray generators opening wide possibilities for physics and technology.

Collective spontaneous emission in the Dicke quasispin model proved to be one of the most difficult for experimental observations collective quantum phenomena. That is why taking into account real conditions of the experiment is of great importance. Thus great quantity of Dicke model generalizations has been considered. There are two factors dependent of temperature, namely the own motion of emitters and their interaction with the media. The both factors are connected with additional chaotic motion, thus they worsen the prospects of self-organizing in a system. The last factor is discussed traditionally as an influence of a cavity (resonator) since experiments in superradiance use laser technology (Kadantseva et al., 1989). The corresponding theoretical analysis is based on modeling the cavity with a system of oscillators (Louisell, 1964). The problem of influence of emitter motion (which is of different nature in different media) can be solved with taking into account this motion via a nonuniform broadening of the working frequency of emitters (Bogolyubov (Jr.) & Shumovsky, 1987). The dispersion of emitter frequencies results in an additional fading in a system and elimination of singularities in kinetic coefficients.

Traditional investigations obtain conclusions about a superfluorescent impulse generation on the basis of calculated behavior of the system of two-level emitters. The problem of light generation in the Dicke model can be investigated in the framework of the Bogolyubov method of eliminating boson variables (Bogolyubov (Jr.) & Shumovsky, 1987) with the suggestion of equilibrium state of field with a certain temperature. The correlation properties of light remain unknown in such picture. Good results can be obtained by applying the Bogolyubov reduced description method (Lyagushyn et al., 2005) to the model. The reduced description method eliminates some difficulties in the Dicke model investigations and allows both to take into account some additional factors (the orientation and motion of emitters, for instance) and to introduce more detailed description of the field. A kind of correlation functions to be used in such approach will be of interest for us.

5. Quantum models for electromagnetic field in media

The main problem of quantum optics is diagnostics of electromagnetic field (f-system) interacting with a medium (m-system). In this connection we have considered a number of models of medium and medium-field interaction. From various points of view the Dicke model of medium consisting of two-level emitters is very useful for such analysis. In the Coulomb gauge it is described by the Hamilton operator (Lyagushyn & Sokolovsky, 2010b)

$$\hat{H} = \hat{H}_{\mathrm{f}} + \hat{H}_{\mathrm{m}} + \hat{H}_{\mathrm{mf}}\,, \quad \hat{H}_{\mathrm{f}} \equiv \sum_{k\alpha} \hbar\omega_k c^{+}_{k\alpha} c_{k\alpha}\,, \quad \hat{H}_{\mathrm{m}} = \hbar\omega \sum_{1\le a\le N} \hat{r}_{az}\,, \tag{23}$$

$$\hat{H}_{\mathrm{mf}} = -\int d^3x \hat{E}^t_n(x) \hat{P}_n(x)$$

Here $\hat{r}_{an}$ is a quasispin operator, a is emitter's number, α is polarization index, $\hat{P}_n(x)$ is the density of electric dipole moment (polarization) of emitters

$$\hat{P}_n(x) = 2 \sum_{1\le a\le N} d_{an}\hat{r}_{ax}\delta\left(x - x_a\right) . \tag{24}$$

We neglect emitter-emitter interaction in (23). Operators of vector potential, transversal electric field and magnetic field are expressed via creation and annihilation boson operators $c^+_{k\alpha}, c_{k\alpha}$ by formulas (9), (10) and commutation relations

$$[\hat{E}^t_m(x), \hat{E}^t_n(x')] = 0 , \quad [\hat{B}_m(x), \hat{B}_n(x')] = 0 , \quad [\hat{B}_m(x), \hat{E}^t_n(x')] = \varepsilon_{mnl} 4\pi i\hbar c \frac{\partial\delta(x-x')}{\partial x_l} \tag{25}$$

are valid (we use the notation $\hat{E}^t_n(x)$ for electric field operator (10) in the discussion of the field-emitters system).

It is very convenient to use operator evolution equations for investigating the dynamics of the system (23). The Maxwell operator equations have a known form

$$\dot{\hat{E}}_n\left(x\right) = c\,\mathrm{rot}_n\,\hat{B}\left(x\right) - 4\pi\hat{J}_n\left(x\right), \qquad \dot{\hat{B}}_n(x) = -c\,\mathrm{rot}_n\,\hat{E}\left(x\right) \tag{26}$$

where total electric field and electromagnetic current

$$\hat{E}_n(x) \equiv \hat{E}^t_n(x) - 4\pi\hat{P}_n(x) , \quad \hat{J}_n(x) \equiv \dot{\hat{P}}_n\left(x\right) = -2\omega\sum_a d_{an}\hat{r}_{ay}\delta\left(x - x_a\right) \tag{27}$$

are introduced. Energy density of emitter medium

$$\hat{\varepsilon}(x) = \hbar\omega \sum_{1\le a\le N} \hat{r}_{az}\delta(x - x_a)$$

obeys the evolution equation

$$\dot{\hat{\varepsilon}}(x) = \hat{J}_n\left(x\right)\hat{E}^t_n\left(x\right) \tag{28}$$

which describes the Joule heat exchange between the emitters and field. Since the field parameters are considered in different spatial points, we obtain the possibility of investigating the field correlation properties.

Also the model of electromagnetic field in plasma medium plays a significant role. The Hamilton operator of such system in the Coulomb gauge was taken in the paper (Sokolovsky & Stupka, 2004) in the form

$$\hat{H} = \hat{H}_{\mathrm{f}} + \hat{H}_{\mathrm{m}} + \hat{H}_{\mathrm{mf}} , \qquad \hat{H}_{\mathrm{f}} \equiv \sum_{k\alpha} \hbar\omega_k c^+_{k\alpha} c_{k\alpha} , \qquad \hat{H}_{\mathrm{mf}} = \hat{H}_1 + \hat{H}_2 , \tag{29}$$

$$\hat{H}_1 = -\frac{1}{c}\int dx\hat{A}_n\left(x\right)\hat{j}_n\left(x\right) , \qquad \hat{H}_2 = \frac{1}{2c^2}\int dx\hat{A}^2(x)\hat{\chi}(x) \qquad \left(\hat{\chi}(x) \equiv \sum_a \frac{e_a^2}{m_a}\hat{n}_a(x)\right).$$

Here $\hat{H}_m$ is the Hamilton operator of plasma particles with account of Coulomb interaction, $\hat{j}_n(x)$ is electric current, $\hat{n}_a(x)$ is density operator of the a th component of the system.

6. Reduced description of electromagnetic field in medium. Role of field correlations

Here we discuss kinetics of electromagnetic field in a medium. This theory must connect dynamics of the field with dynamics of the medium. The problem can be solved only on the basis of the reduced description of a system. One has to choose a set of microscopic quantities in such way that their average values describe the system completely. Therefore, the Bogolyubov reduced description method (Akhiezer & Peletminskii, 1981) can be a basis for the general consideration of the problem. In this approach its starting point is a quantum Liouville equation for the statistical operator $\rho(t)$ of a system including electromagnetic field and a medium

$$\partial_t \rho(t) = -\frac{i}{\hbar}[\hat{H},\rho(t)] , \qquad \hat{H} = \hat{H}_{\mathrm{f}} + \hat{H}_{\mathrm{m}} + \hat{H}_{\mathrm{mf}} . \tag{30}$$

The method is based on the functional hypothesis describing a structure of the operator $\rho(t)$ at large times (Bogolyubov, 1946)

$$\rho(t) \xrightarrow[t \gg \tau_o]{} \rho(\xi(t,\rho_0),\eta(t,\rho_0)) \equiv \rho^{(+)}(t) \qquad (\, \rho_0 \equiv \rho(t=0) \,) \tag{31}$$

where reduced description parameters of the field $\xi_\mu(t,\rho_0)$ and matter $\eta_a(t,\rho_0)$ are defined in a natural way

$$\xi_\mu(t,\rho_0) = \mathrm{Sp}\, \rho^{(+)}(t)\hat{\xi}_\mu , \qquad \eta_a(t,\rho_0) = \mathrm{Sp}\, \rho^{(+)}(t)\hat{\eta}_a \tag{32}$$

(τ_0 is a characteristic time determined by an initial state of the system ρ_0 and a used set of reduced description parameters). The set of parameters $\xi_\mu(t,\rho_0)$, $\eta_a(t,\rho_0)$ is determined by the possibilities and traditions of experiments as well as by theoretical considerations (for simplicity we will drop ρ_0 in the parameters). The development of the problem investigation has resulted in finding the main approximation for the statistical operator $\rho(\xi,\eta)$, so called a quasiequilibrium statistical operator $\rho_q(Z(\xi),X(\eta))$ (though it describes states which are far from the equilibrium) defined by the relations

$$\rho_q(Z,Z_m) = \rho_{\mathrm{f}}(Z)\rho_{\mathrm{m}}(Z_{\mathrm{m}}) ; \tag{33}$$

$$\rho_{\mathrm{f}}(Z) = \exp\{\Phi(Z) - \sum_\mu Z_\mu \hat{\xi}_\mu\} , \quad \mathrm{Sp}_{\mathrm{f}}\, \rho_{\mathrm{f}}(Z) = 1 , \quad \mathrm{Sp}_{\mathrm{f}}\, \rho_{\mathrm{f}}(Z(\xi))\hat{\xi}_\mu = \xi_\mu ; \tag{34}$$

$$\rho_{\mathrm{m}}(X) = \exp\{\Omega(X) - \sum_a X_a \hat{\eta}_a\} , \quad \mathrm{Sp}_{\mathrm{m}}\, \rho_{\mathrm{m}}(X) = 1 , \quad \mathrm{Sp}_{\mathrm{m}}\, \rho_{\mathrm{m}}(X(\eta))\hat{\eta}_a = \eta_a . \tag{35}$$

According to the common idea, electromagnetic field in medium is usually described by average values of electric $E_n(x,t)$ and magnetic $B_n(x,t)$ fields. So, it seems possible to choose operators $\hat{\xi}_\mu$ in (32) as $\hat{\xi}_{\alpha n}(x)$: $\hat{\xi}_{1n}(x)=\hat{E}_n^t(x)$, $\hat{\xi}_{2n}(x)=\hat{B}_n(x)$. However, in this case the statistical operator $\rho_{\mathrm{f}}(Z)$ does not exist (its exponent contains only linear in Bose amplitudes form and $\rho_{\mathrm{f}}(Z)$ is non-normalized). Therefore, one has to use a wider set of parameters $\hat{\xi}_\mu$ in conformity with the observation made in (Peletminkii et al., 1975). At least, exponent in (34) should contain quadratic terms. So the simplest quasiequilibrium statistical operator of the field can be written as

$$\rho_{\mathrm{f}}(Z)=\exp\{\Phi(Z)-\sum_{\alpha k,\alpha' k'} Z_{kk'}^{\alpha\alpha'} c_{\alpha k}^{+} c_{\alpha' k'} -(\sum_{\alpha k,\alpha' k'} \tilde{Z}_{kk'}^{\alpha\alpha'} c_{\alpha k}^{+} c_{\alpha' k'}^{+} + \sum_{\alpha k} Z_k^\alpha c_{\alpha k}^{+} + h.c.)\} \tag{36}$$

Kinetics of the field based on this statistical operator describes states with zero average fields at $Z_k^\alpha = 0$. Quadratic terms in (36) correspond to binary fluctuation of the field $\langle \xi_{\alpha m}^x \xi_{\alpha' n}^{x'} \rangle_t$ (or two binary correlations $(\xi_{\alpha m}^x \xi_{\alpha' n}^{x'})_t$) as additional reduced description parameters

$$\mathrm{Sp}\,\rho^{(+)}(t)\frac{1}{2}\{\hat{\xi}_{\alpha m}(x),\hat{\xi}_{\alpha' n}(x')\} = \langle \xi_{\alpha m}^x \xi_{\alpha' n}^{x'} \rangle_t\,, \qquad (\xi_{\alpha m}^x \xi_{\alpha' n}^{x'})_t = \langle \xi_{\alpha m}^x \xi_{\alpha' n}^{x'} \rangle_t - \xi_{\alpha m}(x,t)\xi_{\alpha' n}(x',t) \tag{37}$$

In other words, the quasiequilibrium statistical operator (34) corresponds to field description by average values of operators

$$\hat{\xi}_\mu: \quad \hat{\xi}_{\alpha n}(x)\,, \quad \frac{1}{2}\{\hat{\xi}_{\alpha m}(x),\hat{\xi}_{\alpha' n}(x')\}\,. \tag{38}$$

The theory can be significantly simplified in the Peletminskii-Yatsenko model (Akhiezer & Peletminskii, 1981) in which

$$\frac{1}{\hbar}[\hat{H}_{\mathrm{f}},\hat{\xi}_\mu] = \sum_{\mu'} c_{\mu\mu'}\hat{\xi}_{\mu'}\,, \qquad \frac{1}{\hbar}[\hat{H}_{\mathrm{m}},\hat{\eta}_a] = \sum_{aa'} c_{aa'}\hat{\eta}_{a'} \tag{39}$$

where $c_{\mu\mu'}$, $c_{aa'}$ are some coefficients. Operators of electromagnetic field $\hat{E}_n^t(x)$, $\hat{B}_n(x)$ and operator $\hat{\varepsilon}(x)$ satisfy these conditions

$$[\hat{H}_{\mathrm{f}},\hat{E}_n^t(x)] = -ic\hbar\,\mathrm{rot}_n\,\hat{B}(x)\,, \qquad [\hat{H}_{\mathrm{f}},\hat{B}_n(x)] = ic\hbar\,\mathrm{rot}_n\,\hat{E}^t(x)\,, \tag{40}$$

$$[\hat{H}_{\mathrm{m}},\hat{\varepsilon}(x)] = 0\,,$$

therefore, relations (39) are valid for all field operators in (38).

In usual kinetic theory nonequilibrium states of quantum system are described by one-particle density matrix $n_{kk'}^{\alpha\alpha'}(t)$

$$n_{kk'}^{\alpha\alpha'}(t) \equiv \mathrm{Sp}\,\rho^{(+)}(t) c_{\alpha k}^{+} c_{\alpha' k'}\,. \tag{41}$$

States, for which parameters

$$\tilde{n}_{kk'}^{\alpha\alpha'}(t) \equiv \mathrm{Sp}\,\rho^{(+)}(t) c_{\alpha k} c_{\alpha' k'}\,, \qquad x_{\alpha k}(t) \equiv \mathrm{Sp}\,\rho^{(+)}(t) c_{\alpha k} \tag{42}$$

are not equal to zero, are considered as states with a broken symmetry. Therefore, $\tilde{n}_{kk'}^{\alpha\alpha'}(t)$ is called an anomalous one-particle density matrix. However, average electromagnetic fields are expressed through $x_{\alpha k}(t)$. Instead of density matrices Wigner distribution functions are widely used (de Groot, S. & Suttorp L., 1972)

$$\mathrm{f}_k^{\alpha\alpha'}(x,t) \equiv \mathrm{Sp}\,\rho^{(+)}(t)\hat{\mathrm{f}}_k^{\alpha\alpha'}(x), \qquad \tilde{\mathrm{f}}_k^{\alpha\alpha'}(x,t) \equiv \mathrm{Sp}\,\rho^{(+)}(t)\hat{\tilde{\mathrm{f}}}_k^{\alpha\alpha'}(x) \tag{43}$$

where

$$\hat{\mathrm{f}}_k^{\alpha\alpha'}(x) = \sum_q c^+_{\alpha,k-q/2} c_{\alpha',k+q/2} e^{iqx}, \qquad \hat{\tilde{\mathrm{f}}}_k^{\alpha\alpha'}(x) = \sum_q c_{\alpha,k+q/2} c_{\alpha',-k+q/2} e^{iqx}. \tag{44}$$

Simple relations between average field, correlations of the field, density matrices and Wigner distribution functions can be established by the formula

$$c_{\alpha k} = (8\pi\omega_k \hbar V)^{-1/2} e^*_{\alpha kn} \int d^3x \{\hat{Z}_n(x)/k - i\hat{E}^t_n(x)\} e^{-ikx}, \qquad \hat{Z}_n(x) \equiv \mathrm{rot}_n \hat{B}(x). \tag{45}$$

Further on kinetics of electromagnetic field in medium consisting of two-level emitters with the Hamilton operator (23) is considered in more detail. According to the general theory (Akhiezer & Peletminskii, 1981), an integral equation for the statistical operator $\rho(\xi,\varepsilon)$ introduced by the functional hypothesis (31) can be obtained (Lyagushyn & Sokolovsky, 2010b)

$$\rho(\xi,\varepsilon) = \rho_{\mathrm{f}}(Z(\xi))\rho_{\mathrm{m}}(X(\varepsilon)) + \int_{-\infty}^{0} d\tau e^{\frac{i}{\hbar}\tau\hat{H}_0} \left\{ \frac{i}{\hbar}[\rho(\xi,\varepsilon), \hat{H}_{\mathrm{mf}}] - \right. \tag{46}$$

$$\left. -\sum_\mu \frac{\partial\rho(\xi,\varepsilon)}{\partial\xi_\mu} M_\mu(\xi,\varepsilon) - \int d^3x \frac{\delta\rho(\xi,\varepsilon)}{\delta\varepsilon(x)} M(x,\xi,\varepsilon) \right\}_{\xi \to e^{-ic\tau}\xi} e^{-\frac{i}{\hbar}\tau\hat{H}_0}$$

where functions $M_\mu(\xi,\varepsilon)$, $M(\xi,\varepsilon)$ are defined as right-hand sides of evolution equations for the reduced description parameters

$$\partial_t \xi_\mu(t) = i\sum_{\mu'} c_{\mu\mu'}\xi_{\mu'}(t) + M_\mu(\xi(t),\varepsilon(t)), \qquad \partial_t\varepsilon(x,t) = M(x,\xi(t),\varepsilon(t)); \tag{47}$$

$$M_\mu(\xi,\varepsilon) \equiv \frac{i}{\hbar}\mathrm{Sp}\,\rho(\xi,\varepsilon)[\hat{H}_{\mathrm{mf}}, \hat{\xi}_\mu], \qquad M(x,\xi,\varepsilon) \equiv \frac{i}{\hbar}\mathrm{Sp}\,\rho(\xi,\varepsilon)[\hat{H}_{\mathrm{mf}}, \hat{\varepsilon}(x)]$$

(see notations in (39)). Quasiequilibrium statistical operator of the emitters

$$\rho_{\mathrm{m}}(X) = w_d(d) w_\sigma(\omega) \exp\{\Omega(X) - \int d^3x X(x)\hat{\varepsilon}(x)\} \tag{48}$$

describes a state of local equilibrium of the emitter medium with temperature $T(x) = X(x)^{-1}$ in the considered case. Function $w_d(d)$ describes distribution of orientations of emitter dipole moments (Lyagushyn et al., 2008). Further it is assumed for simplicity that correlations of dipole orientations are absent and their distribution is isotropic one. Function $w_\sigma(\omega)$ is defined by formulas

$$w_\sigma(\omega) = c(\sigma)\frac{\sigma}{(\omega-\omega_0)^2+\sigma^2}\,, \qquad \int_0^{+\infty} d\omega w_\sigma(\omega) = 1 \qquad (\sigma \ll 1) \tag{49}$$

and phenomenologically accounts for non-resonant interaction between the field and emitters.

The obtained integral equation is solved in perturbation theory in emitter-field interaction $\hat{H}_{\mathrm{mf}} \sim \lambda$ ($\lambda \ll 1$). Important convenience is provided by the structure of $\rho_{\mathrm{f}}(Z(\xi))$ allowing to use the Wick--Bloch--de Dominicis theorem. However, one needs this theorem for calculating contributions of the third and higher orders of the perturbation theory to the statistical operator $\rho(\xi,\varepsilon)$. Averages that are linear and bilinear in the field can be calculated on the basis of relations:

$$\mathrm{Sp}_{\mathrm{f}}\rho_{\mathrm{f}}(Z(\xi))\hat{\xi}_{\alpha n}(x) = \xi_{\alpha n}(x)\,, \tag{50}$$

$$\mathrm{Sp}_{\mathrm{f}}\rho_{\mathrm{f}}(Z(\xi))\hat{\xi}_{\alpha m}(x)\hat{\xi}_{\alpha' n}(x') = (\xi^{x}_{\alpha m}\xi^{x'}_{\alpha' n}) + \xi_{\alpha m}(x)\xi_{\alpha' n}(x') + \frac{1}{2}[\hat{\xi}_{\alpha m}(x),\hat{\xi}_{\alpha' n}(x')]\,.$$

Moreover, according to the general theory of the Peletminskii-Yatsenko model (Akhiezer & Peletminskii, 1981) the same formulas are valid for calculations with the statistical operator $\rho(\xi,\varepsilon)$:

$$\mathrm{Sp}\rho(\xi,\varepsilon)\hat{\xi}_{\alpha n}(x) = \xi_{\alpha n}(x)\,, \tag{51}$$

$$\mathrm{Sp}\rho(\xi,\varepsilon)\hat{\xi}_{\alpha m}(x)\hat{\xi}_{\alpha' n}(x') = (\xi^{x}_{\alpha m}\xi^{x'}_{\alpha' n}) + \xi_{\alpha m}(x)\xi_{\alpha' n}(x') + \frac{1}{2}[\hat{\xi}_{\alpha m}(x),\hat{\xi}_{\alpha' n}(x')]\,.$$

Averages with a quasiequilibrium statistical operator of the medium are calculated by the method developed for spin systems (Lyagushyn et al., 2005). It gives, for example, an expression for energy density of emitter medium via its temperature $T(x)$ and density $n(x)$

$$\varepsilon(x) = -\frac{\hbar\omega}{2}n(x)\mathrm{th}\frac{\hbar\omega}{2T(x)} \qquad (\,n(x) \equiv \sum_{1\le a\le N}\delta(x-x_a)\,). \tag{52}$$

Integral equation (46) solution gives evolution equations for all parameters of the reduced description. Average electric and magnetic fields satisfy the Maxwell equations

$$\partial_t E_n(x,t) = c\,\mathrm{rot}_n\, B(x,t) - 4\pi J_n(x,\xi(t),\varepsilon(t))\,, \qquad \partial_t B_n(x,t) = -c\,\mathrm{rot}_n\, E(x,t) \tag{53}$$

where average current density in terms of the total electric field is given by the relation

$$J_n(x,\xi,\varepsilon) = \int dx'\sigma(x-x',\varepsilon(x))E_n(x') + c\int dx'\chi(x-x',\varepsilon(x))Z_n(x') + O(\lambda^3) \tag{54}$$

$$E_n(x,t) = E^t_n(x,t) + O(\lambda^2)\,, \qquad Z_n(x,t) \equiv \mathrm{rot}_n B(x,t)$$

(for all parameters $A(\xi,\varepsilon) \equiv \mathrm{Sp}\,\rho(\xi,\varepsilon)\hat{A}$). This material equation takes into account spatial dispersion and Fourier transformed functions $\sigma(x,\varepsilon)$, $\chi(x,\varepsilon)$ give conductivity $\sigma(k,\varepsilon)$ and magnetic susceptibility $\chi(k,\varepsilon)$ of the emitter medium

$$\sigma(k,\varepsilon) = -\frac{2\pi}{3}\frac{\varepsilon d^2}{\hbar^2} w_\sigma(\omega_k), \qquad \chi(k,\varepsilon) = -\frac{4}{3}\frac{\varepsilon d^2}{\hbar^2}\int_0^{+\infty} d\omega\, w_\sigma(\omega) \mathrm{P}\frac{1}{\omega^2 - \omega_k^2} . \tag{55}$$

Average density of the dipole moment of emitters is given by expression

$$P_n(x,\varepsilon,\gamma) = \int dx' \kappa(x - x', \varepsilon(x)) E_n(x') + +c\int dx' \alpha(x - x', \varepsilon(x)) Z_n(x') + O(\lambda^3) \tag{56}$$

where

$$\kappa(k,\varepsilon) = \chi(k,\varepsilon), \qquad \alpha(k,\varepsilon) = -\sigma(k,\varepsilon)/\omega_k^2 . \tag{57}$$

Evolution equation for energy density $\varepsilon(x,t)$ of emitters has the form

$$\partial_t \varepsilon(x,t) = L(x, \xi(t), \varepsilon(t)) , \tag{58}$$

$$L(x,\xi,\varepsilon) = \int dx' \sigma(x - x', \varepsilon(x))\{(E_n^x E_n^{x'}) + E_n(x)E_n(x')\} +$$

$$+c\int dx' \chi(x - x', \varepsilon(x))\{(E_n^x B_n^{x'}) + E_n(x)B_n(x')\} + R(n(x)) + O(\lambda^3) .$$

The last term describes dipole radiation of the emitters

$$R(n) \equiv -\frac{2d^2}{3\pi c^3} n \int_0^{+\infty} d\omega \omega^4 w_\sigma(\omega) \tag{59}$$

and for small σ gives a known expression

$$R(n) = -\frac{2d^2\omega_0^4}{3\pi c^3} n . \tag{60}$$

Evolution equations for correlation functions of electromagnetic field in terms of the total electric field can be written in the form

$$\partial_t (E_m^x E_n^{x'})_t = c\,\mathrm{rot}_m (B^x E_n^{x'})_t + c\,\mathrm{rot}'_n (E_m^x B^{x'})_t - 4\pi (J_m^x E_n^{x'})_t - 4\pi (E_m^x J_n^{x'})_t , \tag{61}$$

$$\partial_t (E_m^x B_n^{x'})_t = c\,\mathrm{rot}_m (B^x B_n^{x'})_t - c\,\mathrm{rot}'_n (E_m^x E^{x'})_t - 4\pi (J_m^x B_n^{x'})_t ,$$

$$\partial_t (B_m^x E_n^{x'})_t = -c\,\mathrm{rot}_m (E^x E_n^{x'})_t + c\,\mathrm{rot}'_n (B_m^x B^{x'})_t - 4\pi (B_m^x J_n^{x'})_t ,$$

$$\partial_t (B_m^x B_n^{x'})_t = -c\,\mathrm{rot}_m (E^x B_n^{x'})_t - c\,\mathrm{rot}'_n (B_m^x E^{x'})_t .$$

Current-field correlation functions are defined analogously to (37). Material equations for these correlations are given by expressions in terms of the total electric field

$$(E_m^x J_n^{x'})_t = \int dx'' \sigma(x'-x'', \varepsilon(x',t))(E_m^x E_n^{x''})_t + \tag{62}$$

$$+c\int dx'' \chi(x'-x'', \varepsilon(x',t))(E_m^x Z_n^{x''})_t + S_{mn}(x-x', n(x')) + O(\lambda^3),$$

$$(B_m^x J_n^{x'})_t = \int dx'' \sigma(x'-x'', \varepsilon(x',t))(B_m^x E_n^{x''})_t +$$

$$+c\int dx'' \chi(x'-x'', \varepsilon(x',t))(B_m^x Z_n^{x''})_t + T_{mn}(x-x', n(x')) + O(\lambda^3),$$

where Fourier transformed functions $S_{mn}(x,n)$, $T_{mn}(x,n)$ are given by expressions

$$S_{mn}(k,n) = -\frac{2\pi}{3} d^2 n(\delta_{mn} - \tilde{k}_m \tilde{k}_n)\omega_k^2 w_\sigma(\omega_k), \tag{63}$$

$$T_{mn}(k,n) = \frac{4\pi i}{3} c d^2 n\, e_{mnl} k_l \int_0^{+\infty} d\omega\, w_\sigma(\omega) \mathrm{P}\frac{\omega}{\omega^2 - \omega_k^2}$$

Quantities $S_{mn}(k,n)$, $T_{mn}(k,n)$ determine equilibrium correlations of the electromagnetic field. Comparing relations (54) and (62) shows that the Onsager principle is valid for the considered system.

Hereafter we consider kinetics of electromagnetic field in plasma medium with the Hamiltonian (29) in more detail. We restrict ourselves by considering equilibrium plasma (Sokolovsky & Stupka, 2004) and states of the field described by average fields $E_n^t(x,t)$, $B_n(x,t)$ and one-particle density matrix $n_{kk'}^{\alpha\alpha'}(t)$ defined in (41). The problem for plasma medium in terms of hydrodynamic states has been investigated in (Sokolovsky & Stupka, 2005). Instead of average fields and matrix $n_{kk'}^{\alpha\alpha'}(t)$ one can use average Bose amplitudes $x_{\alpha k}(t)$ defined in (42) and correlation function

$$g_{kk'}^{\alpha\alpha'}(t) = n_{kk'}^{\alpha\alpha'}(t) - x_{\alpha k}^*(t) x_{\alpha' k'}(t). \tag{64}$$

So, for this problem in above notations we have parameters ξ_μ: $n_{kk'}^{\alpha\alpha'}$, $x_{\alpha k}$, $x_{\alpha k}^*$ and corresponding operators $\hat{\xi}_\mu$: $c_{\alpha k}^+ c_{\alpha' k'}$, $c_{\alpha k}$, $c_{\alpha k}^+$. A statistical operator of the system introduced by the functional hypothesis depends in this case only on the field variables and satisfies the integral equation

$$\rho(\xi) = \rho_{\mathrm{f}}(\mathrm{Z}(\xi)) w_{\mathrm{m}} + \int_{-\infty}^{0} d\tau e^{\frac{i}{\hbar}\tau \hat{H}_0} \left\{ \frac{i}{\hbar}[\rho(\xi), \hat{H}_{\mathrm{mf}}] - \sum_\mu \frac{\partial \rho(\xi)}{\partial \xi_\mu} M_\mu(\xi) \right\}_{\xi \to e^{-ic\tau}\xi} e^{-\frac{i}{\hbar}\tau \hat{H}_0}, \tag{65}$$

where quasiequilibrium statistical operator $\rho_{\mathrm{f}}(\mathrm{Z})$ is given by formula (36) with $\tilde{Z}_{kk'}^{\alpha\alpha'} = 0$, w_{m} is a statistical operator of equilibrium plasma

$$w_{\mathrm{m}} = e^{(\Omega - \hat{H}_{\mathrm{m}} + \sum_a \mu_a \hat{N}_a)/T} \qquad (\hat{N}_a \equiv \int dx \hat{n}_a(x)). \tag{66}$$

Functions $M_\mu(\xi)$ define the right-hand sides of evolution equations for the reduced description parameters

$$\partial_t \xi_\mu(t) = i \sum_{\mu'} c_{\mu\mu'} \xi_{\mu'}(t) + M_\mu(\xi(t)), \qquad M_\mu(\xi) \equiv \frac{i}{\hbar} \mathrm{Sp}\, \rho(\xi)[\hat{H}_{\mathrm{mf}}, \hat{\xi}_\mu]. \tag{67}$$

Integral equation (65) is solvable in a perturbation theory in plasma-field interaction based on estimations $\hat{H}_1 \sim \lambda^1$, $\hat{H}_2 \sim \lambda^2$ (see (29)). As a result, evolution equations for the reduced description parameters take the form (Sokolovsky & Stupka, 2004)

$$\partial_t g_{kk'}^{\alpha\alpha'} = i(\Omega_k - \Omega_{k'}) g_{kk'}^{\alpha\alpha'} - (\nu_k + \nu_{k'})(g_{kk'}^{\alpha\alpha'} - n_k \delta_{\alpha\alpha'} \delta_{kk'}) + O(\lambda^3), \tag{68}$$

$$\partial_t x_{\alpha k} = -(i\Omega_k + \nu_k) x_{\alpha k} + (\nu_k + i\omega_k \chi_k) x_{\alpha,-k}^* + O(\lambda^3)$$

where Ω_k is photon spectrum in the plasma, n_k is the Planck distribution with the plasma temperature, ν_k is a frequency of photon emission and absorption. These quantities are given by formulas

$$\Omega_k \equiv \omega_k \{1 - 2\pi\chi(k)\}, \quad \nu_k \equiv 2\pi\sigma(k). \tag{69}$$

The second equation in (68) is a form of the Maxwell equations (53) with similar to (54) material equation

$$J_n(x,\xi) = \int dx' \sigma(x - x') E_n(x') + c \int dx' \chi(x - x') Z_n(x') + O(\lambda^3). \tag{70}$$

This material equation takes into account spatial dispersion and Fourier transformed functions $\sigma(x)$, $\chi(x)$ give conductivity $\sigma(k)$ and magnetic susceptibility $\chi(k)$ of the plasma medium. Their values are given by relations

$$\sigma(k) = -\frac{\mathrm{Im}\, G(k, \omega_k)}{\omega_k}, \qquad \chi(k) = -\frac{\mathrm{Re}\, G(k, \omega_k) + \chi}{c\, \omega_k}, \tag{71}$$

where $G(k,\omega)$ is a transversal part of current-current Green function:

$$G(k,\omega) \equiv \frac{1}{2} G_{mn}(k,\omega)(\delta_{mn} - \tilde{k}_m \tilde{k}_n), \qquad G_{mn}(x,t) = -\frac{i}{\hbar} \theta(t) \mathrm{Sp}_{\mathrm{m}}\, w_{\mathrm{m}} [\hat{j}_m(x,t), \hat{j}_n(0)]; \tag{72}$$

$$\chi \equiv \sum_a \frac{n_a e_a^2}{m_a} \equiv \frac{\Omega^2}{4\pi}.$$

In fact, the obtained results are valid for $kc \gg \Omega$ where Ω is Langmuir frequency.

7. Connection between correlation functions of different nature and some suitable representations for them

One can notice that simultaneous correlation functions of field amplitudes of (37) type arise in a natural way in the framework of the reduced description method. At the same time Glauber correlation functions of (19) type (including positive-frequency and negative-frequency parts of the electric field operator (11) in the interaction picture) seem to be observable quantities from the point of view of experimental possibilities. The most interesting effects of quantum optics can be described with non-simultaneous Glauber functions (Lyagushyn & Sokolovsky, 2010a; Lyagushyn et al., 2011). Nevertheless we can insist that there are no real contradictions between the approaches. Correlation functions (19) characterize properties of electromagnetic field described by the statistical operator ρ. In the previous section we have been constructed a reduced description for electromagnetic field in emitter medium and in plasma medium. These theories lead not only to equations for the reduced description parameters but also to the expression for corresponding nonequilibrium statistical operators. For the field-emitters system a nonequilibrium statistical operator has the form

$$\rho(\xi,\varepsilon)=\rho_{\mathrm{f}}(Z(\xi))\rho_{\mathrm{m}}(X(\varepsilon))-\frac{i}{\hbar}\int\limits_{-\infty}^{0}d\tau\int dx\,[\rho_{\mathrm{f}}(Z(\xi))\rho_{\mathrm{m}}(X(\varepsilon)),\hat{E}_n^t(x,\tau)\hat{P}_n(x,\tau)]+O(\lambda^2)\,. \quad (73)$$

where $\hat{E}_n^t(x,\tau)$, $\hat{P}_l(x,\tau)$ are operators $\hat{E}_n^t(x)$, $\hat{P}_n(x)$ in the interaction picture. Analogously, a nonequilibrium statistical operator for the field-plasma system is given by the formula

$$\rho(\xi)=\rho_{\mathrm{f}}(Z(\xi))w_{\mathrm{m}}-\frac{i}{c\hbar}\int\limits_{-\infty}^{0}d\tau\int dx[\rho_{\mathrm{f}}(Z(\xi))w_{\mathrm{m}},\hat{A}_n(x,\tau)\hat{j}_n(x,\tau)]+O(\lambda^2) \quad (74)$$

where $\hat{A}_n(x,\tau)$, $\hat{j}_n(x,\tau)$ are operators $\hat{A}_n(x)$, $\hat{j}_n(x)$ in the interaction picture. According to general theory of the Peletminskii-Yatsenko model (Akhiezer & Peletminskii, 1981), the following relations for the field-emitters system

$$\mathrm{Sp}\,\rho(\xi,\varepsilon)c_{\alpha k}=\mathrm{Sp}\,\rho_{\mathrm{f}}(Z(\xi))c_{\alpha k}\,,\qquad \mathrm{Sp}\,\rho(\xi,\varepsilon)c_{\alpha k}^{+}c_{\alpha' k'}=\mathrm{Sp}\,\rho_{\mathrm{f}}(Z(\xi))c_{\alpha k}^{+}c_{\alpha' k'}\,, \quad (75)$$

$$\mathrm{Sp}\,\rho(\xi,\varepsilon)c_{\alpha k}c_{\alpha' k'}=\mathrm{Sp}\,\rho_{\mathrm{f}}(Z(\xi))c_{\alpha k}c_{\alpha' k'}$$

and for the field-plasma system

$$\mathrm{Sp}\,\rho(\xi)c_{\alpha k}=\mathrm{Sp}_{\mathrm{f}}\,\rho_{\mathrm{f}}(Z(\xi))c_{\alpha k}\,,\qquad \mathrm{Sp}\,\rho(\xi)c_{\alpha k}^{+}c_{\alpha' k'}=\mathrm{Sp}_{\mathrm{f}}\,\rho_{\mathrm{f}}(Z(\xi))c_{\alpha k}^{+}c_{\alpha' k'} \quad (76)$$

are valid. Average of products of three and more Bose operators should be calculated with taking into account the second term in expressions (73), (74) and using the Wick–Bloch–de Dominicis theorem. It is convenient to perform the calculation of correlation functions (23) for the field-plasma system through using formulas (11), (74). For the field-emitters system the following formula

$$\hat{E}_n^{(+)}(x,t)=i\int dx'\{D_{+}(x-x',t)\hat{Z}_n(x')+\frac{1}{c}\dot{D}_{+}(x-x',t)\hat{E}_n^t(x')\} \quad (77)$$

can be useful. Here $D_+(x,t)$ is a standard function widely used in electromagnetic theory (Akhiezer A. & Berestetsky V., 1969) and defined by expression

$$D_+(x,t) \equiv \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\frac{1}{k}e^{i(kx-\omega_k t)} . \tag{78}$$

Calculation of the simplest correlation function $G_{nl}^{(1,1)}(y_1,y_1')$ can be done according to (75), (76) exactly. For example, for the field-plasma system one has

$$G_{mn}^{(1,1)}(x,t;x',t') = \sum_{k\alpha,k'\alpha'} \frac{2\pi\hbar c}{V}(kk')^{1/2} e^*_{\alpha km} e_{\alpha' k' n} e^{i(\omega_k t - kx)} e^{-i(\omega_{k'} t' - k'x')} n_{kk'}^{\alpha\alpha'} \tag{79}$$

An exact expression for this correlation function of the field-emitters system is given by the formula

$$\begin{aligned} G_{mn}^{(1,1)}(x,t;x',t') = \int dx_1 dx_1' \Big\{ & D_+^*(x-x_1,t) D_+(x'-x_1',t') \langle Z_m^{x_1} Z_n^{x_1'} \rangle + \\ & + \frac{1}{c^2} \dot{D}_+^*(x-x_1,t) \dot{D}_+(x'-x_1',t') \langle E_m^{x_1} E_n^{x_1'} \rangle + \\ & + [D_+^*(x-x_1,t)\dot{D}_+(x'-x_1',t') + \frac{1}{c}\dot{D}_+^*(x-x_1,t) D_+(x'-x_1',t')] \times \\ & \times [\langle E_m^{x_1} Z_n^{x_1'} \rangle + 2i\pi\hbar c(\delta_{mn}\Delta_1 - \frac{\partial^2}{\partial x_{1m}\partial x_{1n}})\delta(x_1 - x_1')] \Big\} . \end{aligned} \tag{80}$$

Correlation function $G_{nl}^{(2,2)}(y_1 y_2, y_1' y_2')$ can be calculated only approximately. For example, for the field-plasma system the formula

$$G_{mn}^{(1,1)}(y_1,y_2;y_1',y_2') = G_{m_1 n_1}^{(1,1)}(y_1,y_1') G_{m_2 n_2}^{(1,1)}(y_2,y_2') + G_{m_1 n_2}^{(1,1)}(y_1,y_2') G_{m_2 n_1}^{(1,1)}(y_2,y_1') + O(\lambda^1) . \tag{81}$$

is obtained.

So, the method of the reduced description of nonequilibrium states allows calculating Glauber correlation functions in important models. It gives possibility to analyze correlation properties of electromagnetic field interacting with emitters and plasma in the considered examples. Such analysis can be performed in terms of average electromagnetic field and binary correlations of the field.

Quantum theory of radiation transfer is an important part of quantum optics (Perina, 1984). The problem is: to choose parameters that describe radiation transfer in a medium and obtain a closed set of equations for such parameters. This problem can be solved in the reduced description method.

In the theory of radiation transfer (Chandrasekhar, 1950) energy fluxes in medium and polarization of the radiation are problems of interest. Operator of energy flux is given by the formula

$$\hat{q}_n(x) = \frac{c}{8\pi}\varepsilon_{nlm}\{\hat{E}_l(x), \hat{B}_m(x)\} . \tag{82}$$

In the developed above theory average values of binary in the field quantities can be calculated exactly. For the field-plasma model the following result can be obtained in terms of the one-particle density matrix and Wigner distribution function

$$q_n(x) = \frac{\hbar c^2}{V}\sum_{kq,\alpha\alpha'} n^{\alpha\alpha'}_{k-q/2,k+q/2}\varphi^{\alpha\alpha'}_n(k,q)e^{iqx} = \frac{\hbar c^2}{V}\sum_{k,\alpha\alpha'}\varphi^{\alpha\alpha'}_n(k,-i\frac{\partial}{\partial x})\mathrm{f}^{\alpha\alpha'}_k(x) \tag{83}$$

where

$$\varphi^{\alpha\alpha'}_n(k,q) \equiv \varphi^{\alpha\alpha'}_n(k-q/2,k+q/2) , \tag{84}$$

$$\varphi^{\alpha_1\alpha_2}_n(k_1,k_2) = \frac{1}{2}(\delta_{nl}\delta_{ms} - \delta_{ml}\delta_{ns})(k_1k_2)^{1/2}\{\tilde{k}_{1l}e^*_{\alpha_1k_1s}e_{\alpha_2k_2m} + \tilde{k}_{2l}e^*_{\alpha_1k_1m}e_{\alpha_2k_2s}\} .$$

For a weakly nonuniform states of the field formula (82) can be simplified and gives (at $V \to \infty$) a classic expression

$$q_n(x) = \sum_\alpha\int\frac{d^3k}{(2\pi)^3}\omega_k\hbar\frac{\partial\omega_k}{\partial k_n}\mathrm{f}^{\alpha\alpha}_k(x) . \tag{85}$$

Formula (83) should be put in the basis of the theory of radiation transfer. The simplest consideration is based on the approximate expression (85). Radiation transfer can be described with specific intensity of radiation in the form

$$I^{\alpha\alpha'}_\omega(n,x) \equiv \frac{\omega^3\hbar}{(2\pi)^3c^2}\mathrm{f}^{\alpha\alpha'}_k(x)\Big|_{k=n\frac{\omega}{c}} \qquad (|n| \equiv 1) \tag{86}$$

Therefore, an equation of radiation transfer can be based on the kinetic equation for the Wigner distribution function of the field. According to definition (43) and equation (68), for weakly nonuniform states in the absence of the average field this kinetic equation is written as follows

$$\partial_t\mathrm{f}^{\alpha\alpha'}_k = -\frac{\partial\Omega_k}{\partial k_n}\frac{\partial\mathrm{f}^{\alpha\alpha'}_k}{\partial x_n} - 2\nu_k(\mathrm{f}^{\alpha\alpha'}_k - n_k\delta_{kk'}\delta_{\alpha\alpha'}) + \frac{1}{4}\frac{\partial^2\nu_k}{\partial k_n\partial k_l}\frac{\partial^2\mathrm{f}^{\alpha\alpha'}_k}{\partial x_n\partial x_l} . \tag{87}$$

The radiation transfer equation follows from the definition (86) and kinetic equation (87)

$$\partial_t I^{\alpha\alpha'}_\omega(n,x) = -c_\omega n_l\frac{\partial I^{\alpha\alpha'}_\omega(n,x)}{\partial x_l} - 2\nu_\omega\{I^{\alpha\alpha'}_\omega(n,x) - I_\omega\delta_{\alpha\alpha'}\} + \{a_\omega n_l n_m + b_\omega\delta_{lm}\}\frac{\partial^2 I^{\alpha\alpha'}_\omega(n,x)}{\partial x_l\partial x_m} , \tag{88}$$

where the notations

$$\frac{\partial\tilde{\omega}_k}{\partial k_l}\Big|_{k=\frac{\omega}{c}n} \equiv c_\omega n_l , \qquad \frac{\partial^2\nu_k}{\partial k_l\partial k_m}\Big|_{k=\frac{\omega}{c}n} \equiv a_\omega n_l n_m + b_\omega\delta_{lm} , \qquad \nu_k\big|_{k=\frac{\omega}{c}n} \equiv \nu_\omega ; \tag{89}$$

$$I_\omega \equiv \frac{\omega^3\hbar}{(2\pi)^3 c^2}\frac{1}{e^{\hbar\omega/T}-1}$$

are introduced. Usually this equation is written for stationary states and given without correction with the last term. So, the reduced description method provides an approach in which it is possible to justify the radiation transfer theory.

In quantum optics functional methods are widely used. Starting point of such methods is a definition of a generating functional (3) for average values calculated with considered statistical operator ρ. This functional gives possibility of calculating all necessary average values

$$\mathrm{Sp}\rho c^+_{\alpha_1 k_1}\ldots c^+_{\alpha_s k_s} c_{\alpha'_1 k'_1}\ldots c_{\alpha'_{s'} k'_{s'}} = (-1)^{s'} \left.\frac{\partial^{s+s'} F(u,u^*)}{\partial u_{\alpha_1 k_1}\ldots\partial u_{\alpha_s k_s}\partial u^*_{\alpha'_1 k'_1}\ldots\partial u^*_{\alpha'_{s'} k'_{s'}}}\right|_{u,u^*=0} . \tag{90}$$

Hence, the generating functional gives complete description of a system and evolution equation for this functional is equivalent to the quantum Liouville equation. Definition (3) shows that the functional obeys the property

$$F(u,u^*)^* = F(-u,-u^*)\cdot \tag{91}$$

Let us suppose that effective photon interaction in a system has the form

$$\hat{H} = \sum_1 \varepsilon_1 c_1^+ c_1 + \sum_{123}\{\Phi(12,3)c_1^+ c_2^+ c_3 + h.c.\} \tag{92}$$

where notations

$$c_i = c_{\alpha_i k_i}, \quad c_i^+ = c^+_{\alpha_i k_i}, \quad \varepsilon_i = \omega_{k_i} c, \quad \Phi(12,3) = \Phi(\alpha_1 k_1, \alpha_2 k_2; \alpha_3 k_3), \quad \sum_i = \sum_{\alpha_i k_i} \tag{93}$$

are introduced. The following evolution equation for $F(u,u^*,t)$ can be easily obtained analogously to (Akhiezer & Peletminskii, 1981) from the Liouville equation

$$i\hbar\partial_t F(u,u^*,t) = \sum_1 \varepsilon_1 (u_1 \frac{\partial}{\partial u_1} - u_1^* \frac{\partial}{\partial u_1^*}) F(u,u^*,t) + \tag{94}$$

$$+\sum_{123}\left\{\Phi(12,3)\left((u_3 - \frac{\partial}{\partial u_3^*})\frac{\partial^2}{\partial u_1 \partial u_2} + (u_1^* - \frac{\partial}{\partial u_1})(u_2^* - \frac{\partial}{\partial u_2})\frac{\partial}{\partial u_3^*}\right) + c.c.\right\} F(u,u^*,t)$$

Instead of the generating functional the Glauber-Sudarshan distribution (Glauber, 1969; Klauder & Sudarshan, 1968)

$$P(z,z^*) = \frac{1}{\pi}\int d^2u F(u,u^*) e^{\sum_{\alpha k}(u^*_{\alpha k} z_{\alpha k} - u_{\alpha k} z^*_{\alpha k})}, \quad F(u,u^*) = \int d^2 z P(z,z^*) e^{\sum_{\alpha k}(u_{\alpha k} z^*_{\alpha k} - u^*_{\alpha k} z_{\alpha k})} \tag{95}$$

is widely used. Formula (95) shows that this distribution is the Fourier transformed generating functional. Note that an evolution equation for the Glauber-Sudarshan distribution can be easily obtained by substituting the second formula in (95) into equation (94). Such evolution equations can be a starting point for constructing the reduced description of a system (Peletminskii, S. & Yatsenko A., 1970). Obtaining the field evolution picture in terms of *P*-function is very attractive from the point of view of analysis of field properties under consideration in quantum optics.

8. Conclusions

Kinetic theory of electromagnetic field in media has choosing a set of parameters describing nonequilibrium states of the field as a starting point with necessity. The minimal set of such parameters includes binary correlations of field amplitudes. The corresponding mathematical apparatus uses different structures of averages: one-particle density matrices, Wigner distribution functions, and conventional simultaneous correlation functions of field operators. All approaches can be connected with each other due to the possibility of expressing the main correlation parameters in various forms. The reduced description method elucidates the construction of kinetic equations in electrodynamics of continuous media (field-plasma, field-emitters systems) and radiation transfer theory. Electromagnetic field properties are discussed in quantum optics in terms of Glauber correlation functions measured in experiments. Theoretical calculation of such functions requires information about the statistical operator of the system under investigation. In the framework of the reduced description method we have succeeded in obtaining the statistical operator of the field in the form that is convenient for calculations in a number of interesting cases.

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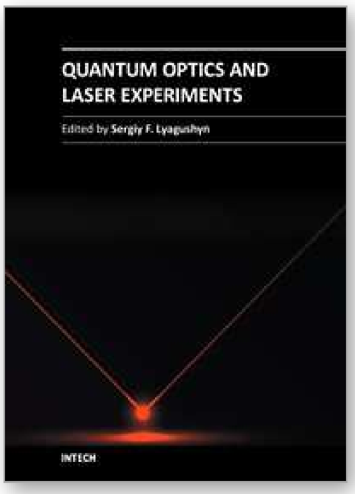

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The book embraces a wide spectrum of problems falling under the concepts of "Quantum optics" and "Laser experiments". These actively developing branches of physics are of great significance both for theoretical understanding of the quantum nature of optical phenomena and for practical applications. The book includes theoretical contributions devoted to such problems as providing a general approach to describe electromagnetic field states with correlation functions of different nature, nonclassical properties of some superpositions of field states in time-varying media, photon localization, mathematical apparatus that is necessary for field state reconstruction on the basis of restricted set of observables, and quantum electrodynamics processes in strong fields provided by pulsed laser beams. Experimental contributions are presented in chapters about some quantum optics processes in photonic crystals - media with spatially modulated dielectric properties - and chapters dealing with the formation of cloud of cold atoms in magneto optical trap. All chapters provide the necessary basic knowledge of the phenomena under discussion and well-explained mathematical calculations.

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