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Modelling of Bound Estimation Laws and Robust Controllers for Robot Manipulators Using Functions and Integration Techniques

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1. Introduction

Some robust control methods have been developed in the past in order to increase tracking performance in the presence of parametric uncertainties. In the presence of parametric uncertainty, unmodelled dynamics and other sources of uncertainties, robust control laws are used. Corless-Leitmann [1] approach is a popular approach used for designing robust controllers for robot manipulators. In early application of Corless-Leitmann [1] approach to robot manipulators [2, 3], it is difficult to compute uncertainty bound precisely. Because, uncertainty bound on parameters depends on the inertia parameters, the reference trajectory and manipulator state vector. Spong [4] proposed a new robust controller for robot manipulators using the Lyapunov theory that guaranties stability of uncertain systems. In this approach, Leithmann [5] or Corless-Leithman [1] approach is used for designing the robust controller. One of the advantage of Spong's approach [4] is that uncertainty on parameter is needed to derive robust controller and uncertainty bound parameters depends only on the inertia parameters of the robots. Yaz [6] proposed a robust control law based on Spong's study [4] and global exponential stability of uncertain system is guaranteed. However, disturbance and unmodelled dynamics are not considered in algorithm of [4, 6]. Danesh at al [7] develop Spong's approach [4] in such a manner that control scheme is made robust not only to uncertain inertia parameters but also to robust unmodelled dynamics and disturbances. Koo and Kim [8] introduce adaptive scheme of uncertainty bound on parameters for robust control of robot manipulators. In [8], upper uncertainty bound is not known as would be in robust controller [4] and uncertainty bound is estimated with estimation law in order to control the uncertain system. A new robust control approach is proposed by Liu and Goldenerg [9] for robot manipulators based on a decomposition of model uncertainty. Parameterized uncertainty is distinguished from unparameterized uncertainty and a compensator is designed for parameterized and unparameterized uncertainty. A decomposition-based control design framework for mechanical systems with model uncertainties is proposed by Liu [10].

In order to increases tracking performance of uncertain systems, design of uncertainty bound estimation functions are considered. For this purpose, some uncertainty bound estimation functions are developed [11-15] based on a Lyapunov function, thus, stability of

uncertain system is guaranteed. In early derivation of uncertainty bound estimation laws [11-13], only a single derivation is possible because selection of variable function is difficult for other derivation and first order differential equation is used. Only exponential function and logarithmic functions are used for derivations because it is diffucut to define variable functions for other derivations.
In previous studies, some robust control laws are introduced, however, a method for derivation of adaptive bound estimation law for robust controllers is not proposed. Recently, a new approach for derivation of bound estimation laws for robust control of robot manipulators is proposed [14, 15]. A general equation is developed based on the Lyapunov theory in order to derive adaptive bound estimation laws and stability of uncertain system is guaranteed. In the approach [15], some functions depending on robot kinematics and control parameters and proper integration techniques can be used for derivation of new bound estimation laws. Then, new bound estimation laws are derived and this derivations also show how the general rule can be used for derivation of different bound estimation laws. After that, four new robust controllers are designed based on each bound estimation law. Lyapunov theory based on Corless-Leitmann [1] approach is used and uniform boundedness error convergence is achieved. This study also shows that bound estimation laws for robust control input do not only include these derivations but also allows derivation of other bound estimation laws for robust controllers provided that appropriate function and proper integration techniques are chosen. In this work, based on the study [15], some appropriate functions are developed and proper integration techniques are chosen. As results, new uncertainty bound estimation laws for robust control input are developed and new robust controllers are proposed. In derivations, some functions and integration techniques are used.

2. A method for derivation of bound estimation laws

In the absence of friction or other disturbances, the dynamic model of an n-link manipulator can be written as [16]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{1}$$

where q denotes generalised coordinates, τ is the n-dimensional vector of applied torques (or forces), $M(q)$ is a positive definite mass matrix, $C(q,\dot{q})\dot{q}$ is the n-dimensional vector of centripetal and Coriolis terms and G(q) is the n-dimensional vector of gravitational terms. Equation (1) can also be expressed in the following form.

$$\tau = Y(q,\dot{q},\ddot{q})\pi \tag{2}$$

where π is a p-dimensional vector of robot inertia parameters and Y is an nxp matrix which is a function of joint position, velocity and acceleration. For any specific trajectory, the desired position, velocity and acceleration vectors are q_d, $\dot{q}_d$ and $\ddot{q}_d$. The measured actual position and velocity errors are $\tilde{q} = q - q_d$, and $\dot{\tilde{q}} = \dot{q} - \dot{q}_d$. Using the above information, the corrected desired velocity and acceleration vectors for nonlinearities and decoupling effects are proposed as:

$$\dot{q}_r = \dot{q}_d - \Lambda\tilde{q}\ ;\ \ddot{q}_r = \ddot{q}_d - \Lambda\dot{\tilde{q}} \tag{3}$$

where Λ is a positive definite matrix. Then the following nominal control law is considered:

$$\begin{aligned}\tau_0 &= M_0(q)\ddot{q}_r + C_0(q,\dot{q})\dot{q}_r + G_0(q) - K\sigma \\ &= Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\pi_0 - K\sigma\end{aligned} \tag{4}$$

where $\pi_0 \in R^p$ represents the fixed parameters in dynamic model and $K\sigma$ is the vector of PD action. The corrected velocity error σ is given as

$$\sigma = q - \dot{q}_r = \dot{\tilde{q}} + \Lambda\tilde{q} \tag{5}$$

The control input τ is defined in terms of the nominal control vector τ_0 as

$$\tau = \tau_0 + Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)u(t) = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)(\pi_0 + u(t)) - K\sigma \tag{6}$$

Where u(t) is the additional robust control input. It is assumed that there exists an unknown bound on parametric uncertainty such that

$$\tilde{\pi} = \pi_0 - \pi \le \rho \tag{7}$$

Since $\rho \in R^{+p}$ is assumed to be unknown, ρ should be estimated with the estimation law to control the system properly. $\hat{\rho}(t)$ shows the estimate of ρ and $\tilde{\rho}(t)$ is

$$\tilde{\rho}(t) = \rho - \hat{\rho}(t) \tag{8}$$

Substituting (6) into (1) and after some algebra yields

$$M(q)\dot{\sigma} + C(q,\dot{q})\sigma + K\sigma = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)(\tilde{\pi} + u(t)) \tag{9}$$

By taking into account above parameters and control algorithm, the Lyapunov function candidate is defined as [15, 16].

$$V(\sigma,q,\tilde{\rho}(t)) = \frac{1}{2}\sigma^T M(q)\sigma + \frac{1}{2}\tilde{q}^T B\tilde{q} + \frac{1}{2}\tilde{\rho}(t)^T \Phi(t)^2 \tilde{\rho}(t) \tag{10}$$

where $B \in R^{nxn}$ is a positive diagonal matrix, $\Phi(t)$ is chosen as a pxp dimensional diagonal matrix changes in time. The time derivative of V along the trajectories is

$$\dot{V} = \sigma^T M(q)\dot{\sigma} + \frac{1}{2}\sigma^T \dot{M}(q)\sigma + \tilde{q}^T B\dot{\tilde{q}} + \tilde{\rho}(t)^T \Phi(t)\dot{\Phi}(t)\tilde{\rho}(t) + \tilde{\rho}(t)^T \Phi(t)^2 \dot{\tilde{\rho}}(t) \tag{11}$$

Taking $B = 2\Lambda K$, using the property $\sigma^T [\dot{M}(q) - 2C(q,\dot{q})]\sigma = 0 \;\; \forall \sigma \in R^n$ [17, 18], and taking time derivative of V of system (9) is

$$\dot{V} = -\dot{\tilde{q}}^T K\dot{\tilde{q}} - \tilde{q}^T \Lambda K \Lambda \tilde{q} + \sigma^T Y u(t) + \sigma^T Y \tilde{\pi} + \tilde{\rho}(t)^T \Phi(t)\dot{\Phi}(t)\tilde{\rho}(t) + \tilde{\rho}(t)^T \Phi(t)^2 \dot{\tilde{\rho}}(t) \tag{12}$$

Equation (12) is arranged as

$$\begin{aligned}\dot{V} = &-\dot{\tilde{q}}^T K\dot{\tilde{q}} - \tilde{q}^T \Lambda K \Lambda\, \tilde{q} + \sigma^T Y u(t) + \sigma^T Y\tilde{\pi} \\ &+ (\rho - \hat{\rho}(t))^T \Phi(t)\dot{\Phi}(t)(\rho - \hat{\rho}(t)) - (\rho - \hat{\rho}(t))^T \Phi(t)^2 \dot{\hat{\rho}}(t) \le 0\end{aligned} \tag{13}$$

$\dot{\tilde{\rho}}(t) = -\dot{\hat{\rho}}(t)$ (since ρ is a constant). Remembering that $\rho \geq \tilde{\pi}$ and if u(t) is taken as the estimated term of uncertainty bound, that is $u(t) = -\hat{\rho}(t)$ then Equation (13 is written as

$$\begin{aligned}\dot{V} = &-\dot{\tilde{q}}^T K \dot{\tilde{q}} - \tilde{q}^T \Lambda K \Lambda \tilde{q} + \sigma^T Y(-\hat{\rho}(t)) + \sigma^T Y \rho \\ &+ [\rho - \hat{\rho}(t)]^T \Phi(t) \dot{\Phi}(t) [\rho - \hat{\rho}(t)] - (\rho - \hat{\rho}(t))^T \Phi(t)^2 \dot{\hat{\rho}}(t)] \leq 0\end{aligned} \tag{14}$$

Equation (14) can be arranged as

$$\begin{aligned}\dot{V} = &-\dot{\tilde{q}}^T K \dot{\tilde{q}} - \tilde{q}^T \Lambda K \Lambda \tilde{q} + \sigma^T Y[\rho - \hat{\rho}(t)] \\ &+ [\rho - \hat{\rho}(t)]^T \Phi(t) \dot{\Phi}(t) [\rho - \hat{\rho}(t)] - (\rho - \hat{\rho}(t))^T \Phi(t)^2 \dot{\hat{\rho}}(t)] \leq 0\end{aligned} \tag{15}$$

Consequently, a suitable expression for the time derivative of V is obtained.

$$\dot{V} = -\dot{\tilde{q}}^T K \dot{\tilde{q}} - \tilde{q}^T \Lambda K \Lambda \tilde{q} + [(\rho - \hat{\rho}(t)]^T [Y^T \sigma + \Phi(t) \dot{\Phi}(t) (\rho - \hat{\rho}(t)) - \Phi(t)^2 \dot{\hat{\rho}}(t)] \leq 0 \tag{16}$$

where $-\dot{\tilde{q}}^T K \dot{\tilde{q}} - \tilde{q}^T \Lambda K \Lambda \tilde{q} \leq 0$. If the rest of Equation (16) is zero, system will be stable. Remaining terms in Equation (16) are

$$[(\rho - \hat{\rho}(t)]^T [Y^T \sigma + \Phi(t) \dot{\Phi}(t) (\rho - \hat{\rho}(t)) - \Phi(t)^2 \dot{\hat{\rho}}(t)] = 0 \tag{17}$$

$[(\rho - \hat{\rho}(t)]$ is considered as a common multiplier then

$$Y^T \sigma + \Phi(t) \dot{\Phi}(t) (\rho - \hat{\rho}(t)) - \Phi(t)^2 \dot{\hat{\rho}}(t) = 0 \tag{18}$$

Hence, we look for the conditions for which the equation

$$Y^T \sigma + \Phi(t) \dot{\Phi}(t) (\rho - \hat{\rho}(t)) - \Phi(t)^2 \dot{\hat{\rho}}(t) = 0$$

is satisfied. Equation (18) can be written as

$$\Phi(t) \dot{\Phi}(t) (\rho - \hat{\rho}(t)) - \Phi(t) \Phi(t) \dot{\hat{\rho}}(t) = -Y^T \sigma \tag{19}$$

Then

$$\Phi(t) \dot{\hat{\rho}}(t) + \dot{\Phi}(t) \hat{\rho}(t) = \Phi(t)^{-1} Y^T \sigma + \dot{\Phi}(t) \rho \tag{20}$$

Equation (20) is arranged as

$$\frac{d}{dt}(\Phi(t) \hat{\rho}(t)) = \Phi(t)^{-1} Y^T \sigma + \dot{\Phi}(t) \rho \tag{21}$$

Integration both side of Equation (21) yields

$$\Phi(t) \hat{\rho}(t) = \int \Phi(t)^{-1} Y^T \sigma dt + \int \dot{\Phi}(t) \rho dt + C \tag{22}$$

Then, a general equation for derivation derivation of bound estimation law is developed as [14, 15]

$$\hat{\rho}(t) = \Phi(t)^{-1} [\int \Phi(t)^{-1} Y^T \sigma dt] + \rho + \Phi(t)^{-1} C \tag{23}$$

The Equation (23) is a general equation for derivation of the bound estimation law and it is derived from Lyapunov function. As a result, $\hat{\rho}(t)$ all derived from Equation (23) guarantess stability of uncertain system. However, $\Phi(t)^{-1}$ and $\hat{\rho}(t)$ are unknown and $\hat{\rho}(t)$ is derived depending on the function $\Phi(t)^{-1}$. For derivation, selection of $\Phi(t)^{-1}$ and integration techniques are very important. There is no certain rule for selection of $\Phi(t)^{-1}$ and integration techniques for this systems. System state parameters and mathematical insight are used to search for appropriate function of $\Phi(t)^{-1}$ as a solution of the Equation (23).

2.1 First choice of $\Phi(t)^{-1}$

For the first derivation of $\hat{\rho}(t)$, $\Phi(t)^{-1}$ is chosen as a time varying function such that

$$\Phi(t)^{-1} = diag(\beta_i \mathrm{e}^{(\alpha_i \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_i} sin(\mathrm{e}^{(\alpha_i \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_i})) \tag{24}$$

Substituting Equation (24) into (23) yields

$$\begin{bmatrix} \hat{\rho}(t)_1 \\ \hat{\rho}(t)_2 \\ \dots\dots \\ \hat{\rho}(t)_p \end{bmatrix} = \Phi(t)^{-1}[\int \begin{bmatrix} \beta_1 \mathrm{e}^{(\alpha_1 \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_1} sin(\mathrm{e}^{(\alpha_1 \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_1})(Y^T\sigma)_1 \\ \beta_2 \mathrm{e}^{(\alpha_2 \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_2} sin(\mathrm{e}^{(\alpha_2 \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_2})(Y^T\sigma)_2 \\ \dots\dots\dots\dots\dots \\ \beta_p \mathrm{e}^{(\alpha_p \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_p} sin(\mathrm{e}^{(\alpha_p \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_p})(Y^T\sigma)_p \end{bmatrix} dt] + \begin{bmatrix} \rho_1 \\ \rho_2 \\ \dots\dots \\ \rho_p \end{bmatrix} + \Phi(t)^{-1} \begin{bmatrix} 1 \\ 1 \\ \dots\dots \\ 1 \end{bmatrix} \tag{25}$$

After integration, the result is

$$\begin{bmatrix} \hat{\rho}(t)_1 \\ \hat{\rho}(t)_2 \\ \dots\dots \\ \hat{\rho}(t)_p \end{bmatrix} = \Phi(t)^{-1} \begin{bmatrix} -(\beta_1 / \alpha_1)cos(\mathrm{e}^{(\alpha_1 \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_1}) \\ -(\beta_2 / \alpha_2)cos(\mathrm{e}^{(\alpha_2 \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_2}) \\ \dots\dots\dots\dots\dots \\ -(\beta_p / \alpha_p)cos(\mathrm{e}^{(\alpha_p \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_p}) \end{bmatrix} + \begin{bmatrix} \rho_1 \\ \rho_2 \\ \dots\dots \\ \rho_p \end{bmatrix} + \Phi(t)C^{-1} \begin{bmatrix} 1 \\ 1 \\ \dots\dots \\ 1 \end{bmatrix} \tag{26}$$

Then

$$\begin{aligned} \begin{bmatrix} \hat{\rho}(t)_1 \\ \hat{\rho}(t)_2 \\ \dots\dots \\ \hat{\rho}(t)_p \end{bmatrix} &= \begin{bmatrix} -(\beta_1^2 / \alpha_1)\mathrm{e}^{(\alpha_1 \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_1} sin(\mathrm{e}^{(\alpha_1 \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_1})cos(\mathrm{e}^{(\alpha_1 \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_1}) \\ -(\beta_2^2 / \alpha_2)\mathrm{e}^{(\alpha_2 \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_2} sin(\mathrm{e}^{(\alpha_2 \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_2})cos(\mathrm{e}^{(\alpha_2 \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_2}) \\ \dots\dots\dots\dots\dots \\ -(\beta_p^2 / \alpha_p)\mathrm{e}^{(\alpha_p \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_p} sin(\mathrm{e}^{(\alpha_p \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_p})cos(\mathrm{e}^{(\alpha_p \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_p}) \end{bmatrix} + \begin{bmatrix} \rho_1 \\ \rho_2 \\ \dots\dots \\ \rho_p \end{bmatrix} \\ &+ C \begin{bmatrix} \beta_1 \mathrm{e}^{(\alpha_1 \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_1} sin(\mathrm{e}^{(\alpha_1 \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_1}) \\ \beta_2 \mathrm{e}^{(\alpha_2 \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_2} sin(\mathrm{e}^{(\alpha_2 \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_2}) \\ \dots\dots \\ \beta_p \mathrm{e}^{(\alpha_p \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_p} sin(\mathrm{e}^{(\alpha_p \int \mathrm{Y}^{\mathrm{T}}\sigma \mathrm{dt})_p}) \end{bmatrix} \end{aligned} \tag{27}$$

If $\hat{\rho}(0)=\rho$ is taken as initial condition, constant C is equivalent to Cos(1). So, the estimation law for the uncertainty bound is derived as.

$$\begin{bmatrix}\hat{\rho}(t)_1\\ \hat{\rho}(t)_2\\ \cdots\\ \hat{\rho}(t)_p\end{bmatrix}=\begin{bmatrix}-(\beta_1^2/\alpha_1)e^{(\alpha_1\int Y^T\sigma dt)_1}sin(e^{(\alpha_1\int Y^T\sigma dt)_1})cos(e^{(\alpha_1\int Y^T\sigma dt)_1})\\ -(\beta_2^2/\alpha_2)e^{(\alpha_2\int Y^T\sigma dt)_2}sin(e^{(\alpha_2\int Y^T\sigma dt)_2})cos(e^{(\alpha_2\int Y^T\sigma dt)_2})\\ \cdots\cdots\\ -(\beta_p^2/\alpha_p)e^{(\alpha_p\int Y^T\sigma dt)_p}sin(e^{(\alpha_p\int Y^T\sigma dt)_p})cos(e^{(\alpha_p\int Y^T\sigma dt)_p})\end{bmatrix}+\begin{bmatrix}\rho_1\\ \rho_2\\ \cdots\\ \rho_p\end{bmatrix}$$
$$+Cos(1)\begin{bmatrix}\beta_1 e^{(\alpha_1\int Y^T\sigma dt)_1}sin(e^{(\alpha_1\int Y^T\sigma dt)_1})\\ \beta_2 e^{(\alpha_2\int Y^T\sigma dt)_2}sin(e^{(\alpha_2\int Y^T\sigma dt)_2})\\ \cdots\\ \beta_p e^{(\alpha_p\int Y^T\sigma dt)_p}sin(e^{(\alpha_p\int Y^T\sigma dt)_p})\end{bmatrix} \quad (28)$$

2.2 Second choice of $\Phi(t)^{-1}$

For the second derivation of $\hat{\rho}(t)$, $\Phi(t)^{-1}$ is defined as

$$\Phi(t)^{-1}=diag(\beta_i\frac{e^{\int(\alpha_i Y^T\sigma dt)_i}}{1+e^{(\int 2\alpha_i Y^T\sigma dt)_i}}) \quad (29)$$

Substituting Equation (29) into (23) yields

$$\begin{bmatrix}\hat{\rho}(t)_1\\ \hat{\rho}(t)_2\\ \cdots\\ \hat{\rho}(t)_p\end{bmatrix}=\Phi(t)^{-1}[\int\begin{bmatrix}\beta_1\frac{e^{\int(\alpha_1 Y^T\sigma dt)_1}}{1+e^{(2\alpha_1\int Y^T\sigma dt)_1}}(Y^T\sigma)_1\\ \beta_2\frac{e^{\int(\alpha_2 Y^T\sigma dt)_2}}{1+e^{(2\alpha_2\int Y^T\sigma dt)_2}}(Y^T\sigma)_2\\ \cdots\cdots\\ \beta_p\frac{e^{\int(\alpha_p Y^T\sigma dt)_p}}{1+e^{(2\alpha_p\int Y^T\sigma dt)_p}}(Y^T\sigma)_p\end{bmatrix}dt]+\begin{bmatrix}\rho_1\\ \rho_2\\ \cdots\\ \rho_p\end{bmatrix}+\Phi(t)^{-1}C\begin{bmatrix}1\\ 1\\ \cdots\\ 1\end{bmatrix} \quad (30)$$

After integration, the result is

$$\begin{bmatrix}\hat{\rho}(t)_1\\ \hat{\rho}(t)_2\\ \cdots\\ \hat{\rho}(t)_p\end{bmatrix}=\Phi(t)^{-1}\begin{bmatrix}(\beta_1/\alpha_1)arctan(e^{\int(\alpha_1 Y^T\sigma dt)_1})\\ (\beta_2/\alpha_2)arctan(e^{\int(\alpha_2 Y^T\sigma dt)_2})\\ \cdots\cdots\\ (\beta_p/\alpha_p)arctan(e^{\int(\alpha_p Y^T\sigma dt)_p})\end{bmatrix}+\Phi(t)\begin{bmatrix}\rho_1\\ \rho_2\\ \cdots\\ \rho_p\end{bmatrix}+C\begin{bmatrix}1\\ 1\\ \cdots\\ 1\end{bmatrix} \quad (31)$$

After multiplication by $\Phi(t)^{-1}$, the result will be

$$\begin{bmatrix}\hat{\rho}(t)_1\\ \hat{\rho}(t)_2\\ .....\\ \hat{\rho}(t)_p\end{bmatrix}=\begin{bmatrix}(\beta_1^2/\alpha_1)\dfrac{e^{\int(\alpha_1 Y^T\sigma dt)_1}}{1+e^{(2\alpha_1\int Y^T\sigma dt)_1}}arctan(e^{\int(\alpha_1 Y^T\sigma dt)_1})\\ (\beta_2^2/\alpha_2)\dfrac{e^{\int(\alpha_2 Y^T\sigma dt)_2}}{1+e^{(2\alpha_2\int Y^T\sigma dt)_2}}arctan(e^{\int(\alpha_2 Y^T\sigma dt)_2})\\ ....\\ (\beta_p^2/\alpha_p)\dfrac{e^{\int(\alpha_p Y^T\sigma dt)_p}}{1+e^{(2\alpha_p\int Y^T\sigma dt)_p}}arctan(e^{\int(\alpha_p Y^T\sigma dt)_p})\end{bmatrix}+\begin{bmatrix}\rho_1\\ \rho_2\\ .....\\ \rho_p\end{bmatrix}+C\begin{bmatrix}\beta_1\dfrac{e^{\int(\alpha_1 Y^T\sigma dt)_1}}{1+e^{(2\alpha_1\int Y^T\sigma dt)_1}}\\ \beta_2\dfrac{e^{\int(\alpha_2 Y^T\sigma dt)_2}}{1+e^{(2\alpha_2\int Y^T\sigma dt)_2}}\\ .....\\ \beta_p\dfrac{e^{\int(\alpha_p Y^T\sigma dt)_p}}{1+e^{(2\alpha_p\int Y^T\sigma dt)_p}}\end{bmatrix} \quad (32)$$

If $\hat{\rho}(0)=\rho$ is taken as initial condition, constant C is equivalent to -arctan(1). So, the estimation law for the uncertainty bound is derived as.

$$\begin{bmatrix}\hat{\rho}(t)_1\\ \hat{\rho}(t)_2\\ .....\\ \hat{\rho}(t)_p\end{bmatrix}=\begin{bmatrix}(\beta_1^2/\alpha_1)\dfrac{e^{\int(\alpha_1 Y^T\sigma dt)_1}}{1+e^{(2\alpha_1\int Y^T\sigma dt)_1}}arctan(e^{\int(\alpha_1 Y^T\sigma dt)_1})\\ (\beta_2^2/\alpha_2)\dfrac{e^{\int(\alpha_2 Y^T\sigma dt)_2}}{1+e^{(2\alpha_2\int Y^T\sigma dt)_2}}arctan(e^{\int(\alpha_2 Y^T\sigma dt)_2})\\ ....\\ (\beta_p^2/\alpha_p)\dfrac{e^{\int(\alpha_p Y^T\sigma dt)_p}}{1+e^{(2\alpha_p\int Y^T\sigma dt)_p}}arctan(e^{\int(\alpha_p Y^T\sigma dt)_p})\end{bmatrix}+\begin{bmatrix}\rho_1\\ \rho_2\\ .....\\ \rho_p\end{bmatrix}-arctan(1)\begin{bmatrix}\beta_1\dfrac{e^{\int(\alpha_1 Y^T\sigma dt)_1}}{1+e^{(2\alpha_1\int Y^T\sigma dt)_1}}\\ \beta_2\dfrac{e^{\int(\alpha_2 Y^T\sigma dt)_2}}{1+e^{(2\alpha_2\int Y^T\sigma dt)_2}}\\ .....\\ \beta_p\dfrac{e^{\int(\alpha_p Y^T\sigma dt)_p}}{1+e^{(2\alpha_p\int Y^T\sigma dt)_p}}\end{bmatrix} \quad (33)$$

2.3 Third choice of $\Phi(t)^{-1}$

For the third derivation of $\hat{\rho}(t)$, $\Phi(t)^{-1}$ is defined as

$$\Phi(t)^{-1}=diag(\beta_i \mathrm{Sin}^2(\alpha_i\int Y^T\sigma dt)_i \mathrm{Cos}(\alpha_i\int Y^T\sigma dt)_i) \quad (34)$$

Substitution of Equation (34) into Equation (23) yields

$$\begin{bmatrix}\hat{\rho}(t)_1\\ \hat{\rho}(t)_2\\ .....\\ \hat{\rho}(t)_p\end{bmatrix}=\Phi(t)^{-1}[\int\begin{bmatrix}\beta_1\mathrm{Sin}^2(\alpha_1\int Y^T\sigma dt)_1\mathrm{Cos}(\alpha_1\int Y^T\sigma dt)_1(Y^T\sigma)_1\\ \beta_2\mathrm{Sin}^2(\alpha_2\int Y^T\sigma dt)_2\mathrm{Cos}(\alpha_2\int Y^T\sigma dt)_2(Y^T\sigma)_2\\ ....\\ \beta_p\mathrm{Sin}^2(\alpha_p\int Y^T\sigma dt)_p\mathrm{Cos}(\alpha_p\int Y^T\sigma dt)_p(Y^T\sigma)_p\end{bmatrix}dt]+\begin{bmatrix}\rho_1\\ \rho_2\\ .....\\ \rho_p\end{bmatrix}$$
$$+\Phi(t)^{-1}C\begin{bmatrix}1\\ 1\\ .....\\ 1\end{bmatrix} \quad (35)$$

After integration, the result is

$$\begin{bmatrix} \hat{\rho}(t)_1 \\ \hat{\rho}(t)_2 \\ \cdots\cdots \\ \hat{\rho}(t)_p \end{bmatrix} = \Phi(t)^{-1} \begin{bmatrix} (\beta_1 / \alpha_1) \dfrac{\mathrm{Sin}^3(\alpha_1 \int \mathrm{Y}^{\mathrm{T}} \sigma \mathrm{dt})_1}{3} \\ (\beta_2 / \alpha_2) \dfrac{\mathrm{Sin}^3(\alpha_2 \int \mathrm{Y}^{\mathrm{T}} \sigma \mathrm{dt})_2}{3} \\ \cdots\cdots\cdots \\ (\beta_p / \alpha_p) \dfrac{\mathrm{Sin}^3(\alpha_p \int \mathrm{Y}^{\mathrm{T}} \sigma \mathrm{dt})_p}{3} \end{bmatrix} + \begin{bmatrix} \rho_1 \\ \rho_2 \\ \cdots\cdots \\ \rho_p \end{bmatrix} + \Phi(t)^{-1} C \begin{bmatrix} 1 \\ 1 \\ \cdots\cdots \\ 1 \end{bmatrix} \tag{36}$$

If $\hat{\rho}(0) = \rho$ is taken as initial condition, constant C is equivalent to zero. So, the estimation law for the uncertainty bound is derived as .

$$\begin{bmatrix} \hat{\rho}(t)_1 \\ \hat{\rho}(t)_2 \\ \cdots\cdots \\ \hat{\rho}(t)_p \end{bmatrix} = \begin{bmatrix} (\beta_1^2 / \alpha_1) \dfrac{\mathrm{Sin}^5(\alpha_1 \int \mathrm{Y}^{\mathrm{T}} \sigma \mathrm{dt})_1 \mathrm{Cos}(\alpha_1 \int \mathrm{Y}^{\mathrm{T}} \sigma \mathrm{dt})_1}{3} \\ (\beta_2^2 / \alpha_2) \dfrac{\mathrm{Sin}^5(\alpha_2 \int \mathrm{Y}^{\mathrm{T}} \sigma \mathrm{dt})_2 \mathrm{Cos}(\alpha_2 \int \mathrm{Y}^{\mathrm{T}} \sigma \mathrm{dt})_2}{3} \\ \cdots\cdots\cdots \\ (\beta_p^2 / \alpha_p) \dfrac{\mathrm{Sin}^5(\alpha_p \int \mathrm{Y}^{\mathrm{T}} \sigma \mathrm{dt})_p \mathrm{Cos}(\alpha_p \int \mathrm{Y}^{\mathrm{T}} \sigma \mathrm{dt})_p}{3} \end{bmatrix} + \begin{bmatrix} \rho_1 \\ \rho_2 \\ \cdots\cdots \\ \rho_p \end{bmatrix} \tag{37}$$

If we substitute Φ, $\dot{\Phi}$, and $\dot{\hat{\rho}}(t)$ into Equation (16), the right terms of Equation (16) $[(\rho\text{-}\hat{\rho}(t)]^{\mathrm{T}}[\mathrm{Y}^{\mathrm{T}}\sigma + \Phi(t)\dot{\Phi}(t)(\rho - \hat{\rho}(t)) - \Phi(t)^2\dot{\hat{\rho}}(t)] = 0$ will be always zero and the derivation of the Lyapunov function will become a negative semidefinite function such that

$$\dot{\mathrm{V}} = -\dot{\tilde{\mathrm{q}}}^{\mathrm{T}}\mathrm{K}\dot{\tilde{\mathrm{q}}} - \tilde{\mathrm{q}}^{\mathrm{T}}\Lambda\mathrm{K}\Lambda\tilde{\mathrm{q}} \leq 0 \tag{38}$$

So, the system is stable for all $\hat{\rho}(t)$ derived from Equation (23).

3. Design of robust contol laws

Based on the uncertainty bound estimation laws derived in section 2, and in [15], it is possible to develop robust control inputs.

3.1 Robust control law 1

In order to define first robust control input, the following theorem is proposed.

Theorem:

Additional control input in control law (6) is

$$(u(t))_i = \begin{cases} -\dfrac{(Y^T\sigma)_i}{\left|(Y^T\sigma)_i\right|}\hat{\rho}(t)_i & if \quad \left|(Y^T\sigma)_i\right| > \varepsilon_i \\ -\dfrac{(Y^T\sigma)_i}{\varepsilon_i}\hat{\rho}(t)_i & if \quad \left|(Y^T\sigma)_i\right| \leq \varepsilon_i \end{cases} \tag{39}$$

Where ε>0. If the control input (39) is substituted into the control law (6) for the control of the model manipulator, then, the control law (6) is continuous and the closed-loop system is uniformly ultimate bounded.

Proof

It is assumed that there exists an unknown bound on parametric uncertainty such that

$$\pi_0 - \pi \le \rho \quad \text{and} \quad \|\pi_0 - \pi\| \le \delta \tag{40}$$

If Φ, $\hat{\rho}(t)$, $\dot{\Phi}$ and $\dot{\hat{\rho}}(t)$ are substituted into (13), the time derivative of the Lyapunov function (13) is written as [14, 15].

$$\begin{aligned}\dot{V} &= -\dot{\tilde{q}}^T K\dot{\tilde{q}} - \tilde{q}^T \Lambda K \Lambda \tilde{q} + \sigma^T Y u(t) + \sigma^T Y \tilde{\pi} + \sigma^T Y \tilde{\rho} \\ &= -\dot{\tilde{q}}^T K\dot{\tilde{q}} - \tilde{q}^T \Lambda K \Lambda \tilde{q} + \sigma^T Y u(t) + \sigma^T Y \tilde{\pi} - \sigma^T Y(\rho - \hat{\rho}(t) \\ &=\le -x^T Q x + \sigma^T Y u(t) + \sigma^T Y \hat{\rho}(t)\end{aligned} \tag{41}$$

Where $x^T = [\tilde{q}^T, \dot{\tilde{q}}^T]$ and Q=diag[$\Lambda^T K \Lambda$, K]. Based on the Leitman [1], we can show that $\dot{V} \le 0$ for ||x||>w where

$$w^2 = \hat{\rho}(t)/2\lambda_{\min}(Q) \tag{42}$$

Where $\lambda_{min}(Q)$ denotes the minimum eigenvalue of Q. Second term in Equation (41), if $||Y^T\sigma||>\varepsilon$ then

$$\begin{aligned}&\le -x^T Q x + \sigma^T Y \hat{\rho}(t) - \sigma^T Y \begin{bmatrix} \dfrac{(Y^T\sigma)_1}{|(Y^T\sigma)_1|}\hat{\rho}(t)_1 \\ \dots\dots\dots\dots\dots \\ \dfrac{(Y^T\sigma)_p}{|(Y^T\sigma)_p|} + \hat{\rho}(t)_p \end{bmatrix} \\ &\le -x^T Q x + \sigma^T Y(\hat{\rho}(t)) - \hat{\rho}(t)) \le 0\end{aligned} \tag{43}$$

From the Cauchy-Schawartz inequality and our assumption on . If $||Y^T \sigma||<\varepsilon$ then

$$\begin{aligned}\sigma^T Y u(t) + \sigma^T Y(\hat{\rho}(t)) &\le (Y^T\sigma)^T \left(\|\hat{\rho}(t)\| \frac{Y^T\sigma}{\|Y^T\sigma\|} + u(t) \right) \\ &\le (Y^T\sigma)^T \left(\|\hat{\rho}(t)\| \frac{Y^T\sigma}{\|Y^T\sigma\|} - \frac{Y^T\sigma}{\varepsilon}\|\hat{\rho}(t)\| \right)\end{aligned} \tag{44}$$

This last term achieves a maximum value of $\varepsilon\|\hat{\rho}(t)\|/4$ when $||Y^T \sigma||=\varepsilon/2$. We have that

$$\dot{V} \le -x^T Q x + \varepsilon\|\hat{\rho}(t)\|/4 \tag{45}$$

Note that $\hat{\rho}(t)$ is bounded. The rest of the proof can be seen in [4, 8].

3.2 Robust control law 2

Based on the bound estimation law $\hat{\rho}(t)$ derived from general Equation (23), additional control input u(t) are defined [15]. The additional control input in control law (6) is defined as [15]

$$\mathrm{u(t)} = \begin{bmatrix} -\hat{\rho}(t)_1 \operatorname{sgn}(\mathrm{Y}^{\mathrm{T}}\sigma)_1 \\ -\hat{\rho}(t)_2 \operatorname{sgn}(\mathrm{Y}^{\mathrm{T}}\sigma)_2 \\ \dots\dots\dots \\ -\hat{\rho}(t)_p \operatorname{sgn}(\mathrm{Y}^{\mathrm{T}}\sigma)_p \end{bmatrix} \tag{46}$$

3.3 Robust control law 3

υ_{i} denote the ith component of the vector $\mathrm{Y}^{\mathrm{T}}\sigma$, ε_{i} choose as the ith component of ε. Then, considering the $\hat{\rho}(t)$ derived from Equation (23), u(t) for each $\hat{\rho}(t)$ is defined as as follows:
For Equation (28), $\mathrm{u(t)_i}$ is

$$\mathrm{u(t)_i} = \begin{cases} -\dfrac{\upsilon_{\mathrm{i}}}{|\upsilon_{\mathrm{i}}|}\,[-(\beta_i^2/\alpha_{\mathrm{i}})\mathrm{e}^{\alpha_i\left|\int \mathrm{v_i dt}\right|}\, sin(\mathrm{e}^{\alpha_i\left|\int \mathrm{v_i dt}\right|})cos(\mathrm{e}^{\alpha_i\left|\int \mathrm{v_i dt}\right|} + Cos(1)\mathrm{e}^{\alpha_i\left|\int \mathrm{v_i dt}\right|}\, sin(\mathrm{e}^{\alpha_i\left|\int \mathrm{v_i dt}\right|}) + \rho_{\mathrm{i}}] & \text{if } |\upsilon_{\mathrm{i}}| > \varepsilon_{\mathrm{i}} \\ -\dfrac{\upsilon_{\mathrm{i}}}{\varepsilon_{\mathrm{i}}}[-(\beta_i^2/\alpha_{\mathrm{i}})\mathrm{e}^{\alpha_i\left|\int \mathrm{v_i dt}\right|}\, sin(\mathrm{e}^{\alpha_i\left|\int \mathrm{v_i dt}\right|})cos(\mathrm{e}^{\alpha_i\left|\int \mathrm{v_i dt}\right|} + Cos(1)\mathrm{e}^{\alpha_i\left|\int \mathrm{v_i dt}\right|}\, sin(\mathrm{e}^{\alpha_i\left|\int \mathrm{v_i dt}\right|}) + \rho_{\mathrm{i}}] & \text{if } |\upsilon_{\mathrm{i}}| \le \varepsilon_{\mathrm{i}} \end{cases} \tag{47}$$

For Equation (33),

$$\mathrm{u(t)_i} = \begin{cases} -\dfrac{\upsilon_{\mathrm{i}}}{|\upsilon_{\mathrm{i}}|}\,[(\beta_{\mathrm{i}}^2/\alpha_i)\dfrac{\mathrm{e}^{\alpha_i\left|\int \mathrm{v_i dt}\right|}}{1+\mathrm{e}^{2\alpha_i\left|\int \mathrm{v_i dt}\right|}} arctan(\mathrm{e}^{\alpha_i\left|\int \mathrm{v_i dt}\right|}) - arctan(1)\dfrac{\mathrm{e}^{\alpha_i\left|\int \mathrm{v_i dt}\right|}}{1+\mathrm{e}^{2\left|\int \mathrm{v_i dt}\right|}} + \rho_{\mathrm{i}})] & \text{if } |\upsilon_{\mathrm{i}}| > \varepsilon_{\mathrm{i}} \\ -\dfrac{\upsilon_{\mathrm{i}}}{\varepsilon_{\mathrm{i}}}\,[(\beta_{\mathrm{i}}^2/\alpha_i)\dfrac{\mathrm{e}^{\alpha_i\left|\int \mathrm{v_i dt}\right|}}{1+\mathrm{e}^{2\alpha_i\left|\int \mathrm{v_i dt}\right|}} arctan(\mathrm{e}^{\alpha_i\left|\int \mathrm{v_i dt}\right|}) - arctan(1)\dfrac{\mathrm{e}^{\alpha_i\left|\int \mathrm{v_i dt}\right|}}{1+\mathrm{e}^{2\left|\int \mathrm{v_i dt}\right|}} + \rho_{\mathrm{i}})] & \text{if } |\upsilon_{\mathrm{i}}| \le \varepsilon_{\mathrm{i}} \end{cases} \tag{48}$$

For Equation (36):

$$\mathrm{u(t)_i} = \begin{cases} -\dfrac{\upsilon_{\mathrm{i}}}{|\upsilon_{\mathrm{i}}|}\,[(\beta_{\mathrm{i}}^2/\alpha_i)\dfrac{sin^5(\alpha_{\mathrm{i}}\left|\int \mathrm{v_i dt}\right|)Cos(\alpha_{\mathrm{i}}\left|\int \mathrm{v_i dt}\right|)}{3} + \rho_{\mathrm{i}}] & \text{if } |\upsilon_{\mathrm{i}}| > \varepsilon_{\mathrm{i}} \\ -\dfrac{\upsilon_{\mathrm{i}}}{\varepsilon_{\mathrm{i}}}\,[(\beta_{\mathrm{i}}^2/\alpha_i)\dfrac{sin^5(\alpha_{\mathrm{i}}\left|\int \mathrm{v_i dt}\right|)Cos(\alpha_{\mathrm{i}}\left|\int \mathrm{v_i dt}\right|)}{3} + \rho_{\mathrm{i}}] & \text{if } |\upsilon_{\mathrm{i}}| \le \varepsilon_{\mathrm{i}} \end{cases} \tag{49}$$

3.4 Robust control law 4

From Equations (43) and (44), it is ease to define the following control law.

$$u(t) = \begin{cases} -\dfrac{Y^T\sigma}{\left\|(Y^T\sigma)\right\|}\left\|\hat{\rho}(t)\right\| & if \ \left\|(Y^T\sigma)\right\| > \varepsilon \\ -\dfrac{Y^T\sigma}{\varepsilon}\left\|\hat{\rho}(t)\right\| & if \ \left\|(Y^T\sigma)\right\| \le \varepsilon \end{cases} \tag{50}$$

4. Simulation results

For illustration, a two-link robot manipulator is given in Figure 1 [4]. Parameterisation of this robot is given by

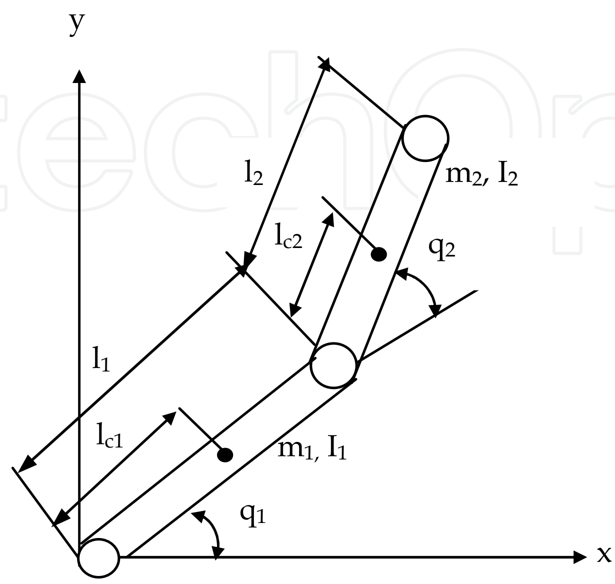


Fig. 1. Two-link planar robot [4].

$$\pi_1=m_1 l_{c1}^2+m_2 l_1^2+I_1, \qquad \pi_2= m_2 l_{c2}^2+I_2, \qquad \pi_3=m_2 l_1 l_{c2},$$

$$\pi_4=m_1 l_{c1}, \qquad \pi_5=m_2 l_1, \qquad \pi_6=m_2 l_{c2}, \tag{51}$$

With this parameterisation, the component y_{ij} of $Y(q,\dot{q},\ddot{q})$ in Equation 2 are given as

$$y_{11} = \ddot{q}_1\,; \quad y_{12} = \ddot{q}_1 + \ddot{q}_2\,; \quad y_{13} = \cos(q_2)(2\ddot{q}_1 + \ddot{q}_2) - \sin(q_2)(\dot{q}_2^{\,2} + 2\dot{q}_1\dot{q}_2)\,;$$

$$y_{14}= g_c\cos(q_1); \quad y_{15}= g_c\cos(q_1); \quad y_{16}= g_c\cos(q_1+q_2)\,;$$

$$y_{21}=0; \quad y_{22} = \ddot{q}_1 + \ddot{q}_2\,; \quad y_{23} = \cos(q_2)\ddot{q}_1 + \sin(q_2)(\dot{q}_1^{\,2})\,;$$

$$y_{24}=0\,; \quad y_{25}=0\,; \quad y_{26}= g_c\cos(q_1+q_2). \tag{52}$$

$Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)$ in Equation (4) have the components

$$y_{11} = \ddot{q}_{r1}\,;\ y_{12} = \ddot{q}_{r1} + \ddot{q}_{r2}\,;$$

$$y_{13} = \cos(q_2)(2\ddot{q}_{r1} + \ddot{q}_{r2}) - \sin(q_2)(\dot{q}_1\dot{q}_{r2} + \dot{q}_1\dot{q}_{r2} + \dot{q}_2\dot{q}_{r2})\,;$$

$$y_{14}=g_c\cos(q_1); \quad y_{15}= g_c\cos(q_1)\,; \quad y_{16}= g_c\cos(q_1+q_2)$$

$$y_{21}=0; \quad y_{22} = \ddot{q}_{r1} + \ddot{q}_{r2}\,; \quad ;\ y_{23} = \cos(q_2)\ddot{q}_{r1} + \sin(q_2)(\dot{q}_1\dot{q}_{r1})$$

$$y_{24}=0\,; \quad y_{25}=0\,; \quad y_{26}= g_c\cos(q_1+q_2). \tag{53}$$

For illustrated purposes let us assume that the parameters of the unloaded manipulator are known and are given by Table 1. Using these values in Table 1, the ith component of π obtained by means of Equation (51) are given in Table 2. It is assumed that the parameters m_2, l_{c2} and I_2 are changed in the intervals

$$0 \le \Delta m_2 \le 10 \ ; \quad 0 \le \Delta l_{c2} \le 0.5 \ ; \quad 0 \le I_2 \le \frac{15}{12} \tag{54}$$

Choosing the mean value for the range of possible π_i in Equation (54) yields the nominal parameter vector and the computed values for ith component of π_0 is shown in Table 3 [4].

m_1	m_2	l_1	l_2	l_{c1}	l_{c2}	I_1	I_2
10	5	1	1	0.5	0.5	10/12	5/12

Table 1. Parameters of the unloaded arm [4].

π_1	π_2	π_3	π_4	π_5	π_6
8.33	1.67	2.5	5	5	2.5

Table 2. π_i for the unloaded arm [4]

π_{01}	π_{02}	π_{03}	π_{04}	π_{05}	π_{06}
13.33	8.96	8.75	5	10	8.75

Table 3. Nominal parameter vector π_0 [4].

With this choice of nominal parameter vector π_0 and uncertainty range given by (54), it is an easy matter to calculate the uncertainty bound ρ as follows:

$$\|\tilde{\pi}\|^2 = \sum_{i=1}^{6} (\pi_{i0} - \pi_i)^2 \le 181.26 \tag{55}$$

and thus $\delta = \sqrt{181.26} = 13.46$.
For explanation, Spong's algorithms are given.

$$u(t) = \begin{cases} -\delta \dfrac{Y^T \sigma}{\left\|Y^T \sigma\right\|} & \text{if} \quad \left\|Y^T \sigma\right\| > \varepsilon \\ -\delta \dfrac{Y^T \sigma}{\varepsilon} & \text{if} \quad \left\|Y^T \sigma\right\| \le \varepsilon \end{cases} \tag{56}$$

As a measure of parameter uncertainty on which the additional control input is based, ρ can be defined as

$$\delta = \left(\sum_{i=1}^{p} \rho_i^{\,2} \right)^{1/2} \tag{57}$$

Having a single number ρ to measure the parameter uncertainty may lead to overly conservative design, higher than necessary gains, etc. For this purpose, different "weights" or gains to the components of u may be assigned. This can be done as follows: Supposing that a measure of uncertainty for each parameter $\tilde{\pi}_i$ can be defined separately as:

$$\lfloor \tilde{\pi}_i \rfloor \le \rho_i \qquad I=1,2,\ldots\ldots\ldots,p \tag{58}$$

Let υ_i denote the *i*tch component of the vector $Y^T\sigma$, ε_i choose as the itch component of ε, and consequently the *i*tch component of the control input u_p is defined as [4].

$$u_i = \begin{cases} -\rho_i \upsilon_i / |\upsilon_i| & \text{if } |\upsilon_i| > \varepsilon_i \\ -(\rho_i/\varepsilon_i)\upsilon_i & \text{if } |\upsilon_i| \le \varepsilon_i \end{cases} \tag{59}$$

Since extended algorithm (56) is used, the uncertainty bounds for each parameter are shown separately in Table 4. The uncertainty bounds ρ_i in Table 4 are simply the difference between values given in Table 3 and Table 2, and the value of ρ is the Euclidean norm of the vector with components ρ_i [4].

ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
5	7.29	6. 25	0	5	6.25

Table 4. Uncertainty bound [4].

For computer simulation, a fifth order polynomial function is used as a reference trajectory for both joints. In order to analyse performance of the proposed controllers, each control law with the same control parameters K and Λ is applied to the same model system using same trajectory. The control matrices Λ and K are chosen to be identical as Λ=diag(10 10) and K=diag(30 30) for all controllers. The obtained results are plotted in Figures 2-4.

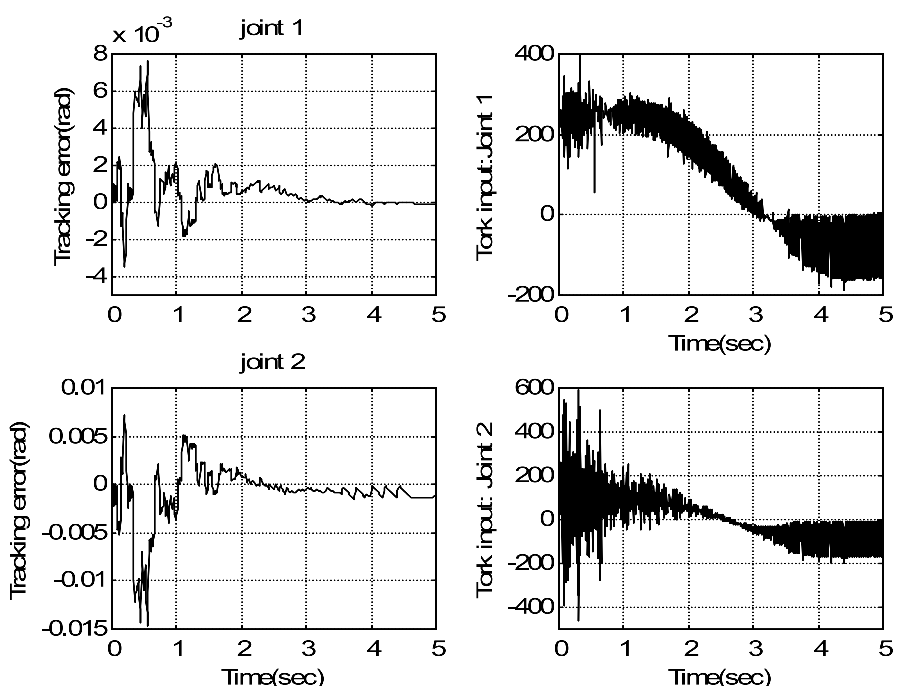


Fig. 2. Response using the robust control law (39) with uncertainty bound estimation law (28) when Λ=diag(10 10), K=diag(30 30), α=1, β=1.

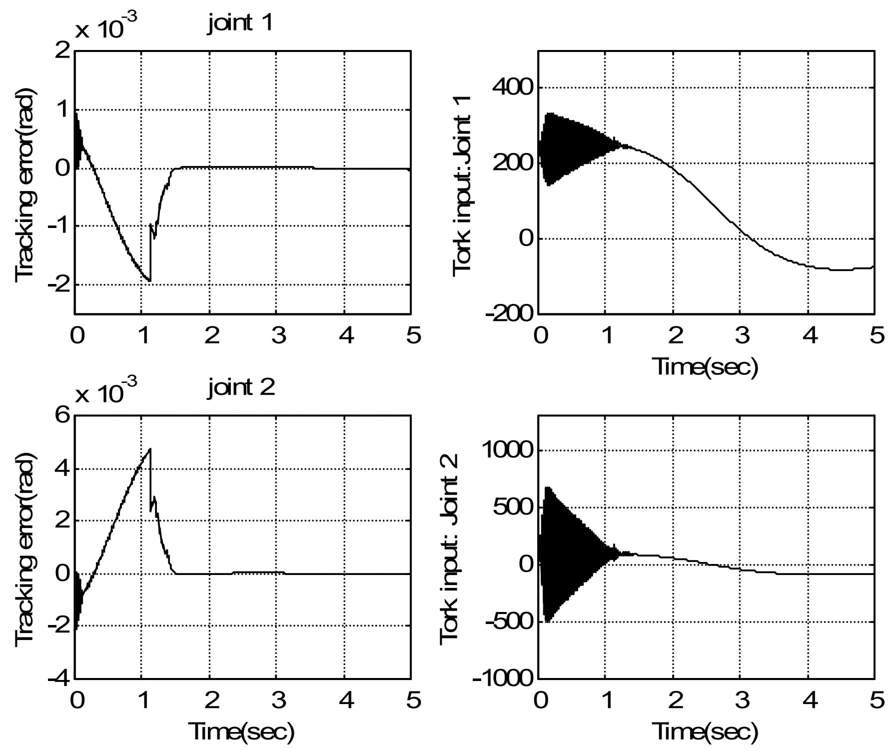


Fig. 3. Response using the robust control law (39) with uncertainty bound estimation law (35) when Λ=diag(10 10), K=diag(30 30), α=0.8, β=0.4.

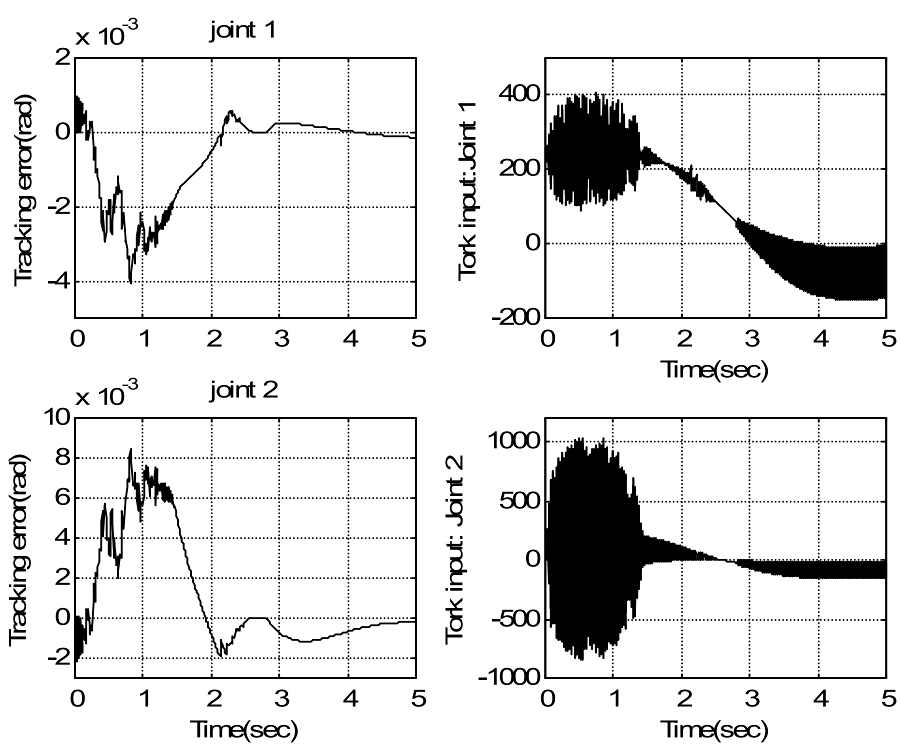


Fig. 4. Response using the robust control law (39) with uncertainty bound estimation law (37) when Λ=diag(10 10), K=diag(30 30), α=0.5, β=2.

As shown in Figures 2-4, tracking error is small and tracking performance changes according to uncertainty bound estimation laws.

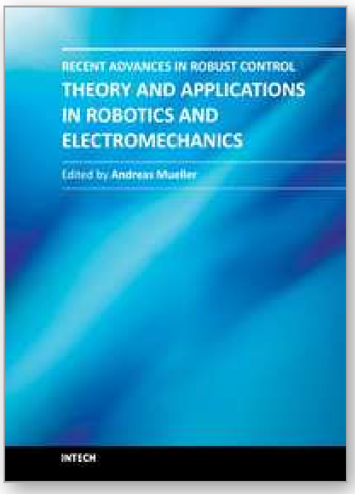
5. Conclusion

In the past, some robust controllers are developed for robot manipulators. Corless-Leitmann [1] approach is a popular approach used for designing robust controllers for robot manipulators. Spong [4] proposed a new robust controller for robot manipulators and Leithmann [5] or Corless-Leithman [1] approach is used for designing the robust controller. In [4], uncertainty bound on parameter is needed to derive robust controller and uncertainty bound parameters depends only on the inertia parameters of the robots. However, constant uncertainty bound parameters cause pure tracking performances. In order to increase tracking performance of the uncetain system, uncertainty bound estimation laws are developed [11-13]. Uncertainty bound estimation laws are updated as a function of exponential [11, 12], logarithmic [13] and trigonometric [14] functions depending on robot kinematics parameters and tracking error. A first order differential equation function is developed for derivation of control parameters and only a single derivation of uncertainty bound estimation law is possible. A new method for derivation of a bound estimation law is not proposed in [11-13], because, definition of a new variable function for other derivation is diffucult.
In the study [14], a general equation is developed from Lyapunov function and uncertainty bound estimation laws depending on trigonometric functions are developed. However, a general method for derivation of uncertainty bound estimation laws is not proposed. In a recent study [15], a general method for derivation of bound estimation laws based on the Lyapunov theory is proposed. In this method, functions and integration techniques are used for derivation of uncertainty bound estimation laws. Then, relations between the bound estimation laws and robust control inputs are established and four new robust control inputs are designed depending on each bound estimation law. It is possible to derive other different uncertainty bound estimation laws from general equation (23) if appropriate functions and integration techniques are defined. In this work, three different variable functions are defined and integration techniques are used in order to derive $\hat{\rho}(t)$ and relations between the uncertainty bouns and robust control laws are established. There is no distinct rule for definition of the $\Phi(t)$ and integration techniques in order to derive $\hat{\rho}(t)$. We use system state parameters and mathematical insight to search for appropriate function of $\Phi(t)$ as a derivtion of $\hat{\rho}(t)$. This study also shows that robust controllers are not limited with these derivations. It will be also possible to derive another bound estimation laws from Equation (23) if appropriate function $\Phi(t)$ and integration techniques are chosen.

6. References

[1] M. Corless, G. Leitmann, “Continuous feedback guaranteeing uniform ultimate boundedness for uncertain dynamic systems”, IEEE Transactions Automatic Control Vol.26, pp. 1139-1144, 1981.
[2] M. Corless, “Tracking controllers for uncertain systems: Applications to a manutec R3 robot. Journal of Dynamic Systems Measurement and Control”, Vol.111, pp. 609-618, 1989.

[3] M.W. Spong, M. Vidyasagar, "Robust linear compensator design for nonlinear robotic control", IEEE Journal of Robotic and Automation, Vol.3(4), pp. 345-350, 1987.
[4] M.W. Spong, "On the robust control of robot manipulators", IEEE Transactions on Automatic Control, Vol.37, pp.1782-1786, 1992.
[5] G. Leitmann, "On the efficiency of nonlinear control in uncertain linear system", Journal of Dynamic Systems Measurement and Control, Vol.102, pp. 95-102, 1981.
[6] E. Yaz, "Comments on the robust control of robot manipulators", IEEE Transactions on Automatic Control, Vol. 38(38), pp. 511-512, 1993.
[7] M. Danesh, M. Keshmiri and F. Sheikholeslam, "Developing a robust control scheme robust to uncertain model parameters and unmodeled dynamics" Proceeding of IEEE Conference on Industrial Electronics and Applications (ICIEA), May 24-26, 2006, Singapore.
[8] K.M. Koo, J.H. Kim, "Robust control of robot manipulators with parametric uncertainty", IEEE Transactions Automatic Control, Vol.39(6), pp. 1230-1233, 1994.
[9] G. Liu, A.A. Goldenberg, "Uncertainty decomposition-based robust control of robot manipulators", IEEE Transactions on Control Systems Technology, Vol.4, pp. 384-393, 1996.
[10]. G. Liu, Decomposition-based control of mechanical systems Electrical and Computer Engineering, 2000 Canadian Conference on Volume: 2 Publication Year: 2000 , Page(s): 966 - 970 vol.2
[11] R. Burkan, New approaches in controlling robot manipulators with parametric uncertainty, Ph.D. Thesis, Erciyes University, Institute of Science, Turkey, 2002.
[12] R. Burkan, İ. Uzmay, "Upper bounding estimation for robustness to the parameter uncertainty in trajectory control of robot arm", Robotics and Autonomous Systems Vol.45, pp.99-110, 2003.
[13] R. Burkan, İ. Uzmay, "Logarithmic based robust approach to parametric uncertainty for control of robot manipulators", International Journal of Robust and Nonlinear Control, Vol.15, pp. 427-436, 2005.
[14] R. Burkan, "Upper bounding estimation for robustness to the parameter uncertainty with trigonometric function in trajectory control of robot arms". Journal of Intelligent & Robotic Systems, Vol.46, pp.263-283, 2006.
[15] R. Burkan, "Modelling of bound estimation laws and robust controllers for robustness to parametric uncertainty for control of robot manipulators", Journal of Intelligent and Robotic Systems, Vol.60, pp. 365-394, 2010.
[16] M.W. Spong, S. Hutchinson, M. Vidyasagar, Robot Dynamics and Control, John Willey&Sons, Inc., 2006.
[17] J.J. Slotine, W. Li, "On the adaptive control of robotic manipulator", The International Journal of Robotics Research, Vol.6(3), pp. 49-59, 1987.
[18] L. Sciavicco, B. Siciliano, Modeling and Control of Robot Manipulators, The McGraw-Hill Companies, 1996.



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