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1. Introduction

Many studies have been made to develop walking anthropomorphic robots able to move in environments well-adapted to human beings and able to cooperate with them. Among the more advanced projects of humanoid robots, one can mention: the Honda robots P2, P3 (Hirai, 1997) (Hirai et al., 1998) and Asimo (Sakagami et al., 2002), the HRP series developed by AIST (Kaneko et al., 1998) (Kajita et al., 2005) (Kaneko et al., 2004) (Morisawa et al., 2005), the small robot QRIO proposed by Sony (Nagasaka et al., 2004), the KHR series developed by KAIST (Kim et al., 2004) (Kim et al., 2005), the last robot of Waseda University having seven degrees of freedom per leg (Ogura et al., 2004), Johnnie (Lohmeier et al., 2004) and H7 (Kagami et al., 2005). These robots are namely able to climb stairs and to carry their own power supply during stable walking. The problem of dynamic locomotion and gait generation for biped robot has been studied theoretically and experimentally with quite different approaches. However, when searchers study the behavior or the design of dynamic walking robots, they inevitably meet a number of intrinsic difficulties related to these kinds of systems: a large number of parameters have to be optimized during the design process or have to be controlled during the locomotion task; the intrinsic stability of walking machines with dynamic behaviors is not robust; the coordination of the legs is a complex task. When human walks, it actively uses its own dynamic effects to ensure its propulsion. Today, some studies exploit the dynamic effects to generate walking gaits of robots. In this research field, four kinds of approaches are used. The first one uses pragmatic rules based on qualitative studies of human walking gaits (Pratt et al., 2001) (Sabourin et al., 2004). The second one focuses on the mechanical design of the robot in order to obtain natural passive dynamic gaits (Collins et al., 2005). The third one deals with theoretical
studies of limit cycles (Westervelt et al., 2004) (Canudas-de-Wit et al., 2002). The fourth one creates various dynamic walking gait with reference trajectories (Bruneau et al., 2001) (Chevallereau et al., 2003). However, all these approaches are not really universal and do not allow online dynamic adaptation of the robot motions as function of the environment and of the real capabilities of the system.

Our objective is to carry out a more adaptive and universal approach based on the dynamic equations in order to generate walking gaits with a high dynamic behavior. Moreover, we do not wish to impose exact temporal desired trajectories. On the contrary, the capabilities of the robot, in term of intrinsic dynamics and actuator torques, are constantly evaluated and exploited as well as possible with an aim of reaching a desired average walking rate of the system. In order to do this, we propose a strategy based on two dynamic criterions, which takes into account, at every moment, the intrinsic dynamics of the system and the capabilities of the actuators torques in order to modify the dynamics of the robot directly. These criterions called the "dynamic propulsion criterion" and the "dynamic propulsion potential" permit to produce analytically the motions of the legs during all phases and all transition phases without specification of the events to perform. Even if this method is universal, we illustrate it for a planar under-actuated robot without foot : RABBIT (Buche, 2006). The particularity of this robot is that it can not be statically stable because the contact area between the legs and the ground is very small. Thus, a control strategy based on dynamic considerations is required.

The organization of this chapter is as follows. In the second part the presentation of the robot Rabbit and its characteristics are given. In part three, dynamics of the biped robot Rabbit are introduced. Then, in part four, the two dynamic criterions are expressed: the "dynamic propulsion criterion" and the "dynamic propulsion potential". In part five, we describe the control strategy, based on these criterions, to generate highly dynamic walking gaits. The sixth part gives the simulation results obtained with this analytical approach.

2. The bipedal robot RABBIT

The robot Rabbit (fig. 1.) is an experimentation of seven French laboratories (IRCCYN Nantes, LAG Grenoble, LMS Poitiers, LVR Bourges, LGIPM Metz, LIRMM Montpellier, LRV (new name : LISV) Versailles) concerning the control of a biped robot for walking and running gaits generation within the framework of a CNRS project ROBEA (Sabourin et al., 2006) (Chevallereau et al., 2003).

This robot is composed of two legs and a trunk and has not foot. The joints are located at the hip and at the knee. This robot has four motors: one for each hip, one for each knee. Rabbit weighs 32 kilograms and measures 1 meter. Its main masses and lengths are given in table 1. Its motions are included in sagittal planes, by using a radial bar link fixed at a central column that allows to guide the direction of progression of the robot around a circle. As Rabbit moves only in the sagittal plane, it can be modeled in two dimensions. This robot represents the minimal system able to generate walking and running gaits. Since the contact between the robot and the ground is just one point (passive dof (degree of freedom)), the robot is under-actuated during the single support phase: there are only two actuators, at the hip and at the knee of the stance leg, to control three parameters:
- the trunk evolution along the moving direction $\mathbf{x}$ at the reference point
- the trunk evolution along the gravity direction $\mathbf{y}$ at the reference point
- the trunk angle evolution around the rotation direction $\mathbf{z}$

Fig. 1. Robot Rabbit.

<table>
<thead>
<tr>
<th></th>
<th>Weight (Kg)</th>
<th>Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trunk</td>
<td>12</td>
<td>0.2</td>
</tr>
<tr>
<td>Thigh</td>
<td>6.8</td>
<td>0.4</td>
</tr>
<tr>
<td>Shin</td>
<td>3.2</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 1. Characteristics of Rabbit.

The configuration of Rabbit is described by the parameters given in fig.2. The reference’s point $R_p$ is selected at the hip of the robot for the definition of the position, velocity and acceleration of the trunk.

Fig. 2. Model parameters of Rabbit.
3. Analytical dynamics of Rabbit

The approach is based on the dynamic equations with the lagrangian form for the robot Rabbit at the reference point \( R_p \).

3.1 Lagrangian Dynamic Equations

The lagrangian motion equations, applied at \( R_p \), are written in two subsystems:

\[
\begin{align*}
\overrightarrow{M_p} \overrightarrow{\ddot{q}} + \overrightarrow{H_p} \overrightarrow{\dot{q}} + \overrightarrow{C_p} + \overrightarrow{G_p} &= D_{\alpha_{12}} \overrightarrow{F_{1}} + D_{\alpha_{12}} \overrightarrow{F_{2}} \\
\overrightarrow{H_{k}} \overrightarrow{\ddot{q}} + \overrightarrow{M_{k}} \overrightarrow{\dot{q}} + \overrightarrow{C_{k}} &= D_{\alpha_{12}} \overrightarrow{F_{1}} + D_{\alpha_{12}} \overrightarrow{F_{2}} + \overrightarrow{\Gamma}
\end{align*}
\]

where \( \overrightarrow{q} = [\dot{x}, \dot{y}, \dot{z}]^T \) is the trunk accelerations vector at \( R_p \), \( \overrightarrow{\dot{q}} = [\ddot{x}, \ddot{y}, \ddot{z}]^T \) the joint accelerations vector, \( \overrightarrow{F_{j}} (j=1,2) \) the contacting forces applied to the feet and \( \overrightarrow{\Gamma} \) the joint torques vector. The index “p” refers to the trunk and the index “k” to the legs. The subsystem (1), coming from the first three motion equations, is related to the trunk. This subsystem is said passive. The subsystem (2), coming from the last four motion equations, is related to the legs. This subsystem is said active.

3.2 Trunk Dynamics and External Generalized Force Expression

The trunk dynamics and the external generalized forces expression, at the reference point, are obtained by isolating \( \overrightarrow{\dot{q}} \) in (2) and by injecting it in (1). The terms which are functions of the trunk configuration uniquely are isolated in the inertia matrix \( \overrightarrow{\overrightarrow{M_p}} \), centrifugal and Coriolis vector \( \overrightarrow{\overrightarrow{C_p}} \) and gravity vector \( \overrightarrow{\overrightarrow{G_p}} \). The other terms related with legs or coupled between legs and trunk are grouped in the matrix \( \overrightarrow{\overrightarrow{M_k}} \) and vectors \( \overrightarrow{\overrightarrow{C_k}} \) and \( \overrightarrow{\overrightarrow{G_k}} \). Thus we can write:

\[
\begin{align*}
\overrightarrow{\overrightarrow{M_p}} &= \overrightarrow{\overrightarrow{M_p}} + \overrightarrow{\overrightarrow{M_k}} \\
\overrightarrow{\overrightarrow{C_p}} &= \overrightarrow{\overrightarrow{C_p}} + \overrightarrow{\overrightarrow{C_k}} \\
\overrightarrow{\overrightarrow{G_p}} &= \overrightarrow{\overrightarrow{G_p}} + \overrightarrow{\overrightarrow{G_k}}
\end{align*}
\]

with \( \overrightarrow{\overrightarrow{C_p}} = \overrightarrow{\overrightarrow{\Gamma}} \)

The following equations system is also obtained:

\[
\begin{align*}
\overrightarrow{\overrightarrow{M_p}} + \overrightarrow{\overrightarrow{C_p}} + \overrightarrow{\overrightarrow{G_p}} &= -\overrightarrow{\overrightarrow{M_p}} - \overrightarrow{\overrightarrow{M_k}} - \overrightarrow{\overrightarrow{M_k}} \overrightarrow{\overrightarrow{F_{1}}} + \overrightarrow{\overrightarrow{F_{2}}} \\
&+ \overrightarrow{\overrightarrow{C_k}} + \overrightarrow{\overrightarrow{G_k}} + \overrightarrow{\overrightarrow{\Gamma}} - \overrightarrow{\overrightarrow{\Gamma}} \\
&+ \sum_{i=1}^{n} D_{\alpha_{12}} \overrightarrow{F_{i}} - \overrightarrow{\overrightarrow{M_k}} \overrightarrow{\overrightarrow{F_{i}}} \\
&+ \left[ \frac{D_{\alpha_{12}}}{D_{\alpha_{12}}} \overrightarrow{F_{i}} + D_{\alpha_{12}} \overrightarrow{F_{i}} \right] \overrightarrow{\overrightarrow{G_k}}
\end{align*}
\]

The left term of (4) represents trunk dynamics whereas the right term is the external generalized forces \( \overrightarrow{\overrightarrow{F}} \) applied to the trunk at \( R_p \).

3.3 Computed Torque Method

The actuator torques can be used on one hand to control the evolution of the trunk and on the other hand to satisfy constraints related to the locomotion (no-contact between the
swing leg and the ground and landing without impact). In the second case, the actuator torques will be function of joint trajectories. The method used to calculate these desired actuator torques is the computed torque method using non linear decoupling of the dynamics. For that, we express the actuator torques $\tau_p$ by isolating $\tau_p$ in (1) and by injecting it in (2):

$$\tau = \{I_m - \Pi_i \Pi_j^{-1} \Pi_k \} \left[ \tau_i - \Pi_i \Pi_j^{-1} \tau_j \right] - \left\{ \frac{D_{ai}}{F_{ai}} + D_{aj} F_{aj} \right\} \left\{ \frac{D_{ai}}{F_{ai}} + D_{aj} F_{aj} \right\}$$

(5)

Then, we separate (5) in two subsystems. A subsystem, named A, composed of the equations of the actuator torques used to generate a desired joint trajectory. A subsystem, named B, composed of the other equations. In the subsystem B, we isolate the joint acceleration, associated to the actuator torques not used to imposed a desired joint trajectory. In this case, if we inject their expressions in the subsystem A, this subsystem A express the actuator torques used to impose joint trajectories as function of the desired joint acceleration and of the other actuator torques. So, knowing the configuration of the robot and the desired joint accelerations, given by the desired joint trajectories, we can calculated the actuator torques.

3.4 Close-Loop Constraint

During the double contact phase, we have to respect the close-loop constraint along the $\dot{y}$ direction when we calculate the actuator torques, in order to avoid a no desired take off of a leg. At first, this geometric constraint is expressed as follows:

$$y_{p1} - y_{p2} = g(q_1, q_2, q_3, q_4)$$

(6)

where $y_{p1}$ and $y_{p2}$ are respectively the foot position of the leg 1 and the leg 2 along the $\dot{y}$ direction. Then, we have to express the close-loop constraint in terms of actuator torques constraint. For that, we derive (6) twice:

$$\ddot{y}_{p1} - \ddot{y}_{p2} = f(q_1, q_2, q_3, q_4, \dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4)$$

(7)

where $\ddot{y}_{p1} - \ddot{y}_{p2}$ is equal to zero. Then, we replace the joint acceleration vector by its expression obtained with $f=g$ (5):

$$\ddot{y}_{p1} - \ddot{y}_{p2} = \left[ \tau - \{I_m - \Pi_i \Pi_j^{-1} \Pi_k \} \left[ \tau_i - \Pi_i \Pi_j^{-1} \tau_j \right] \right]$$

(8)

Thus, we obtain the close-loop constraint as function of the joint torques:

$$0 = f(\tau, \dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4, \dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4)$$

(9)

4. Dynamic criterions

Based on the previous equations, two dynamic criterions are proposed : the dynamic propulsion criterion and the dynamic propulsion potential.
4.1 Dynamic Propulsion Criterion

To start, we define the dynamic propulsion criterion. The idea of this criterion is to quantify exactly the acceleration required instantaneously to the robot to generate the desired trajectory, for the three dof. We note that the desired trajectory can be expressed just as an average desired speed of the robot at $R_p$. Then, knowing the configuration of the robot, the aim is to calculate the joint torques to generate these desired accelerations.

To do this, we define two generalized forces. Let $\mathbf{F}_r$ be the external generalized force applied to the trunk at $R_p$ by the two legs and generating the real trajectory. Let $\mathbf{F}_d$ be the external generalized force which should be applied to the trunk by the two legs at $R_p$ to generate the desired trajectory. Thus, the dynamic propulsion of the robot will be ensured if $\mathbf{F}_r$ is equal to $\mathbf{F}_d$. Finally, the difference $\mathbf{F}_r - \mathbf{F}_d$ represents the dynamic propulsion criterion that we have to minimize. Knowing the configuration of the robot, $\mathbf{F}_d$ and the expression of the real generalized forces (right term of eq. 4), we can calculate the desired joint torques $\mathbf{\Gamma}^d$ to generate the desired trajectory:

\[
\mathbf{F}_d = \mathbf{C}_d \Gamma^d + \sum_{i=1}^{3} \mathbf{D}_{di} \left( \mathbf{F}_r - \mathbf{F}_i \right)
\]

To calculate $\mathbf{F}_d$, we use the expression of the trunk dynamics obtained in left term of (4), where we replace the real values $\dot{x}$, $\dot{x}$, $\dot{y}$, $\dot{y}$, $\dot{q}$, $\dot{q}$, and $\dot{q}$ of the trajectory by their desired values $\dot{x}^d$, $\dot{x}^d$, $\dot{y}^d$, $\dot{y}^d$, $\dot{q}^d$, $\dot{q}^d$, and $\dot{q}^d$. (In paragraph 5.1, we will see how obtain these desired values). The desired trajectory being defined in acceleration terms as well as in velocity and position terms, we replace:

\[
\pi_p^d = \left[ \dot{x}^d \quad \dot{y}^d \quad \dot{q}^d \right] 
\]

by:

\[
\pi_p^d = \left[ \begin{array}{c}
\dot{x}^d + K_x (\dot{x}^d - \dot{x}) + K_y \left( \dot{x}^d - \dot{x} \right) \\
\dot{y}^d + K_y \left( \dot{y}^d - \dot{y} \right) + K_x \left( \dot{x}^d - \dot{x} \right) \\
\dot{q}^d + K_q \left( \dot{q}^d - \dot{q} \right) + K_q \left( \dot{q}^d - \dot{q} \right)
\end{array} \right]
\]

what gives us the following expression of $\mathbf{F}_d$:

\[
\mathbf{F}_d = \mathbf{C}_d \mathbf{\pi}_p^d + \mathbf{D}_{di} \left( \mathbf{F}_r - \mathbf{F}_i \right)
\]

4.2 Dynamic Propulsion Potential

With the dynamic propulsion criterion, we know the required acceleration to generate the desired trajectory and the actuator torques we must apply. However, does the robot have the capability to generate this required acceleration?

To answer this question, we introduce the dynamic propulsion potential $\mathbf{\tilde{F}}$. This criterion represents the capability of a system to generate a desired generalized force, i.e. the minimum and the maximum generalized force that it can generate.
Analytical Criterions for the Generation of Highly Dynamic Gaits for Humanoid Robots: Dynamic Propulsion Criterion and Dynamic Propulsion Potential

For a planar system (Oxy plane), $\tilde{P}$ is written as follows:

$$
\tilde{P} = \begin{pmatrix}
\frac{F_{mn}^\text{mu}}{M_{mn}^\text{mu}}, & \frac{F_{mn}^\text{mu}}{M_{mn}^\text{mu}} \\
M_{mn}^\text{mu}, & \frac{M_{mn}^\text{mu}}{M_{mn}^\text{mu}}
\end{pmatrix}
$$

(14)

This criterion integrates the technology used as well as the constraints related to the system’s design. In the case of Rabbit, we define three criterions, one for each leg ($\tilde{P}_1$, $\tilde{P}_2$, $\tilde{P}_3$) and one for the robot ($\tilde{P}$).

The expressions of $\tilde{P}_1$ and $\tilde{P}_2$ are based on the expression of the external generalized force generated by each leg and the field of variation of the actuator torques. The expression of the external generalized force performed by each leg is obtained with the decomposition of (4):

$$
\tilde{F}_j = \begin{pmatrix}
\frac{\ddot{p} - \ddot{r}_j}{\ddot{r}_j} \\
\frac{\ddot{p} - \ddot{r}_j}{\ddot{r}_j}
\end{pmatrix}
$$

(15)

where $\ddot{r}_j$ are respectively the hip and knee actuator torque of the leg j.

Then, we have to determine the field of variation of the actuator torques. For that, the idea is to limit the field of variation of the actuator torques, given by the actuator technology, such as the joint actuators have the capability to stop the motion before reaching the joint limits. To determine the limits of the field of variation of each actuator torque, we use three relations. The first relation (16) integrates the actuator technology. The joint velocity and the actuator torque give the maximum and the minimum actuator torque that the joint actuator can generate for the next sample time:

$$
(q_j, \Gamma_j) \Rightarrow (\Gamma^\text{max}, \Gamma^\text{min})
$$

(16)

The second relation (17), based on (8), gives the expression of the joint acceleration as function of the actuator torque:

$$
\Gamma_j \Rightarrow \dot{q}_j
$$

(17)

The third relation (18) gives the new joint velocity ($\dot{q}^\text{new}$) and position ($q^\text{new}$), knowing the new joint acceleration ($\ddot{q}^\text{new}$) and the old angular velocity ($\dot{q}_j$) and position ($q_j$):

$$
\dot{q}^\text{new} = \dot{q}_j + \ddot{q}^\text{new} \delta t
$$

$$
q^\text{new} = q_j + \dot{q}_j \delta t + \ddot{q}^\text{new} \delta t^2 / 2
$$

(18)

where $\delta t$ is the sample time.

With these three relations and the maximum and minimum joint limits ($\dot{q}_{\text{max}}, q_{\text{max}}$), we limit the field of variation of the actuator torque. Here is explained the method to determinate the maximum torque that the joint actuator can generate, but the principle is the same for the minimum torque. This method is given by fig. 3.

With (16), we determine the maximum torque $\Gamma^\text{max}$ that the technology of the joint actuator allows to generate. Then, we check if the joint actuator has the capability to stop the joint motion before reaching the joint limit $q_{\text{max}}$, if we apply $\Gamma^\text{max}$. In order to perform that, we
calculate, with (17) and (18), the new joint configuration that we would have obtained if we had applied \( \Gamma_{\text{ex}} \). Then, in order to stop the joint motion, we apply the minimum torque that the joint actuator can generate in this new joint configuration. We continue this loop as long as the joint velocity is positive. And the minimum torque is recalculated at each step of time as function of the new joint configuration with (16). Then, we compare the joint position obtained with the joint limit \( q_{\text{max}}^i \). If it is lower than \( q_{\text{max}}^i \), \( \Gamma_{\text{min}}^i \) can be applied because the joint actuator has the capability to stop the joint motion before reaching the joint limit. Otherwise, \( \Gamma_{\text{max}}^i \) is reduced until obtaining the maximum value allowing to stop the joint motion before reaching the joint limit.

![Diagram](image)

**Fig. 3. Determination of the real maximum torque.**

To summarize, the algorithm of fig. 3. permits to calculate the lower and upper limit of the effective torque that we can really apply, because it takes into account the configuration of the robot, the actuator technology and the joint limits:

\[
\Gamma_{\text{eff min}} = \left[ \Gamma_{\text{max}}^i, \Gamma_{\text{ex}} \right]
\]

So, knowing the field of variation of the torque that the joint actuators can generate (19) and using (15), the dynamic propulsion potential of each leg and the dynamic propulsion potential of the robot are computed.

Related to the design of Rabbit, one joint actuator per leg has an important influence on the propulsion along the moving direction. It is the knee joint for the stance leg and the hip joint for the swing leg. So, we decide to consider only the field of variation of one actuator torque.
Analytical Criterions for the Generation of Highly Dynamic Gaits for Humanoid Robots:
Dynamic Propulsion Criterion and Dynamic Propulsion Potential

per leg to calculate its dynamic propulsion potential. Consequently, the dynamic propulsion potential $\tilde{P}$ of the leg $i$ is:

$$
\tilde{P} = \begin{cases} 
\min\left\{ F_c\left[r_{i,x}, T_{i,x}\right] F_c\left[r_{i,x}, T_{i,x}\right] \right\} & \max\left\{ F_c\left[r_{i,x}, T_{i,x}\right] F_c\left[r_{i,x}, T_{i,x}\right] \right\} \\
\min\left\{ M_c\left[r_{i,y}, \Gamma_{i,y}\right] M_c\left[r_{i,y}, \Gamma_{i,y}\right] \right\} & \max\left\{ M_c\left[r_{i,y}, \Gamma_{i,y}\right] M_c\left[r_{i,y}, \Gamma_{i,y}\right] \right\}
\end{cases}
$$

(20)

If the leg $i$ is the swing leg, and:

$$
\tilde{P} = \begin{cases} 
\min\left\{ F_c\left[r_{i,x}, T_{i,x}\right] F_c\left[r_{i,x}, T_{i,x}\right] \right\} & \max\left\{ F_c\left[r_{i,x}, T_{i,x}\right] F_c\left[r_{i,x}, T_{i,x}\right] \right\} \\
\min\left\{ M_c\left[r_{i,y}, \Gamma_{i,y}\right] M_c\left[r_{i,y}, \Gamma_{i,y}\right] \right\} & \max\left\{ M_c\left[r_{i,y}, \Gamma_{i,y}\right] M_c\left[r_{i,y}, \Gamma_{i,y}\right] \right\}
\end{cases}
$$

(21)

If the leg $i$ is the stance leg.

Knowing the potential of each leg, we calculate the dynamic propulsion potential $\tilde{P}$ of the robot. During the single contact phase, with no close-loop constraints to respect we have directly:

$$
\tilde{P} = \tilde{P}_i + \tilde{P}_j
$$

(22)

During the double contact phase, we have to respect the close-loop constraint along the $\dot{y}$ direction. So, we limit again the field of variation of the actuator torques at the knee joint by taking into account this constraint. Equation (9) establishing a direct relation between the two actuator torques, we recalculate the minimum and the maximum torque of each joint actuator compatible with the minimum and the maximum torque of the other joint torque. This gives the new field of variation of the torque in (19) in the case of the double contact phase. Then, we recalculate with (21) the dynamic propulsion potential of each leg and, in the same way as during the single contact phase, the potential of the robot with (22).

5. Control Strategy

Based on the dynamic propulsion of the two legs ($\tilde{P}_i, \tilde{P}_j$) and of the robot ($\tilde{P}$), we are able to produce for a desired average walking speed ($v_a$) the motion during the different locomotion phases and to detect the necessity to perform a transition between one phase to another. To perform this, we have to define on one hand the trajectory of $R_p$ as function of $v_a$ and on the other hand we have to define the control strategy to generate this desired trajectory.

5.1 Trajectory definition

The desired trajectory is defined by $v_a$ and the desired trunk angle, is fixed equal to zero. With these two parameters, the nine parameters $x^d, x^e, x^v, y^d, y^e, y^v, q^d_p, q^e_p$, and $q^v_p$ are computed:

$$
x^d = x + v_a X ; \quad x^e = v_a t
\quad \dot{x}^d = \frac{v_a - \dot{x}}{\Delta t} ; \\
y^d = y ; \quad y^e = y ; \quad \dot{y}^d = \dot{y} ; \\
q^d_p = 0 ; \quad q^e_p = \frac{q^e_p - q^d_p}{\Delta t} ; \quad q^v_p = \frac{q^e_p - q^d_p}{\Delta t} ;
$$

where $\Delta t$ is the sample time. We note that the evolution of the height $y^v$ is free.
5.2 Control Strategy based on generalized forces

We wish to generate the desired trajectory along the $\hat{x}$ direction and around the $\hat{z}$ direction. For that, with (13), we determine $F_x$ and $M_z$ that the robot has to perform. Then, we verify if the robot has the capability to generate $F_x$ and $M_z$. In order to do that, we compare $F_x$ to the lower and upper bounds of $P_x$. If $F_x$ is included in $P_x$, the robot has the capability to generate $F_x$. Otherwise, $F_x$ is limited to the maximum force that the robot can generate along the $\hat{x}$ direction if $F_x$ is larger than the upper limit of $P_x$ and limited to the minimum force if $F_x$ is lower than the lower limit of $P_x$. We use the same algorithm for $M_z$. We note that the direction with the most important constraint is along the $\hat{x}$ direction. Thus, with the dynamic propulsion criterion (10) along the $\hat{x}$ direction and around the $\hat{z}$ direction, we calculate the actuator torques we must apply to generate $F_x$ and $M_z$:

\[
F_x = f(T) = f(V_{x1}, \Gamma_{x1}, \Gamma_{x2}, \Gamma_{x3})
\]

(24)

\[
M_z = f(T) = f(\Gamma_{z1}, \Gamma_{z1}, \Gamma_{z2}, \Gamma_{z3})
\]

(25)

However, the constraints related to the Rabbit’s design and to the generation of walking gaits have to be taken into account. So, we complete the calculation of the desired actuator torques by taking into account these constraints as function of the different phases of the walking gaits.

1) Double Contact Phase: During the double contact phase, we have to respect the close-loop constraint (9). As we have three equations ((24), (25) and (9)) to check, we use a generalized inverse to calculate the four desired joint torques.

2) Transition from the Double to the Single Contact Phase: In order to not exceed the capability of the legs, we initiate the single contact phase when the dynamic propulsion potential of one leg is weak, i.e. when the force generated by the leg along or around a direction reaches the maximum or the minimum forces that it can generate. In the case of walking gaits, the dynamic propulsion potential concerned is that of the back leg along the $\hat{x}$ direction. During this phase, we have to ensure the take off the back leg. For that, we define a desired height for the tip of the back leg, then a trajectory to joint it. The time to reach this desired height is not fixed arbitrarily. We determine the shortest time to reach this height such as the capability of the knee actuator of the back leg is not exceeded. So, this time and consequently the trajectory is recalculated at each step of time. We decide to use the knee actuator of the back leg because it has a very weak influence on the propulsion of the robot. With inverse kinematics, we calculate the desired joint acceleration at the knee joint and, with the computed torque method, the desired actuator torque at the knee joint.

With the three other joints, we perform the dynamic propulsion. As the swing leg has a weak influence on the propulsion around $\hat{z}$, we decide to distribute $F_x$ on each leg, as function of their dynamic propulsion potential along the $\hat{x}$ direction, that gives us the following decomposition for (24):

\[
F_{x1} = f(\Gamma_{x1}, \Gamma_{x1})
\]

(26)

\[
F_{x2} = f(\Gamma_{x2})
\]

(27)

if the leg 1 is the stance leg and the leg 2 the swing leg. So, as we have three equations ((26), (27) and (25)), we can calculate the last three desired actuator torques directly.
3) Single Contact Phase: During this phase, we use the same strategy as during the transition phase. The only difference is that we use the knee actuator of the swing leg to preserve the desired height for the tip of the swing leg and not to join it.

4) Transition from the Single to the Double Contact Phase: In order to not exceed the capability of the legs, we initiate the return to the double contact phase when the dynamic propulsion potential of one leg is weak.

During this transition, we have to ensure the landing of the swing leg, which has to avoid a strong impact. So, we must control the trajectory of the tip of the swing leg along the $\mathbf{x}$ and $\mathbf{y}$ directions. In order to carry out this, we use the same strategy as this one used to control the height of the swing leg's tip during the single contact phase. However, we have to control the trajectory along two directions. Thus, we decide to use the two joint actuators of the swing leg.

With the two other joint actuators we perform the dynamic propulsion. As we do not use the swing leg to ensure the dynamic propulsion, we compare the force and the torque, that the stance leg has to perform to generate $F_x$ and $M_z$ respectively, with its dynamic propulsion potential. If the stance leg does not have the capability to generate them, we limit them. So, as we have two equations ((24) and (25)), we can calculate the last two desired actuator torques directly.

6. Simulation results

With these criterions, we develop a strategy to control RABBIT and we validate it with the simulation of the first step of a walking gait. To start, we define the desired average velocity at $p_R$. As an illustration, linear velocity $\mathbf{v}_x$ is imposed and the desired trunk angle $\mathbf{q}_p$ is equal to zero. The evolution of the height $y$ is free. So, we ensure the dynamic propulsion along the x direction and around the z direction only. The next idea is to modify, via the joint torques, the dynamic effects generated by the legs in order to make $\mathbf{r}_F$ converge towards $\mathbf{r}_F^i$. With the equations along the x direction and around the z direction, we can calculate $\mathbf{T}'$ directly. However, in this case, we do not take into account the constraints related to the robot’s design and to the generation of walking gaits. These constraints are introduced in the next section as functions of locomotion phases.

6.1 Double support phase

During this phase, we have four joint actuators to ensure the dynamic propulsion and the closed loop constraint (Fig. 2. with $\mathbf{y}_a=0$). As we have three equations, we use a generalized inverse to calculate $\mathbf{T}'$. In order to not exceed the capability of the robot, we use the dynamic propulsion potential of each leg. We compare the force generated by the legs to their dynamic propulsion potential along the $\mathbf{x}$ direction. If a leg reaches its maximum capability, we decide to start its take off to perform the single support phase. We see that the robot generates the desired moving velocity at $p_R$ (Fig. 4.), without exceeding its capability nor the capability of each leg along the $\mathbf{x}$ direction (Fig.5). After 1.3 seconds, the maximum capability of the back leg along the $\mathbf{x}$ direction reduces. Indeed, the knee joint of the back leg approaches its joint stop (Fig.6). So, when the back leg reaches its maximum capability, the take off starts in order to perform the single support phase.
Fig. 4. Desired and real Moving velocity at $R_p$.

Fig. 5. Dynamic propulsion potential and real force of the back leg along the x direction at $R_p$.

Fig. 6. Angle and limit angle of the back leg knee.
6.2 Single support phase

During this phase, we have four joint actuators to ensure the dynamic propulsion and to avoid the contact between the tip of the swing leg and the ground. In order to do this, the swing leg’s knee is used to avoid the contact while the three other joints perform the dynamic propulsion of the robot.

The joint torque of the swing leg’s knee is calculated with a computed torque method using non-linear decoupling of the dynamics. The desired joint acceleration, velocity and position of the swing leg knee are calculated with inverse kinematics. We express the joint torque of the swing leg’s knee as function of the three other joint torques and of the desired control vector components of the swing leg’s knee.

With the three other joints we perform the dynamic propulsion. It is possible that the robot does not have the capability to propel itself along the x direction dynamically. In this case, we limit the desired force with its dynamic propulsion potential. Then, we distribute this desired force with the dynamic propulsion potential of each leg. In order to keep a maximum capability for each leg, the desired force generated by each leg is chosen to be as further as possible from the joint actuators limits. In this case, we have three equations, one for the desired force along the x direction for each leg and one for the desired force around the z direction, to calculate the three joint torques. The joint torque of the swing leg’s knee is replaced by its expression in function of the three other joint torques. So, we calculate the three joints torque performing the dynamic propulsion, then the joint torque avoiding the contact between the tip of the swing leg and the ground.

We see that the robot can ensure the propulsion along the x direction (Fig. 7) and generates the desired moving velocity (single contact phase in Fig. 4.). Moreover, the control strategy involves naturally the forward motion of the swing leg (Fig. 8). After 1.675 seconds, the robot can not ensure exactly the desired propulsion along the x direction. Indeed, the swing leg is passed in front of the stance leg and the system is just like an inverse pendulum submitted to the gravity and for which rotational velocity increases quickly.

Fig. 7. Dynamic propulsion potential and real force of the robot along the x direction at $R_p$. 

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**Analytical Criterions for the Generation of Highly Dynamic Gaits for Humanoid Robots: Dynamic Propulsion Criterion and Dynamic Propulsion Potential**

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7. Conclusions and future work

In this paper, we present an analytical approach for the generation of walking gaits with a high dynamic behavior. This approach, using the dynamic equations, is based on two dynamic criterions: the dynamic propulsion criterion and the dynamic propulsion potential. Furthermore, in this method, the intrinsic dynamics of the robot as well as the capability of its joint torques are taken into account at each sample time.

In this paper, in order to satisfy the locomotion constraints, for instance the no-contact between the tip of the swing’s leg and the ground, we selected joint actuators, for instance the knee to avoid this contact during the single contact phase. Our future work will consist in determining the optimal contribution of each joint actuator by using the concept of dynamic generalized potential in order to satisfy at the same time the dynamic propulsion and the locomotion constraints. In this case, just one desired parameter will be given to the robot: the average speed. Thus, all the decisions concerning the transitions between two phases or the motions during each phase will be fully analytically defined.

8. References


Analytical Criteria for the Generation of Highly Dynamic Gaits for Humanoid Robots:
Dynamic Propulsion Criterion and Dynamic Propulsion Potential


For many years, the human being has been trying, in all ways, to recreate the complex mechanisms that form the human body. Such task is extremely complicated and the results are not totally satisfactory. However, with increasing technological advances based on theoretical and experimental researches, man gets, in a way, to copy or to imitate some systems of the human body. These researches not only intended to create humanoid robots, great part of them constituting autonomous systems, but also, in some way, to offer a higher knowledge of the systems that form the human body, objectifying possible applications in the technology of rehabilitation of human beings, gathering in a whole studies related not only to Robotics, but also to Biomechanics, Biomimetics, Cybernetics, among other areas. This book presents a series of researches inspired by this ideal, carried through by various researchers worldwide, looking for to analyze and to discuss diverse subjects related to humanoid robots. The presented contributions explore aspects about robotic hands, learning, language, vision and locomotion.

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