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# Advanced Control Techniques for the Transonic Phase of a Re-Entry Flight

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## 1. Introduction

New technological developments in space engineering and science require sophisticated control systems with both high performance and reliability. How to achieve these goals against various uncertainties and off-nominal scenarios has been a very challenging issue for control system design over the last years.

Several efforts have been spent on control systems design in aerospace applications, in order to conceive new control approaches and techniques trying to overcome the inherent limitations of classical control designs.

In fact, the current industrial practice for designing flight control laws is based on Proportional Integral Derivative (PID) controllers with scheduled gains. With this approach, several controllers are designed at various points in the operative flight envelope, considering local time-invariant linear models based on small perturbations of a detailed nonlinear aircraft model. Although these techniques are commonly used in control systems design, they may have inherent limitations stemming from the poor capability of guaranteeing acceptable performances and stability for flight conditions different from the selected ones, especially when the scheduling parameters rapidly change.

This issue becomes very critical when designing flight control system for space re-entry vehicles. Indeed, space reentry applications have some distinctive features with respect to aeronautical ones, mainly related to the lack of stationary equilibrium conditions along the trajectories, to the wide flight envelope characterizing missions (from hypersonic flight regime to subsonic one) and to the high level of uncertainty in the knowledge of vehicle aerodynamic parameters.

Over the past years, several techniques have been proposed for advanced control system development, such as Linear Quadratic Optimal Control (LQOC), Eigenstructure Assignment, Robust control theory, Quantitative feedback theory (QFT), Adaptive Model Following, Feedback Linearization, Linear Parameter Varying (LPV) and probabilistic approach. Hereinafter, a brief recall of the most used techniques will be given.

Linear Quadratic Optimal Control (LQOC) allows finding an optimal control law for a given system based on a given criterion. The optimal control can be derived using Pontryagin's maximum principle and it has been commonly applied in designing Linear Quadratic Regulator (LQR) of flight control system (see Xing, 2003; Vincent et al., 1994).

The Eigenstructure Assignment consists of placing the eigenvalues of a linear system using state feedback and then using any remaining degrees of freedom to align the eigenvectors as accurately as possible (Konstantopoulos & Antsaklis, 1996; Liu & Patton, 1996; Ashari et al., 2005). Nevertheless there are several limitations, since only linear systems are considered and moreover the effects of uncertainty have been not extensively studied.

Robust analysis and control theory is a method to measure performance degradation of a control system when considering system uncertainties (Rollins, 1999; Balas, 2005). In this framework a concept of structured singular value (i.e  $\mu$ -Synthesis) is introduced for including structured uncertainties into control system synthesis as well as for checking robust stability of a system.

Adaptive Model Following (AMF) technique has the advantage of strong robustness against parameter uncertainty of the system model, if compared to classical control techniques (Bodson & Grosziewicz, 1997; Kim et al., 2003). The model following approach has interesting features and it may be an important part of an autonomous reconfigurable algorithm, because it aims to emulate the performance characteristics of a target model, even in presence of plant's uncertainties.

Another powerful nonlinear design is Feedback Linearization which transforms a generic non linear system into an equivalent linear system, through a change of variables and a suitable control input (Bharadwaj et al., 1998; Van Soest et al., 2006). Feedback linearization is an approach to nonlinear control design which is based on the algebraic transformation of nonlinear systems dynamics into linear ones, so that linear control techniques can be applied.

More recently an emerging approach, named Linear Parameter Varying (LPV) control, has been developed as a powerful alternative to the classical concept of gain scheduling (Spillman, 2000; Malloy & Chang, 1998; Marcos & Balas, 2004). LPV techniques are well suited to account for on-line parameter variations such that the controllers can be designed to ensure performance and robustness in all the operative envelope. In this way a gain-scheduling controller can be achieved without interpolating between several design points.

The main effort (and also main drawback) required by the above techniques is the modelling of a nonlinear system as a LPV system. Several techniques exist but they may require a huge effort for testing controller performances on the nonlinear system. Other modelling techniques try to overcome this problem at the expense of a higher computational effort.

Finally in the last decades, a new philosophy has emerged, that is, probabilistic approach for control systems analysis and synthesis (Calafiore et al., 2007; Tempo et al., 1999; Tempo et al., 2005). In this approach, the meaning of robustness is shifted from its usual deterministic sense to a probabilistic one. The new paradigm is then based on the probabilistic definition of robustness, by which it is claimed that a certain property of a control system is "almost" robustly satisfied, if it holds for "most" instances of uncertainties. The algorithms based on probabilistic approach, usually called randomized algorithms (RAs), often have low complexity and are associated to robustness bounds which are less conservative than classical ones, obviously at the expense of a probabilistic risk.

In this chapter the results of a research activity focused on the comparison between different advanced control architectures for transonic phase of a reentry flight are reported. The activity has been carried out in the framework of Unmanned Space Vehicle (USV) program of Italian Aerospace Research Centre (CIRA), which is in charge of developing unmanned space Flying Test Beds (FTB) to test advanced technologies during flight. The first USV

Dropped Transonic Flight Test (named DTFT1) was carried out in February 2007 with the first vehicle configuration of USV program (named FTB1) (see Russo et al., 2007 for details). For this mission, a conventional control architecture was implemented. DTFT1 was then used as a benchmark application for comparison among different advanced control techniques. This comparison aimed at choosing the most suited control technique to be used for the subsequent, more complex, dropped flight test, named DTFT2, successfully carried out on April 2010. To this end, three techniques were selected after a dedicated literature survey, namely:

- $\mu$ -Control with Fuzzy Logic Gain-Scheduling
- Direct Adaptive Model Following Control
- Probabilistic Robust Control Synthesis

In the next sections, the above techniques will be briefly described with particular emphasis on their application to DTFT1 mission. In sec. 5 the performance analysis carried out for comparison among the different techniques will be presented.

## 2. Fuzzy scheduled MU-controller

### 2.1 The $H^\infty$ control problem

The  $H^\infty$  Control Theory (Zhou & Doyle, 1998) rises as response to the deficiencies of the classical Linear Quadratic Gaussian (LQG) control theory of the 1960s applications. The general problem formulation is described through the following equations:

$$\begin{bmatrix} z \\ y \end{bmatrix} = P(s) \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \quad (1)$$

$$u = K(s)y$$

where  $P$  is the nominal plant,  $u$  is the control variable,  $y$  is the measured variable,  $w$  is an exogenous signals (such as disturbances) and  $z$  is the error signal to be minimized. The generic control scheme is depicted in Fig 1.

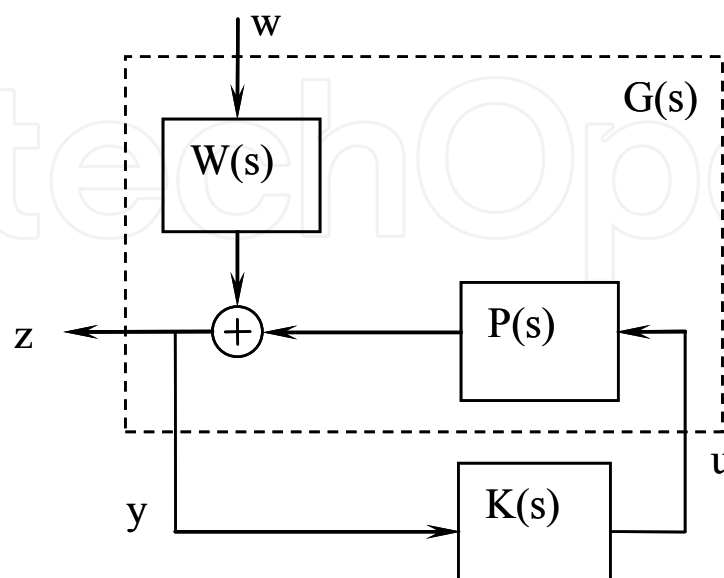


Fig. 1. Nominal Performance Scheme

It can be shown that closed-loop transfer function from  $w$  to  $z$  can be obtained via lower linear fractional transformation (Zhou & Doyle, 1998).

Therefore  $H_\infty$  control problem is to find a stabilizing controller,  $K$ , which minimizes

$$\|F_l(P, K)\|_\infty = \sup_{\omega} \bar{\sigma}(F_l(P, K)(j\omega)) = \gamma_M \quad (2)$$

where  $F_l$  is the lower linear fractional transformation from  $w$  to  $z$  and  $\bar{\sigma}$  is the singular value of specified transfer function.

For what concerns Nominal Performance Problem, it is required that error  $z$  is kept as small as possible. To this end, a new generalized plant can be considered (see the dashed line). The weighting function penalizes the infinite-norm of new plant to achieve required performances.

In the same way, Robust Stability Problem can be solved applying Small Gain Theorem (Zhou & Doyle, 1998) to the following new generalized plant selected (see the dashed line):

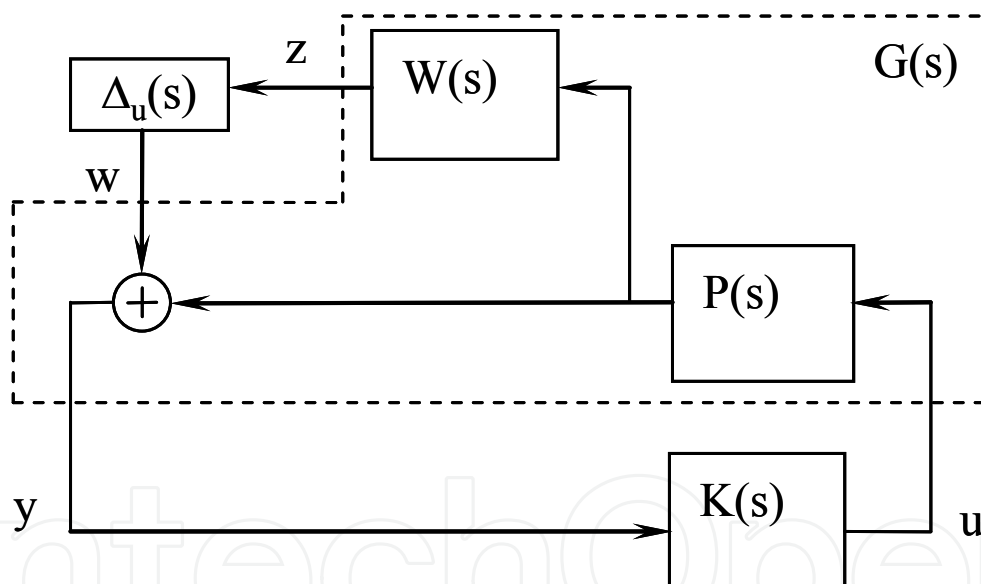


Fig. 2. Robust Stability Scheme

A more general problem to solve is Robust Performance Problem that takes into account both Nominal Performance and Robust Stability Problems. It is worth noting that a Nominal Performance Scheme allows to find a stabilizing controller that satisfies Small Gain Theorem in presence of a fictitious uncertainty block  $\Delta_f(s)$  (with  $\|\Delta_f(s)\|_\infty < 1 / \gamma_M$ ). Hence a general scheme for Robust Performance Problem is the following one:

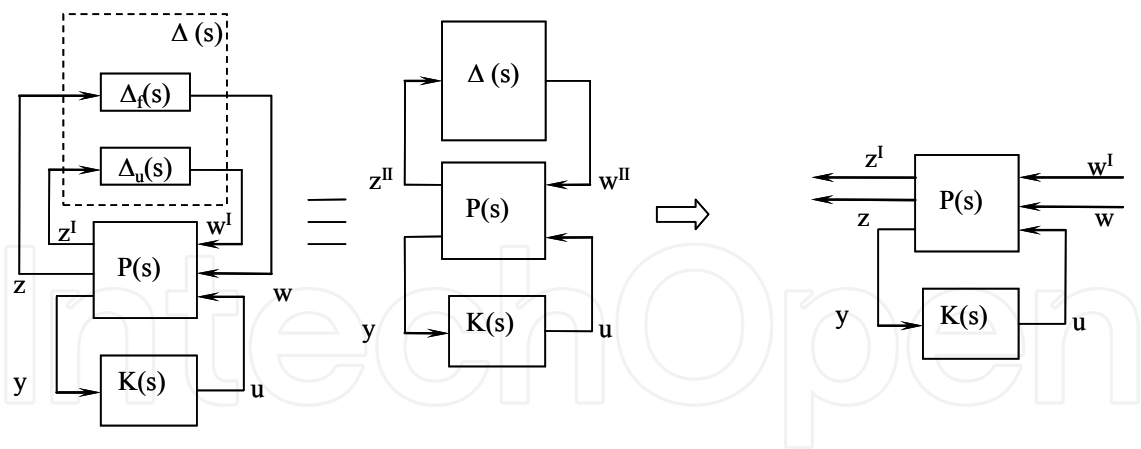


Fig. 3. Robust Performance Scheme

where:

$$\Delta(s)=\begin{bmatrix} \Delta_u(s) & 0 \\ 0 & \Delta_f(s) \end{bmatrix} \tag{3}$$

It is clear from the figures that all above problem formulations can be always rearranged to solve the same general  $H_\infty$  problem. It is worth noting that for what concerns Robust Performance Case,  $\Delta(s)$  matrix has a diagonal block structure. Plant uncertainties can be structured like mixed (real and complex) uncertainties. Unfortunately  $H_\infty$  problem only deals with unstructured full complex  $\Delta(s)$ , so optimal (or sub-optimal) controller might be very conservative.  $\mu$ -analysis and synthesis try to solve this issue by dealing with structured uncertainties.

2.2  $\mu$ -synthesis framework

The brief discussion of previous paragraph has shown how to design an  $H_\infty$  controller starting from a generalized plant. Required performances are achieved through optimal (or sub-optimal) controller by means of weighting functions of a generalized plant. In the same way, robust stability is achieved together with performances by solving Robust Performance Problem. Let  $M = F_l(P,K)$ , then a general scheme for  $\mu$ -analysis is the following one:

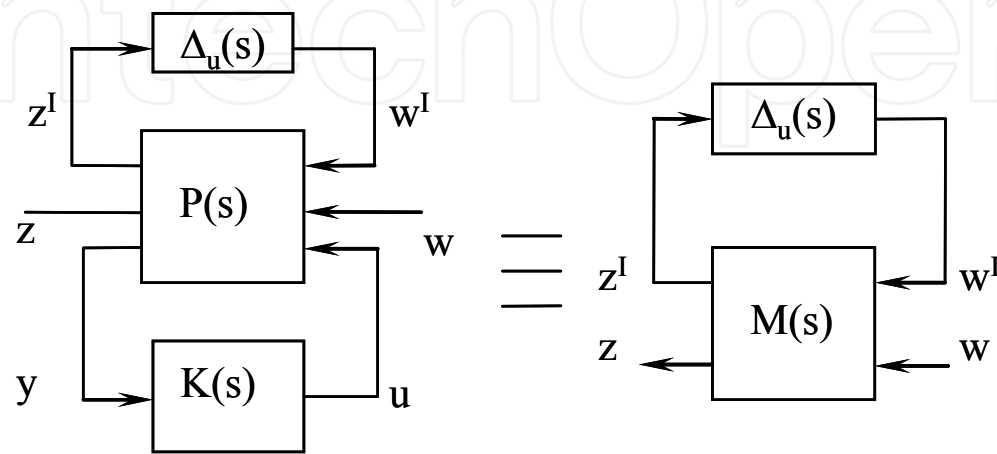


Fig. 4.  $\mu$ -Analysis Scheme

Introducing structured singular value:

$$\mu_{\Delta}(M) := \frac{1}{\min\{\bar{\sigma}(\Delta) : \det(I - M\Delta) = 0\}} \quad (4)$$

(where  $\Delta_u$  is the structured uncertainty mentioned earlier), for all  $\Delta_u(s)$  with  $\|\Delta_u(s)\|_{\infty} < 1/\beta$ , and  $\beta > 0$ , the loop of previous figure is internally stable and  $\|F_l(M, \Delta_u)\|_{\infty} < \beta$  if and only if

$$\sup_{\omega \in R} \mu_{\Delta}(M(j\omega)) \leq \beta \quad (5)$$

Therefore, given a controller  $K(s)$ ,  $\mu$ -bound  $\beta$  can be numerically computed. Finally  $\mu$ -synthesis framework can be represented through the following scheme.

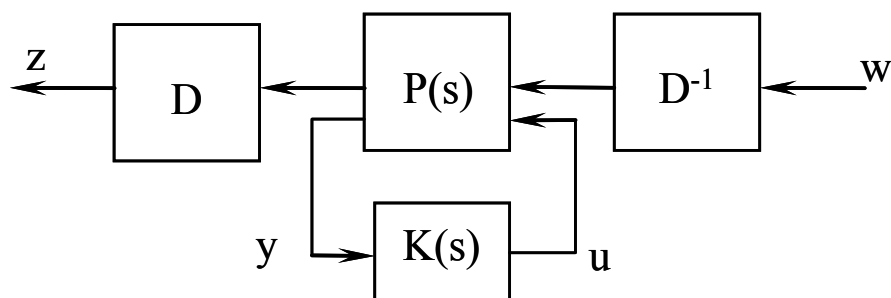


Fig. 5.  $\mu$ -Synthesis via scaling

where  $D$  matrix allows scaling the process taking into account structured uncertainties from  $z$  to  $w$ . A commonly used methodology to solve the above problem is DK-iterations algorithm (Zhou & Doyle, 1998) that sequentially performs two parameter minimization: first minimizing over  $K$  with  $D$  fixed, then minimizing over  $D$  with  $K$  fixed, then again over  $K$ , and again over  $D$ , etc. The algorithm runs until a fixed bound is achieved and final  $K$  is the desired controller.

### 2.3 Fuzzy scheduling

Each controller developed using the technique described in the previous sections can be considered as a “local” controller, since it might not guarantee the same performances “far away” from design point (or outside a given region of flight envelope). If that region does not cover flight envelope of interest, a controller scheduling is necessary.

Many techniques and methodology have been investigated in literature (Nichols et al., 1993; Pedrycz & Peters, 1997; Hyde & Glover, 1993), but no one guarantees that scheduled controller provides robust performance to be achieved by closed loop system.

In (Pedrycz & Peters, 1997), authors present a general approach to a fuzzy interpolation of different LTI controllers. Although controllers are PID with different gains, the technique can be easily generalized to more complex LTI systems.

For what concerns application of fuzzy scheduling for the proposed DTFT application, a sort of fuzzy gain scheduling technique has been implemented using an approach similar to the one described in (Pedrycz & Peters, 1997).



Considering a system with dynamics described by the following equations:

$$\begin{aligned}\dot{x} &= f(x(t), u(t)) \\ y &= g(x(t), u(t)) \\ x &\in R^n, u \in R^r, y \in R^m\end{aligned}\quad (6)$$

and a family of equilibrium points  $(x_{ei}, y_{ei}, u_{ei})$ , with  $i=1, 2, \dots, c$ . System linearization leads to the following linear system:

$$\begin{aligned}\dot{\tilde{x}} &= A_i \tilde{x} + B_i \tilde{u} \\ \tilde{y} &= C_i \tilde{x} + D_i \tilde{u}\end{aligned}\quad (7)$$

For each linearized model it is possible to design a local (linear) controller,  $L_1, L_2, \dots, L_c$ , on which the overall control laws will be based. Let  $\Omega_1, \Omega_2, \dots, \Omega_c$  be fuzzy relations whose activation levels require specific control actions; the computations of control are then regulated by smooth, centre of gravity type of switching:

$$u = \frac{\sum_{i=1}^c L_i \left( y * \frac{\Omega_i(x, u_{ei})}{\sum_{i=1}^c \Omega_i(x, u_{ei})} \right) * \Omega_i(x, u_{ei})}{\sum_{i=1}^c \Omega_i(x, u_{ei})} \quad (8)$$

### 3. Adaptive control system

Direct Adaptive Model Following (DAMF) is a Model Reference Control Strategy with strong robustness properties obtained through the use of direct adaptation of control loop gains in order to achieve a twofold objective: zero error between output of reference model and output of real plant and furthermore minimization of control effort. The proposed adaptation algorithm is based on Lyapunov theory. Hereinafter a brief mathematical description of the method, fully reported in (Kim et al., 2003), will be given. Starting from generic linear model of a plant:

$$\begin{aligned}\dot{x} &= Ax + Bu + d \\ y &= Cx\end{aligned}\quad (9)$$

where  $x \in \mathbb{R}^n$  is the state vector,  $y \in \mathbb{R}^l$  the output vector,  $u \in \mathbb{R}^m$  the control vector,  $A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times m}$ ,  $C \in \mathbb{R}^{l \times n}$  and the term  $d$  represents the trim data, reference system dynamics are written in term of desired input-output behaviour:

$$\dot{y}_m = A_m y_m + B_m r \quad (10)$$

where  $y_m$  is the desired output for the plant,  $r$  is the reference signal,  $A_m$  and  $B_m$  represent the reference linear system dynamics. Control laws structure is defined as:



$$u = C_0(G_0x + v + r + K_0y_m) \quad (11)$$

where  $G_0$ ,  $C_0$  and  $v$  are adaptive control gains, while  $K_0$  is a feed-forward gain matrix off-line computed. It is possible to demonstrate that the following adaptation rules for control laws parameters:

$$\begin{aligned} \dot{G}_0 &= -\gamma_1 B_m^T P e x^T \\ \dot{C}_0 &= -\gamma_2 C_0 B_m^T P e u^T C_0 \\ \dot{v}_0 &= -\gamma_3 B_m^T P e \end{aligned} \quad (12)$$

imply the non-positiveness of Lyapunov candidate function derivative:

$$\dot{V} = -e^T P e \leq 0 \quad (13)$$

which guarantees the asymptotical stability for the error dynamic system (Kim et al., 2003). Matrix  $P$  is the solution of Lyapunov equation:

$$A_e^T P + P A_e = -Q; \quad \text{with } Q > 0 \quad (14)$$

With reference to the implementation of adaptive technique for DTFT1 benchmark application, detailed scheme of MIMO controller is reported in Fig. 6. The design parameters of both inner and outer loops consist of a few number of matrices. First of all, Reference Dynamics are expressed by means of two matrices  $A_m$  and  $B_m$  with limitation that  $B_m$  must be chosen invertible. Desired error dynamic is regulated by means of  $A_e$ . Through this matrix it is also possible to modify system capability to reject noise and disturbances, thus defining the shape of closed loop system bandwidth. The matrix  $Q$  in Eq. (14) is used to specify tracking performance requirements of output variables. Finally, parameters  $\gamma_1$ ,  $\gamma_2$  and  $\gamma_3$  are used to regulate the adaptive capability of control gains. Large values imply quick adaptivity and vice versa.

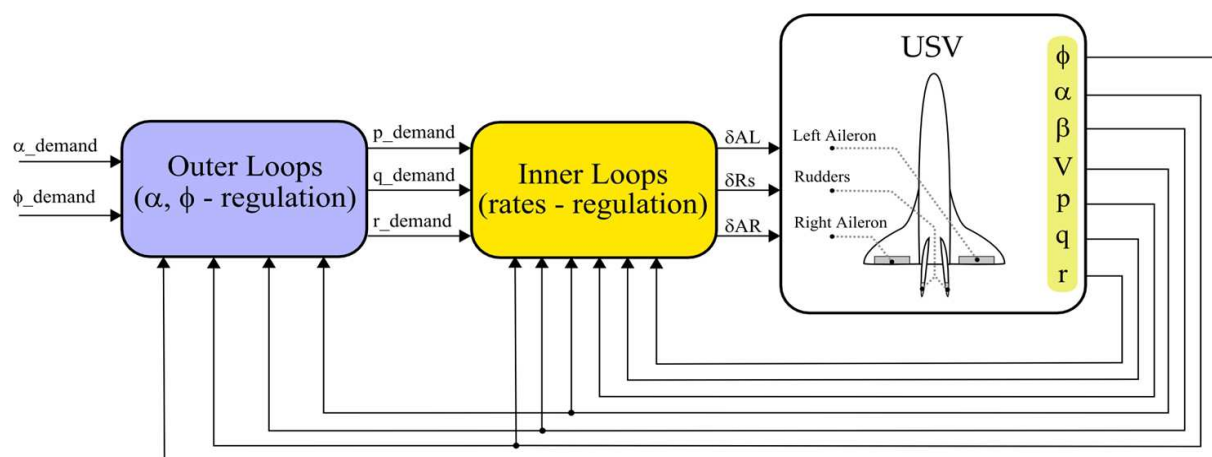


Fig. 6. The general scheme of control system architecture

Control architecture depicted in Fig. 6 is made of two MIMO control loops. The inner one is referred to the rates ( $p$ ,  $q$ ,  $r$ ) regulation, while the outer one is used to control both angle of

attack ( $\alpha$ ) and roll angle ( $\phi$ ). Either MIMO controllers are designed with Adaptive Model Following (AMF) control technique above described.

In the following table a brief description of which variables have been used for the design of both inner and outer loops is given.

	$y$	$x$	$r$	$u$
inner loops	$p, q, r$	$v_{TAS}, \alpha, \beta, p, q, r$	$p_{DEM}, q_{DEM}, r_{DEM}$	$\delta_{Al}, \delta_{AR}, \delta_R$
outer loops	$\alpha, \phi$	$v_{TAS}, \alpha, \beta, \phi$	$\alpha_{DEM}, \phi_{DEM}$	$p_{DEM}, q_{DEM}, r_{DEM}$

Table 1. Controller Variables

4. Probabilistic robust controller

Within the stochastic paradigm for control system design/analysis, the meaning of robustness is moved from its well known deterministic sense to a probabilistic one. Indeed, it is claimed that a certain property of a control system is “almost” robustly satisfied, if it holds for “most” of the occurrences of uncertain variables. In other words, a risk that this property is violated by a set of uncertainties with small probability measure is considered acceptable.

Nevertheless, from a computational point of view, assessing probabilistic robustness of a given property may be more difficult than the deterministic case, since it requires the computation of a multi-dimensional probability integral. This problem is overcome by means of randomized algorithms which estimate performance probability by randomly sampling the uncertainty space, and computing bounds on the estimation error. Since estimated probability is itself a random quantity, this method always entails a certain risk of failure, i.e. there exists a nonzero probability of making an erroneous estimation. These algorithms have low complexity and are associated to robustness bounds which are less conservative than classical ones, obviously at the expense of a non deterministic result.

Randomization can be effectively used for control synthesis, by means of two different approaches. The first one aims at designing controllers that satisfy a given performance specification for most values of uncertainties, i.e. that are robust in a probabilistic sense, while the second one aims at finding a controller that maximizes the mean value of performance index, thus in the latter case the objective is to obtain a controlled system that guarantees the best performance on average (Tempo et al., 2005). For what concerns the use of this technique for DTFT1 benchmark application, the second approach has been used.

The approach used for controller synthesis was to look for a controller that (probabilistically) minimizes the mean value of the performance index, thus the objective was to obtain a controlled system that guarantees the best performance *on average*.

Performance function for the uncertain system is first defined:

$$u(\Delta) : \Delta \rightarrow \Re$$

(15)

the above function gives a measure of system performance for a given value of uncertainty  $\Delta$ . In this application the function  $u$  is the following Boolean function which represents the “failure” of a given controller, that is,

$$u(\Delta) = \begin{cases} 1 & \text{if a given system property is not satisfied} \\ 0 & \text{otherwise} \end{cases}$$

(16)

Controller  $C_{opt}$  will be the one guaranteeing that the expected value of performance function  $u(\Delta, C)$  is minimized:

$$C_{prob} = \arg \min_{c \in C} E[u(\Delta, C)] \quad (17)$$

An approximate solution can be obtained by means of randomized algorithms which are based on sampling both uncertainty set  $\Delta$  and controller set  $C$ . To this end, two separate problem need to be solved: first, an estimate of the expected value is computed, then this estimate is minimized.

Computation of  $E[u(\Delta, C)]$  is carried out through randomization, that is,  $M$  independent, identically distributed (i.i.d.) controllers  $C^1, C^2, \dots, C^M \in C$  are extracted according to their probability density function  $f_C(C)$ ; an estimation of the minimum value is then given by:

$$\min_{i=1,2,\dots,M} E[u(\Delta, C^i)] \quad (18)$$

It is possible to demonstrate (Tempo et al., 2005) that, given  $\varepsilon_1 \in [0, 1]$ , and  $\delta \in [0, 1]$ , if

$$M \geq \frac{\log \frac{1}{\delta}}{\log \frac{1}{1 - \varepsilon_1}} \quad (19)$$

then:

$$\text{Prob} \left\{ \text{Prob} \left\{ E[u(\Delta, C)] \leq \min_{i=1,2,\dots,M} E[u(\Delta, C^i)] \right\} \leq \varepsilon_1 \right\} \geq 1 - \delta \quad (20)$$

As it can be noticed, computation of  $E[u(\Delta, C^i)]$  for each  $C^i$  requires the execution of a multidimensional integral, that is very difficult in general; also this problem is solved by randomization approach.

For what concerns estimation of the expected value  $E[u(\Delta, C^i)]$  of performance function,  $N$  i.i.d. samples  $\Delta^1, \Delta^2, \dots, \Delta^N$  are extracted from  $\Delta$ , according to their density function  $f_\Delta$ ; performance functions  $u(\Delta^1, C^1), \dots, u(\Delta^N, C^1)$  are then computed for a fixed controller  $C^1 \in C$ , and an estimation  $\hat{E}_N[u(\Delta, C^i)]$  of the expected value  $E[u(\Delta, C^i)]$  is given by:

$$\hat{E}_N[u(\Delta, C^i)] = \frac{1}{N} \sum_{k=1}^N u(\Delta^k, C^i) \quad (21)$$

It can be demonstrated (Tempo et al., 2005) that, if:

$$N \geq \frac{\log \frac{2}{\delta}}{2\varepsilon_2^2} \quad (22)$$

then:

$$\text{Prob}\left\{\left|E\left[u(\Delta, C^i)\right] - \hat{E}_N\left[u(\Delta, C^i)\right]\right| \leq \varepsilon\right\} \geq 1 - \delta \quad (23)$$

In order to compute a probabilistic controller, equations (20) and (23) must be put together. To this end, it can be shown that, for any  $\varepsilon_1, \varepsilon_2 \in [0,1]$  and  $\delta \in [0,1]$ , if

$$M \geq \frac{\log \frac{1}{\delta}}{\log \frac{1}{1 - \varepsilon_1}} \quad \text{and} \quad N \geq \frac{\log \frac{2M}{\delta}}{2\varepsilon_2^2} \quad (24)$$

then

$$\text{Prob}\left\{\text{Prob}\left\{E\left[u(\Delta, C)\right] \leq \min_{i=1,2,\dots,M} \hat{E}_N\left[u(\Delta, C^i)\right] - \varepsilon_2\right\} \leq \varepsilon_1\right\} \geq 1 - \frac{\delta}{2} \quad (25)$$

The randomized probabilistic controller is given by:

$$\hat{C}_{NM} = \arg \min_{i=1,\dots,M} \frac{1}{N} \sum_{k=1}^N u(\Delta^k, C^i) \quad (26)$$

Eq. (25) states that the estimated minimum  $\min_{i=1,2,\dots,M} \hat{E}_N\left[u(\Delta, C^i)\right]$  is “close” to the actual one  $E\left[u(\Delta, C_{prob})\right]$  within  $\varepsilon_2$  in terms of probability, and this is guaranteed with an accuracy  $\varepsilon_1$  and a confidence level at least  $\delta/2$ .

For what concerns the implementation of the above technique to the benchmark application, a fixed control system architecture (inherited from GNC system of DTFT1) has been chosen and its parameters have been optimized according to a stochastic technique. Since controller gains are scheduled with dynamic pressure, controller design have been carried out by optimizing scheduling parameters through stochastic synthesis.

In particular, once the controller structure is defined, stochastic optimization allows selecting the optimum controller parameters also accounting for all the uncertain parameters, mainly vehicle and environment ones. For each candidate vector of control parameters, success rate is computed according to a pre-specified figure of merit and the applied uncertainties. To this end, a first test is performed considering nominal conditions, i.e. no uncertainties applied. If the considered controller passes the test in nominal conditions, uncertainty region is sampled and, for each uncertainty sample, the nonlinear test is repeated and success rate is computed. For this application, the following test success criteria have been used:

- No instability (identified as commands oscillation along the trajectory) occurring during the trajectory.
- No Out-of-Range commands deflection (Max.  $\pm 25^\circ$  for the elevons,  $\pm 20^\circ$  for the rudder);
- Satisfactory tracking performances for tracked variables ( $1^\circ$  RMS in  $\alpha$  and  $\beta$ ,  $3^\circ$  RMS in  $\phi$ );
- Valid aerodynamic data during the trajectory ( $\alpha \in [-5 \div 18^\circ]$ ,  $\beta \in [-8 \div 8^\circ]$ ).

5. Numerical analysis

In this section the results of a numerical analysis carried out in order to compare performance and robustness of the developed control systems will be presented. The activity has been carried out in the framework of Unmanned Space Vehicle (USV) program of Italian Aerospace Research Centre (CIRA), which is in charge of developing unmanned space Flying Test Beds (FTB) to test advanced technologies during flight. The first USV Dropped Transonic Flight Test (named DTFT1) was accomplished in February 2007 with the first vehicle configuration of USV program (named FTB1) (Russo et al., 2007). In this mission, a conventional control architecture was implemented. DTFT1 was then used as a benchmark application for comparison among different advanced control techniques. This comparison aimed at choosing the most suited control technique to be used for subsequent dropped flight test, named DTFT2, carried out on April 2010.

In the figure below the trajectory of DTFT1 mission in the plane Mach-altitude is depicted.

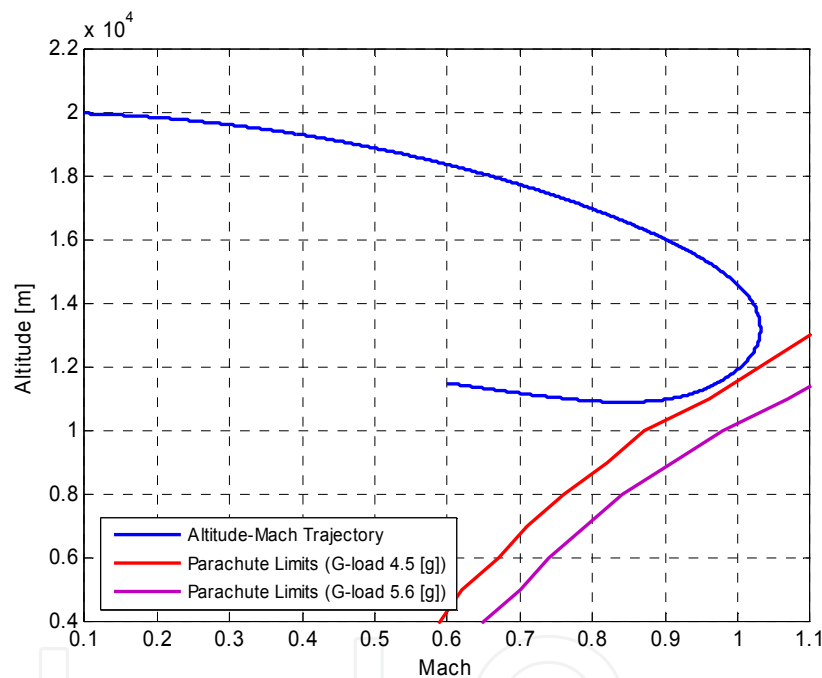


Fig. 7. Mach-Altitude Trajectory

For what concerns control performances, they are specified in the next table and they are valid only for Mach>0.7 (transonic regime):

Variable Tracked	Tracked Value [deg]	RMS Accuracy [deg]
Angle of Attack	7	1
Angle of Sideslip	0	1
Roll angle	0	3

Table 2. Tracked Variables

In order to compare the advanced controllers described in previous sections, the following scenarios have been identified:

- robustness to parametric uncertainties, namely aerodynamic ones
- robustness to initial state displacement
- robustness to navigation errors (sensor noise)
- robustness to actuator failures

Within this framework, control laws robustness is the capability of guaranteeing performances and stability of control systems in presence of above uncertainties.

Numerical evaluation reported in this section have been carried out using a complete 6 Dof model of FTB1 vehicle together with Atmospheric Model, wind model, Hydraulic Actuator System, Air Data System, Inertial Measurement System and control laws.

Parametric uncertainties have been accounted for by considering a particular aerodynamic configuration (hereafter called the worst configuration) which was identified as the aerodynamic uncertainty configuration leading to worst dynamic behaviour of FTB1 vehicle, in terms of stability, damping and control derivatives that mainly affect stability properties.

Two test cases, named  $C_0$ , and  $\hat{C}_0$ , have been accomplished with both nominal conditions (nominal initial state, zero navigation errors, no failure, etc.) and worst aerodynamic configuration respectively.

For what concerns robustness to initial state displacement, several off-nominal conditions in terms of Euler angles and angular rates have been considered (see the following table). Nominal DTFT1 initial state in terms of attitude, heading and angular rates is:  $\phi_0 = 0$  deg,  $\theta_0 = -90$  deg,  $\psi = 0$  deg,  $p = 0$  deg/s,  $q = 0$  deg/s,  $r = 0$  deg/s.

Initial State Displacement	$\phi_0$ [deg]	$\theta_0$ [deg]	$\psi_0$ [deg]	$p_0$ [deg/s]	$q_0$ [deg/s]	$r_0$ [deg/s]
$I_1$	-20	-89.9	0	-5	3	-3
$I_2$	20	-89.9	0	2	2	-2
$I_3$	20	-89.9	0	5	2	1
$I_4$	0	-85	0	-5	2	0

Table 3. Initial State Displacement

Furthermore for each case of the above table, nominal (case  $C_1 - C_4$ ) and worst aerodynamic configuration (case  $C_5 - C_8$ ) have been simulated.

For what concerns navigation errors, simulations have been performed with both nominal (case  $C_9$ ) and worst aerodynamic configuration (case  $C_{10}$ ) without any initial state have displacement.

As far as robustness to actuator failures is concerned, a rudder failure occurring after 30 s from vehicle's drop has been simulated, in particular a jam of the right rudder. It is worth noting that in this case ( $C_{11}$ ) both initial state and aerodynamic configuration are nominal. All the benchmark scenarios are summarized in the following table.

Case	Aerodynamic Configuration	Initial State	Navigation Errors	Actuator Failure
$C_0$	Nominal	Nominal	No errors	No failure
$\hat{C}_0$	Worst-Aero 1	Nominal	No errors	No failure
$C_1$	Nominal	Off-nominal $I_1$	No errors	No failure
$C_2$	Nominal	Off-nominal $I_2$	No errors	No failure
$C_3$	Nominal	Off-nominal $I_3$	No errors	No failure
$C_4$	Nominal	Off-nominal $I_4$	No errors	No failure
$C_5$	Worst-Aero	Off-nominal $I_1$	No errors	No failure
$C_6$	Worst-Aero	Off-nominal $I_2$	No errors	No failure
$C_7$	Worst-Aero	Off-nominal $I_3$	No errors	No failure
$C_8$	Worst-Aero	Off-nominal $I_4$	No errors	No failure
$C_9$	Nominal	Nominal	Errors	No failure
$C_{10}$	Worst-Aero	Nominal	Errors	No failure
$C_{11}$	Nominal	Nominal	No errors	Right Rudder jamming at 30 s

Table 4. Benchmark Scenarios for Performance Evaluation

For what concerns robust performance indicators, tracking accuracy of trajectory has been defined as a performance parameter. Performance requirements are given in the table below:

Variable Tracked	RMS Tracking error [deg]
Angle of Attack	$\pm 1$
Angle of Sideslip	$\pm 1$
Roll angle	$\pm 3$

Table 5. Tracking Performance requirements



In the next figures the results of scenarios  $C_0$  and  $\hat{C}_0$  are reported.

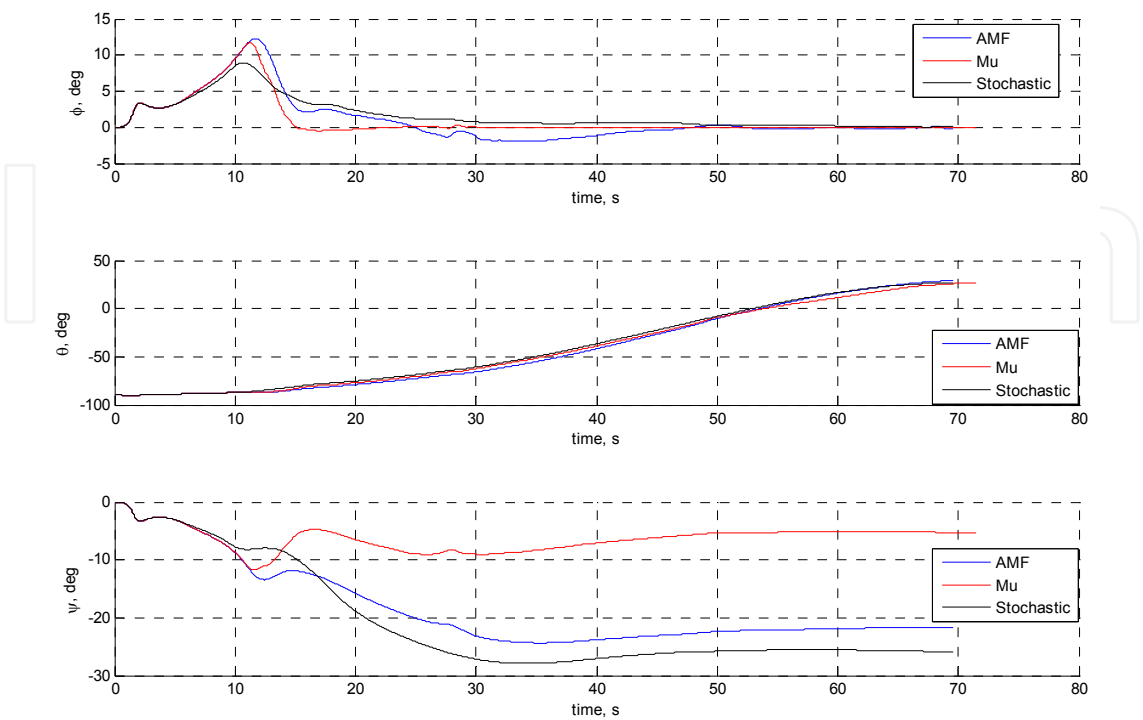


Fig. 8. Case  $C_0$  -Euler Angles

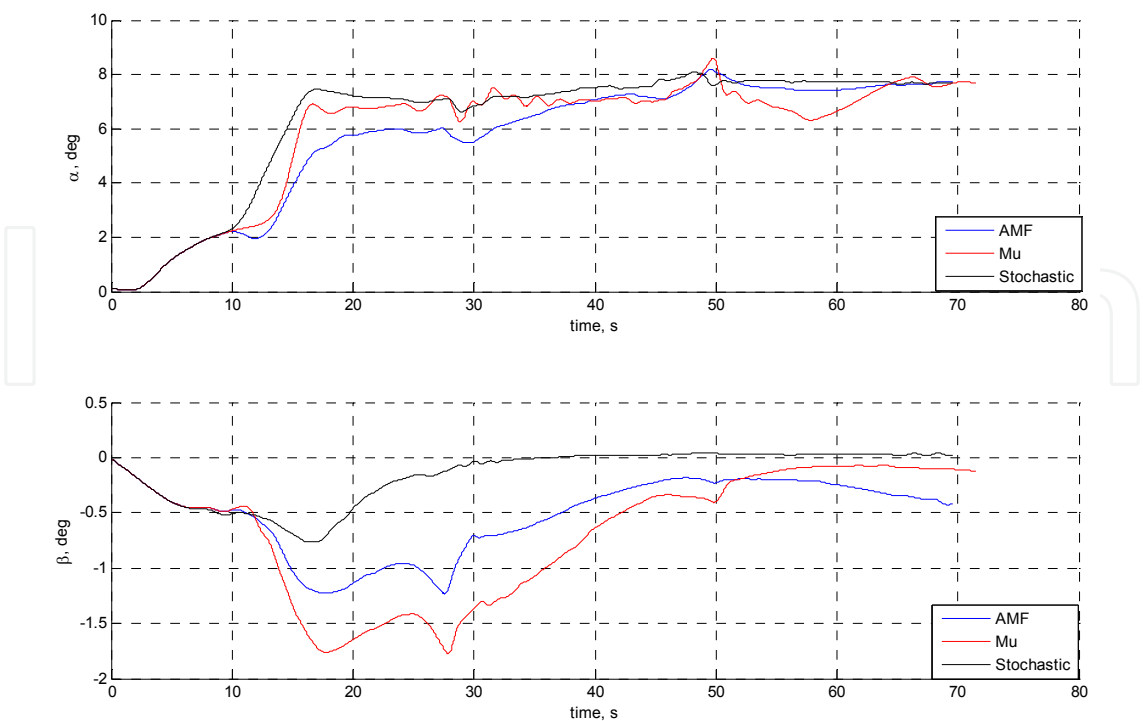


Fig. 9. Case  $C_0$  -Incidence Angles

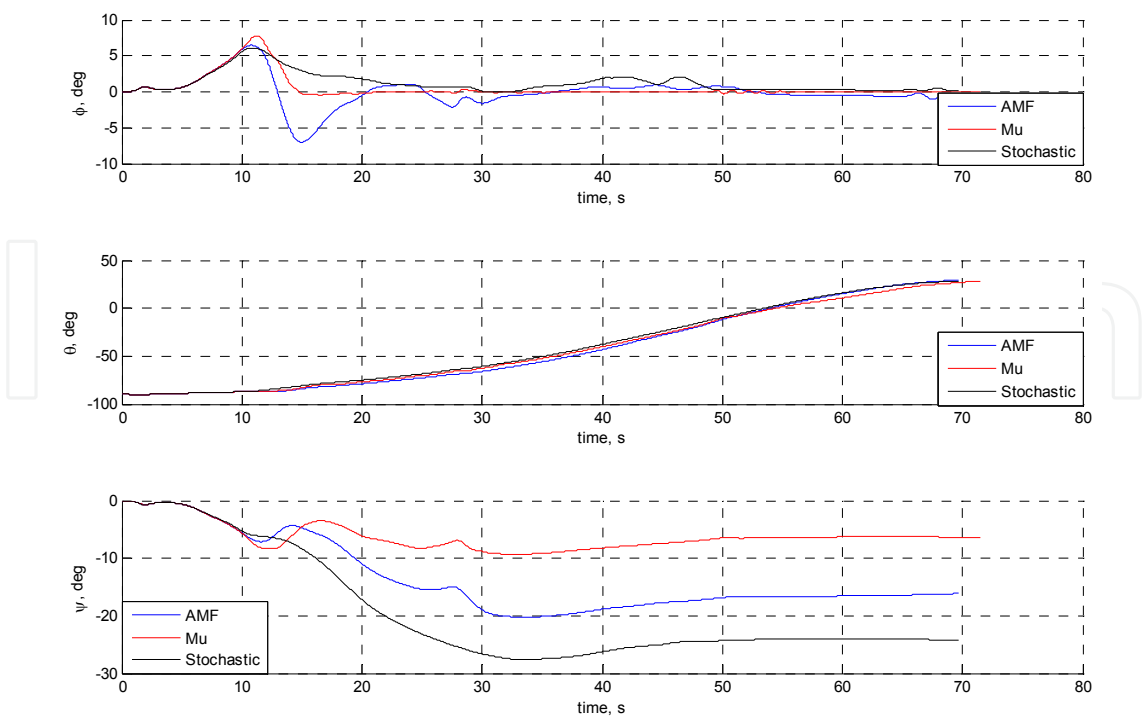


Fig. 10. Case  $\hat{C}_0$  -Euler Angles

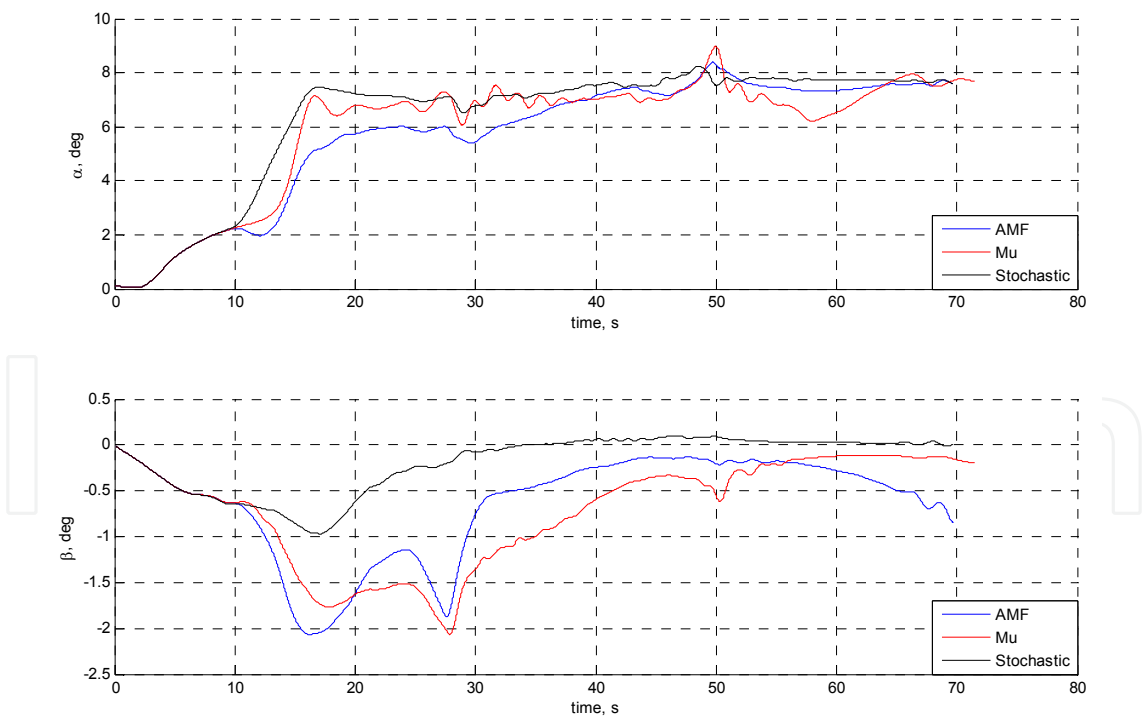


Fig. 11. Case  $\hat{C}_0$  -Incidence Angles

As it can be seen from the figures, performances of all control algorithms are globally satisfactory. It is worth noting that Mu-controller shows a light unstable behaviour on sideslip angle around 50 s in worst case configuration, but it is however rapidly damped in

few seconds. For what concerns uncertainties to initial state displacement, several cases have been considered with different attitude and angular velocity at vehicle drop. In the following figures, for sake of brevity, only the cases  $C_5$  and  $C_8$  are reported. They refer to initial state conditions  $I_1$  and  $I_4$  with worst aerodynamic configuration (see Table 4).

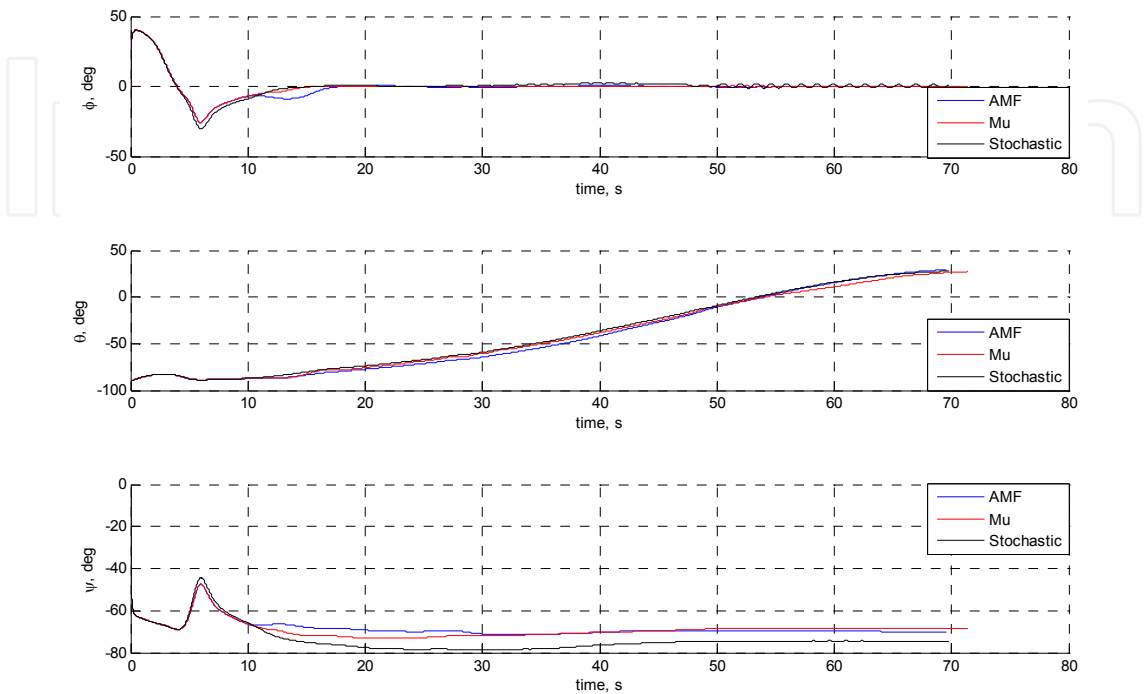


Fig. 12. Case  $C_5$  -Euler Angles

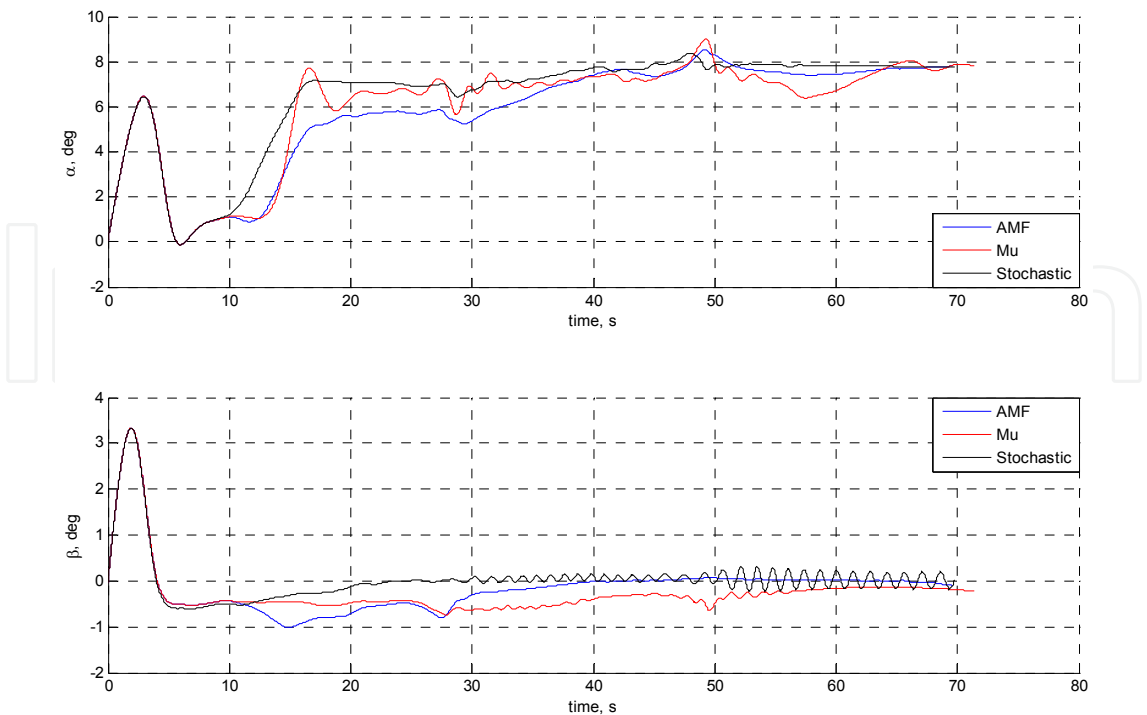


Fig. 13. Case  $C_5$  -Incidence Angles

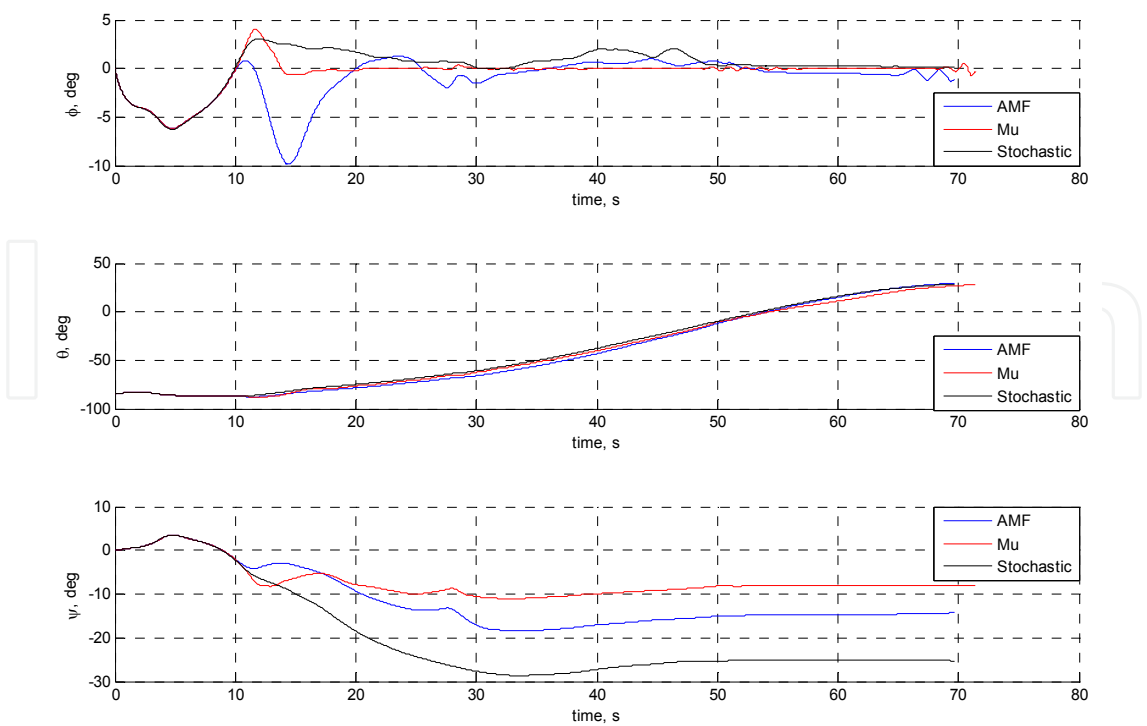


Fig. 14. Case  $C_8$  -Euler Angles

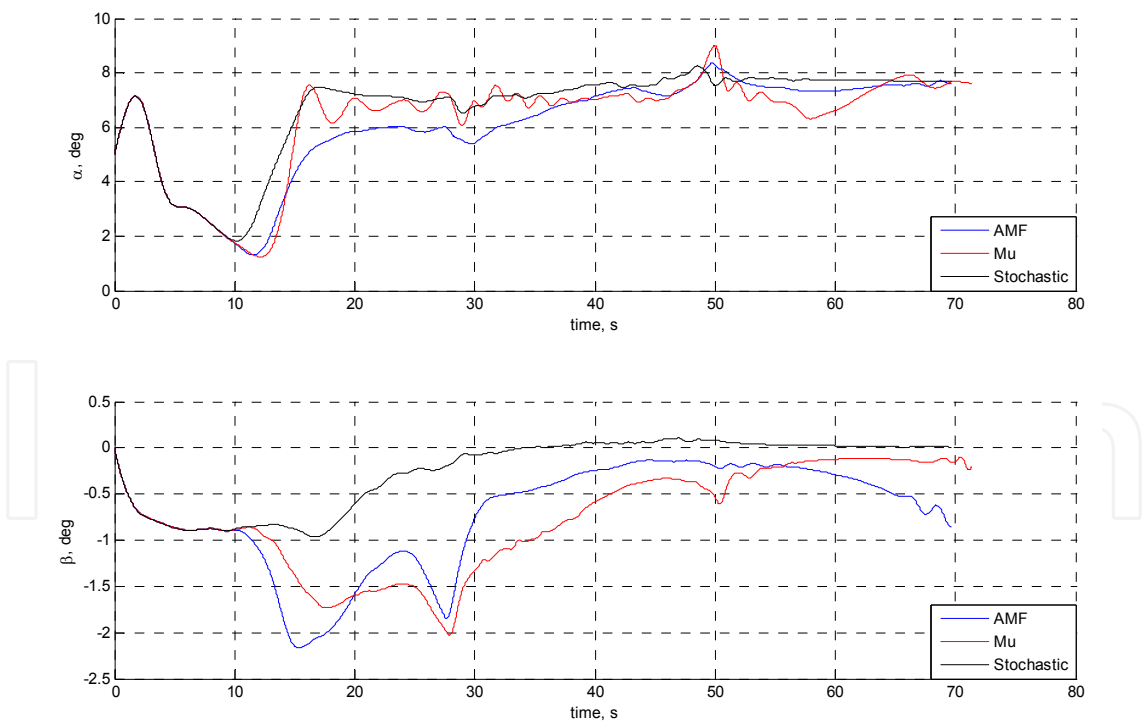


Fig. 15. Case  $C_8$  -Incidence Angles

All controllers satisfactory work in presence of initial state displacement. In spite of a light oscillatory mode on sideslip and roll angles, stochastic controller guarantees very good performances for what concerns tracking of sideslip angle and angle of attack.

As mentioned earlier, in order to evaluate control algorithms capabilities to face disturbances such as navigation errors, two test cases have been considered, i.e. nominal and worst aerodynamic configuration (cases  $C_9$  and  $C_{10}$ ) without any initial state displacement. The comparison between controllers is reported in the following figure, only with reference to the case  $C_{10}$  for sake of simplicity.

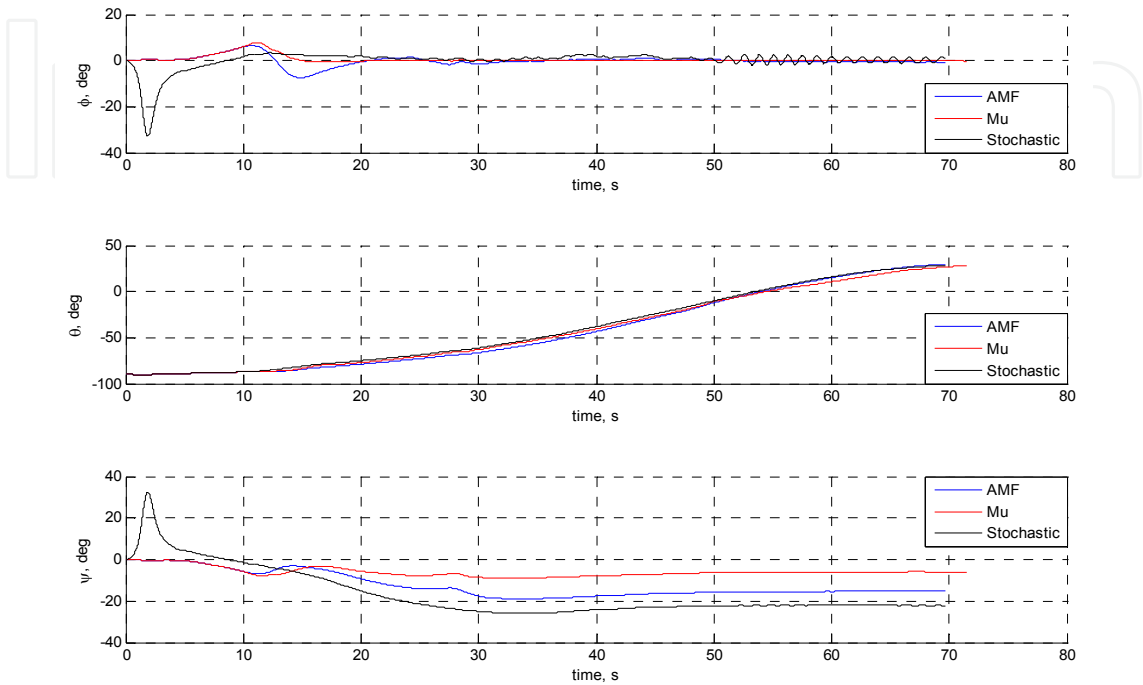


Fig. 16. Case  $C_{10}$  -Euler Angles

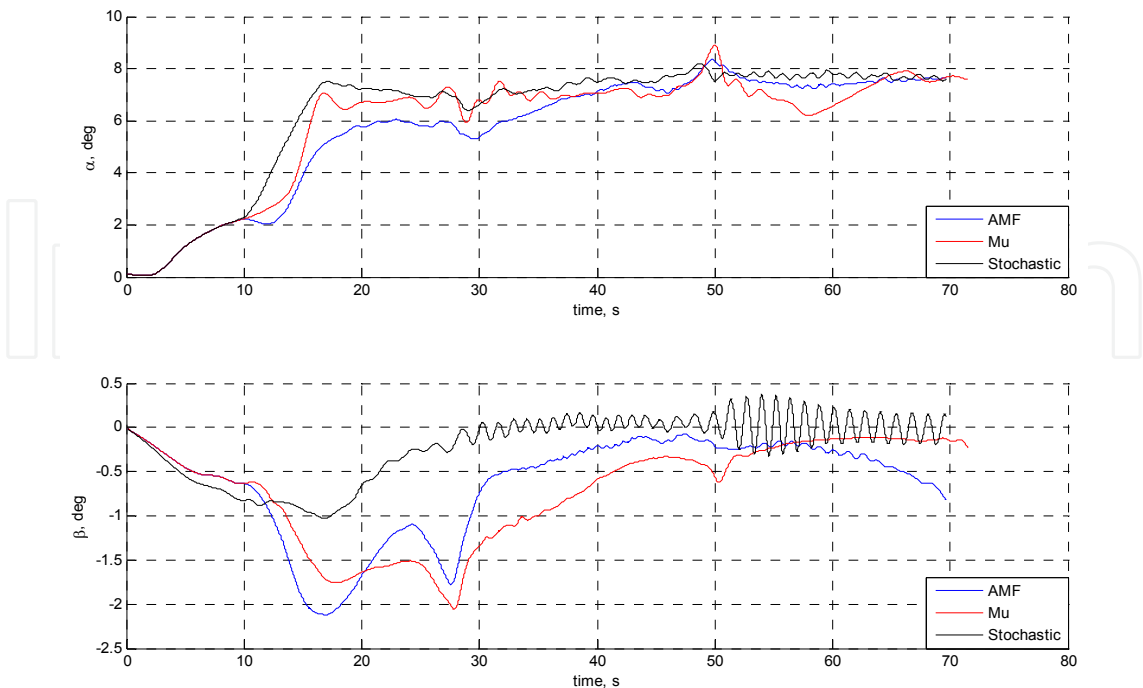


Fig. 17. Case  $C_{10}$  -Incidence Angles

Simulations show an acceptable robustness to sensor noise. A small effect on incidence angles, in terms of reduced damping, is shown by stochastic controller. Finally algorithms robustness to an actuator failure has been evaluated. In particular a rudder jamming at  $t=30$  s has been simulated.

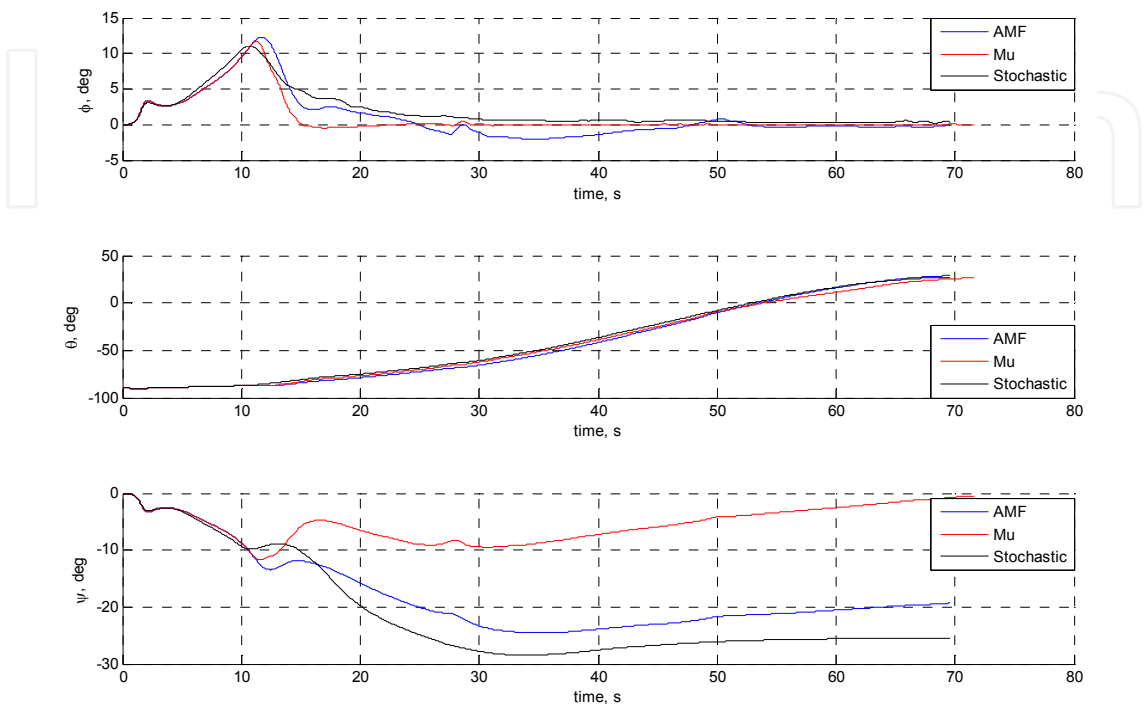


Fig. 18. Case  $C_{11}$  -Euler Angles

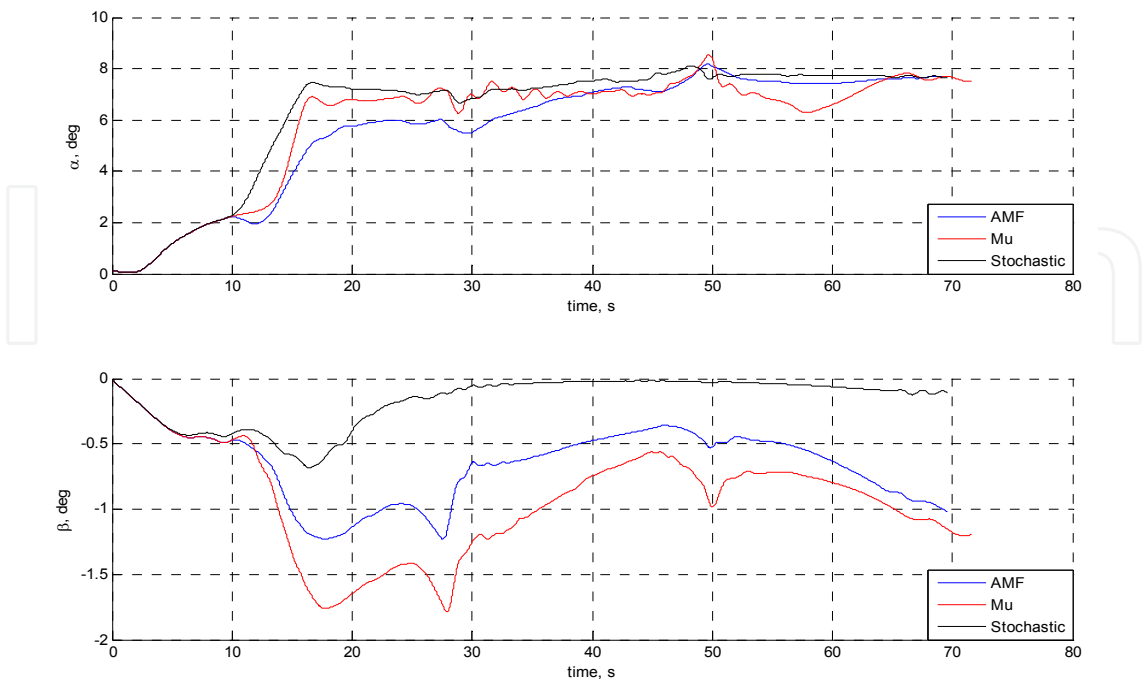


Fig. 19. Case  $C_{11}$  -Incidence Angles

The above figures show that rudder jam failure is well tolerated by all the controllers. The following table summarizes the performances achieved by the controllers for all considered scenarios.

Case	Adaptive Controller AoA-AoS-Phi accuracy [deg]	MU-Controller AoA-AoS-Phi accuracy [deg]	Stochastic Controller AoA-AoS-Phi accuracy [deg]
$C_0$	0.87676	0.57129	0.21606
	0.5969	0.88441	0.13512
	0.72448	0.053638	0.61134
$\hat{C}_0$	0.90016	0.62284	0.24501
	0.74635	0.91081	0.18806
	0.59708	0.067061	0.8251
$C_1$	0.88835	0.5642	0.21624
	0.26603	0.63046	0.066145
	0.45265	0.061391	0.42644
$C_2$	0.88043	0.57188	0.21585
	0.58375	0.97939	0.14444
	0.75665	0.089979	0.64976
$C_3$	0.87282	0.57878	0.21536
	0.62874	0.83847	0.13685
	0.71016	0.038177	0.63915
$C_4$	0.86873	0.53892	0.21588
	0.593	0.87025	0.13086
	0.71128	0.05387	0.59952
$C_5$	0.91649	0.62481	0.2452
	0.29559	0.40438	0.1115
	0.35717	0.14435	1.1254
$C_6$	0.91255	0.62265	0.24415
	0.75401	1.0448	0.20172
	0.69885	0.13026	0.82289
$C_7$	0.89565	0.6301	0.24409
	0.73756	0.84141	0.18635
	0.66584	0.044825	0.76552
$C_8$	0.8919	0.59116	0.24443
	0.74122	0.89455	0.18392
	0.59849	0.10033	0.82521
$C_9$	0.87611	0.57183	0.22299
	0.60085	0.88728	0.15436
	0.7007	0.053653	0.61589
$C_{10}$	0.90059	0.62011	0.25904
	0.74182	0.91444	0.23867
	0.57556	0.061636	1.2075
$C_{11}$	0.8759	0.57115	0.20854
	0.72793	1.0664	0.12598
	0.80693	0.066715	0.65093

Table 6. Controller Performances



The comparison between the controllers has shown that:

1. Stochastic controller guarantees the best performances for what concerns the tracking of AoA and AoS and it always meets performance requirements. Anyway, it presents some light oscillations on lateral and directional dynamics in worst aerodynamic configuration.
2.  $\mu$ -controller guarantees the best tracking performance in terms of roll angle, much better than required (3 deg error), but in some cases it fails to meet performance requirements on AoA and AoS (see  $C_6$  and  $C_{11}$ ).
3. Adaptive Controller guarantees almost good tracking performances even though they are never better than the other two controllers. In any case, performance requirements are always met.

Based on the above considerations, the controller obtained by means of stochastic synthesis was considered the most suited for the DTFT2 scopes, so it was selected (after a fine tuning) as a part of an advanced GNC system for DTFT2 mission, successfully carried out on April 2010.

In fact, despite the limitation of using an 'a priori' fixed control structure, control laws obtained through the stochastic synthesis have the following good features:

- excellent performances and good stability properties in spite of large uncertainties affecting the system;
- simple control structures while guaranteeing robust performances and stability as well as low computational effort and implementation simplicity.

## 6. Conclusions

Over the last decades many efforts have been spent to develop advanced control techniques for aerospace applications, aimed at overcoming limitations of commonly used control techniques, mainly lack of robustness against various uncertainties affecting the system to be controlled. The importance of a robust control system is readily understood when space reentry applications are considered. Indeed, these applications have some distinctive features, mainly related to the lack of stationary equilibrium conditions along the trajectories, to the wide flight envelope characterizing the missions (from hypersonic flight regime to subsonic one) and to the high level of uncertainty in the knowledge of vehicle aerodynamic parameters.

The development of an advanced control system having robustness capabilities is one of the goals of research activities carried out by Italian Aerospace Research Centre in the framework of USV program. In order to select a control strategy having the advantages above discussed, three candidate control techniques have been compared with the aim of selecting the most suited one for the second dropped flight test of USV program, named DTFT2, successfully carried out on April 2010. The three techniques are:

- $\mu$ -Controller with Fuzzy Logic Gain-Scheduling
- Direct Adaptive Model Following Control
- Probabilistic Robust Control Synthesis

In order to evaluate the robustness capabilities of proposed control algorithms, a numerical robustness analysis has been performed. Performances and stability of candidate control techniques have been evaluated in presence of several sources of uncertainties (aerodynamics, initial state, etc.) and failure scenarios.

Robustness analysis showed that all three techniques are well suited to accomplish robust control in USV DTFT1 mission, in presence of large parameters uncertainty (the vehicle mostly flies in transonic regime, where accurate aerodynamic prediction is very difficult to obtain).

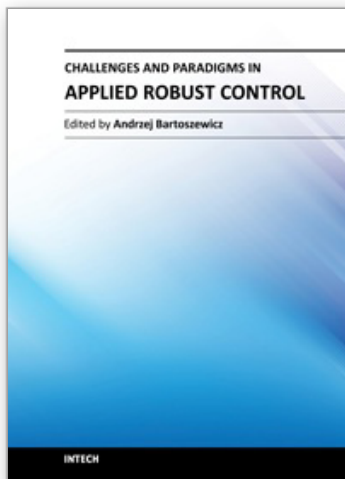
Nevertheless the controller obtained by means of stochastic synthesis was selected as a part of on-board advanced GNC system for DTFT2 mission, due to its good performances and relatively simple implementation.

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## **Challenges and Paradigms in Applied Robust Control**

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The main objective of this book is to present important challenges and paradigms in the field of applied robust control design and implementation. Book contains a broad range of well worked out, recent application studies which include but are not limited to H-infinity, sliding mode, robust PID and fault tolerant based control systems. The contributions enrich the current state of the art, and encourage new applications of robust control techniques in various engineering and non-engineering systems.

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